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Usability evaluation of an augmented reality system for teaching Euclidean vectors

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ABSTRACT

Augmented reality (AR) is one of the emerging technologies that has demonstrated to be an efficient technological tool to enhance learning techniques. In this paper, we describe the development and evaluation of an AR system for teaching Euclidean vectors in physics and mathematics. The goal of this pedagogical tool is to facilitate user's understanding of physical concepts, such as magnitude, direction and orientation, together with basic vector-related operations like addition, subtraction and cross product. The result of the system usability scale showed our system's usability and learnability. The system merges a real-world scenario with virtual elements controlled with a practical body-interactive interface.

KEYWORDS
Educational technology; mathematics education; augmented reality; system usability scale; interactive learning environments

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Introduction

For over a decade, emerging technologies have had a great impact on education, encouraging the creation of new types of tools, applications, media and environments in education to promote learning. Innovations such as virtual environments, wireless mobile devices, digital teaching platforms and augmented reality (AR) systems support improved teaching and learning, along with the students' motivation and interest on learning (Lee, 2012; Nincarean, Alia, Halim, & Rahman, 2013; Roussos et al., 1999).

AR is defined as a technology which enhances the real-world perception of the user by superimposing computer-generated objects (Azuma, 1997; Azuma et al., 2001). Contrary to virtual reality, where the user is completely immersed in a computer-generated virtual environment, augmented reality allows the user to see the real world merged with virtual objects, and interact with them.

As reported by Cai, Chiang, and Wang (2013), an AR learning environment coincides with concepts in education theories, supporting, for instance, that learning is the result of associations formed between stimuli and responses. Moreover, an AR-based learning platform provides students with model constructing tools and scenarios, designed to be easily used by them. In recent years, learning by interacting with virtual objects integrated into reality has become a modern digital learning type (Lin, Chen, Hsieh, Lee, & Huang, 2012).

Various educative AR applications have been proposed to assist different academic fields. In astronomy, for instance, an AR system was developed to learn about the relationship between the earth and the sun by using tri-dimensional (3D) rendered earth and sun shapes (Shelton & Hedley, 2002). In chemistry, implementation of a tangible user interface called augmented chemistry to show students 3D molecular models via AR was achieved (Fjeld et al., 2007; Fjeld & Voegtli, 2002). Moreover, an

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augmented reality-based 3D user interface was evaluated by Maier and Klinker (2013), to enhance the 3D understanding of molecular chemistry. In biology, the anatomy and body structures can be studied by means of an AR system, like the Mirracle, which is an augmented reality magic mirror that shows virtual organs intuitively (Blum, Kleeberger, Bichlmeier, & Navab, 2012). In mathematics and geometry education, the Construct3D system was designed to construct geometric virtual shapes by multiple users (Kaufmann & Schmalstieg, 2002). In physics, Duarte, Cardoso, and Lamounier (2005) used augmented reality to improve learning on kinematics properties.

Teaching basic science has always been a major challenge in higher education, especially for topics in mathematics and physics. This is mainly because the student needs to achieve a cognitive level that permits the understanding of abstract concepts. One example of such concepts is a vector, which can model real-world phenomena like fluid mechanics, force, dynamics, etc.

In general, Euclidean vectors and, in particular, their arithmetic operations (e.g. addition, subtraction, cross product, dot product) are explained in the classroom through lines drawn in a two-dimensional (2D) Cartesian plane (e.g. whiteboard, paper sheet), and in a gradual manner, according to the intellectual capacities of students.

