Homework 2

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EE 157/L
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Problem 1: Compare and contrast the mixing products from a Gilbert Cell mixer, like the SA602, and a single high frequency silicon diode, like the 1N916. In your discussion, develop the theory necessary to explain how they operate.

A single high-frequency diode implemented as a mixer may most commonly be used as a switching diode as shown in Figure 1a). For this case, the RF signal amplitude should be small enough such that it does not turn on the diode by itself. In this case, the circuit behaves like a switch opening and closing at f_{LO} assuming a constant voltage model for the diode. This results in the RF signal being multiplied by the square-wave local oscillator, which consists of the fundamental f_{LO} as well as the odd harmonics of f_{LO}. On the output, the RF signal is present along with the RF frequency plus/minus the fundamental as well as the sum/difference of the RF signal and the local oscillators harmonics. This results in an output signal which contains the original signal as well as many other harmonics.

The Gilbert Cell implemented as a mixer, shown in Figure 2, consists of a matched differential pair, each collector of which is attached to the joint emitter of another differential pair for a total of 3 pairs. The circuit operates as two differential pairs (3,4 and 5,6) having their gains controlled by two controlled current sources Q1 and Q2 which are controlled by the differential RF input. The gain from the LO input to the IF output can be represented simply (assuming the differential output is loaded with some resistance) as $V_{IF} = K * V_{LO}$ where the gain of the split differential pair K is proportional to the difference between the currents through Q1 and Q2, which are then proportional to the differential RF input. You then end up with a theoretical relationship $V_{IF} = K * V_{RF} * V_{LO}$ which looks like an analogue multiplier.

The mixing products of these two circuits are quite different. The single-diode switching mixer, in addition to the first-harmonic products will feed through the attenuated input signal as well as the products of the input signal and the odd-harmonics of the local oscillator frequency. In contrast, the gilbert cell should only output the sum and difference products of the local oscillator signal and the RF input signal with no harmonic content.

Problem 2: The Smith Chart can be used for both distributed and lumped circuit analysis and design. Moving clockwise about the Chart "toward the generator" describes a space phase for a distributed circuit, but what does it describe for a lumped circuit?

In a lumped circuit, there is no concept of a space phase, because there is no physical space between components. This also means there is no reflection coefficient because the reflection coefficient is defined as the ratio of the forward and reverse travelling wave amplitudes and phases, and travelling waves don't exist in lumped circuit land. The only thing I can think is that moving around the center of the smith chart indicates a changing relative phase looking into and out of

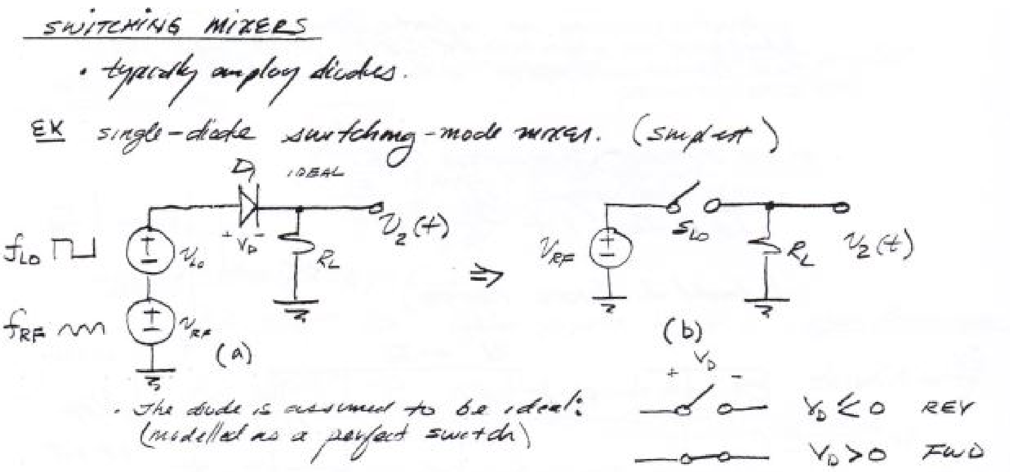

Figure 1: a) Single diode implemented as a switching mixer
b) effective behavior of diode mixer (Source: EE157 lecture notes pg 22)

