Homework 1

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EE 157/L
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Problem 1: Explain when impedance matching is necessary and when it's not in RF engineering

Impedance matching is necessary when the input signal to a stage is expected to be of small amplitude, e.g. at the input to an RF receiver where the antenna is trying to pick up a low-power transmitted signal. If the power of a signal is down near the noise floor, then any lost power may result in the signal becoming unrecoverable. Thus, impedance matching to ensure max-power transfer is necessary. Impedance matching is also necessary in distributed circuit models where un-transferred energy will be reflected back into the source impedance. Impedance matching is not necessary in circuits where signal amplitude is not a concern, such as after a power amplifier stage. Alternatively, sometimes you want a mismatched impedance, such as a high-impedance input to a buffer, where you want to reduce the loading effect of subsequent stages on your previous stage.

Problem 2: Study the references noted above (Bowick Ch 2,4) and discuss the following concepts. a) Explain what Quality Factor, "Q" is and why it's used as a parameter of analysis and design in RF circuits.

The Q factor of a resonant circuit is defined as the ratio of the center frequency and the half-power bandwidth ($\frac{f_c}{f_2-f_1}$) and is a measure of the circuit's 'selectivity'. But what the hell does 'selectivity' mean? A resonant circuit with a high selectivity is resonant at only the center frequency and a very small domain around the center frequency, whereas a low-Q resonant circuit will be resonant at an attenuated amplitude further out from the center frequency. This is bad, because when we design a resonant circuit, we want it to be resonant for only the specified frequency. For example, when you design a 10MHz oscillator, you want the generated signal to be ONLY a 10MHz sinusoid. If you have a low-Q oscillator, the instantaneous frequency will end up drifting around inside the relatively large bandwidth and you end up with phase noise. A low-Q band-pass filter will let through undesired frequencies at attenuated amplitudes, making it a crappy filter.

b) Develop the perspectives of source and load impedance. What do they tell us and why are they important?

source vs load impedance is a useful paradigm for impedance matching. By assuming that the source drives the load, it gives us an engineering design goal: to match the subsequent load impedance to the previous source impedance. The 'source driving load' paradigm also implies a directionality of signal flow, it tells us which direction the information is coming from. This becomes more important in distributed circuits, where there is actually a time delay for the travelling signal and direction of flow becomes relevant.

c) Consider a 2-port LTI network consisting of a passive impedance matching circuit. It could be a simple untuned transformer or a tuned resonant circuit. Discuss

what is meant by the concept of reflected impedance when looking into the 2-port network from either side and how it relies on the concept of solid terminations. What is a solid termination and why is it important?

'Reflected impedance' is the effective or 'transformed' impedance looking into the one of the ports at the **terminated** second port. It's important that this second port be terminated. Explained simply, if the second port is not terminated have a non-zero non-infinite impedance then there is no impedance to be scaled by the intermediate passive network, so there's no impedance to be reflected back when looking through the network.

Problem 3: My lecture note discusses only the parallel resonant LC tank circuit. Bowick in CH 4 expands on the topic of many "infinitely many" circuits by discussing how to use them as simple low or high-pass "L Networks" that can also be used to match impedances. Explain what he is doing in fig. 4-6 and 4-7 to realize a filter design and why it works.

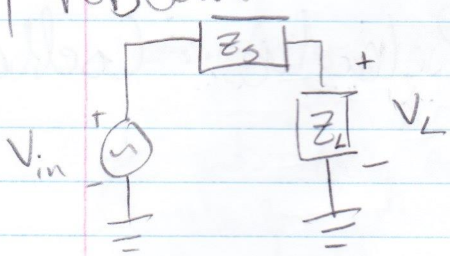
In figures 4-6 and 4-7, Bowick is using a capacitor in parallel with the 1k ohm resistor to create an equivalent series impedance by using a parallel-series transform. The parallel capacitor has the effect of lowering the resistance seen looking into the matching network. This is similar to putting two resistive loads in parallel. When you do that, you're lowering the effective resistance seen looking across them. In this case, when you put two impeditive components in parallel they lower the total magnitude of the seen complex impedance.