In mathematics, physics and engineering, Euclidean vectors play an important role since they are used to characterise physical quantities having magnitude and direction (e.g. force, velocity, magnetic or electric fields), opposite to scalar quantities that have no direction (e.g. time, temperature, speed). A diversity of mathematical operations can be applied on vectors. One of them is addition, which is the sum of two vectors, and represents, for example, the net force experienced by an object (i.e. the vector sum of all the individual forces acting upon that object). The inverse operation of addition is the subtraction of vectors, which results from the sum of a vector with another vector in its opposite direction; in other words, the difference $\mathbf{a} - \mathbf{b}$ of two vectors $\mathbf{a}$ and $\mathbf{b}$ can be defined as

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) \tag{1}$$

where $-\mathbf{b}$ represents vector $\mathbf{b}$ in opposite direction. Another operation is the cross product of two vectors, $\mathbf{a} \times \mathbf{b}$, which is the perpendicular vector to both $\mathbf{a}$ and $\mathbf{b}$ vectors, and it is defined as

$$\mathbf{a} \times \mathbf{b} = ||\mathbf{a}||\,||\mathbf{b}|| \sin\,(\theta)\mathbf{n} \tag{2}$$

where θ is the measure of the angle between $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{n}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. The magnitude of the cross product is the area of the parallelogram with sides $\mathbf{a}$ and $\mathbf{b}$. The orientation of the cross product is orthogonal to the plane containing this parallelogram.

Some vector properties (e.g. orthogonality, direction, orientation, projection) involve the concept of a 3D space. Thus, such properties may be difficult to understand by visualising them in a 2D plane. In this sense, the use of new technologies such as augmented reality can enhance the limitations of current educational resources to facilitate learning of these subjects by virtualising objects (e.g. vectors), along with their properties, visualised in a 3D real world. Therefore, the primary focus of this study is on the development and evaluation of an augmented reality tool designed to enable the educator to use modern techniques for teaching concepts in mathematics and physics (i.e. vector properties and basic operations), thus to help students to learn more completely through AR environments.

Methods

System set-up

Our system set-up consists of a display device, a colour and a depth camera (see Figure 1). The display device is used to visualise the real-world scenario, captured by the colour camera, and the corresponding augmentations (virtual objects). The colour and depth camera belong to the Microsoft™ Kinect sensor, developed for the Xbox 360 game console, which enables the use of gestures and body movement as a controller for the system. The main workstation for general processing has an AMD Phenom™ II Quad-Core Processor at 2.20 GHz with 4 GB RAM and an integrated graphics card.

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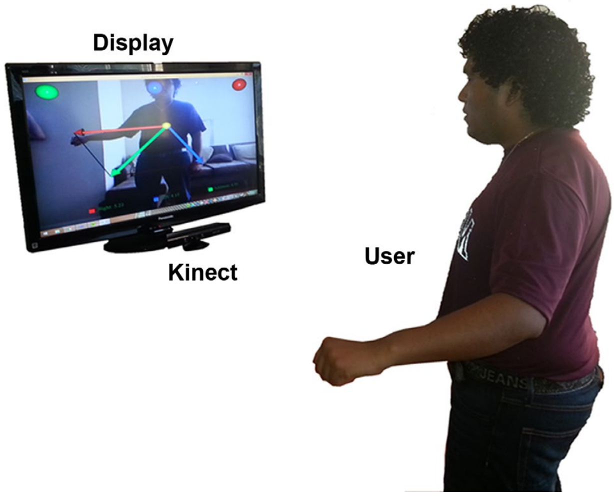

Figure 1. AR system set-up in a common scenario. The user visualises the augmented outcome on the display screen while moving the hands to change vectors' magnitude and/or direction.

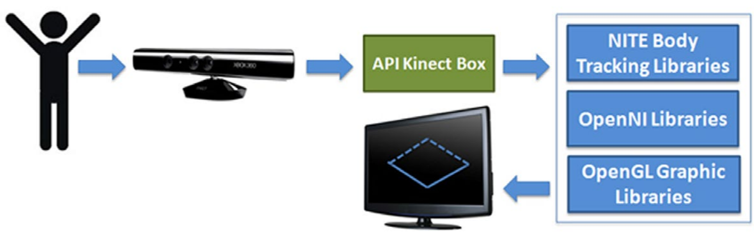

Figure 2. Functional architecture of the AR system.

