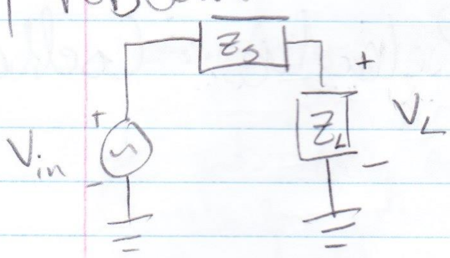
# Problem 5.




$$\overline{V}_L = \overline{V}_{in} \frac{Z_L}{Z_S + Z_L}$$

Apparent Power dissapated over the load, $S_L$:

$$S_L = \frac{\overline{V}_L^2}{Z_L} = \frac{\overline{V}_{in}^2 \frac{Z_L^{\cancel{2}}}{(Z_S+Z_L)^2}}{\cancel{Z_L}} = V_{in}^2 \frac{Z_L}{(Z_S+Z_L)^2}$$

$$= V_{in}^2 \frac{R_L + jX_L}{(R_S + jX_S + R_L + jX_L)^2}$$

We're only concerned with Real power dissapation so we can ignore the imaginary term

$$P_L = V_{in}^2 \frac{R_L}{[R_S + R_L + j(X_S + X_L)]^2}$$

Now, we can take the denominator as a magnitude and ignore the phase.

$$|P_L| = V_{in}^2 \frac{R_L}{\left[\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}\right]^2}$$

$$\Rightarrow |P_L| = V_{in}^2 \frac{R_L}{(R_S+R_L)^2 + (X_S+X_L)^2} \quad (1)$$

Now we take the derivative with respect to $X_L$

$$\frac{d|P_L|}{dX_L} = \frac{-2(X_S+X_L)}{(1+(X_S+X_L)^2)^2}$$

which is made zero only when $X_L = -X_S$

Plugging this relation back into equation ①

$$|P_L| = V_{in}^2 \frac{R_L}{(R_S+R_L)^2}$$

which is the same relation we get from the proof of DC MPT from page 63 of Bowick, which concluded with the following condition for MPT:

$$R_L = R_S$$

So, for max power transfer in an AC impeditive circuit,

$$X_L = -X_S \quad \text{and} \quad R_L = R_S$$

$$\boxed{\therefore \quad Z_L = Z_S^*}$$