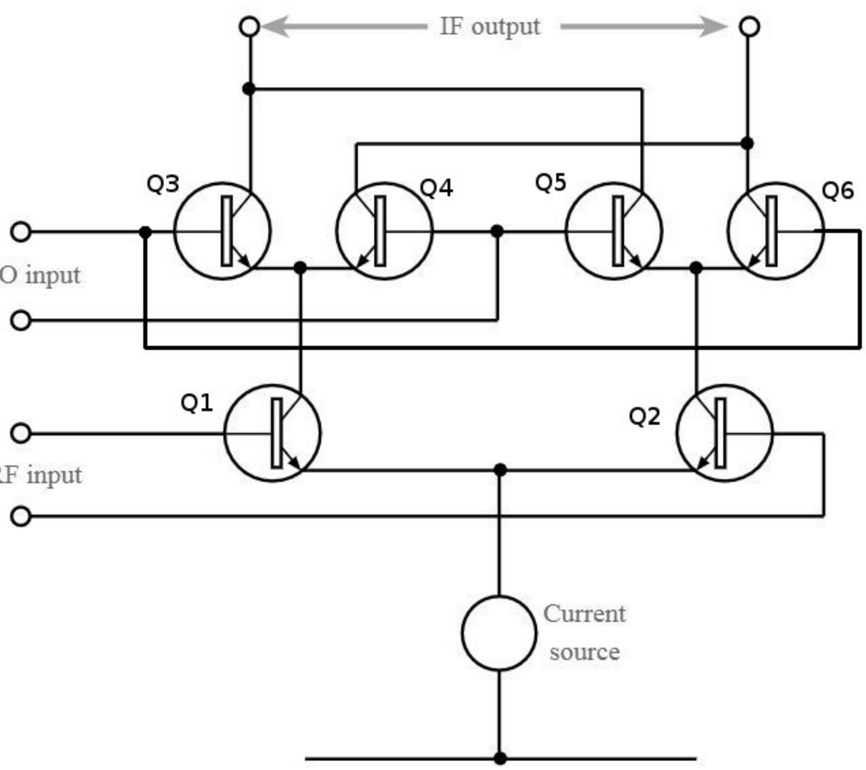

Figure 2: Engineering schematic of a Gilbert cell mixer

a reactive component. For example, if you sent a wave into a non-resonant tank circuit, the wave travelling out will be attenuated and have a time-phase shift, and that might be analogous to moving around the center of a smith chart in a distributed circuit.

Problem 3: Show at least three different passive circuits that can impedance match a 50 Ohm source to a load impedance $Z_L = 1500 - j20$ Ohms. Discuss the practical and theoretical characteristics of each one

One way to match a source impedance to a load impedance is with a simple LC low pass filter, such as the one shown in Figure 3. This circuit matches a 50 Ohm source impedance to a $1500 - j20$ Ohm load using just a low pass filter. The major practical limitations of this circuit are that for a given resonant frequency (10MHz in this case) there is exactly 1 combination of L and C that gives you perfect impedance matching at that frequency. This means that any deviation in component values will throw off the resonance and matching. Also of note is that this does not AC couple the source from the load, meaning if you need to DC isolate the two parts, then an additional blocking capacitor is necessary. One theoretical benefit to this circuit is that it performs as both an impedance matching circuit and a low pass filter, which may be useful depending on your filtering needs.

A second, similar way of passively matching impedances is with a high-pass filter, as shown in Figure 4. This has a similar limitation to the low-pass topology in that it is perfectly constrained and so there is only one possible set of LC values that will give you a perfect match at a given frequency, so again your values must be very precise. This topology AC couples your source and your load, but because of the shunt inductor, if your load is DC biased, you might short your bias to ground without another coupling cap on the load side. This circuit has the benefit of also behaving as a high-pass filter, which may be useful.

Figure 5 shows a third passive impedance matching circuit, the tapped C resonant circuit. For a high-circuit Q, the impedance transformation of this circuit is approximately $Z_{out} = Z_G * \alpha = Z_G(1 + \frac{C_1}{C_2}$, sourced from the EE157 lecture notes page 105. From playing around in sim-smith, I found that this relation becomes less true as the inductor increases in value. For This circuit, $\alpha = 1.5k/50 = 30$. The capacitor ratio is then $\frac{C_1}{C_2} = 4.47$. It can be seen in the simsmith circuit that the capacitor ratio is about 2 instead of 4.47. However, as I decreased the inductance to 1-2 microHenries, and matched the impedance, the capacitor ratio went up to more like 4.1, much closer to the predicted value. This circuit does have the theoretical luxury of having one more degree of freedom for impedance matching. For the previous two circuits, there was only one combination of components that matched impedances. Here, for each inductance there is a pair of values for the capacitors that produces a matched impedance at resonance. This circuit also acts as a bandpass filter in addition to matching impedances.

