The logic of randomisation methods in statistics

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Two probability models

1. General idea of the key concept
2. Step by step example demonstrating the process
3. Related methods and tests

Assumptions on which statistical inference is made [1]

Population model

- Assumes random sampling from population
- Assumes population distribution
- Compares test statistic to distribution (e.g., t, F)
- Can generalise to population of interest

[1] Ernst, M D. 2004. *Stat. Sci.* 19:676-685.

[2] Ludbrook, J, & H Dudley. 1998. *Am. Stat.*, 52:127-132.

[3] Manly, B F J. 2007. *Randomization, Bootstrap and Monte Carlo Methods in Biology* (3rd ed.). Chapman & Hall/CRC.

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- Randomises data to build a null distribution
- Statistical inference limited to sample*

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The general idea of the randomisation approach

1. Randomly and repeatedly shuffle data in some way to generate a null distribution that is neutral with respect to our test statistic.

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2. Compare the observed test statistic with the null distribution of test statistics generated in step 1.

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The general idea of the randomisation approach

1. Randomly and repeatedly shuffle data in some way to generate a null distribution that is neutral with respect to our test statistic.
2. Compare the observed test statistic with the null distribution of test statistics generated in step 1.
3. Determine the *probability of observing a test statistic at least as extreme as the one we actually observed given the null distribution.*

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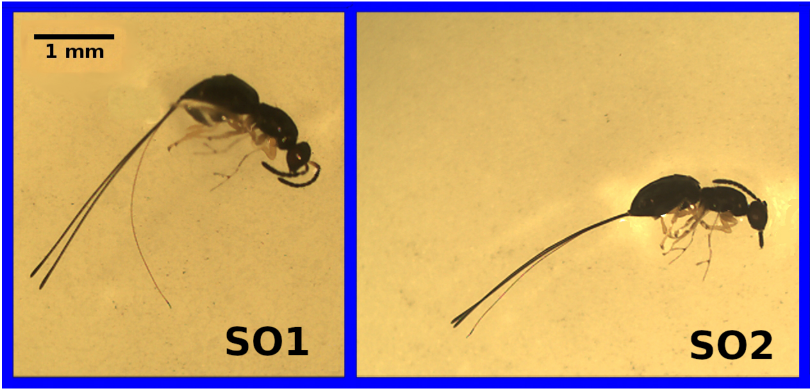
Empirical data from fig-fig wasp associations

Figs sampled from *Ficus petiolaris* in Baja, Mexico

Do SO1 and SO2 fig wasps differ in ovipositor length?



Cryptic species of *Idarnes* non-pollinating fig wasps

[1] Duthie, A B, et al. 2015. *Am. Nat.* 186:151-158.

Ovipositor measurements for SO1 and SO2

Species	Ovipositor (mm)
SO1	3.256
SO1	3.133
SO1	3.071
SO1	2.299
SO1	2.995
SO1	2.929
SO1	3.291
SO1	2.658
SO1	3.406
SO1	2.976
SO1	2.817
SO1	3.133
SO1	3.000
SO1	3.027
SO1	3.178
SO1	3.133
SO1	3.210

Species	Ovipositor (mm)
SO2	3.014
SO2	2.790
SO2	2.985
SO2	2.911
SO2	2.914
SO2	2.724
SO2	2.967
SO2	2.745
SO2	2.973
SO2	2.560
SO2	2.837
SO2	2.883
SO2	2.668
SO2	3.063
SO2	2.639

[1] Duthie, A B, et al. 2015. *Am. Nat.* 186:151-158.

Measurement data for two fig wasp species

Summary statistics for SO1

```
##  Ovipositor (mm)
##  Min.   :2.299
##  1st Qu.:2.976
##  Median :3.071
##  Mean   :3.030
##  3rd Qu.:3.178
##  Max.   :3.406
```

$$\hat{\sigma}_{SO1} = 0.260018$$

Summary statistics for SO2

```
##  Ovipositor (mm)
##  Min.   :2.560
##  1st Qu.:2.735
##  Median :2.883
##  Mean   :2.845
##  3rd Qu.:2.970
##  Max.   :3.063
```

$$\hat{\sigma}_{SO2} = 0.151241$$

Null hypothesis that ovipositor length does not differ with respect to species ($\mathbf{H_0 : E[SO1] = E[SO2]}$)

[1] Duthie, A B, et al. 2015. *Am. Nat.* 186:151-158.

Calculating a p-value from randomised data

The difference between mean ovipositor lengths was:

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Assume that you randomly shuffle group identity (i.e., SO1 & SO2) and recalculate the difference between means 9999 times, finding that 19 random resuffles are as or more extreme (i.e., larger absolute value) than the observed difference. **How do you calculate a p-value?**

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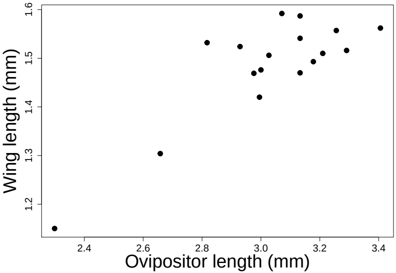
$$\hat{P} = \frac{19 + 1}{9999 + 1} = 0.002.$$

Randomisation not restricted to categorical variables

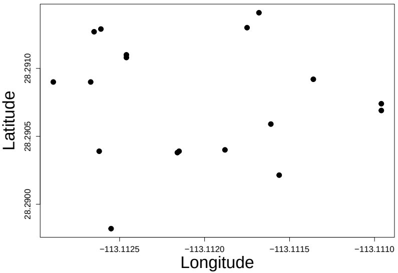
Species	Ovipositor (mm)	Wing (mm)
SO1	3.256	1.557
SO1	3.133	1.470
SO1	3.071	1.592
SO1	2.299	1.150
SO1	2.995	1.420
SO1	2.929	1.524
SO1	3.291	1.516
SO1	2.658	1.304
SO1	3.406	1.562
SO1	2.976	1.469
SO1	2.817	1.532
SO1	3.133	1.587
SO1	3.000	1.476
SO1	3.027	1.506
SO1	3.178	1.493
SO1	3.133	1.541
SO1	3.210	1.510



Compare estimated r = 0.834 with null distribution by randomly shuffling ovipositor length

Null distribution not restricted to existing data

Tree	Latitude	Longitude
T34	28.29021	-113.1116
T01	28.29141	-113.1117
T02	28.29130	-113.1118
T03	28.29129	-113.1126
T04	28.29127	-113.1127
T05A	28.29110	-113.1125
T05B	28.29108	-113.1125
T06	28.29090	-113.1127
T07	28.29090	-113.1129
T08	28.29039	-113.1126
T09	28.28982	-113.1125
T10	28.29038	-113.1122
T11	28.29039	-113.1121
T12	28.29040	-113.1119
T13	28.29059	-113.1116
T14	28.29092	-113.1114
T15A	28.29074	-113.1110
T15B	28.29069	-113.1110



Compare estimated nth nearest neighbour with null distribution by placing 18 trees in new locations.

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