Problem 4: What does Bowick mean by "the absorption" technique?

parasitic capacitances and inductances can be 'absorbed' into the passive impedance matching network by manipulating them such that the parasitic capacitances are in parallel with the network capacitance and the parasitic inductance is in series with the network inductance. This way the network and parasitic values add together to produce an effective parallel capacitance and series inductance.

Problem 5: Prove the maximum power transfer theorem for AC source and load impedances using phasors. Support your mathematical derivation with appropriately drawn schematics.

Problem 5.



$$\overline{V}_L = \overline{V}_{in} \frac{Z_L}{Z_S + Z_L}$$

Apparent Power dissapated over the load, S_L:

$$S_L = \frac{\overline{V}_L^{\,2}}{Z_L} = \frac{\overline{V}_{in}^{\,2} \frac{Z_L^{\cancel{2}}}{(Z_S+Z_L)^2}}{\cancel{Z_L}} = V_{in}^2 \frac{Z_L}{(Z_S+Z_L)^2}$$

$$= V_{in}^2 \frac{R_L + jX_L}{(R_S + jX_S + R_L + jX_L)^2}$$

We're only concerned with Real power dissapation so we can ignore the imaginary term

$$P_L = V_{in}^2 \frac{R_L}{[R_S + R_L + j(X_S + X_L)]^2}$$

Now, we can take the denominator as a magnitude and ignore the phase.

$$|P_L| = V_{in}^2 \frac{R_L}{\left[\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}\right]^2}$$

$$\Rightarrow |P_L| = V_{in}^2 \frac{R_L}{(R_S+R_L)^2 + (X_S+X_L)^2} \quad (1)$$

Now we take the derivative with respect to X_L

$$\frac{d|P_L|}{dX_L} = \frac{-2(X_S+X_L)}{(1+(X_S+X_L)^2)^2}$$

which is made zero only when $X_L = -X_S$

Plugging this relation back into equation ①

$$|P_L| = V_{in}^2 \frac{R_L}{(R_S+R_L)^2}$$

which is the same relation we get from the proof of DC MPT from page 63 of Bowick, which concluded with the following condition for MPT:

$$R_L = R_S$$

So, for max power transfer in an AC impeditive circuit,

$$X_L = -X_S \quad \text{and} \quad R_L = R_S$$

$$\boxed{\therefore \quad Z_L = Z_S^*}$$

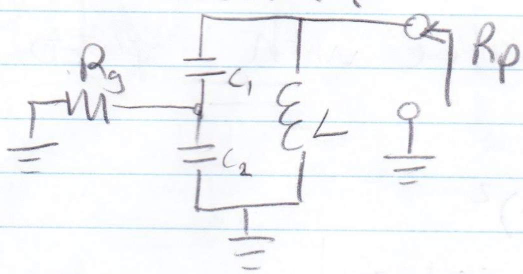
Problem 6: Explain the basic circuit theory concept of resistance and how it is related to impedance and admittance.

Resistance is the ratio of the real voltage across an element over the real current through an element. It is a measure of the current a resistive element draws given an applied DC voltage. Reactance is also a V/I ratio, but it is the imaginary component of the ratio of a voltage phasor over a current phasor. Impedance is the complete complex value consisting of a real resistance and an imaginary reactance. It is the measure of the magnitude attenuation and phase shift of a voltage phasor and its corresponding current phasor across and through an impeditive component represented as a phasor V/I ratio. Admittance is simply the reciprocal of the impedance, and describes the magnitude and phase of a voltage phasor that appears across an impeditive element if a current phasor is forced through the component.

Problem 7: In my RLC lecture note, pg. 4 (Impedance Matching Circuits), I showed the common tapped-C resonant impedance matching circuit as having an impedance transformation ratio given by: $R_p = R_g(1 + \frac{C_2}{C_1})^2$, but I didn't prove it. Carefully discuss any assumptions that are needed to use this equation and proceed to prove it.

