# **Mathematics**

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#### (Mathematics, Statistics, and Computer Science)

Mathematics, the study of patterns and order, is a creative art, a language, and a science. The practice of mathematics combines the aesthetic appeal of creating patterns of ideas with the utilitarian appeal of applications of these same ideas. Long seen as the language of physical science, mathematics is also used increasingly to model phenomena in the biological and social sciences. Mathematical literacy is indispensable in today's society. As members of the Department of Mathematics, Statistics, and Computer Science (MSCS), mathematics faculty strive to help students understand natural connections among these related but distinct disciplines.

Mathematics at St. Olaf is interesting, exciting, accessible, and an appropriate area of study for all students. Each year, seven to ten percent of graduating seniors complete mathematics majors. The department offers courses representing various mathematical perspectives: theoretical and applied, discrete and continuous, algebraic and geometric, and more. Our faculty also teach courses in statistics, computer science, and mathematics education.

A concentration in statistics and a major in computer science are also available. Courses in these areas are taught by faculty from the Department of Mathematics, Statistics, and Computer Science. For more information on these, consult the separate listings under Statistics (http://catalog.stolaf.edu/academic-programs/statistics) and Computer Science (http://catalog.stolaf.edu/academic-programs/computer-science).

# **Overview of the Major**

Students arrange a major in mathematics by developing an Individualized Mathematics Proposal (IMaP). An IMaP outlines a complete, coherent program of study consistent with the goals of the individual student. The courses included in a student's IMaP are determined after consultation with an MSCS faculty member and approved by the department chair. About ten courses are normally required.

# Intended Learning Outcomes for the Major (http://wp.stolaf.edu/curriculum-committee/mathematics-major-ilos)

# **Special Programs and Opportunities**

Mathematical experiences inside and outside the classroom are important parts of St. Olaf mathematics. Following are some of the many possibilities. For more information consult the mathematics website (http://wp.stolaf.edu/math) or a mathematics faculty member.

- Research: An invigorating way to explore mathematics; research opportunities exist both on and off campus.
- Experiential learning: Internships, independent studies, and various courses (including MATH 390 Mathematics Practicum)

- allow students to apply mathematical knowledge beyond the classroom.
- Mathematical biology concentration: Mathematics students who are interested in biology should consider a concentration in this exciting interdisciplinary field.
- Study abroad: The IMaP's flexibility allows some study-abroad programs to fit into a student's mathematics major. Students interested in a program focused on upper-level mathematics should consider the Budapest Semesters in Mathematics.
- Problem solving and competitions: The department organizes
  problem-solving activities and sponsors student participation in
  regional and national competitions. St. Olaf also periodically hosts
  its own mathematics competition, the Carlson Contests.
- Student organizations: The department has an active student chapter of the national organization Pi Mu Epsilon.

# **Distinction**

See Academic Honors (http://catalog.stolaf.edu/academic-regulations-procedures/academic-honors/#distinction)

Distinction in mathematics is awarded to graduating seniors for distinguished work that goes beyond the minimum requirements for the major. Information is available in the MSCS department and on the mathematics website (http://wp.stolaf.edu/math).

# Recommendations for Graduate Study

Students planning graduate work in the mathematical sciences should pursue both depth and breadth in their majors. A broad range of courses will help students prepare for the Graduate Record Exam subject test in mathematics. Fifty percent of the GRE subject test covers single and multivariable calculus, and differential equations; 25 percent covers material from linear algebra, abstract algebra, and elementary number theory; 25 percent covers more advanced topics: real analysis, topology, combinatorics, complex analysis, probability, statistics, etc. Taking many level III courses will help students prepare for graduate study. Real Analysis II, Abstract Algebra II, Topology, Combinatorics, and Complex Analysis are especially recommended for students going into "pure" mathematics; Differential Equations II, Complex Analysis, Real Analysis II, and Applied Mathematics seminars are especially recommended for students going into applied mathematics. Research experiences (on or off campus) and independent studies will also help students assess and explore their interest in further mathematical study. Students considering graduate school should consult early with a mathematics faculty member about planning an appropriate IMaP.

