Hand ControlAR: An Augmented Reality Application for Learning 3D Geometry

*Rui Cao[1] †Yue Liu[1, 2]

[1]Beijing Engineering Research Center of Mixed Reality and Advanced Display

Beijing Institute of Technology

[2]AICFVE of Beijing Film Academy

Abstract

The traditional way of learning geometry cannot provide a great support for novice students since the geometric figures are 2D on the blackboard or the book. In consideration that Augmented Reality(AR) provides an intuitive way to learn geometry, an interactive AR system that enables students to naturally and directly manipulating 3D objects through hand gesture-based interactions and intuitively explore the spatial relationship between spheres and polyhedrons is proposed in this paper. The proposed gesture-based interaction enables the user manipulate AR objects in the real 3D space instead of 2D space. We design three levels of study to enable students to learn the geometric concepts as well as an experiment to evaluate the effectiveness of the AR system. Analysis of experimental results showed that the proposed system is easy to use, attractive, and helpful for students.

Keywords: Augmented Reality; Hand Gesture Interaction; User Defined Targets; 3D Objects Manipulation; Geometry Education.

Index Terms: H.5.1 [Information Interfaces and Presentation]: Multimedia Information Systems—Artificial, augmented, and virtual realities; H.5.2 [Information Interfaces and Presentation]: User Interfaces—Interaction styles; J.2 [Computer Applications]: Physical Sciences and Engineering—Mathematics and statistics

1 Introduction

Geometry learning requires skills of spatial thinking, spatial imagination and visualization to deal with 3D shapes, which has always been one of the most difficult parts of mathematics for students to learn and understand. It requires students to possess the ability to picture 3D shapes mentally from the 2D projection picture of 3D models. In the teaching of geometry in high school, the combination of spheres and polyhedrons is an important part of geometry, especially the circumscribed sphere of triangular pyramid. It is also a great challenge that calls for students' spatial thinking and logical analysis ability. However, it is difficult for students to imagine the spatial relationship between the sphere and the polyhedron (such as inscribed sphere, edge-tangent's sphere and circumscribed sphere), which results in the fact that the students cannot draw 2D images correctly. In addition, it is difficult for students to determine the position of the spherical center and radius of the sphere such as the circumscribed sphere of the polyhedron on the premise of drawing graphics as shown in Fig.1.

Development of spatial skills can be obtained indirectly through sketching tasks, creating and reading orthographic and axonometric projections in class [1]. Spatial abilities can be improved after specific training. The research of Potter and Van der Merwe [2] shows that spatial thinking can be increased through instruction focusing on using perception and mental imagery in 3D representation. Presenting learning materials through visual resources is of positive significance for fully understanding scientific phenomena and building conceptual models. The traditional way of learning geometry from the blackboard or the book can't meet students' requirements. Nowadays the development of such new technologies as AR has brought new ways to deal with mathematical knowledge, especially geometry [3]. AR can place virtual objects in real environments and many researchers believe that AR technology promotes the creation of interesting teaching content and the development of spatial thinking.

Hand gestures are one of the most basic characteristics of human beings and an indispensable part of interpersonal communication. Compared with the direct interaction on the 2D screen, gesture interaction is a more natural way of interaction [4]. A user can directly see the results of the interaction, which can provide users with an intuitive and natural way to interact with 3D shapes and fully improve the operability of the AR classroom.

The main objective of this paper is to develop an AR system using hand gesture interactions with the aim of improving students' spatial ability about the spatial relationship between spheres and polyhedrons.

The remaining part of this paper is organized as follows: Section II introduces the existing applications and related researches. The key technical elements and functional operations of the proposed system are presented in Section III. Section IV evaluates the results of user studies, which is followed by discussions, suggestions and improvements. Finally, the work is concluded with possible future extensions.

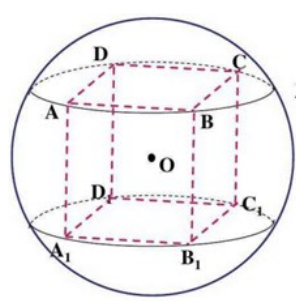


Figure 1: The circumscribed sphere of the cube.

