47.

由题可知：$E(X_i)=0.5$　$var(X_i)=0.01$

$(i=1,2,\cdots 5000)$

$$P(\sum_{i=1}^{5000}X_i \geqslant 2510) = P\left(\frac{\sum_{i=1}^{5000}X_i-\sum_{i=1}^{5000}E(X_i)}{\sqrt{\sum_{i=1}^{5000}var(X_i)}} \geqslant \frac{2510-2500}{\sqrt{50}}\right)$$

∴由中心极限定理可知：

$$P(\sum_{i=1}^{5000}X_i \geqslant 2510) = 1-\Phi(\sqrt{2}) \approx 0.0793$$

48.

(1) 由题可知：$X_i \sim B(1, 0.9)$　$(i=1,2,\cdots 100)$

∴ $EX_i = 0.9$　$varX_i = 0.09$　$(i=1,2,\cdots 100)$

∴ $$P(\sum_{i=1}^{100}X_i \geqslant 85) = P\left(\frac{\sum_{i=1}^{100}X_i-\sum_{i=1}^{100}EX_i}{\sqrt{\sum_{i=1}^{100}varX_i}} \geqslant \frac{85-90}{3}\right)$$

∴由中心极限定理可知

$$P(\sum_{i=1}^{100}X_i \geqslant 85) = 1-\Phi(-\frac{5}{3}) \approx 0.9525$$

(2) $P(\sum_{i=1}^{n} X_i \geq 80) = P\left(\frac{\sum_{i=1}^{n} X_i - \sum_{i=1}^{n} EX_i}{\sqrt{\sum_{i=1}^{n} VarX_i}} \geq \frac{0.8n-0.9n}{0.3\sqrt{n}}\right)$

$\therefore P(\sum_{i=1}^{n} X_i \geq 80) = 1 - \Phi(-\frac{\sqrt{n}}{3}) = 0.95$

$\therefore \frac{\sqrt{n}}{3} \geq 1.65$ 即 $n \geq 25$

14. $X_1 \sim N(0,1)$ $X_2 \sim N(0,1)$

$\because$ X_1与X_2相互独立

$\therefore P(x_1, x_2) = \frac{1}{2\pi} e^{-\frac{x_1^2+x_2^2}{2}}$

$\therefore$ 令 $U = X_1 - X_2$ $V = X_1 + X_2$

即 $\begin{cases} X_1 = \frac{U+V}{2} \\ X_2 = \frac{V-U}{2} \end{cases}$

$$\therefore P(u,v) = \frac{1}{2\pi} \cdot e^{-\frac{u^2+v^2}{4}} \cdot \frac{\partial(X_1, X_2)}{\partial(u,v)}$$

$$= \frac{1}{4\pi} e^{-\frac{u^2+v^2}{4}}$$

$\therefore$ $X_1 - X_2$与$X_1 + X_2$相互独立.

$\because$ $X_1 - X_2 \sim N(0,2)$ $X_1 + X_2 \sim N(0,2)$

$$\therefore \left(\frac{\frac{X_1-X_2}{\sqrt{2}}}{\frac{X_1+X_2}{\sqrt{2}}}\right)^2 = \frac{(X_1-X_2)^2}{(X_1+X_2)^2} \sim F(1,1)$$

15、 $T = a(X_1-2X_2)^2 + b(3X_3-4X_4)^2$

$\because$ X_1、X_2，X_3，X_4 是来自 $N(0,2^2)$ 的简单随机样本

$\therefore$ $(X_1-2X_2)^2$ 与 $(3X_3-4X_4)^2$ 相互独立

$\therefore$ $X_1-2X_2 \sim N(0,20)$ $3X_3-4X_4 \sim N(0,10^2)$

$$\therefore \frac{(X_1-2X_2)^2}{20} \sim \chi^2(1) \qquad \frac{(3X_3-4X_4)^2}{100} \sim \chi^2(1)$$

$\therefore$ 当 $a = \frac{1}{20}$，$b = \frac{1}{100}$ 时 $T \sim \chi^2(2)$

16.

$\because X_1, X_2, \cdots X_9$ 为独立同分布正态随机变量，即 $X_i \sim N(\mu, \sigma^2)$ $(i=1,2,\cdots 9)$

且 $Y_1 = \dfrac{X_1+X_2+\cdots+X_6}{6}$ $\quad Y_2 = \dfrac{X_7+X_8+X_9}{3}$

$\therefore Y_1 - Y_2 \sim N(0, \frac{\sigma^2}{2})$

$X_i - Y_2 \sim N(0, \frac{2\sigma^2}{3})$ $(i=7,8,9)$

$\therefore \dfrac{\sqrt{2}(Y_1-Y_2)}{\sigma} \sim N(0,1)$

$\dfrac{\sqrt{3}(X_i-Y_2)}{\sqrt{2}\sigma} \sim N(0,1)$, $(i=7,8,9)$

$\therefore \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(2)$

$\therefore \dfrac{\sqrt{2}(Y_1-Y_2)}{S} \sim t(2)$