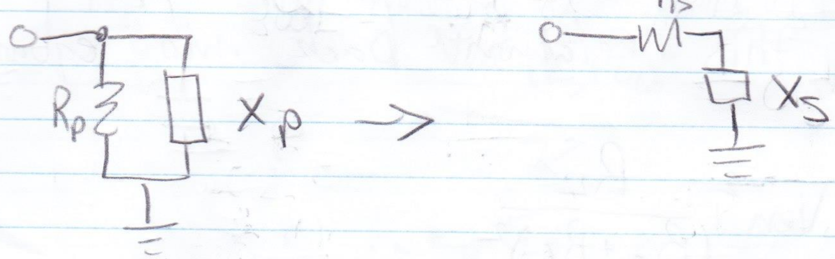
When you use this impedance matching equation, the biggest assumption made is that the opposite port from where you're looking in is terminated, in this case with R_g. If the port is not terminated, then there's no reflected impedance and the only impedance you see is from the resonant network itself. In using this equation, we are also assuming the reactive components are ideal.

Problem 7 Proof



Prove: $R_p = R_g\left(1 + \frac{C_2}{C_1}\right)^2$

Series parallel transform:



$$Q_p = \frac{R_p}{X_p}, \quad Q_s = \frac{X_s}{R_s}$$

$$Z_p = \frac{jR_pX_p}{R_p + jX_p} = \frac{R_pX_p^2}{R_p^2 + X_p^2} + j\frac{X_pR_p^2}{R_p^2 + X_p^2}$$

and $Z_s = R_s + jX_s$

$$Z_p = Z_s \implies R_s = \frac{R_pX_p^2}{R_p^2 + X_p^2} \text{ and } X_s = \frac{X_pR_p^2}{R_p^2 + X_p^2}$$

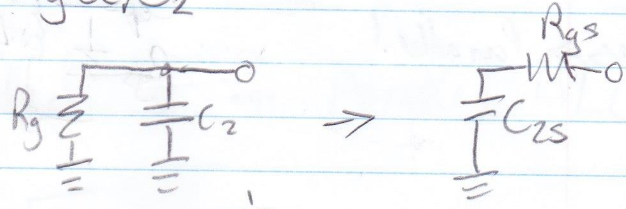
Using $Q_p = \frac{R_p}{X_p}$; $\implies R_s = \frac{R_p}{1 + Q_p^2}$ and $X_s = \frac{X_p}{1 + \frac{1}{Q_p^2}}$

Continuing w/ proof

Q - factor of R_g & C_2

$$Q_2 = \frac{R_g}{X_2} = jR_g\omega_r C_2$$

Parallel - series transform

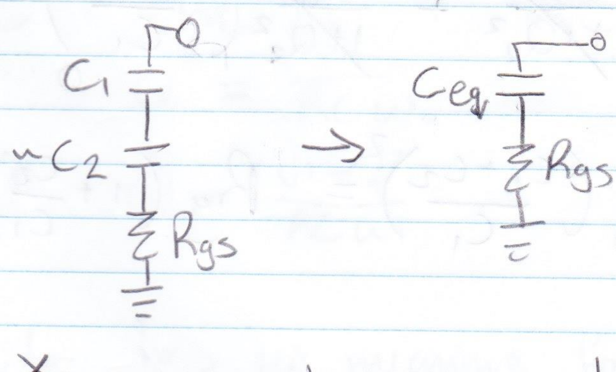


$$R_{gs} = \frac{R_g}{1+Q_2^2}, \quad \frac{1}{C_{2s}} = \frac{\frac{1}{C_2}}{1+\frac{1}{Q_2^2}}$$

$$\Rightarrow C_{2s} = \frac{1+\frac{1}{Q_2^2}}{1/C_2} = C_2\left(1+\frac{1}{Q_2^2}\right)$$

$$\Rightarrow C_{2s} \approx C_2 \text{ if } Q_2^2 \gg 1$$

Now we have



$$C_{eq} \approx \frac{C_1 C_2}{C_1 + C_2}$$

Q factor of C_1, C_2, R_{gs} branch

$$Q_{1,2} = \frac{X_{eq}}{R_{gs}} = \frac{1}{\omega_0 R_{gs} C_{eq}} = \frac{1}{\omega_0 \frac{R_g}{1+Q_2^2} C_{eq}} = \frac{1}{\cancel{\omega_0}\frac{Q_2/\cancel{\omega_r} C_2}{1+Q_2^2} C_{eq}}$$