*e-mail: 2120170523@bit.edu.cn
† e-mail: liuyue@bit.edu.cn

978-1-7281-4765-9/19/$31.00 ©2019 IEEE
DOI 10.1109/ISMAR-Adjunct.2019.00-60

2 RELATED WORKS

Researchers have long been studying the ways to apply AR in the field of education. According to the study performed by Kerawalla [5], with the development of AR technology, various AR projects for training and education are implemented both in formal and informal education. Kaufmann [6] presented a 3D geometric construction application Construct3D which is designed for mathematics and geometry education. Construct3D can support various teacher-student interaction scenarios, however, it needs sophisticated and expensive equipment such as tracking systems, head mounted displays and stereoscopic video projections, which may reduce the comfort and interest of learning.

AR-Dehaes[3] is an augmented book for spatial abilities development in engineering students during a short remedial course. Compared with traditional books, augmented books provide students with added value contents. Unlike Construct3D, AR-Dehaes only required simple and low cost equipment such as a computer with a webcam. However, the application simply superimposed virtual objects on the real environment without corresponding interaction.

In 2009, Kaufmann [7] outlined an AR application for differential geometry learning which enables its users to intuitively explore the properties of curves and construct some special spheres such as Meusnier’ s sphere. The work presented by Castillo et al [8] illustrates how to use mobile AR in quadratic equations and how to create a complete framework to interact with quadratic equations. An interesting work on AR application for calculus has been conducted by Salinas [9] and Quintero [10], which is divided into three levels of learning parabolic, circular and sinusoidal forms. They emphasized that more attentions should be paid to the visualization process when building objects, rather than just presenting them in AR mathematics education. In their work a little acrylic was used to generate and cut solids. There are also other AR applications designed for learning conic curves. Salinas [11] discussed an AR environment that allows students to interact with cones and planes in real time using virtual objects and can dynamically recreate mathematical proofs. Furthermore, math anxiety that affects the learning process can be effectively reduced through such learning method.

In addition to the above-mentioned AR applications for geometry education such as Construct3D and AR-Dehaes, several other new approaches have also been applied into the classroom to improve the teaching and learning of geometry. The Geometer's Sketchpad [12] is the first software packages that can dynamically show the position relationship and operation variation rules of geometric objects. It takes points, lines and circles as the basic elements, through transformation, construction, calculation, calculation, animation and tracking of these basic elements, it can display or construct relatively complex graphics. Teachers often use it to create animations for geometry learning. However, it only supports 2D geometry.

As a novel human computer interface, gestures have the characteristics of naturalness, intuition and easy learning, and have been widely used in AR teaching system. Whie Jung [13] proposed BoostHand, a freehand, distance-free object manipulation system that supports simple trigger gestures for AR Classrooms. Teachers and students can use virtual textbooks and handle virtual objects conveniently without the limitation of space and distance. Hong-Quan Le and Jee-In Kim [14] also discussed an AR application with hand gestures for learning 3D geometry. It was suggested that hand gestures could provide students with an easier way of studying geometry. However, this application can only be used to learn simple geometry.

3 PROPOSED SYSTEM

This section introduces the design of hand gesture interaction as well as the functional operations of the proposed AR system to help students learn the spatial combination of spheres and polyhedrons, especially the circumscribed sphere of triangular pyramid. The objective of our research is to present a novel method to effective learn geometry knowledge in classroom. Both AR and hand gesture recognition are major components of the proposed system. With AR, students can intuitively perceive the spatial relationship between 3D objects and the dynamic change process of 3D objects, thus improving students’ spatial thinking and logical thinking ability. Using gestures to interact directly with virtual objects in 3D space can deepen students’ understanding of geometric concepts.

3.1 System Design

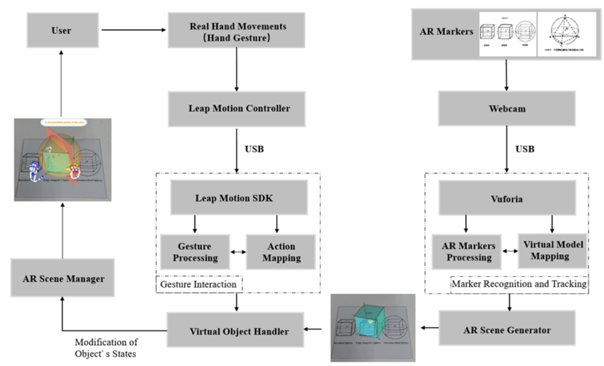


Figure 2: Overview of the proposed system.