The AR system presented in this work has mainly been developed for assisting the education of concepts in physics and mathematics, particularly within the classroom. The AR system focuses on the understanding of properties related to the Euclidian vectors (magnitude and direction), and some operations of vectors (addition, subtraction and cross product). Moreover, the system is portable, as the set-up requires only a small area of space (2 × 2 m room), and is available at low cost.

User interface

The AR system permits the user to interact with three different basic operations of vectors: addition, subtraction and cross product. Every operation modality can be selected through virtual buttons displayed at a fixed location (upper part of the screen, approximately). Depending on the desired operation, the user has to place a hand at the corresponding button location, in other words, the user has to touch the virtual button. The system modality will change according to the user selection.

The body of the user is continuously tracked based on the depth image from the Kinect sensor using the NITE skeleton tracking software (www.openni.org). The user pose includes the position, in 3D coordinates, from different parts of the user skeleton (i.e. head, neck, torso, shoulders, elbows, hands, etc.) The real-world 3D coordinates obtained by the Kinect sensor are transformed to the monitor 2D coordinate system through the OpenNI libraries. Finally, all graphics are generated through the OpenGL libraries (www.opengl.org). The system's architecture is shown in Figure 2.

The 3D coordinates of the subject torso obtained by the Kinect sensor are used as a common origin to form two angular vectors $\mathbf{l}$ and $\mathbf{r}$. The 3D positions of the subject left and right hand are continuously tracked in order to get the ending points of the corresponding $\mathbf{l}$ and $\mathbf{r}$ vectors. The outcome vector of every operation is virtually generated and superimposed to the real-world video image of the user, creating an AR environment.

Participants

Study participants included 18 undergraduate students and two lecturers. Students' average age was 20 years old, ranging from 18 to 22 years (two women). The lecturers mean age was 35 years with a standard deviation of 1 year. Subjects represented mainly the computer undergraduate discipline, and they had the requirement of being able to use both hands for controlling the system. Subjects were recruited from the Facultad de Matemáticas from the Universidad Autónoma de Yucatán.

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Experiment design

Before each trial, the Kinect sensor needs to be calibrated for the user with a specific posture so that the sensor correctly tracks the user body. For this purpose, the user has to stand straight in front of the Kinect cameras holding the hands up ('Psi' pose, Ψ). Once the user is detected, the AR system initiates the body tracking.

After the user body is successfully detected by the Kinect sensor, an instructor introduces the user with the AR system by explaining its main purpose and its interactive interface to control it. Once this initial explanation is given, the instructor invites the user to freely try each of the three available operations of vectors (addition, subtraction and cross product), and also to vary vectors' magnitudes and directions by moving the hands at different locations so the user can observe the effect on each operation. Afterwards, the instructor starts asking the user a set of open-ended questions to measure his/her degree of knowledge about vectors. If the user does not remember how to operate vectors, the instructor briefly explains the operations with the help of the AR system visualisation. Otherwise, the instructor will point out vector properties on the virtual outcomes to make the user confirm his/her knowledge.

Furthermore, in order to evaluate the system, we applied the system usability scale (SUS) to the user once the system trial was finished. The SUS was developed by John Brooke (Brooke, 1986), and it is a questionnaire (see Appendix 1) of Likert scale to assess usability, learnability and users' subjective satisfaction. It consists of 10 items with five response options (scales) for respondents from 'Strongly disagree' (position 1) to 'Strongly agree' (position 5). The questionnaire has odd-numbered items worded positively and even-numbered items worded negatively. For positively worded items (1, 3, 5, 7 and 9), their score contribution is equal to their scale position minus 1; and for negatively worded items (2, 4, 6, 8 and 10), it is 5 minus their scale position. In this way, each item's score contribution will range from 0 to 4, where 4 is the best possible item's score contribution. The overall SUS score is obtained by multiplying the sum of the item score contributions by 2.5. Thus, SUS scores range from 0 to 100, where a 100 score indicates the highest level of usability and learnability.