Problem 4: On page 11 of my lecture notes on Receivers I show you the Dual Noise Model. Explain why two noise sources are necessary to account for the input – output relationship of an RF amplifier.

For an amplifier stage, two random variables must be introduced to account for the noise the stage introduces to the signal: a voltage noise V_n and a current noise I_n this is the 'dual noise model'. The reason why both of these are needed is that the voltage noise and current noise introduced by the amplifier are uncorrelated random variables. If they were correlated you could determine one from the other by using ohm's law across the source impedance.

Problem 5: Explain why scattering parameters can be converted to h, γ, or z parameters and vice-versa

scattering parameters are defined in terms of travelling 'power waves' specified as a complex value with an amplitude and phase, effectively a Volt-Amp product. Hybrid, γ and z parameters are defined in terms of ratios of voltage and current at the ports of the 2-port network. The

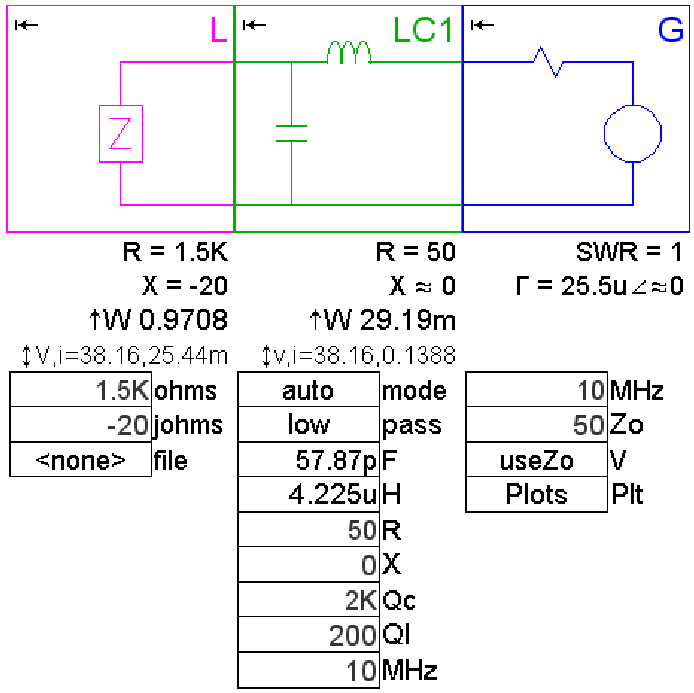

Figure 3: A low-pass LC impedance matching circuit generated in sim-smith

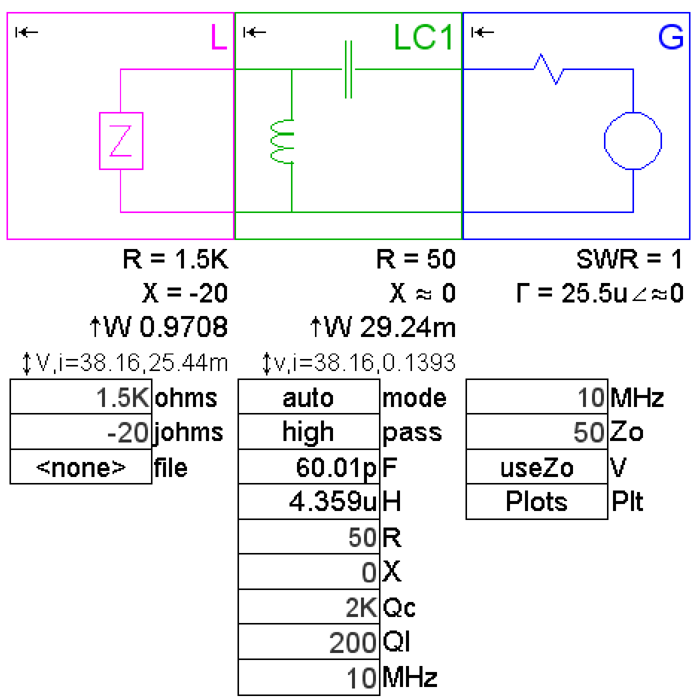

Figure 4: A high-pass LC impedance matching circuit generated in sim-smith

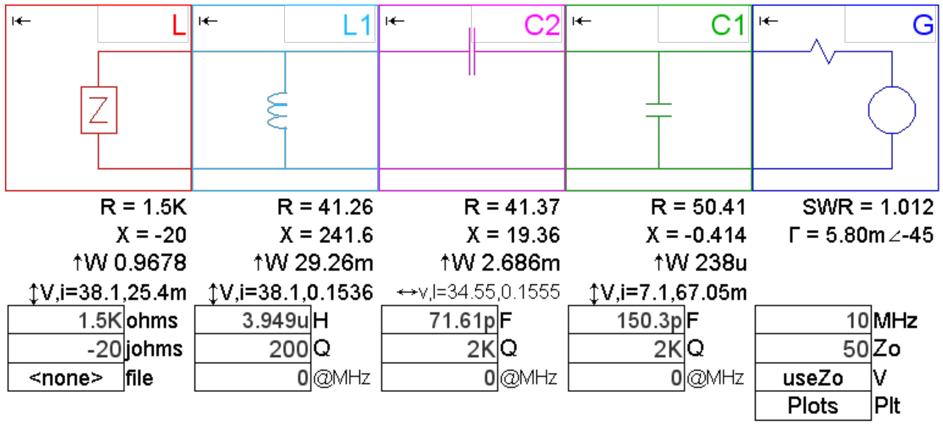

Figure 5: Tapped-C passive impedance matching circuit generated in sim-smith

voltage across and current through one port contains the same information as the power wave seen entering and reflected to that same port, the S-parameters are just specified in terms of gain, and H-parameters etc. are specified in terms of impedances and admittances. They are both ways of describing the phasor values of voltage and current seen at the two ports, and so they contain the same information expressed differently.

Problem 6: Explain the difference between "maximum unilateral transducer power gain" and "transducer power gain in 50 Ohm system"

The 'transducer power gain in 50 Ohm system' describes purely the gain of the amplifying component and does not take into account the reflection coefficients due to impedance matching. It's merely the power gain that the amplifying system produces. The 'maximum unilateral transducer power gain' describes the unilateral ($S_{12} = 0$, so assumed no feedback gain) gain of the system assuming terminated inputs and outputs and that both the output and input impedances are matched, giving maximum power transfer into and out of the amplifier.

