$$\operatorname{var}(W_n) = EW_n^2 - (EW_n)^2$$

$$EW_n^2 = E[E(W_n^2 \mid W_{n-1})]$$ （条件期望公式）

$\because$

$W_n^2 \mid W_{n-1}$	$(W_{n-1}+1)^2$	W_{n-1}^2
P	$\frac{a}{a+b}$	$\frac{b}{a+b}$

$$\therefore E(W_n^2 \mid W_{n-1}) = \frac{a}{a+b}(W_{n-1}+1)^2 + \frac{b}{a+b}W_{n-1}^2$$

$$= W_{n-1}^2 + \frac{2a}{a+b}W_{n-1} + \frac{a}{a+b}$$

$$\therefore EW_n^2 = E(W_{n-1}^2) + \frac{2a}{a+b}E(W_{n-1}) + \frac{a}{a+b}$$

$$\because EW_{n-1} = \frac{(n-1)a}{a+b}$$

$$\therefore EW_n^2 = EW_{n-1}^2 + 2(n-1)\cdot\left(\frac{a}{a+b}\right)^2 + \frac{a}{a+b}$$

$$EW_{n-1}^2 = EW_{n-2}^2 + 2(n-2)\cdot\left(\frac{a}{a+b}\right)^2 + \frac{a}{a+b}$$

$$\vdots$$

依此类推可得

$$EW_2^2 = EW_1^2 + \frac{a}{a+b}$$

$$\because EW_1^2 = \frac{a}{a+b}$$

$$\therefore EW_n^2 = EW_1^2 + \frac{n(n-1)}{2} \cdot \frac{2a^2}{(a+b)^2} + \frac{(n-1)a}{a+b}$$

$$\therefore EW_n^2 = \frac{na}{a+b} + n(n-1) \cdot \left(\frac{a}{a+b}\right)^2$$

$$\therefore var(W_n) = EW_n^2 - (EW_n)^2$$

$$= \frac{na}{a+b} + n(n-1) \cdot \left(\frac{a}{a+b}\right)^2 - \left(\frac{na}{a+b}\right)^2$$

$$\therefore var(W_n) = \frac{na}{a+b} - \frac{na^2}{(a+b)^2}$$

$$\therefore var(W_n) = \frac{nab}{(a+b)^2}$$