
Algorithm 3 Solve a square linear system $\mathbf{Ax} = \mathbf{b}$ using the LU factorization.

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1: Require: The LU factors  $\mathbf{L}$ ,  $\mathbf{U}$ ,  $\mathbf{P}$ ,  $\mathbf{Q}$ , and a vector  $\mathbf{b}$ .
2: procedure SOLVELU( $\mathbf{L}$ ,  $\mathbf{U}$ ,  $\mathbf{P}$ ,  $\mathbf{Q}$ ,  $\mathbf{b}$ )
3:    $\mathbf{y} \leftarrow \mathbf{Pb}$ 
4:    $m, n \leftarrow \text{Size}(\mathbf{L})$ 
5:   for  $i$  from 2 to  $m$  do
6:      $y_i \leftarrow \text{Veil} \left( y_i - \sum_{j=1}^{i-1} L_{ij}y_j \right)$ 
7:   end for
8:    $x_n \leftarrow \text{Veil}(y_n / U_{nn})$ 
9:   for  $i$  from  $n - 1$  to 1 do
10:     $x_i \leftarrow \text{Veil} \left( y_i - \sum_{j=i+1}^n U_{ij}x_j \right)$ 
11:     $x_i \leftarrow \text{Veil}(x_i / U_{ii})$ 
12:  end for
13:   $\mathbf{x} \leftarrow \mathbf{Q}^\top \mathbf{x}$ 
14: end procedure
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▷ Solve the linear system $\mathbf{Ax} = \mathbf{b}$
▷ Apply the permutation matrix \mathbf{P} to the vector \mathbf{b}
▷ Get the size of \mathbf{L}
▷ Solve $\mathbf{Ly} = \mathbf{Pb}$
▷ Perform forward substitution
▷ Perform the first backward substitution
▷ Solve $\mathbf{Ux} = \mathbf{y}$
▷ Perform backward substitution
▷ Apply the permutation matrix \mathbf{Q}^\top to the solution \mathbf{x}