According to Bangor, Kortum, and Miller (2008), SUS has reasonable reliability (coefficient alpha of 0.91) for usability. More recently, Lewis and Sauro (2009) show that SUS provides a global measure of system satisfaction and sub-scales of usability (items 1–3, and 5–9) and learnability (items 4 and 10) when multiplied their summed score contributions by 3.125 and 12.5, respectively.

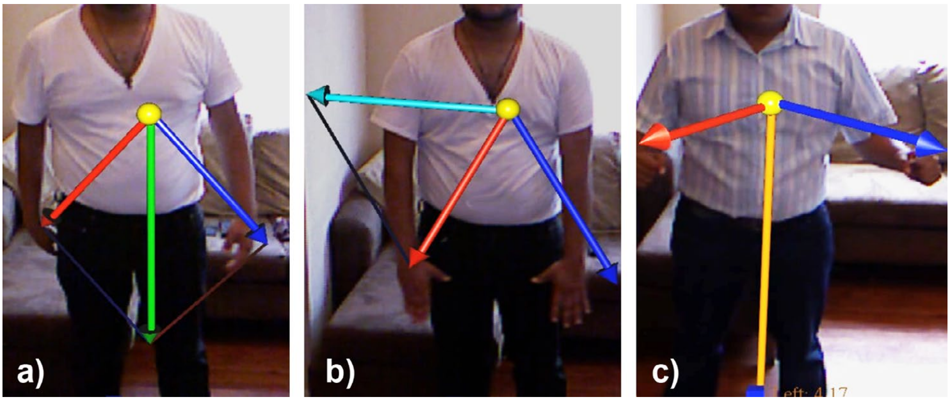

Figure 3. Augmented view of the AR system: (a) addition, (b) subtraction and (c) cross product.

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Results

System interface

The augmented reality view of the system under the modality addition of vectors is shown in Figure 3(a). Positions of the left and right hand of the user define ending points of vectors $\mathbf{l}$ (blue) and $\mathbf{r}$ (red) on which the addition will be computed. The resultant vector $\mathbf{l} + \mathbf{r}$ (vector sum) is visualised in green. Position of the user torso is displayed as a yellow sphere, indicating the common origin of vectors. For this operation, the user can also move the hands back and forward to change magnitude and orientations of vectors, and observe different addition results. In order to illustrate the so-called parallelogram method for vector addition, extra lines (i.e. vectors $\mathbf{l}$ and $\mathbf{r}$ placed head to tail) are visualised with darker colours to form the parallelogram, thus, the vector sum $\mathbf{l} + \mathbf{r}$ is the diagonal of the parallelogram.

Subtraction of vectors is visualised in Figure 3(b). The system subtracts the $\mathbf{l}$ vector (in blue colour) from the $\mathbf{r}$ vector (red colour). The final vector $\mathbf{r} - \mathbf{l}$ (subtraction result) is visualised in cyan colour. In order to show the so-called triangle method for vector subtraction, a copy of the $\mathbf{l}$ vector (visualised as a line with a darker colour) is placed after the terminal point of the $\mathbf{r}$ vector, such that, the subtraction result is the vector drawn from the initial point of the $\mathbf{r}$ vector to the terminal point of the duplicated $\mathbf{l}$ vector.

The system's augmented view of the cross product of vectors can be observed in Figure 3(c). The cross-product vector (in orange colour) is the outcome of applying Equation (2) to the vectors $\mathbf{l}$ (blue) and $\mathbf{r}$ (red) defined by the positions of each hand. We can observe the orthogonality property of the cross product in the augmented scenario.

Open-ended questions

In order to observe students' learning, the instructor challenged students, while trying the system, with a set of guide questions. Such questions were designed to make the subject analyse in detail the properties and operations of vectors, and thus, to observe if the user obtained the proper analytical understanding of the concepts.

