4.

$X \sim N(\mu, \sigma^2)$ $\quad \bar{X} = 2.1322$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ $\quad S^2 = 0.00039$

(1) $\sigma = 0.01$ $\quad \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \sim N(0,1)$

$$P(-u_{0.05} < \frac{\sqrt{n}(\bar{x} - \mu)}{\sigma} < u_{0.05}) = 0.9$$

$$\therefore \mu \in [\bar{x} \pm \frac{u_{0.05}\sigma}{\sqrt{n}}]$$

即 $\mu \in [2.0774, 2.1870]$

(2) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$$T = \frac{\cancel{\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma}}}{\sqrt{\frac{S^2}{\sigma^2}}} = \frac{\sqrt{n}(\bar{x} - \mu)}{S} \sim t(n-1)$$

$$P(|T| < t_{0.05}(8)) = 0.9$$

$$\Rightarrow \mu \in \left[\bar{X} \pm \frac{t_{0.05}(8) \cdot S}{\sqrt{n}} \right]$$

即 $\mu \in [2.1199, 2.1445]$

6.

$$\bar{X} \sim N\left(3, \frac{5^2}{n}\right) \quad \frac{\sqrt{n}(\bar{X}-3)}{5} \sim N(0,1)$$

$$P(1 < \bar{X} < 5) \geq 0.95$$

$$\Rightarrow P\left(-\frac{2\sqrt{n}}{5} < \frac{\sqrt{n}(\bar{X}-3)}{5} < \frac{2\sqrt{n}}{5}\right) = 2\Phi\left(\frac{2\sqrt{n}}{5}\right) - 1 \geq 0.95$$

$\Rightarrow n \geq 24.01$ $\quad \therefore n \geq 25$

7. $\bar{X} \sim N(0, \frac{\sigma^2}{n})$

$P(|\bar{X}| < 2)$

$= P(|\frac{\sqrt{n}\bar{X}}{\sigma}| < \frac{2\sqrt{n}}{\sigma})$

$= 2\Phi(\frac{2\sqrt{n}}{\sigma}) - 1 \geq 0.99$

$\Phi(\frac{2\sqrt{n}}{\sigma}) \geq 0.995$

$\frac{2\sqrt{n}}{10} \geq 2.575$

$n \geq 165.7656$

$\therefore n \geq 166$

8.

(1) $X = e^{Y}$

$$EX = \int_{-\infty}^{+\infty} e^{y} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(y-\mu)^2}{2}} dy = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2-2\mu y+\mu^2-2y}{2}} dy$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{[y-(1+\mu)]^2+\mu^2-(1+\mu)^2}{2}} dy$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{[y-(1+\mu)]^2}{2}} dy \cdot e^{\frac{1+2\mu}{2}}$$

$$= e^{\mu+\frac{1}{2}}$$

$$\therefore a = e^{\mu+\frac{1}{2}}$$

(2) $\bar{Y} \sim N(\mu, \frac{1}{4})$ $\quad u = 2(\bar{Y} - \mu) \sim N(0,1)$

又 $\bar{Y} = \frac{1}{4}\sum_{i=1}^{4} \ln x_i = 0.0578$

$P(|u| < u_{0.025}) = 0.95$

$\Rightarrow \mu \in [\bar{Y} \pm \frac{1.96}{2}]$

μ的95%置信区间为：$[-0.9222, 1.0378]$

$P(|u| < u_{0.05}) = 0.90$

$\Rightarrow \mu \in [\bar{Y} \pm \frac{1.645}{2}]$

μ的90%置信区间为：$[-0.7647, 0.8803]$

(3) $a = e^{n+\frac{1}{2}}$

$\therefore a \in [e^{-0.2647}, e^{1.3803}]$