Problem 7: Prove Friis Link Equation using aperture theory and explain the physical significance of both the receive and transmit antenna's apertures, A_{et} and A_{er}. Solution shown in Figure 6 through Figure 9. The explanation for the physical significance of the aperture of the receive antenna in Figure 8 and Figure 9 is my digestion of the derivation of the isotropic receive antenna aperture equation from the wikipedia page on antenna apertures. I realize now after writing it that it's not necessarily an intuitive explanation of the physical significance, but more a simplified model of the circuit that offers an intuition on how the equation is derived. To add to that explanation, the receive antenna's aperture is analogous to the aperture of a camera. The camera aperture captures only a percentage of the available light moving towards the lens proportional to the diameter of the aperture. The receive antenna captures an amount of the available radio EM radiation moving past it, proportional to the antenna's aperture and gain. The aperture can be represented as a power density over a surface area, the size of which corresponds to the amount of power received by the antenna. The reason why the aperture is actually proportional to the square of the signal wavelength is not obvious and I don't really understand the reason yet even after reading the wikipedia article and several explanations of the derivation.

Problem 8: Explain the physics of how an antenna is able to successfully radiate a

self-propagating electromagnetic wave.

When an antenna is subjected to a time-variant current through it, in a resonant antenna the current reaches the end of the antenna and bounces back, creating a standing current wave, which generates a standing voltage wave which is 90 degrees out of phase of the current wave.The varying voltage creates an electric field that radiates out into space and the current creates a magnetic field that radiates out into space. This is the near field, where the voltage and current in the antenna are independently generating electric and magnetic waves. In the far field, these radiated waves couple, begin to transfer energy between each other and become perpendicular and in-phase with respect to time. This effective storage and intertransferrence of energy between the far-field electric and magnetic waves are what allows the wave to self-propagate.