Fig.2 shows the framework of the proposed system, which consists of marker recognition and tracking subsystem as well as hand gesture interaction subsystem. Firstly, the webcam captures the real scene and identifies the printed AR markers. Then, the system measures the distance and estimates the orientation of the markers. The captured and aligned markers are matched with the database to retrieve the corresponding 3D models. The virtual 3D models are overlaid onto the markers in the real scene. The students can interact with the 3D models in real-time by hands using the Leap Motion controller.

3.1.1 Marker Recognition and Tracking

Vuforia is an open-software library that enables real-time tracking for AR applications. In our system, we used Vuforia for AR marker recognition and tracking processes, and Unity3D for rendering the 3D models. Once the image target is detected, Vuforia Engine will track the image and place the corresponding virtual model on the image.

We use markers with a 2D projection of 3D models that help students to construct links between the physical models and their representations on paper like Fig.3.

3.1.2 Hand Gesture Interaction Subsystem

This section introduces the design of hand gesture interaction and the function.

In order to interact with the help of gesture interaction, the Leap Motion device is used to track hand positions and trigger gestures using a detection module whenever users make certain gestures. The Leap Motion device is connected to computer via a USB 3.0 port. Whenever the students try to interact with the 3D geometrical objects, the proposed system recognizes their hand gestures using the Leap Motion controller and maps them into appropriate commands of the system.

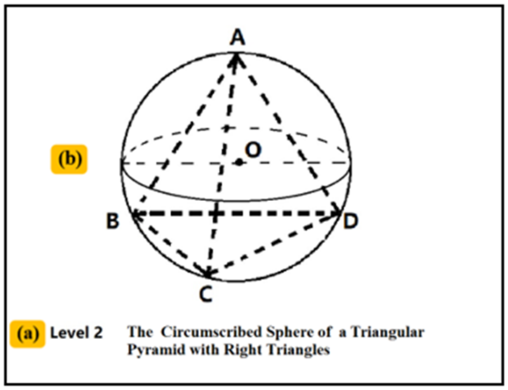


Figure 3: AR marker in level 2. Each marker includes:(a) shape name and (b) geometric shape.

In order to enable the virtual hand to interact with the 3D model, we set the initial position of the virtual hand in the middle of the virtual 3D model. The Action Mapping function is an interface that consists of a set of functions for controlling geometrical objects as shown in Table 1. The Leap Motion device detects user's hands in each frame. If the user performs the gesture consistent with the triggering gesture of relevant functions, the virtual 3D geometric model will change correspondingly.

Table 1 describes all of functions which are used for controlling the objects in our system.

Table 1: The circumscribed sphere of the cube.

Function	Hand Gestures Descriptions
Hand UI	Users perform this action by using the left hand to face the camera to display the UI, and then clicking the corresponding button with the right hand, including displaying answers, displaying hints, changing transparency and locking rotation, moving and zooming.
Rotation	Users perform this action by using their hand to drive a set of rotating handles on 3D object to rotate the object in 6 DOF.
Zooming	User perform this action by using both hands and scale the objects in three axes of object's coordinates – x, y and z. User holds his forefinger and thumb together, similar to stretching an object, and the hands close to and away from each other to control the scale of the object.
Moving	Users perform this actions by using their hand to drive a set of moving handles on 3D object to move the object around.
Cutting	Users perform this action by turning left hand over, pointing the thumb at the camera and waving the left hand down to cut the object.
Recovery	Users perform this action by clenching the left hand into a fist.

Fig.4 shows the gestures that we set and their corresponding functions. In our system, the gestures are specified as simple as possible for the students to remember and perform easily and conveniently. For such operations as changing transparency and displaying answers, there is no natural gesture suitable, so we use "Hand UI" gesture that users can click the button on the UI to select the operation. Users can use virtual hands to move and rotate objects directly. The gesture for zooming and cutting is similar to stretching and cutting objects in our everyday life.

3.2 Description of the Didactic Content

It's important that learners don't always passively reproduce the information provided by teachers, but can construct and reflect on the knowledge that they have learned as well as apply the knowledge to solve problems [15]. According to this teaching idea, the training is made up of three levels and each one corresponds to a different point of knowledge in our system to help students learn the circumscribed sphere of triangular pyramid.

