

HM2 Praktikum Serie 9

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1 Aufgabe 2

$$\int_0^{\pi} \cos(x^2) dx$$

Berechne manuell mit der Trapezregel $Tf(h)$ für die Schrittweiten $h_j = \frac{b-a}{2^j}$, ($j = 0, \dots, 4$)

Achtung: die erste Spalte enthält also fünf Werte und extrapolieren Sie diese mittels Romberg-Extrapolation so weit wie möglich.

1.1 Lösung

1.1.1 j 0

$$j_0 = 0, h_0 = \frac{b-a}{2^0} = \frac{b-a}{1} = b-a, n_0 = 2^0 = 1$$

$$T_{00} = h_0 \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^0 f(x_i) \right)$$

$$T_{00} = h_0 \cdot \frac{f(a) + f(b)}{2}$$

$$T_{00} = h_0 \cdot \frac{f(0) + f(\pi)}{2}$$

$$T_{00} = \pi \cdot \frac{\cos(0^2) + \cos(\pi^2)}{2}$$

$$T_{00} = \pi \cdot \frac{1 + \cos(\pi^2)}{2}$$

1.1.2 j 1

$$j_1 = 1, h_1 = \frac{b-a}{2^1} = \frac{b-a}{2}, n_1 = 2^1 = 2$$

$$T_{10} = h_1 \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^1 f(x_i) \right)$$

$$T_{10} = h_1 \cdot \left(\frac{f(a) + f(b)}{2} + f\left(\frac{\pi}{2}\right) \right)$$

$$T_{10} = \frac{\pi}{2} \cdot \left(\frac{1 + \cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{2}\right)^2\right) \right)$$

1.1.3 j 2

$$j_2 = 2, h_2 = \frac{b-a}{2^2} = \frac{b-a}{4}, n_2 = 2^2 = 4$$

$$T_{20} = h_2 \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^3 f(x_i) \right)$$

$$T_{20} = h_2 \cdot \left(\frac{f(a) + f(b)}{2} + f\left(\frac{\pi}{4}\right) + f\left(\frac{2\pi}{4}\right) + f\left(\frac{3\pi}{4}\right) \right)$$

$$T_{20} = h_2 \cdot \left(\frac{f(a) + f(b)}{2} + f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{2}\right) + f\left(\frac{3\pi}{4}\right) \right)$$

$$T_{20} = \frac{\pi}{4} \cdot \left(\frac{1 + \cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{4}\right)^2\right) + \cos\left(\left(\frac{\pi}{2}\right)^2\right) + \cos\left(\left(\frac{3\pi}{4}\right)^2\right) \right)$$

1.1.4 j 3

$$j_3 = 3, h_3 = \frac{b-a}{2^3} = \frac{b-a}{8}, n_3 = 2^3 = 8$$

$$T_{30} = h_3 \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^7 f(x_i) \right)$$

$$T_{30} = h_3 \cdot \left(\frac{f(a) + f(b)}{2} + f\left(\frac{\pi}{8}\right) + f\left(\frac{2\pi}{8}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{4\pi}{8}\right) + f\left(\frac{5\pi}{8}\right) + f\left(\frac{6\pi}{8}\right) + f\left(\frac{7\pi}{8}\right) \right)$$

$$T_{30} = h_3 \cdot \left(\frac{f(a) + f(b)}{2} + f\left(\frac{\pi}{8}\right) + f\left(\frac{\pi}{4}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{\pi}{2}\right) + f\left(\frac{5\pi}{8}\right) + f\left(\frac{3\pi}{4}\right) + f\left(\frac{7\pi}{8}\right) \right)$$

$$T_{30} = \frac{\pi}{8} \cdot \left(\frac{1 + \cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{8}\right)^2\right) + \cos\left(\left(\frac{\pi}{4}\right)^2\right) + \cos\left(\left(\frac{3\pi}{8}\right)^2\right) + \cos\left(\left(\frac{\pi}{2}\right)^2\right) + \cos\left(\left(\frac{5\pi}{8}\right)^2\right) + \cos\left(\left(\frac{3\pi}{4}\right)^2\right) + \cos\left(\left(\frac{7\pi}{8}\right)^2\right) \right)$$