# Requirements

Students arrange a major in mathematics by developing an Individualized Mathematics Proposal (IMaP). An IMaP outlines a complete, coherent program of study consistent with the goals of the individual student. The courses included in a student's IMaP are determined after consultation with an MSCS faculty member and approved by the department chair.

A path through the major as described by a student's IMaP normally includes:

Calculus (two semesters)

2.00

Linear algebra (one semester)

1.00

At least seven intermediate or advanced mathematics courses:

Select two of the f	2.00	
MATH 242	Modern Computational Mathematics	
MATH 244	Real Analysis I	
MATH 252	Abstract Algebra I	
Select an intermed the following diffe	3.00	
Computation/n		
Continuous/an		
Discrete/combi		
Axiomatic/algebraic		
Select at least two which must be pa sequence	2.00	
Total Credits		10

An IMaP may include up to two related courses from statistics or computer science; a current listing of such courses is available on the mathematics web page (http://wp.stolaf.edu/math). A student may also find a course outside of MSCS that contributes significantly to a mathematical path of study and may petition to have the course included in his or her IMaP.

The mathematics web page (http://wp.stolaf.edu/math) offers lists of courses that satisfy each perspective, lists of sequences, and other useful information. Students are encouraged to consult early with a mathematics faculty member about planning an appropriate IMaP.

# Recommendations for Mathematics Teaching Licensure

Mathematics majors who intend to teach grades 5-12 mathematics must meet all of the normal requirements for the major while including specific courses required for State of Minnesota licensure. Their IMaPs must include:

# Basic requirements for all majors

	MATH 232	Discrete Mathematics	1.00
	MATH 244	Real Analysis I	1.00
	MATH 252	Abstract Algebra I	1.00
	MATH 262	Probability Theory	1.00
	MATH 356	Geometry	1.00
	Course in statistics		1.00
	EDUC 350	Teaching of Mathematics, 5-12	1.00
	Several Education courses <sup>1</sup>		

As listed in the Education (http://wp.stolaf.edu/education) description

Students should consult the mathematics licensure advisor for advice on planning mathematics and education course sequences.

# **Courses**

# MATH 109: Preparation for Calculus

This course is designed for students who need additional preparation before taking calculus. The course emphasizes functions, including polynomial, exponential, logarithmic, and trigonometric. Enrollment requires permission of the Mathematics Placement Director.

# MATH 117: Gateways to Mathematics

This course explores the nature of mathematics and its role in contemporary society. The content and format of the course vary depending on the instructor's interests. In particular, the course may focus on one or two mathematical topics in depth or survey a wider range of topics. Recent topics have included mathematics of voting, probability, game theory, finance, and connections between mathematics and the arts. Offered annually.

**Prerequisites:** high school algebra and geometry.

#### MATH 119: Calculus I with Review

This course combines a full semester of differential calculus with reviews of algebra and pre-calculus topics for students who did not complete a pre-calculus course in high school, or are otherwise not prepared to enroll in Mathematics 120. Review of algebraic techniques and the language of functions are integrated with topics from the calculus of functions of a single real variable, including trigonometric, exponential, and logarithmic functions. Topics are explored graphically, symbolically, and numerically. Class meets four times per week, including one laboratory meeting. Counts toward mathematics major. Offered in the fall semester.

Prerequisite: mathematics placement recommendation.

# MATH 120: Calculus I

This course introduces differential and integral calculus of functions of a single real variable, including trigonometric, exponential, and logarithmic functions. Derivatives, integrals, and differential equations are explored graphically, symbolically, and numerically. Applications of these topics are emphasized throughout the course. Offered each semester.

**Prerequisite:** mathematics placement recommendation.

# MATH 126: Calculus II

This course covers methods and applications of integration, geometric and Taylor series, and introduces partial derivatives and double integrals. Credit may be earned for either MATH 126 or MATH 128, but not both. Offered each semester.

**Prerequisite:** MATH 119 or MATH 120 or equivalent, or mathematics placement recommendation.

# MATH 128: Honors Calculus II

This course covers the material in MATH 126 in greater depth and breadth. Credit may be earned for either MATH 126 or MATH 128, but not both. Offered annually in the fall semester.