For the addition operation, the instructor asked the subject to choose a specific pair of $\mathbf{l}$ and $\mathbf{r}$ vectors to obtain a resultant vector (vector sum). Then, the following question was asked to the subject: 'Is it possible to have the same resultant vector by subtracting two other $\mathbf{l}$ and $\mathbf{r}$ vectors?' With this approach, students were trying several configurations of vectors by moving their hands, until they figured out that by changing the sign of the vector (inverting one vector orientation) they obtained the requested resultant vector (see Figure 4). The instructor also asked the subject to explain the logical reasoning to get the outcome.

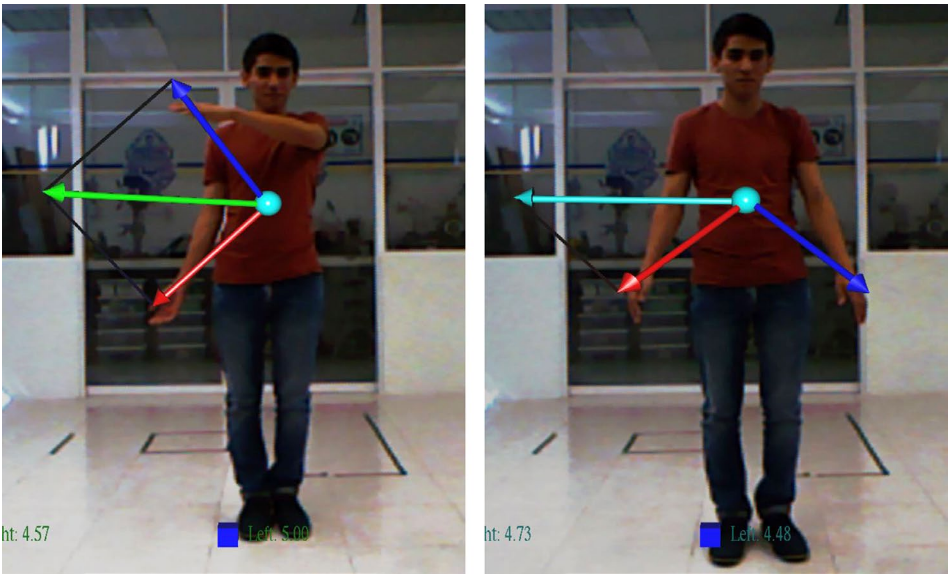

Figure 4. A subject using the AR system to obtain a similar resultant vector for addition (green vector on the left image) and subtraction (cyan vector on the right image) of vectors.

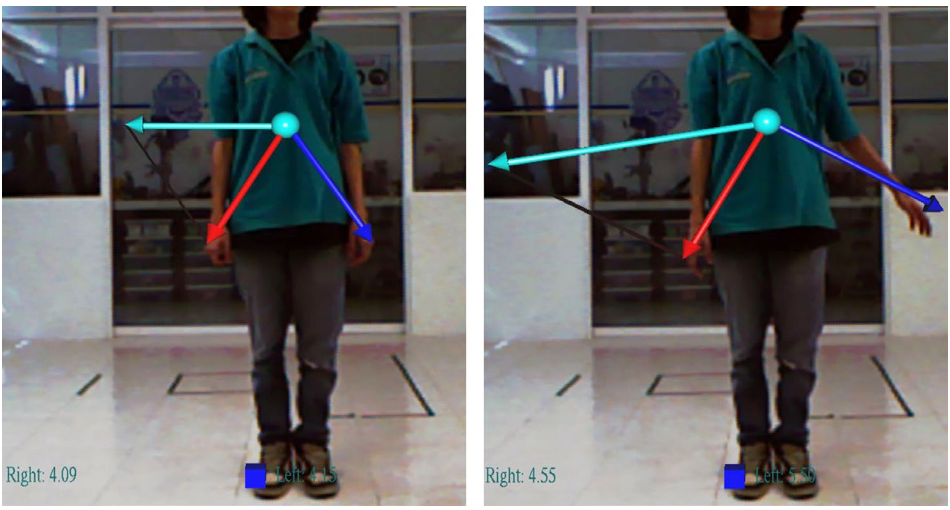

Figure 5. A subject using the AR system to confirm that by increasing the blue vector, the difference (cyan vector) increases.

