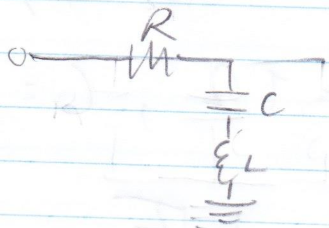
Problem 8 derivation

Series Resonant circuit



$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$ loaded Q for parallel RLC circuit

damping factor $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$

natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

damped frequency: $\omega_1 = \omega_0\sqrt{1-\zeta^2}$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{RC}\sqrt{LC} = \frac{1}{RC\omega_0} = \frac{\sqrt{1-\zeta^2}}{RC\omega_1}$$

$$\therefore \quad Q = \frac{1}{RC\omega_0}$$

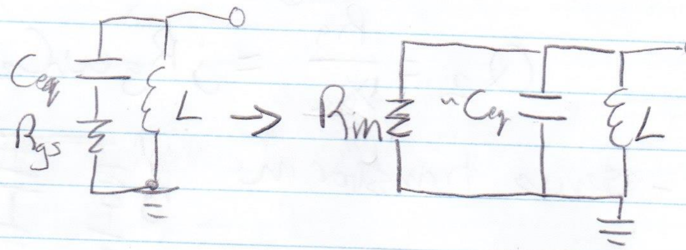
$$\text{and} \quad Q = \frac{\sqrt{1-\zeta^2}}{RC\omega_1}$$

But Q-factor has no meaning for an undamped circuit, because it relies on there being some resistance, so Q-factor can only be actually related to damped frequency.

So, $Q_{1,2} \simeq Q_2\left(\frac{C_1+C_2}{C_1}\right)$

want to transform

series -> Parallel



$$R_{in} = R_{gs}\left(1+Q_{1,2}^2\right)$$

$$= \frac{R_g}{1+Q_2^2}\left(1+Q_{1,2}^2\right) = R_g\,\frac{1+Q_{1,2}^2}{1+Q_2^2}$$

$$\simeq R_g\,\frac{1+Q_2^2\left(\frac{C_1+C_2}{C_1}\right)^2}{1+Q_2^2}$$

$$= R_g\left(\cancelto{0}{\frac{1}{1+Q_2^2}} + \cancelto{1}{\frac{Q_2^2}{1+Q_2^2}}\left(\frac{C_1+C_2}{C_1}\right)^2\right) \quad \text{if } Q_2^2 \gg 1$$

$$R_{in} \simeq R_g\left(\frac{C_1+C_2}{C_1}\right)^2 = R_g\left(1+\frac{C_2}{C_1}\right)^2$$