**Prerequisite:** 4 or 5 on AP Calculus AB exam or permission of the Mathematics Placement Director.

# MATH 200: Topics in Mathematics (0.25)

Students explore special topics in mathematics. Topics may vary from year to year. Counts toward mathematics major. Offered periodically. **Prerequisite:** permission of instructor.

# MATH 201: Topics in Mathematics (0.50)

Students explore special topics in mathematics. Topics may vary from year to year. Counts toward mathematics major. Offered periodically. **Prerequisite:** permission of instructor.

# MATH 218: Geometry and Decorative Art in Morocco (abroad)

Islamic art is decorative and based on geometry. Students study this art, its origins, and its significance, along with cultural topics related to Moroccan life, in the imperial city of Fes. Field work includes identification and analysis of distinct geometrical patterns found on buildings, monuments, and artifacts. Students also use geometry to create their own art. Mosaic designs are still created in Fes, a center for Islamic geometric patterns. Students stay with Moroccan families while in Fes. Field trips visit sites in and around Fes, with day-long visits to Meknès, Moulay Idriss, and Volubis, and a longer excursion to Marrakech and Casablanca. Offered periodically during Interim.

# MATH 220: Elementary Linear Algebra

This course beautifully illustrates the nature of mathematics as a blend of technique, theory, abstraction, and applications. The important problem of solving systems of linear equations leads to the study of matrix algebra, determinants, vector spaces, bases and dimension, linear transformations, and eigenvalues. Offered each semester and periodically during Interim. Counts toward management studies concentration.

**Prerequisite:** MATH 119 or MATH 120.

#### MATH 226: Multivariable Calculus

This course extends important ideas of single-variable calculus (derivatives, integrals, graphs, approximation, optimization, fundamental theorems, etc.) to higher-dimensional settings. These extensions make calculus tools far more powerful in modeling the (multi-dimensional) real world. Topics include partial derivatives, multiple integrals, transformations, Jacobians, line and surface integrals, and the fundamental theorems of Green, Stokes, and Gauss. Offered each semester.

Prerequisites: MATH 126 or MATH 128, and MATH 220.

# MATH 230: Differential Equations I

This course introduces differential equations and analytical, numerical, and graphical techniques for the analysis of their solutions. First- and second-order differential equations and linear systems are studied. Applications are selected from areas such as biology, chemistry, economics, ecology, and physics. Laplace transforms or nonlinear systems may be covered as time permits. Students use computers extensively to calculate and visualize results. Offered each semester. Counts toward neuroscience, management studies, and mathematical biology concentrations.

Prerequisite: MATH 126 or MATH 128 and MATH 220.

# MATH 232: Discrete Mathematics

Using problem solving techniques, students study topics from non-continuous mathematics, including basic counting principles, mathematical induction, recursion, efficiency of algorithms, and graph theory. Offered alternate years. Students may not take this course after MATH 244, MATH 252, or MATH 282 without permission of instructor. Counts toward management studies concentration.

Prerequisite: MATH 119 or MATH 120 or CSCI 121 or CSCI 125 or permission of instructor.

# MATH 236: Mathematics of Biology

This course introduces students to the essential modeling techniques of formulation, implentation, validation, and analysis. Students engage in these areas by combining experiment, mathematical theory, statistics, and computation to better understand a wide variety of biological systems. Offered annually in the spring semester. Counts toward neuroscience and mathematical biology concentrations.

Prerequisite: MATH 126 or MATH 128, and MATH 220.

# MATH 239: Number Theory - Budapest (abroad)

This course introduces the study of patterns and relationships satisfied by natural numbers. Topics include divisibility, modular arithmetic, prime numbers, congruences, primitive roots, and quadratic residues. The course is offered in Budapest, Hungary, a world center for mathematics research. The course includes several hours of Hungarian language instruction and lectures by Hungarian mathematicians and scholars. Offered annually during Interim.

Prerequisite: MATH 220 or permission of instructor.

#### MATH 242: Modern Computational Mathematics

Modern mathematics is characterized by the interaction of theoretical and computational techniques. In this course, students study topics from pure and applied mathematics with the aid of computation. Symbolic, graphical, and numerical computational techniques are introduced. Students develop computational skills sufficient to investigate mathematical questions independently. No previous programming experience is required. Offered annually. Counts toward mathematical biology concentration.

Prerequisite: MATH 220.