One question posed to the subjects regarding the subtraction operation was: 'How can you increase the magnitude of the final vector $\mathbf{r} - \mathbf{l}$ which comes out from the difference of vectors $\mathbf{r}$ and $\mathbf{l}$?' Subjects figured out that if they keep the $\mathbf{r}$ vector fixed, the $\mathbf{l}$ vector is directly proportional to the difference vector, in other words, the larger the $\mathbf{l}$ vector the larger the difference will be (see Figure 5). Another question in relation to vector subtraction was: 'How can you produce a difference vector with a magnitude equal to the magnitude of any of vectors $\mathbf{l}$ or $\mathbf{r}$?' After the students gave their answers (e.g. 'we need to make the $\mathbf{r}$ vector equal to zero'), the instructor asked them to use the AR system to illustrate their solution with an example, and thus, corroborate visually their answers. The next question was: 'What happens to the resultant vector of a sum if we apply the same procedure as in the subtraction?'

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Table 1. SUS items' mean score contribution (range 0–4).

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Mean	3.1	3.2	3.8	2.8	3.6	3.2	3.6	3.8	3.5	3.1
SD	0.7	1.1	0.6	1.2	0.7	0.9	0.7	0.4	0.8	1.2

Table 2. SUS scores.

Mean	SD	Min	Max	Median	Usability	Learnability
83.4	11.1	60	100	85	86.1	72.5

For this question, the same technique for the previous question was used; first, the students said their answers (e.g. 'resultant vector and $\mathbf{l}$ vector are the same in magnitude and orientation'), and afterwards, they confirmed if they were right or wrong by using the AR system.

Taking into account the benefits of a 3D visualisation, one interesting property of the cross product of two vectors is that it is perpendicular to both vectors and therefore, normal to the plane containing them. Based on this property, the instructor asked the subjects to choose a specific pair of $\mathbf{l}$ and $\mathbf{r}$ vectors to obtain a cross-product vector with a specific orientation. Afterwards, the instructor posed the following question: 'How can you get the same cross-product vector, but in the opposite direction?' It took subjects some time before realising that they need to interchange hands position (cross their hands) to change the cross-product orientation, but they were very satisfied when they found the solution.

SUS score

Participants finished operating the system and completed the SUS questionnaire of 10 items with five optional answers, where the answer 'Strongly disagree' occupied position 1 and 'Strongly agree' position 5. Responses to individual SUS items are presented in Table 1. For each item, the mean score contribution and corresponding standard deviation are depicted. The positively worded item 3 ('I thought the system was easy to use') received the most extreme of all responses, with a mean absolute score of 3.8 (SD = 0.6). Negatively worded item 8 ('I found the system very cumbersome to use') had the lowest mean absolute score of 1.2, (SD = 0.4). As summarised in Table 2, the average SUS score obtained is 83.4 with 11.1 of standard deviation.

Discussion

According to Bangor, Kortum, and Miller (2008), with the rule of thumb on the interpretation of SUS scores on products, SUS scores less than 50 should be cause for significant concern and are judged to be unacceptable, SUS scores between 50 and 70 are marginally acceptable, and SUS scores above 70 are adequate. In this study, the mean SUS score of 83.4 indicated that the AR system was generally perceived to be above acceptable. Also, the usability and learnability scores were acceptable. Based on the opinion of the majority of the volunteers (19 out of 20), interacting with our system for the first time is a great tool to understand physical concepts like vector operations. Every user mentioned that the system interface is very easy to use, understand and learn by interacting.