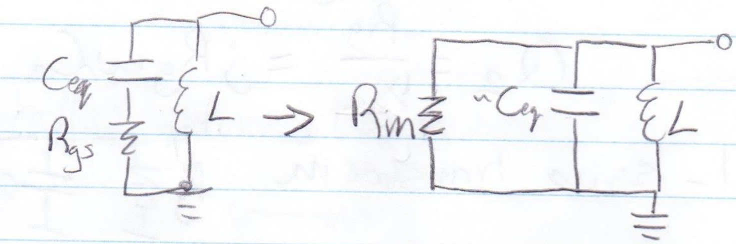
$$= \frac{(1+Q_2^2)C_2}{Q_2 C_{eq}} = \frac{C_2}{Q_2 C_{eq}} + \frac{Q_2 C_2}{C_{eq}}$$

$$= Q_2 \frac{C_2}{C_{eq}}\left(\frac{1}{Q_2^2}+1\right) \approx Q_2 \frac{C_2}{C_{eq}} \text{ when } Q_2 \ll 1$$

$$\text{So}, \; Q_{1,2} \simeq Q_2\left(\frac{C_1 + C_2}{C_1}\right)$$

want to transform

Series → Parallel



$$R_{in} = R_{gs}\left(1 + Q_{1,2}^2\right)$$

$$= \frac{R_g}{1+Q_2^2}\left(1 + Q_{1,2}^2\right) = R_g \, \frac{1 + Q_{1,2}^2}{1 + Q_2^2}$$

$$\simeq R_g \, \frac{1 + Q_2^2\left(\frac{C_1 + C_2}{C_1}\right)^2}{1 + Q_2^2}$$

$$= R_g\left(\cancelto{0}{\frac{1}{1+Q_2^2}} + \cancelto{1}{\frac{Q_2^2}{1+Q_2^2}}\left(\frac{C_1 + C_2}{C_1}\right)^2\right) \quad \text{if } Q_2^2 \gg 1$$

$$R_{in} \simeq R_g\left(\frac{C_1 + C_2}{C_1}\right)^2 = R_g\left(1 + \frac{C_2}{C_1}\right)^2$$

Problem 8: Drive a series or parallel RLC resonant circuit with a step (voltage or current respectively; you choose) and show how the damped and undamped resonant frequencies are related to circuit Q; if you didn't derive this relationship, justify it by showing its derivation.