# MATH 244: Real Analysis I

Students encounter the theory of calculus and develop tools for communicating mathematical ideas with technical accuracy and sophistication. The goal is mastery of the concepts (e.g., limit, continuity, derivatives, and integrals) necessary to verify such important results as the Fundamental Theorem of Calculus, the Mean Value Theorem, and the Bolzano-Weierstrass Theorem. Emphasis is on theory and on developing the ability to write proofs. Offered each semester. Counts toward computer science mjaor.

**Prerequisite:** MATH 126 or MATH 128, and MATH 220. Permission of instructor required for first-year students.

# MATH 252: Abstract Algebra I

Algebra is concerned with sets of objects and operations that satisfy a few basic properties. Using the properties we study axiomatic systems such as groups, rings, and fields, covering topics such as homomorphisms, cosets, quotient structures, polynomial rings, and finite fields. Emphasis is on theory and on developing the ability to write proofs. Offered each semester. Counts toward computer science major.

**Prerequisite:** MATH 220. Permission of instructor required for first-year students.

# MATH 262: Probability Theory

This course introduces the mathematics of randomness. Topics include probabilities on discrete and continuous sample spaces, conditional probability and Bayes' Theorem, random variables, expectation and variance, distributions (including binomial, Poisson, geometric, normal, exponential, and gamma) and the Central Limit Theorem. Students use computers to explore these topics. Offered each semester. Counts toward management studies concentration.

Prerequisite: MATH 126 or MATH 128.

# MATH 266: Operations Research

Students are introduced to modeling and mathematical optimization techniques (e.g., linear programming, network flows, discrete optimization, constrained and unconstrained nonlinear programming, queuing theory). Students use computers to explore these topics, but prior computer experience is not assumed. Offered annually. Counts toward management studies concentration.

**Prerequisites:** MATH 126 or MATH 128, and MATH 220; MATH 226 and/or MATH 262 recommended.

# MATH 282: Topics in Mathematics

Students work intensively on a special topic in mathematics or its applications. Topics vary from year to year. May be repeated if topic is different. Offered periodically.

# MATH 294: Academic Internship MATH 298: Independent Study

# MATH 330: Differential Equations II

This course covers partial differential equations from an applied perspective and emphasizes simple models involving phenomena such as wave motion and diffusion. Topics and techniques such as separation of variables, boundary value problems, Fourier series, and orthogonal functions are developed carefully. Mathematical computing is used freely. Offered alternate years. Counts toward neuroscience and mathematical biology concentrations.

# Prerequisite: MATH 230. MATH 340: Complex Analysis

Complex analysis treats the calculus of complex-valued functions of a complex variable. Familiar words and ideas from ordinary calculus (limit, derivative, integral, maximum and minimum, infinite series) reappear in the complex setting. Topics include complex mappings, derivatives, and integrals; applications focus especially on the physical sciences. Offered annually.

Prerequisite: MATH 220, and MATH 226 or MATH 244.

# MATH 344: Real Analysis II

The main topics of this course are measure theory on the real line, the Lebesgue integral and its relation to the Riemann integral, and convergence theorems for the Lebesgue integral. Applications to probability and harmonic analysis may be included. Offered alternate years.

Prerequisite: MATH 244.

MATH 348: Topology

This course is an introduction to topological spaces and their structures mainly from the point-set perspective. Standard topics include separation axioms, compactness, and connectedness. Other topics from geometric and algebraic viewpoints may be included. Offered alternate years.

Prerequisite: MATH 244.

MATH 352: Abstract Algebra II

This course is a continuation of the study of the theory of groups, rings, and fields. Topics include group actions, Sylow theory, and Galois theory. Other topics may include representation theory, module theory, and others. Offered alternate years.

**Prerequisite:** MATH 252. **MATH 356:** *Geometry* 

Properties of axiomatic systems are illustrated with finite geometries and applied in a synthetic examination of Euclid's original postulates, well-known Euclidean theorems, and non-Euclidean geometries. Euclidean, similarity, and affine transformations are studied analytically. These transformations are generalized to obtain results in hyperbolic geometry and used to generate fractals in an exploration of fractal geometry. Dynamic geometry software and hands-on labs are used to explore both the transformations and properties of these geometries. Offered annually during Interim.