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Problem 7 solution

Friis's link equation

$$P_r = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2}$$

where

P_r : Power recieved by R_x antenna
P_T : Power delivered to transmitt antenna
G_T : gain of transmit antenna
G_R : gain of recieve antenna
λ : wavelength of carrier
R : distance between T_x & R_x antennae

The transmitting antenna has P_T watts of power delivered to it, which radiates isotropically outward as a power density

$$\rho = \frac{P_T}{\underbrace{S}_{\text{surface area of a sphere}}} = \frac{P_T}{4\pi R^2}$$

but the transmitting antenna will have some gain G_T which focuses the signal power, represented as a scalar in a specific direction (towards the R_x antenna)

$$\rho = \frac{P_T}{4\pi R^2} G_T \left[\frac{P}{m^2}\right]$$

Figure 6: Problem 7 solution part 1

56
52

The aperature of the recieving antenna, if it's isotropic can be expressed as

$$A_{R_{iso}} = \frac{\lambda^2}{4\pi}$$

but, the R_x antenna has some gain G_R

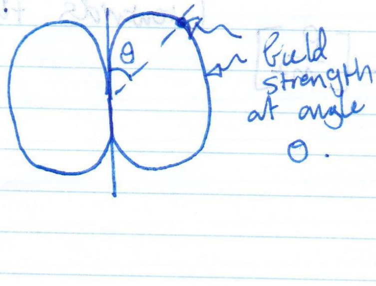
so,

$$A_R = \frac{\lambda^2}{4\pi} G_R \; [m^2]$$

then, the amount of power recieved by the ~~aperat~~ recieving antenna is equal to the power density at that location multiplied by the aperature and gain of the antenna

$$P_R = \rho \cdot A_R = \frac{P_T G_T}{4\pi R^2} \cdot \frac{\lambda^2}{4\pi} \cdot G_R = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2}.$$

The physical signibicance of the ~~recieve~~ transmit antenna is that an isotropic antenna would radiate a far-field signal ~~spheric~~ radially outward, but the gain pattern of the antenna focuses that power radiation. eg. for a dipole:

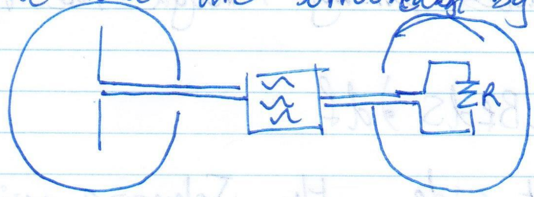


so, $A_{eT} = \frac{P_T \cdot G_T}{4\pi R^2}$

Figure 7: Problem 7 solution part 2

The physical significance of the recieve antenna's aperature is more complicated. The aperature of a recieving isotropic antenna is $\frac{\lambda^2}{4\pi}$.

This comes from modelling the antenna as an antenna and its matched load R. connected through a transmission line with a band pass filter. the antenna and the load are surrounded by walls emitting black body radiation.



The antenna collects the radiation from the black body which transmits only the frequencies of interest through the filter to the resistor.

The Black body radiates according to the Rayliegh-Jeans ~~eqn~~ Law.

$$B_f = \frac{2kT}{\lambda^2}\left[\frac{W}{sr\cdot m^2\cdot Hz}\right]$$

The power seen by the antenna will be

$$P = \int_0^\infty \left(\int_{4\pi} A_e B_f \, dS\right) df$$

with the quantity inside the parens being the surface integral of all collected power by the antenna.

Figure 8: Problem 7 solution part 3

Because the radiation is unpolarized, the antenna can ~~dt~~ collect half of the total power. ~~From~~

So the total power collected is

$$P = \int_0^\infty \left(\frac{1}{2}\int_{4\pi} A_e B_f \, dS\right) df.$$

which on the right side is seen only in the frequencies passing through the filter

$$P = \left(\frac{1}{2}\int_{4\pi} A_e B_f \, dS\right) df$$

On the right side, the Johnson noise of the resistor in the frequency band of interest is represented according to KTB

$$P_T = KT\,df$$

Since the cavities are at the same temperature, there can be no net power flow can happen between the two black bodies, so

$$\frac{1}{2}\left(\int_{4\pi} A_e B_f \, dS\right) \cancel{df} = KT\cancel{df} \quad \text{and } B_f = \frac{2kT}{\lambda^2}$$

$$\Rightarrow \frac{1}{\cancel{2}} A_e \frac{\cancel{2}K\cancel{T}}{\lambda^2} \int_{4\pi} dS = K\cancel{T}$$

$$\Rightarrow \cancel{A_e} = A_e \frac{1}{\lambda^2} \cdot 4\pi = 1$$

$$\Rightarrow A_e = \frac{\lambda^2}{4\pi}$$

This is then focused by the gain of the antenna, G_R

$$A_{ER} = \frac{\lambda^2}{4\pi} G_R$$

Figure 9: Problem 7 solution part 4