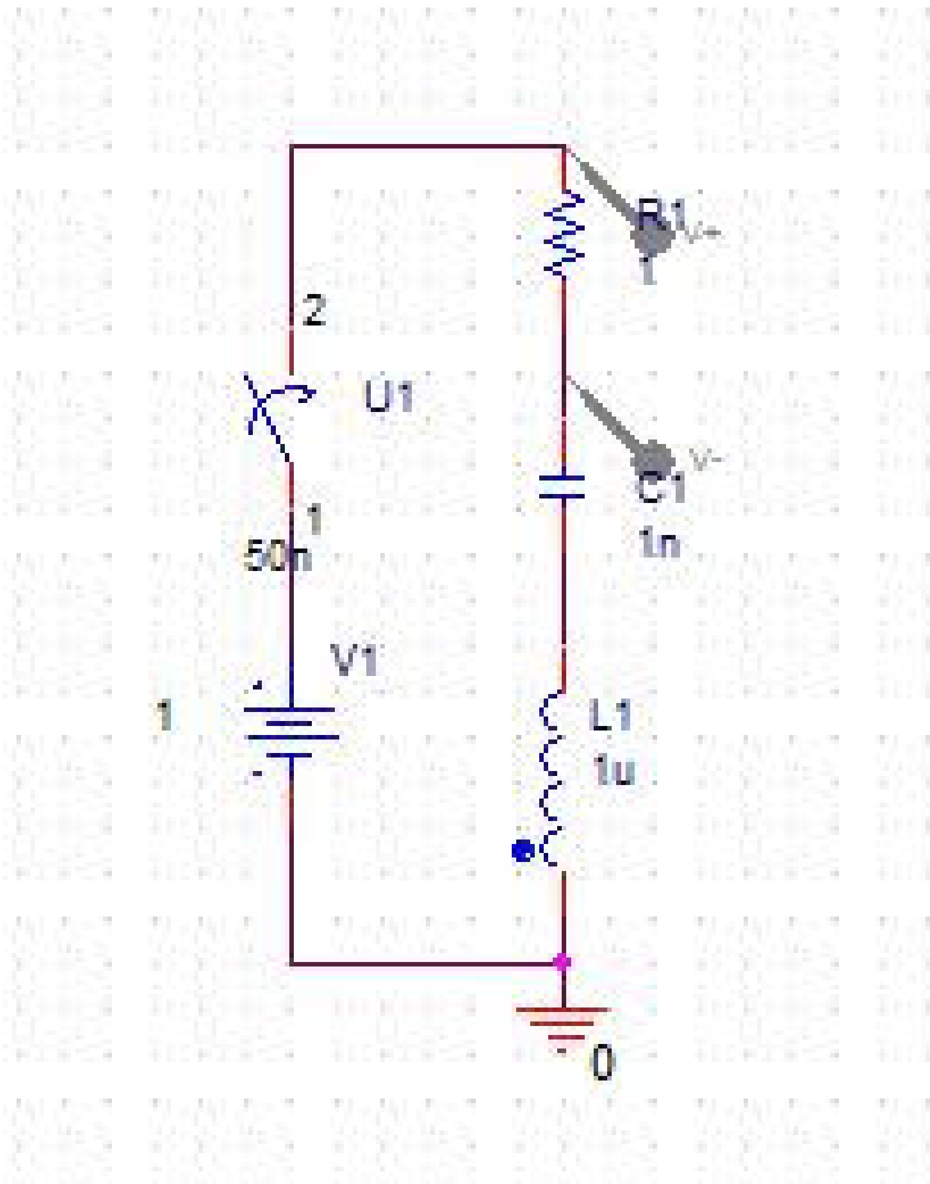


Figure 1: Problem 8: Series resonant circuit implemented in Cadence Pspice

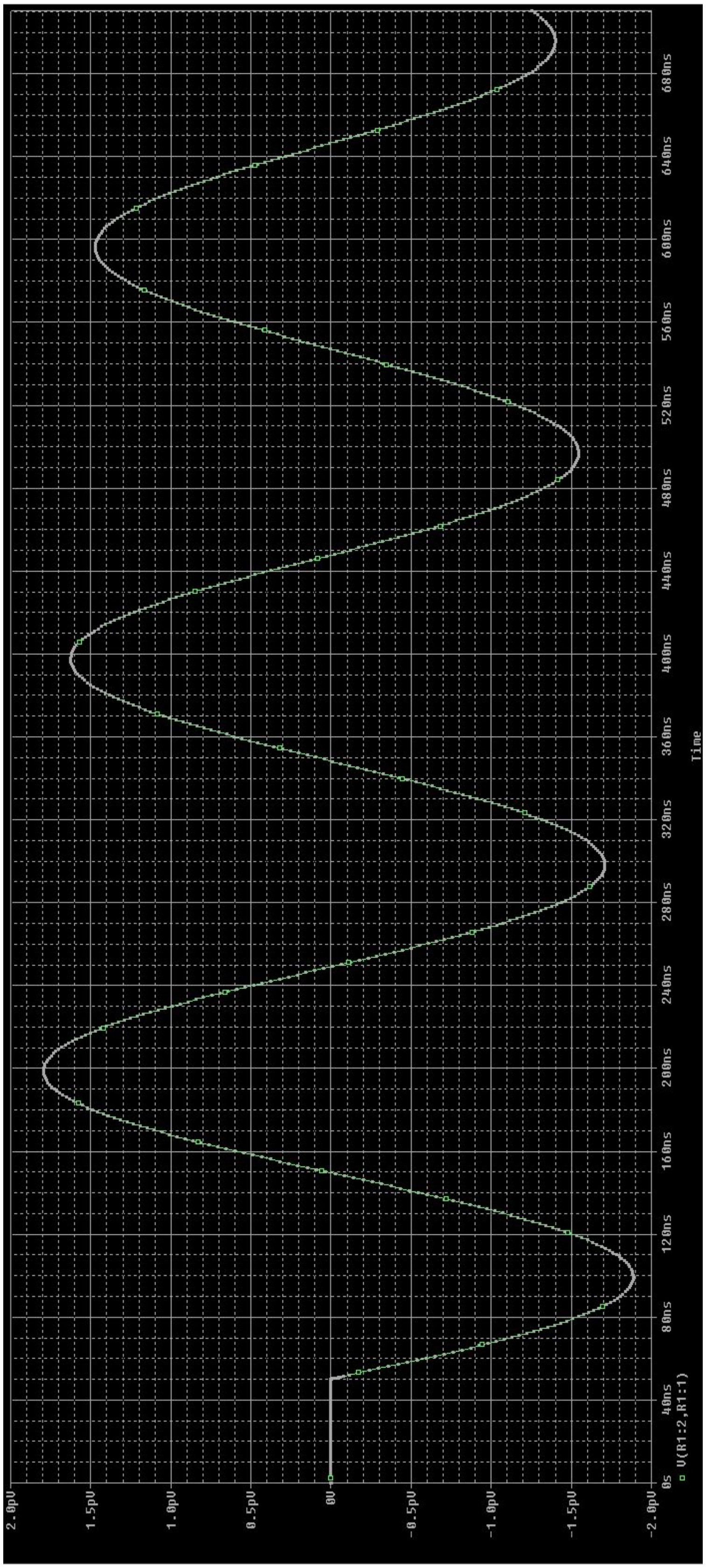
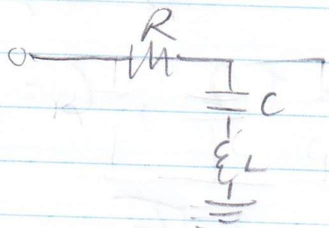