Prerequisite: MATH 220, and MATH 244 or MATH 252.

## **MATH 364: Combinatorics**

This course covers basic enumeration, including generating functions, recursion, and the inclusion-exclusion principle. Basic combinatorial objects such as set partitions, permutations, integer partitions, and posets are discussed. Making conjectures and proving theorems combinatorially are emphasized. Students also explore topics in graph theory, matrix theory, and representation theory. Offered alternate years.

**Prerequisite:** MATH 252; some previous exposure to counting methods (e.g., counting permutations and combinations) is helpful but not required.

### MATH 382: Topics in Mathematics

Students work intensively on a special topic in mathematics. Topics vary from year to year. May be repeated if topic is different. Offered annually.

### MATH 384: Topics in Applied Mathematics

Students work intensively on a special topic in applied mathematics. Topics vary from year to year. May be repeated if topic is different. Offered periodically.

#### MATH 390: Mathematics Practicum

Students work in groups on substantial problems posed by, and of current interest to, area businesses and government agencies. The student groups decide on promising approachesto their problem and carry out the necessary investigations with minimal faculty involvement. Each group reports the results of its investigations with a paper and an hour-long presentation to the sponsoring organization. Offered annually during Interim.

# MATH 394: Academic Internship

# MATH 396: Directed Undergraduate Research

This course provides a comprehensive research opportunity, including an introduction to relevant background material, technical instruction, identification of a meaningful project, and data collection. The topic is determined by the faculty member in charge of the course and may relate to his/her research interests. Offered based on department decision. May be offered as a 1.00 credit course or .50 credit course.

**Prerequisite:** determined by individual instructor. **MATH 398:** *Independent Research* 

# **Related Courses**

# **CSCI 315: Bioinformatics**

Students study computational problems arising from the need to store, access, transform, and utilize DNA-related data. Topics from computer science include: exhaustive search; algorithms (including dynamic programming, divide-and-conquer, graph and greedy algorithms) for fragment reassembly, sequence alignment, phylogenetic trees; combinatorial pattern matching; clustering and trees; and hidden Markov models. Offered alternate years. Counts toward neuroscience, biomolecular studies, and mathematical biology concentrations and mathematics major.

**Prerequisites:** CSCI 253, or one of CSCI 121 or CSCI 125, and one of BIO 125 or MATH 220, or permission of instructor.

# **CSCI 333: Theory of Computation**

Students learn about formal languages, automata, and other topics concerned with the theoretical basis and limitations of computation. The course covers automata theory including regular languages and context-free languages, computability theory, complexity theory including classes P and NP, and cryptographic algorithms. Offered alternate years. Counts toward neuroscience and linguistic studies concentrations.

**Prerequisite:** a proof writing course (such as MATH 244 or MATH 252) or permission of instructor.

# MSCS 389: Math, Statistics, and Computer Science Research Methods (0.50)

Students focus on writing scientific papers, preparing scientific posters, and giving presentations in the context of a specific, yearlong, interdisciplinary research project. In addition, this weekly seminar series builds collaborative research skills such as working in teams, performing reviews of math, statistics, and computer science literature, consulting effectively, and communicating proficiently. Exposure to post-graduate opportunities in math, statistics, and computer science disciplines is also provided. Open to students accepted into the Center for Interdisciplinary Research.

#### STAT 212: Statistics for the Sciences

A first course in statistical methods for scientists, this course addresses issues for proposing/designing an experiment, as well as exploratory and inferential techniques for analyzing and modeling scientific data. Topics include probability models, exploratory graphics, descriptive techniques, statistical designs, hypothesis testing, confidence intervals, and simple/multiple regression. Offered each semester. Enrollment limited for seniors. STAT 110, STAT 212, and ECON 263 all provide an introduction to statistics and students should not take more than one; they all can serve as a prerequisite for further courses. Counts toward environmental studies major (natural science and social science emphases), exercise science major, management studies concentration, and mathematical biology concentration.