1.1.5 j 4

$$j_4 = 4, h_4 = \frac{b-a}{2^4} = \frac{b-a}{16}, n_3 = 2^4 = 16$$

$$T_{40} = h_4 \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^{15} f(x_i) \right)$$

$$\begin{aligned} T_{40} &= h_4 \cdot \left(\frac{f(a)+f(b)}{2} + f\left(\frac{\pi}{16}\right) + f\left(\frac{2\pi}{16}\right) + f\left(\frac{3\pi}{16}\right) + f\left(\frac{4\pi}{16}\right) + f\left(\frac{5\pi}{16}\right) + f\left(\frac{6\pi}{16}\right) + \right. \\ &\quad \left. f\left(\frac{7\pi}{16}\right) + f\left(\frac{8\pi}{16}\right) + f\left(\frac{9\pi}{16}\right) + f\left(\frac{10\pi}{16}\right) + f\left(\frac{11\pi}{16}\right) + f\left(\frac{12\pi}{16}\right) + f\left(\frac{13\pi}{16}\right) + f\left(\frac{14\pi}{16}\right) + f\left(\frac{15\pi}{16}\right) \right) \\ T_{40} &= h_4 \cdot \left(\frac{f(a)+f(b)}{2} + f\left(\frac{\pi}{16}\right) + f\left(\frac{\pi}{8}\right) + f\left(\frac{3\pi}{16}\right) + f\left(\frac{\pi}{4}\right) + f\left(\frac{5\pi}{16}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{7\pi}{16}\right) + \right. \\ &\quad \left. f\left(\frac{\pi}{2}\right) + f\left(\frac{9\pi}{16}\right) + f\left(\frac{5\pi}{8}\right) + f\left(\frac{11\pi}{16}\right) + f\left(\frac{3\pi}{4}\right) + f\left(\frac{13\pi}{16}\right) + f\left(\frac{7\pi}{8}\right) + f\left(\frac{15\pi}{16}\right) \right) \\ T_{40} &= \frac{\pi}{16} \cdot \left(\frac{1+\cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{16}\right)^2\right) + \cos\left(\left(\frac{\pi}{8}\right)^2\right) + \cos\left(\left(\frac{3\pi}{16}\right)^2\right) + \cos\left(\left(\frac{\pi}{4}\right)^2\right) + \right. \\ &\quad \left. \cos\left(\left(\frac{5\pi}{16}\right)^2\right) + \cos\left(\left(\frac{3\pi}{8}\right)^2\right) + \cos\left(\left(\frac{7\pi}{16}\right)^2\right) + \cos\left(\left(\frac{\pi}{2}\right)^2\right) + \cos\left(\left(\frac{9\pi}{16}\right)^2\right) + \cos\left(\left(\frac{5\pi}{8}\right)^2\right) + \right. \\ &\quad \left. \cos\left(\left(\frac{11\pi}{16}\right)^2\right) + \cos\left(\left(\frac{3\pi}{4}\right)^2\right) + \cos\left(\left(\frac{13\pi}{16}\right)^2\right) + \cos\left(\left(\frac{7\pi}{8}\right)^2\right) + \cos\left(\left(\frac{15\pi}{16}\right)^2\right) \right) = \end{aligned}$$

1.2 Rhomberg Tabelle

J	h	T _{j0}	T _{j1}	T _{j2}	T _{j3}	T _{j4}
0	π	0.15...	-1.58...	-1.58...	0.52...	0.5641876002784855
1	$\frac{\pi}{2}$	-1.15...	1.24...	1.24...	0.56...	
2	$\frac{\pi}{4}$	0.64...	0.58...	0.58...		
3	$\frac{\pi}{8}$	0.60...	0.56...			
4	$\frac{\pi}{16}$	0.57...				