22. $\bar{X} \sim N(\mu_1, \frac{\sigma^2}{n})$ $\bar{X} = 31.75$ $S_x^2 = 3.7447^2$

$\bar{Y} \sim N(\mu_2, \frac{\sigma^2}{n})$ $\bar{Y} = 27.9167$ $S_y^2 = 3.5537^2$

$n = 12$

$\bar{X} - \bar{Y} \sim N(\mu_1 - \mu_2, \frac{2\sigma^2}{n})$

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

当 H_0 成立时.

$\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{2\sigma^2}{n}}} \sim N(0, 1)$, $\frac{(n-1)(S_x^2 + S_y^2)}{\sigma^2} \sim \chi^2(2n-2)$

$$T = \frac{\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{2\sigma^2}{n}}}}{\sqrt{\frac{S_x^2 + S_y^2}{2\sigma^2}}} = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{(S_x^2 + S_y^2)}{n}}} \sim t(2n-2)$$

即 $t(22)$

拒绝域 $W = \{|T| \geq t_{0.025}(22)\}$

2.0739

$$T = \frac{3.83333}{\sqrt{\frac{26.6516}{12}}} = 2.5722$$

∴ 拒绝原假设. 可认为这两种钢筋有显著差异.

28. 令 $d_i = x_i - y_i$

$\{d_i\} = \{10.3, 7.4, 7.2, 4.7, 9, 7.1, 10.7, 7.7, 8.7\}$

$\bar{d} = 8.0889 \quad S_d^2 = 1.8299^2 \quad n = 9$

$\bar{d} \sim N(\mu_d, \frac{\sigma_d^2}{n})$

$H_0: \mu_d \geq 8$ vs $H_1: \mu_d < 8$

当H_0成立时 $\bar{d} \sim N(8, \frac{\sigma_d^2}{n})$ $\quad \frac{(n-1)S_d^2}{\sigma_d^2} \sim \chi^2(n-1)$

$$T = \frac{\frac{\bar{d}-8}{\sqrt{\frac{\sigma_d^2}{n}}}}{\sqrt{\frac{S_d^2}{\sigma_d^2}}} = \frac{\sqrt{n}(\bar{d}-8)}{S_d} \sim t(n-1)$$

即 $t(8)$

拒绝域 $W = \{T < t_{0.05}(8)\}$ $\quad t_{0.05}(8) = -1.8595$

$T = \frac{3 \times 0.0889}{1.8299} = 0.1457$

∴ 不拒绝原假设，认为宣传可信

31. $\bar{x}=5407.14$ $\quad S_x^2=1798.9415^2$ $\quad n=7$

$\bar{y}=7281.25$ $\quad S_y^2=2127.3621^2$ $\quad m=8$

① $\dfrac{(n-1)S_x^2}{\sigma_x^2}\sim\chi^2(n-1)$ $\qquad H_0:\sigma_x^2=\sigma_y^2$ vs $H_1:\sigma_x^2\neq\sigma_y^2$

$\dfrac{(m-1)S_y^2}{\sigma_y^2}\sim\chi^2(m-1)$ $\qquad$ 当 H_0 成立时

$$F=\frac{S_x^2}{S_y^2}\sim F(6,7)$$

拒绝域 $W=\{F>\overset{5.12}{F_{0.025}(6,7)}$ 或 $F<\overset{0.1754}{F_{0.975}(6,7)}\}$

$F=0.7151$ $\quad$ ∴ 接受原假设，认为 $\sigma_x^2=\sigma_y^2=\sigma^2$

② 又 $\bar{x}-\bar{y}\sim N(\mu_x-\mu_y,\frac{\sigma^2}{n}+\frac{\sigma^2}{m})$

$H_0:\mu_x\geq\mu_y$ vs $H_1:\mu_x<\mu_y$

当 H_0 成立时 $\dfrac{\bar{x}-\bar{y}}{\sqrt{\frac{(m+n)\sigma^2}{m\cdot n}}}\sim N(0,1)$

$\dfrac{(n-1)S_x^2}{\sigma^2}\sim\chi^2(n-1)$

$\dfrac{(m-1)S_y^2}{\sigma^2}\sim\chi^2(m-1)$

$\dfrac{(n-1)S_x^2+(m-1)S_y^2}{\sigma^2}\sim\chi^2(m+n-2)$

$$\Rightarrow T=\frac{\dfrac{\bar{x}-\bar{y}}{\sqrt{\frac{m+n}{mn}\sigma^2}}}{\sqrt{\dfrac{(n-1)S_x^2+(m-1)S_y^2}{(m+n-2)\sigma^2}}}\sim t(m+n-2)$$

拒绝域 $W=\{T < t_{0.95}(13)\}$ -1.7709

$$T=\frac{\bar{x}-\bar{y}}{\sqrt{\frac{m+n}{mn}}\cdot\sqrt{\frac{(n-1)S_x^2+(m-1)S_y^2}{m+n-2}}} = -1.8265$$

不拒绝原假设.

可认为甲企业产品工资低于乙企业.

33. $\bar{x}_甲 = 19.9989$ $S_甲^2 = 0.0392^2$ $n=9$

$\bar{x}_乙 = 19.9822$ $S_乙^2 = 0.0930^2$

$\frac{(n-1)S_甲^2}{\sigma_甲^2} \sim \chi^2(n-1)$ $H_0: \sigma_1^2 \geq \sigma_2^2$ vs $H_1: \sigma_1^2 < \sigma_2^2$

H_0成立时.

$\frac{(n-1)S_乙^2}{\sigma_乙^2} \sim \chi^2(n-1)$ $F=\frac{S_甲^2}{S_乙^2} \sim F(8,8)$

拒绝域 $W=\{F < F_{0.95}(8,8)\}$ 0.2907

$F=0.1777$ 不拒绝原假设