As it was shown in the results section, the user can easily generate vectors of various magnitudes and directions by translating the hands to different 3D locations within the Kinect tracking area. Also, a student can use the AR system to confirm his/her knowledge about the subject of interest.

The AR system can be used to discover unexplored solutions of new problems. This was evidenced, for example, through the subject's exploration of the problem 'How can you get the same cross-product vector, but in the opposite direction?' Subjects began by analysing the generated virtual vectors they

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were controlling with their hands. They tried several hand and body postures to watch and understand the effects on the operation outcome. Finally, they discovered that by crossing the hands, they observed how the cross-product vector changed its direction by 180°.

The AR system also permits the user to interact with the elements to be understood. For example, by changing vectors' magnitudes and/or directions, students were able to solve questions related to addition and subtraction of vectors. According to Liarokapis et al. (2004), contextually enriched interaction using AR technology can make difficult theories accepted and understood by students. In this sense, the AR system can be very useful for users to enhance their learning in an interactive and easy manner. We do not mean to imply that only by using the AR system students will understand completely all properties and operations of vectors by themselves but, we do think it might facilitate a richer and more useful understanding of such abstract concepts after a concise analytical explanation given by the teacher in a classroom.

In the case of concepts like the cross product, which can only exist in a 3D space, our proposed AR system seems to be efficient for an easier understanding of such kinds of concepts and their properties compared to traditional pedagogical methods that enclose only 2D spaces.

In the traditional teaching methods, static materials do not show any information in a dynamic way such as motion or continuous movement (Craig & McGrath, 2007; Kühl, Scheiter, Gerjets, & Gemballa, 2011). On the other hand, in science and mathematics education, there has been significant research about visual methods to support mathematical problem solving (Lean & Clements, 1981; Presmeg, 1992; Webb, 1979; Zazkis, Dubinsky, & Dautermann, 1996). Authors suggest that visual and analytic thinking may need to be present and integrated in order to construct a full understanding of mathematical concepts. In this regard, our AR system can be easily integrated in a classroom to provide the students with visual 3D instances of examples the teacher mentioned during the lecture. Also, the AR system permits the students to directly interact and play, as a serious game, with the concepts to be learnt, increasing their motivation and possibly their achievement in physics and mathematics. Therefore, we consider our AR system as an effective pedagogical strategy to assist a lecturer in teaching, and to help students reach a higher cognition level for abstract concepts.

To develop a more complete AR system for teaching vectors, more properties and operations of vectors (e.g. vector projection, dot product) need to be considered, which is recommended for future work. Even when existing learning methods work often adequately, there is always an increasing interest in developing more useful and practical methods to improve teaching experiences.

Conclusion

In this work, we presented the development and evaluation of an augmented reality tool with a real-time body-interactive interface to learn Euclidean vectors in mathematics and physics. Users were able to virtually create vectors of several magnitudes and directions, and visualise its properties and operations. Through its 3D visualisation and interaction, the AR system facilitates explanation and conception of abstract concepts that cannot be easily visualised with traditional pedagogical strategies. Our augmented reality interface was designed and implemented in a practical and efficient manner for teachers and students. Most users had a positive attitude towards using our AR system for their learning in Euclidean vectors. According to the SUS scores of usability and learnability, our system was acceptable.

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Disclosure statement

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Appendix 1. System usability scale

Instructions: please indicate how true each statement is for you by making a choice between: '1. Strongly disagree', '2. Disagree', '3. Not sure', '4. Agree' or '5. Strongly agree'.

(1) I think that I would like to use this system frequently.
(2) I found the system unnecessarily complex.
(3) I thought the system was easy to use.
(4) I think that I would need the support of a technical person to be able to use this system.
(5) I found the various functions in this system were well integrated.
(6) I thought there was too much inconsistency in this system.
(7) I would imagine that most people would learn to use this system very quickly.
(8) I found the system very cumbersome to use.
(9) I felt very confident using the system.
(10) I needed to learn a lot of things before I could get going with this system.

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