Dynamic Mirror Descent based Model Predictive Control for Accelerating Robot Reinforcement Learning

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APPENDIX

Let us consider $\mathcal{M}$ and $\mathcal{M}_r$ as the approximated and real MDP with dynamics model f_ϕ and f respectively. Let the total variation distance between them be bounded by ϵ_f (see [1]). This dynamics model predicts both the next state distribution and rewards. The corresponding MPC objective is represented as J and J_r respectively. Here, J denotes that the costs are calculated from the approximated reward function setting whereas J_r is obtained from rollouts in the true MDP. Now, we will derive the bounds on the performance improvement in a similar way as demonstrated in [1] and [2], however with consideration and assumptions related to the convexity of the losses.

Proof: [Proof of Lemma **??**] For any stochastic dynamics model f and reward function r, considering the cost of a trajectory in an MDP with policy π_η and value function V_ζ is given by,

$$C\left(\mathbf{x_t}, \mathbf{u_t}\right) = \sum_{h=0}^{H-1} \gamma^h c(x_{t,h}, u_{t,h}) + \gamma^H c_H(x_{t,H}) \quad (1)$$

where, γ is the discount factor, $c(x_{t,h}, u_{t,h}) = -r(x_{t,h}, u_{t,h})$ and c_H is the terminal cost calculated as $-V_\zeta(x_{t,H})$. Let c_{max} be the bound on this cost.

Now, to realize the maximum improvement in the approximated MDP while using the policy parameters ($\tilde{\eta}_t$), obtained from the shift model, we use a formulation motivated by the bound formulated in Lemma B.3 in [1]. We consider p_ϕ as the discounted state-action visitation corresponding to f_ϕ (similarly p for f) and superscript h to resemble the notations of [1].

$$\begin{aligned}
&J\left(x_t, \tilde{\eta}_t\right) - J_r\left(x_t, \tilde{\eta}_t\right) \\
&= \mathbb{E}_{\mathbf{u_t}\sim\pi_{\tilde{\eta}_t}, \mathbf{x_t}\sim f_\phi}\left[\sum_{h=0}^{H} \gamma^h c(x_{t,h}, u_{t,h}) + \gamma^H c_H(x_{t,H})\right] \\
&\quad - \mathbb{E}_{\mathbf{u_t}\sim\pi_{\tilde{\eta}_t}, \mathbf{x_t}\sim f}\left[\sum_{h=0}^{H} \gamma^h c(x_{t,h}, u_{t,h}) + \gamma^H c_H(x_{t,H})\right] \\
&= \sum_{\mathbf{x_t}, \mathbf{u_t}} \left(p_\phi(x,u) - p(x,u)\right)\, c(x,u) \\
&\leq \sum_{\mathbf{x_t}, \mathbf{u_t}} \sum_{h=0}^{H-1} \gamma^h \, \left(p_\phi^h(x_{t,h}, u_{t,h}) - p^h(x_{t,h}, u_{t,h})\right)\, c(x_{t,h}, u_{t,h}) \\
&\quad + \gamma^H \, \left(p_\phi^H(x_{t,H}, u_{t,H}) - p^H(x_{t,H}, u_{t,H})\right) V_\zeta(x_{t,H}) \\
&\leq 2\, c_{max} \sum_{h=0}^{H-1} \gamma^h \, h\epsilon_f + \gamma^H 2\, V_{max} H\epsilon_f \\
&= 2\, c_{max} \frac{(H-1)\gamma^{H+1} - H\gamma^H + \gamma}{(1-\gamma)^2}\, \epsilon_f + \gamma^H 2\, V_{max} H\epsilon_f
\end{aligned}$$

where, $|(p^h(x,u) - p_\phi^h(x,u))| \leq h\epsilon_f$ is inherited from Lemma B.2 in [1], the uncertainty in dynamics approximation. ■

Proof: [Proof of Theorem **??**] From Lemma-1, we know that,

$$J\left(\tilde{\eta}_t\right) \leq J_r\left(\tilde{\eta}_t\right) + R_{f,H} \quad (2)$$

and subtracting $J_r\left(\eta_t^\star\right)$ from both sides of Eq (4) results in

$$J\left(\tilde{\eta}_t\right) - J_r\left(\eta_t^\star\right) \leq J_r\left(\tilde{\eta}_t\right) - J_r\left(\eta_t^\star\right) + R_{f,H} \quad (3)$$

where LHS corresponds to the instantaneous regret incurred by rollouts on approximate MDP (with J) using shifted parameters ($\tilde{\eta}_t$) and on true MDP (with J_r) using the DMD-optimized parameters (η_t).

Now, to get the cumulative regret for T decision steps, both sides of Eq (5) should be summed over T and can be shown as,

$$\sum_{t=0}^{T} \left(J\left(\tilde{\eta}_t\right) - J_r\left(\eta_t^\star\right)\right) \leq \sum_{t=0}^{T} \left(J_r\left(\tilde{\eta}_t\right) - J_r\left(\eta_t^\star\right)\right) + \sum_{t=0}^{T} R_{f,H} \quad (4)$$

$$Re_T\left(\boldsymbol{\eta}_T\right) \leq \sum_{t=0}^{T} \left(J_r\left(\tilde{\eta}_t\right) - J_r\left(\eta_t^\star\right)\right) + T\; R_{f,H} \quad (5)$$

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Based on [3], the DMD update rule directly results in

$$\sum_{t=0}^{T}\left(J\left(\tilde{\eta}_t\right)-J_r\left(\eta_t^{\star}\right)\right)\leq\frac{D_{\max}}{\alpha_{T+1}}+\frac{4M}{\alpha_T}W_{\Phi_t}\left(\boldsymbol{\eta}_T\right)+\frac{G_\ell^2}{2\sigma}\sum_{t=1}^{T}\alpha_t \tag{6}$$

Substituting Eq (8) in Eq (7), we finally get the bound on the maximum regret as

$$Re_T\left(\boldsymbol{\eta}_T\right)\leq\frac{D_{\max}}{\alpha_{T+1}}+\frac{4M}{\alpha_T}W_{\Phi_t}\left(\boldsymbol{\eta}_T\right)+\frac{G_\ell^2}{2\sigma}\sum_{t=1}^{T}\alpha_t+T\ R_{f,H},$$

which completes the proof. ■

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