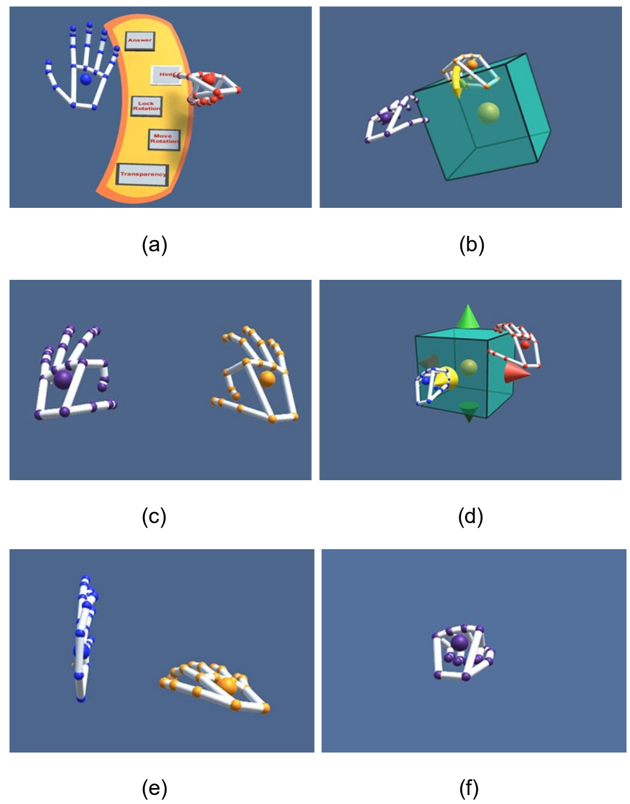


Figure 4: Hand Gesture. (a) Hand UI. (b) Gesture for rotation. (c) Gesture for zooming. (d) Gesture for moving. (e) Gesture for cutting. (f) Gesture for recovery.

3.2.1 Level 1 – The Combination of Spheres and Cubes

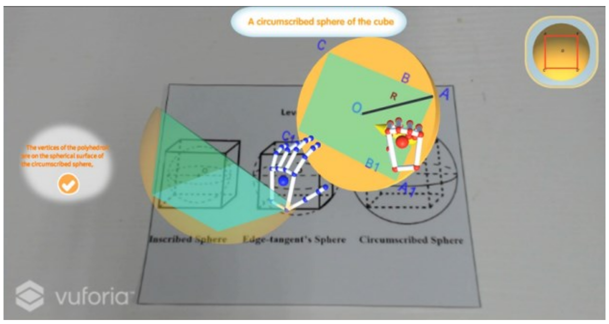
In this level, the center of the sphere coincides with the cube. Students can observe the spatial relationship between the sphere and the cube by controlling the size and transparency of the sphere. When the sphere is the cube's inscribed sphere, edge-tangent's sphere or circumscribed sphere, the corresponding prompt will be displayed on the screen. At this point, students can perform the "Cutting" operation to cut the models, so they can have a more intuitive sense of the spatial relationship between the two models as shown in Fig.5(1). A cross section image of sphere and polyhedron is displayed in real time in the upper right corner of the screen.

The purpose of this training is to enable students to have a full understanding of the inscribed sphere, edge-tangent's sphere and circumscribed sphere through the spatial combination of the sphere and the simple cube, laying a solid foundation for the later learning.

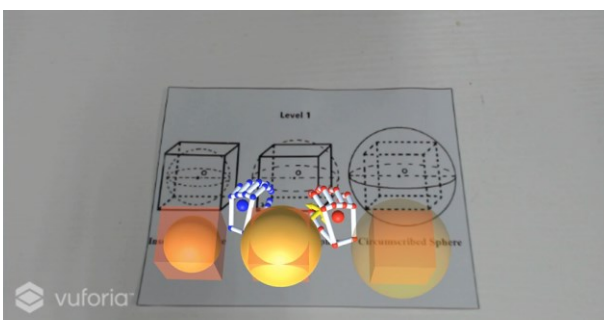
3.2.2 Level 2 – The Combination of Spheres and Triangular Pyramids with Right Triangles

The key to solve the problem of combination of spheres and polyhedrons is to determine the center and the radius of the sphere. This level allows students to move and scale the sphere, so that the problem can be solved in a 3D visualization interface. When the students move the sphere to the right position and the radius is appropriate, the successful interface will be displayed and the corresponding explanation will be displayed. In the process of finding the correct circumscribed sphere, students can get the hint or answer with the corresponding gestures. The prompt is designed according to the corresponding teaching requirements.

