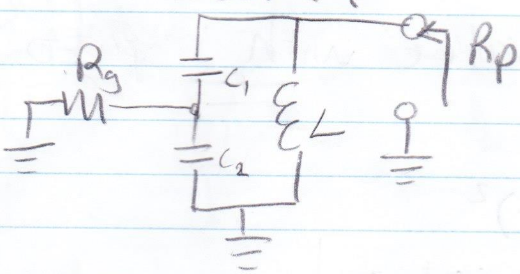
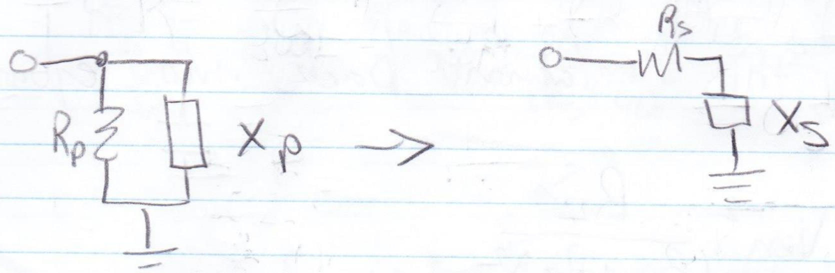
Problem 7 Proof



Prove: $R_p = R_g\left(1 + \frac{C_2}{C_1}\right)^2$

Series parallel transform:



$$Q_p = \frac{R_p}{X_p} \quad , \quad Q_s = \frac{X_s}{R_s}$$

$$Z_p = \frac{jR_pX_p}{R_p + jX_p} = \frac{R_pX_p^2}{R_p^2 + X_p^2} + j\frac{X_pR_p^2}{R_p^2 + X_p^2}$$

and $Z_s = R_s + jX_s$

$$Z_p = Z_s \implies R_s = \frac{R_pX_p^2}{R_p^2 + X_p^2} \text{ and } X_s = \frac{X_pR_p^2}{R_p^2 + X_p^2}$$

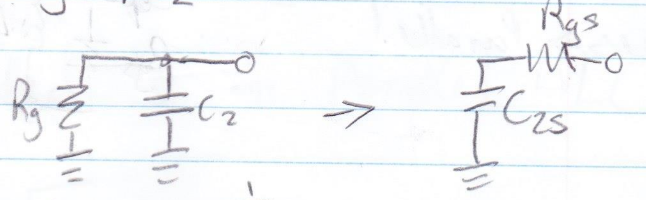
Using $Q_p = \frac{R_p}{X_p}$; $\implies R_s = \frac{R_p}{1 + Q_p^2}$ and $X_s = \frac{X_p}{1 + \frac{1}{Q_p^2}}$

Continuing w/ proof

Q - factor of R_g & C_2

$$Q_2 = \frac{R_g}{X_2} = jR_g\omega_r C_2$$

Parallel - series transform

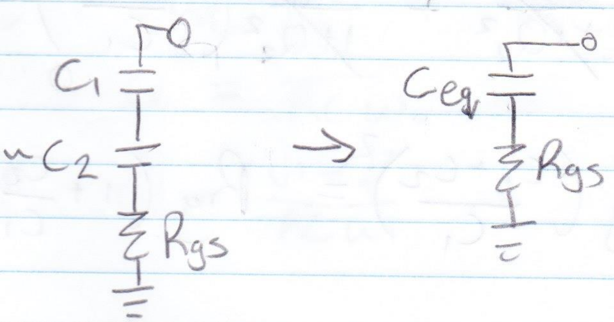


$$R_{gs} = \frac{R_g}{1+Q_2^2} \quad , \quad \frac{1}{C_{2s}} = \frac{\frac{1}{C_2}}{1+\frac{1}{Q_2^2}}$$

$$\Rightarrow C_{2s} = \frac{1+\frac{1}{Q_2^2}}{1/C_2} = C_2\left(1+\frac{1}{Q_2^2}\right)$$

$$\Rightarrow C_{2s} \approx C_2 \text{ if } Q_2^2 \gg 1$$

Now we have



$$C_{eq} \approx \frac{C_1 C_2}{C_1 + C_2}$$

Q factor of C_1, C_2, R_{gs} branch

$$Q_{1,2} = \frac{X_{eq}}{R_{gs}} = \frac{1}{\omega_0 R_{gs} C_{eq}} = \frac{1}{\omega_0 \frac{R_g}{1+Q_2^2} C_{eq}} = \frac{1}{\cancel{\omega_0} \frac{Q_2/\cancel{\omega_r} C_2}{1+Q_2^2} C_{eq}}$$

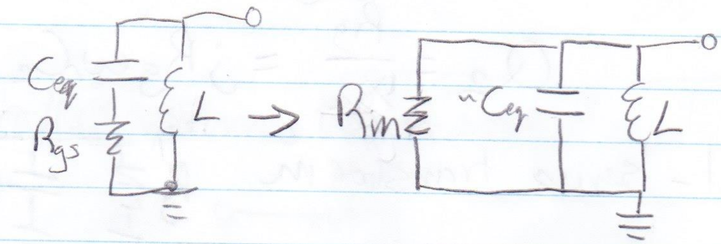
$$= \frac{(1+Q_2^2)C_2}{Q_2 C_{eq}} = \frac{C_2}{Q_2 C_{eq}} + \frac{Q_2 C_2}{C_{eq}}$$

$$= Q_2 \frac{C_2}{C_{eq}}\left(\frac{1}{Q_2^2}+1\right) \approx Q_2 \frac{C_2}{C_{eq}} \text{ when } Q_2 \ll 1$$

So, $Q_{1,2} \approx Q_2\left(\frac{C_1+C_2}{C_1}\right)$

want to transform

Series -> Parallel



$$R_{in} = R_{gs}\left(1+Q_{1,2}^2\right)$$

$$= \frac{R_g}{1+Q_2^2}\left(1+Q_{1,2}^2\right) = R_g\,\frac{1+Q_{1,2}^2}{1+Q_2^2}$$

$$\approx R_g\,\frac{1+Q_2^2\left(\frac{C_1+C_2}{C_1}\right)^2}{1+Q_2^2}$$

$$= R_g\left(\cancelto{0}{\frac{1}{1+Q_2^2}} + \cancelto{1}{\frac{Q_2^2}{1+Q_2^2}}\left(\frac{C_1+C_2}{C_1}\right)^2\right) \quad \text{if } Q_2^2 \gg 1$$

$$R_{in} \approx R_g\left(\frac{C_1+C_2}{C_1}\right)^2 = R_g\left(1+\frac{C_2}{C_1}\right)^2$$