Figure 2: Problem 8: V_R transient time response

Problem 8 derivation

Series Resonant circuit



$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$ loaded Q for parallel RLC circuit

damping factor $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$

natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

damped frequency : $\omega_1 = \omega_0\sqrt{1-\zeta^2}$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{RC}\sqrt{LC} = \frac{1}{RC\omega_0} = \frac{\sqrt{1-\zeta^2}}{RC\omega_1}$$

$$\therefore \quad Q = \frac{1}{RC\omega_0}$$

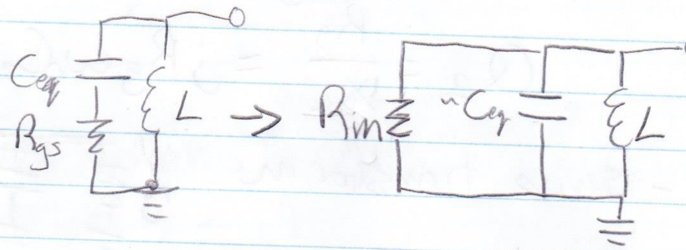
and $$Q = \frac{\sqrt{1-\zeta^2}}{RC\omega_1}$$

But Q-factor has no meaning for an undamped circuit, because it relies on there being some resistance, so Q-factor can only be actually related to damped frequency.

$$\text{So, } Q_{1,2} \simeq Q_2\left(\frac{C_1 + C_2}{C_1}\right)$$

want to transform

series -> Parallel



$$R_{in} = R_{gs}\,(1 + Q_{1,2}^2)$$

$$= \frac{R_g}{1+Q_2^2}\,(1 + Q_{1,2}^2) = R_g\,\frac{1 + Q_{1,2}^2}{1 + Q_2^2}$$

$$\simeq R_g\,\frac{1 + Q_2^2\left(\frac{C_1 + C_2}{C_1}\right)^2}{1 + Q_2^2}$$

$$= R_g\left(\cancelto{0}{\frac{1}{1+Q_2^2}} + \cancelto{1}{\frac{Q_2^2}{1+Q_2^2}}\left(\frac{C_1 + C_2}{C_1}\right)^2\right) \quad \text{if } Q_2^2 \gg 1$$

$$R_{in} \simeq R_g\left(\frac{C_1 + C_2}{C_1}\right)^2 = R_g\left(1 + \frac{C_2}{C_1}\right)^2$$