Level 2 mainly examines whether students can determine the circumscribed sphere of a triangular pyramid according to the characteristics of the right triangle and the definition of the circumscribed sphere, and learn the transformation idea from part to whole.

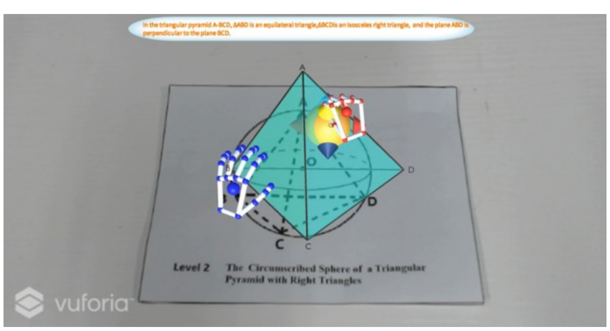


(a)

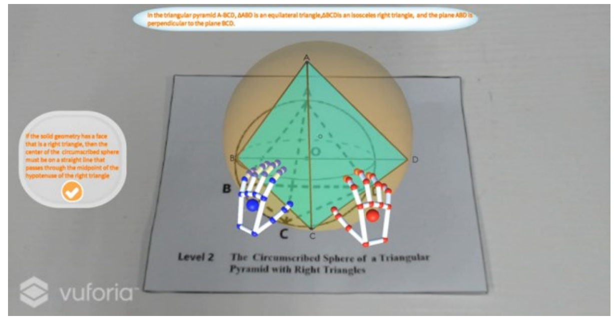


(b)

Figure 5: Example of Level 1 interface. (a) Users can cut the model and view its internal section more intuitively. (b) The answer interface in Level 1.



(a)



(b)

Figure 6: Example of Level 2 interface. (a) A user is moving the ball. (b) The answer interface in Level 2.

3.2.3 Level 3 – The Combination of Spheres and Special Triangular Pyramids

The operation process of this level is the same as that of level 2. However, this level mainly focuses on the circumscribed sphere of a special triangular pyramid, whose four vertices are part of the vertices of a cube or cuboid. Finding the circumscribed sphere of this triangular pyramid requires students to have a stronger spatial ability, because the center of the circumscribed sphere is not on any surface of the pyramid.

This level teaches the students an important method to solve the problem of the circumscribed sphere - Geometric Complement Method. In addition, students can find the circumscribed sphere of some special pyramids, prisms and even circular cylinders according to a cuboid or a cube. Through this study, students can build a sense of space and understand the mathematical ideas of transformation.

4 Experiment and Results

4.1 Experiment

This section evaluates the performance of the proposed application. The validation study was carried out a PC with a 4.00 GHz Intel i7 processor, 16.0 Gb of RAM, and a Windows 10 operative system. The PC had a webcam. We adopted Unity3D with Vuforia extension to construct 3D objects and build the augmented environment, and the Leap Motion Device for gesture input detection.

Five students were invited for the evaluation, three of whom were Senior One students and two of them were junior school students. Only one student knows about inscribed sphere, edge-tangent's sphere and circumscribed sphere. Three students have used AR technology before.

At first, a brief introduction of inscribed sphere, edge-tangent's sphere and circumscribed sphere was conducted by the instructor. Later on, students were asked to accomplish a test, which asked students to calculate the radius of inscribed sphere, edge-tangent's sphere and circumscribed sphere of the cube respectively. The total score of the test is 3 points.

In the experiment, students learn Level 1, Level 2 and Level 3 according to instructions. Each level is associated to a unique training session.

In the last, an achievement test and an attitude questionnaire which is used to evaluate the effects of learning performance and attitudes were implemented. The achievement test is the same as the pretest. The questionnaire is composed of three blocks as shown in Table 2. Parts A and B contain questions on the AR application design and didactic contents from the system. Part C is used to collect user's suggestions on the system, including interface, gesture and content. In Parts A and B, participants used a Likert scale to provide their opinion. A Likert scale is a psychometric scale commonly used for scaling in survey research. Five ordered response levels were used, where number five corresponds to strongly agree and number one corresponds to strongly disagree.