# STAT 214: Honors Statistics for the Sciences

This course engages students with the material in STAT 212 in greater depth and breadth. Topics such as bootstrapping and multiple regression will be introduced in addition to core material from STAT 212. No previous experiences with statistics is assumed; this course provides an introduction to statistics, but it can also be taken by students coming from STAT 110, ECON 263, or AP Statistics who would like a transition into the statistics concentration. Counts toward environmental studies major (social science emphasis), exercise science major, and mathematical biology concentration.

**Prerequisites:** MATH 119 or MATH 120 or equivalent, or CSCI 125, or some previous statistics (e.g., AP Statistics or STAT 110 or ECON 263).

# STAT 272: Statistical Modeling

This course takes a case-study approach to the fitting and assessment of statistical models with application to real data. Specific topics include multiple regression, model diagnostics, and logistic regression. The approach focuses on problem-solving tools, interpretation, mathematical models underlying analysis methods, and written statistical reports. Offered each semester. Counts toward environmental studies major (natural science and social science emphases) and management studies, mathematical biology, and neuroscience concentrations.

**Prerequisite:** STAT 110, STAT 212, or STAT 214, or ECON 263, or equivalent preparation, or permission of instructor.

# STAT 316: Advanced Statistical Modeling

This course extends and generalizes methods introduced in STAT 272 by introducing generalized linear models (GLMs) and correlated data methods. GLMs cover logistic and Poisson regression, and more. Correlated data methods include longitudinal data analysis and multi level models. Applications are drawn from across the disciplines. Offered annually in the spring semester. Counts toward neuroscience concentration.

# Prerequisite: STAT 272. STAT 322: Statistical Theory

This course is an investigation of modern statistical theory along with classical mathematical statistics topics such as properties of estimators, likelihood ratio tests, and distribution theory. Additional topics include Bayesian analysis, bootstrapping, Markov Chain Monte Carlo, and other computationally intensive methods. Offered annually in the fall semester. Counts toward neuroscience concentration.

### Prerequisite: STAT 272 and MATH 262.

# **Faculty**

# Chair, 2016-2017 Kristina C. Garrett

Professor of Mathematics, Statistics, and Computer Science enumerative and algebraic combinatorics; partition theory; q-series

# Richard J. Allen

Professor of Mathematics, Statistics, and Computer Science logic programming; intelligent tutoring systems; computer science; Islamic geometric patterns; bioinformatics

# Joseph J. Benson

Visiting Assistant Professor of Mathematics, Statistics, and Computer

differential geometry; differential equations; lie groups; symmetries; moving frames

# Adam H. Berliner

Associate Professor of Mathematics, Statistics, and Computer Science combinatorial matrix theory; linear algebra; graph theory

#### Peter F. Blanchard

Visiting Assistant Professor of Mathematics

# Debra L. Czarneski

# Jill Dietz

Professor of Mathematics, Statistics, and Computer Science algebraic topology; group theory

# Kosmas J. Diveris

Assistant Professor of Mathematics, Statistics, and Computer Science commutative algebra; homological algebra

# Kathryn Ziegler Graham

Associate Professor of Mathematics, Statistics, and Computer Science biostatistics

# **Bruce Hanson**

Professor of Mathematics, Statistics, and Computer Science real analysis; complex analysis

### Paul Humke (on leave)

Professor of Mathematics, Statistics, and Computer Science real analysis; dynamical systems

# Ryota Matsuura (on leave Interim and spring)

Associate Professor of Mathematics, Statistics, and Computer Science mathematics education; algebraic number theory

# **Steven McKelvey**

Professor of Mathematics, Statistics, and Computer Science, Registrar and Assistant Vice President for Academic Affairs operations research; wildlife modeling

# **Bruce E. Pell**

Instructor in Mathematics, Statistics, and Computer Science

# Marju Purin

Assistant Professor of Mathematics homological algebra; representation theory

# Matthew P. Richey

Professor of Mathematics, Statistics, and Computer Science applied and computational mathematics

# Paul J. Roback (on leave)

Professor of Mathematics, Statistics, and Computer Science statistics

# Kay E. Smith

Associate Professor of Mathematics, Statistics, and Computer Science logic; discrete mathematics

# **Matthew Wright**

Visiting Assistant Professor of Mathematics, Statistics, and Computer Science

# **Paul Zorn**

Professor of Mathematics, Statistics, and Computer Science complex analysis; mathematical exposition