§3.

3.

$X \backslash Y$	1	3
0	0	1/8
1	3/8	0
2	3/8	0
3	0	1/8

6.

$$P(X=i, Y=j) = (1-p)^{i-1} \cdot p\,(1-p)^{j-i-1} \cdot p = (1-p)^{j-2} \cdot p^2 \quad (i=1,2,\cdots, j=i+1, i+2, \cdots)$$

$$P(X=i) = \sum_{j=i+1}^{+\infty} P(X=i, Y=j) = p^2 \sum_{j=i+1}^{+\infty} (1-p)^{j-2} = (1-p)^{i-1} \cdot p \quad (i=1,2,\cdots)$$

$$P(Y=j) = \sum_{i=1}^{j-1} P(X=i, Y=j) = (j-1)(1-p)^{j-2} \cdot p^2 \quad (j=2,3,\cdots)$$

10.
$$F(x,y) = P(X \le x, Y \le y) = \begin{cases} 0, & x<0 \text{ 或 } y<0 \\ \int_0^y \int_0^x \cos u \cdot \cos v \, du\, dv, & 0 \le x < \frac{\pi}{2} \text{ 且 } 0 \le y < \frac{\pi}{2} \\ \int_0^{\frac{\pi}{2}} \int_0^x \cos u \cos v \, du\, dv, & 0 \le x < \frac{\pi}{2},\ y \ge \frac{\pi}{2} \\ \int_0^y \int_0^{\frac{\pi}{2}} \cos u \cos v \, du\, dv, & x \ge \frac{\pi}{2},\ 0 \le y < \frac{\pi}{2} \\ 1, & x \ge \frac{\pi}{2} \text{ 且 } y \ge \frac{\pi}{2} \end{cases}$$

$$F(x,y)=\begin{cases} 0, & x<0 \text{或} y<0 \\ \sin x \sin y, & 0\le x<\frac{\pi}{2} \text{且} 0\le y<\frac{\pi}{2} \\ \sin x, & 0\le x<\frac{\pi}{2},\ y\ge\frac{\pi}{2} \\ \sin y, & x\ge\frac{\pi}{2},\ 0\le y<\frac{\pi}{2} \\ 1, & x\ge\frac{\pi}{2} \text{且} y\ge\frac{\pi}{2} \end{cases}$$

(2) $P(0<X<\pi/4, \pi/4<Y<\pi/2)=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_{0}^{\frac{\pi}{4}}\cos x\cos y\,dx\,dy$

$=\frac{\sqrt{2}-1}{2}$

17.

(1) $f(x,y)=f_X(x)\cdot f_{Y|X}(y|x)=\frac{9y^2}{x}, \quad 0<y<x<1$

(2) $f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dx$

$=\int_y^1\frac{9y^2}{x}dx=-9y^2\cdot\ln y, \quad 0<y<1$

18. $Y\sim U(0,x)$

(1) $f(y|x)=\frac{1}{x}, \quad 0<y<x$

$f(x,y)=f(y|x)\cdot f_X(x)=\frac{1}{x}\cdot x\cdot e^{-x}=e^{-x}, \quad 0<y<x$

$$F(x,y)=P(X\le x, Y\le y)=\begin{cases} 0, & x\le 0 \text{或} y\le 0 \\ \int_0^x\int_0^u e^{-u}\,dv\,du, & 0<x\le y \\ \int_0^y\int_v^x e^{-u}\,du\,dv, & 0<y<x \end{cases}$$

$\therefore F(x,y) = \begin{cases} 0, & x \le 0 \text{ 或 } y \le 0 \\ 1-e^{-x}-xe^{-x}, & 0 < x \le y \\ 1-e^{-y}-y\cdot e^{-x}, & 0 < y < x \end{cases}$

(2). $f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx$

$= \int_{y}^{+\infty} e^{-x}\,dx = e^{-y}, \quad y > 0$

$F_Y(y) = P(Y \le y) = \begin{cases} 0 & y \le 0 \\ 1-e^{-y} & y > 0 \end{cases}$