4.2 Results

4.2.1 Learning performance

The average score of the pretest is 1.4. No student is able to calculate the radius of the edge-tangent's sphere and only two students can calculate the radius of the circumscribed sphere. The average score of the posttest is 2.8, 1.4 points higher than that of the pretest. All students can calculate the radius of the circumscribed sphere and four students can calculate the radius of the edge-tangent's sphere. Such results show that the students' knowledge of inscribed sphere, edge-tangent's sphere and circumscribed sphere is improved after learning the system.

Table 2: Questionnaire.

Assess your agreement in the following questions				
1 Strongly disagree	2 Disagree	3 Undecided	4 Agree	5 Strongly agree
Part A AR application design				
A1 This system allows me to intuitively perceive the relationship between 3D objects and their 2D projections.				
A2 Marker recognition is fast and accurate.				
A3 The familiarization with gestures is easy.				
A4 Using gesture interaction to manipulate virtual 3D model is very natural and simple，and there is no jitter and delay in the operation of the 3D model.				
A5 I prefer this method to the traditional one.				
Part B AR course contents				
B1 The learning activities are interesting.				
B2 The system can help me better understand the spatial combination of spheres and polyhedrons.				
B3 The design of three levels of teaching content is reasonable.				
B4 Tips and answers are necessary for the teaching process.				
B5 I have confidence in resolving this type of problem again.				

4.2.2 Attitudes toward learning

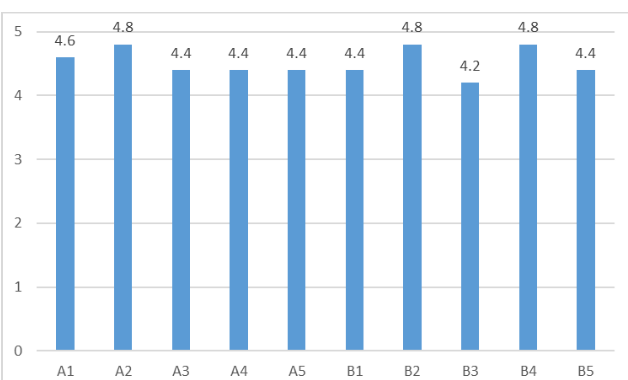


Figure 7: User satisfaction questionnaire: results of A and B blocks.

With respect to the satisfaction survey, the results are summarized in Fig.7. In general, all students are satisfied with the AR application and didactic contents - the mean score of each question is around 4.2 to 4.8 (agree). No student chooses to disagree or strongly disagree.

For the AR application design, the result of question A1(mean=4.6) indicated that the system can help students visualize geometric shapes intuitively and understand the knowledge of geometry. Participants think that the marker recognition is fast and accurate (question A2，mean=4.8). The result of question A3 (mean=4.4) and A4 (mean=4.4) shows that the hand gestures are simple for the participants to interact with the 3D geometrical shapes. Participants confirm the design of the teaching content of the system (question B2，mean=4.8; question B4，mean=4.8) and have confidence in resolving this type of problem again (question B5，mean=4.4).

In Part C, participants gave their suggestions after the experiment. They believe that combining AR technology with gesture interaction is of great help in learning solid geometry. For the teaching content of the system, two students proposed that they want to change the shape of triangular pyramid themselves and observe the change of its circumscribed sphere in real time.

4.3 Improvement

Based on the experimental results, we add Level 4--users can change the four vertex positions of the triangular pyramid to get the circumscribed sphere of any triangular pyramid. The system obtains the coordinate information of each vertex, calculates the center coordinate and radius of the circumscribed sphere of the triangular pyramid in real time, and superimposes them on the triangular pyramid.

The specific calculation process is as follows:

(1) Calculating coordinates of four vertices in real time;

(2) Calculating the center of circumcircle of any two surfaces in a triangular pyramid called O1, O2;

(3) Calculating the intersection of two straight lines passing through O1 and O2 and perpendicular to the corresponding surface. The intersection point is the center of the circumscribed sphere;

(4) Calculating radius;

(5) Displaying circumscribed sphere and corresponding inform--ation.

5 Conclusions

In this paper, we propose a system for learning the spatial combination of sphere and polyhedron based on AR and hand gesture recognition, which could help students to learn the geometry more easily. The teaching content of this system focuses on the circumscribed sphere of the triangular pyramid. We have designed three levels to enable students to learn geometry knowledge step by step. We use markers with a 2D projection of 3D models that help students to establish links between the physical models and their representations on paper. We have also implemented a hand gesture interaction editor that leads students to a more natural interaction with 3D models in AR environments, giving full play to students' initiative. The results of user studies suggested that the proposed application provides an intuitive method for students to learn geometry and can help them better understand the spatial relationship between spheres and polyhedrons.

We added a new level to the system based on user studies. In the future we will first find more students to test the system so as to compare the performance of this method with that of the traditional method. Then, we will extend this system into practical teaching. Moreover, we will adopt this system to multiple devices such as mobile phones or tablets that would enable students to study this geometry education wherever they are. In this system, in order not to let the hand occlude the marker and affect the tracking quality, we use the virtual hand. In the future we will improve it to enable users to interact with real hands.

ACKNOWLEDGEMENTS

This work was supported by the National Key Research and Development Program of China (No. 2016YFB0401202) and the National Natural Science Foundation of China (No.61661146002) and the 111 Project (B18005).

REFERENCES

[1] Maizam, A., Black, T. R., & Gray, D. E. (2002). Effect of instruction on spatial visualization ability in civil engineering students. *International Educational Journal*, 3(1), 1-12,2002.

[2] Potter, C., & Van der Merwe, E. (2003). Perception, imagery, visualization and engineering graphics. *European journal of engineering education*, 28(1), 117-133,2003.

[3] Martín-Gutiérrez, J., Saorín, J. L., Contero, M., Alcañiz, M., Pérez-López, D. C., & Ortega, M. (2010). Design and validation of an augmented book for spatial abilities development in engineering students. *Computers & Graphics*, 34(1), 77-91, 2010.

[4] Kim, M., & Lee, J. Y. (2016). Touch and hand gesture-based interactions for directly manipulating 3D virtual objects in mobile augmented reality. *Multimedia Tools and Applications*, 75(23), 16529-16550,2016.

[5] Kerawalla, L., Luckin, R., Seljeflot, S., & Woolard, A. (2006). "Making it real": exploring the potential of augmented reality for teaching primary school science. *Virtual reality*, 10(3-4), 163-174,2006.

[6] Kaufmann H, Schmalstieg D. Mathematics and geometry education with collaborative augmented reality. In: *ACM SIGGRAPH 2002 conference abstracts and applications*. ACM, 2002: 37-41, July 2002.

[7] Kaufmann, Hannes. *Dynamic differential geometry in education*. na, 2009.

[8] Barraza Castillo, R. I., Cruz Sánchez, V. G., & Vergara Villegas, O. O. (2015). A pilot study on the use of mobile augmented reality for interactive experimentation in quadratic equations. *Mathematical Problems in Engineering*, 2015.

[9] Salinas, P., González-Mendívil, E., Quintero, E., Ríos, H., Ramírez, H., & Morales, S. (2013). The development of a didactic prototype for the learning of mathematics through augmented reality. *Procedia Computer Science*, 25, 62-70, 2013.

[10] Quintero, E., Salinas, P., González-Mendívil, E., & Ramírez, H. (2015). Augmented reality app for calculus: A proposal for the development of spatial visualization. *Procedia Computer Science*, 75, 301-305, 2015.

[11] Salinas, P., & Pulido, R. (2017). Understanding the Conics through Augmented Reality. *Eurasia Journal of Mathematics, Science & Technology Education*, 13(2),2017.

[12] Jackiw, R. (1993). The geometer's sketchpad: programming by geometry. *Watch what I do: programming by demonstration.*pp.293-308,1993.

[13] JUNG, Whie, et al. BoostHand: Distance-free Object Manipulation System with Switchable Non-linear Mapping for Augmented Reality Classrooms. In: *2017 IEEE International Symposium on Mixed and Augmented Reality (ISMAR-Adjunct).* IEEE, 2017. p. 321-325, October 2017.

[14] LE, Hong-Quan; KIM, Jee-In. An augmented reality application with hand gestures for learning 3D geometry. In: *2017 IEEE International Conference on Big Data and Smart Computing (BigComp)*. IEEE, 2017. p. 34-41, February 2017.

[15] Chung, C. J., Hwang, G. J., & Lai, C. L. (2019). A review of experimental mobile learning research in 2010–2016 based on the activity theory framework. *Computers & Education*, 129, 1-13,2019.