HM2 Praktikum Serie 11

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1 Aufgabe 2

$$\frac{dy}{dx} = \frac{x^2}{y}, x \leq 0 \leq 1.4, y(0) = 2$$

1.1 Exakte Lösung h

$$f(x) = \sqrt{\frac{2 * x^3}{3} + 4}$$

$$x_0 = 0 => \sqrt{\frac{2 * 0^3}{3} + 4} = 2$$

$$x_1 = 0.7 => \sqrt{\frac{2 * 0.7^3}{3} + 4} = 2.05637221015$$

$$x_2 = 0.7 => \sqrt{\frac{2 * 1.4^3}{3} + 4} = 2.41440123702$$

1.2 Euler Verfahren h = 0.7

$$x_0 = 0, y_0 = 2$$

$$x_1 = 0.7; y_1 = 2 + h * f(0, 2) = 2 + 0.7 * \frac{0^2}{2} = 2$$

$$x_2 = 1.4; y_2 = 2 + h * f(0, 2) = 2 + 0.7 * \frac{0.7^2}{2} = 2.1715$$

$$|y(x_0) - y_0| = 2 - 2 = 0$$

$$|y(x_1) - y_1| = 2 - 2.05637221015 = 0.05637221015$$

$$|y(x_2) - y_2| = 2.1715 - 2.41440123702 = 0.24290123702$$

1.3 Mittelpunk Verfahren h = 0.7

$$x_{i+1} = x_i + h;$$

$$y_{h/2} = y_i + h/2 * f(x_i, y_i);$$

$$x_{h/2} = x_i + h/2;$$

$$y_{i+1} = y_i + h * f(x_{h/2}, y_{h/2});$$

i	x_i	y_i	$x_{h/2}$	$y_{h/2}$	x_{i+1}	y_{i+1}
0	0	2	0.35	2 + 0.35 * f(0,2) = 2	0.7	2 + 0.7 * f(0.35, 2) = 2.042875
1	0.7	2.042875	1.05	2.12682531512	1.4	2.40573977997

$$|y(x_0) - y_0| = 2 - 2 = 0$$

$$|y(x_1) - y_1| = 2.042875 - 2.05637221015 = 0.01349721015$$

$$|y(x_2) - y_2| = 2.40573977997 - 2.41440123702 = 0.00866145705$$

1.4 Modifizierten Euler Verfahren h = 0.7

$$x_{i+1} = x_i + h$$

$$y_{i+1} = y_i + h * f(x_i, y_i)$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_{i+1}, y_{i+1})$$

$$y_{i+1} = y_i + h * \frac{k1 + k2}{2}$$

i	x_i	y_i	k_1	y_{i+1}	x_{i+1}	k_2	y_{i+1}
0	0	2	f(0,2) = 0	2 + 0.7 * 0 = 2	0.7	f(0.7,2) = 0.245	2.08575
1	0.7	2.08575	0.23492748411	2.25019923888	1.4	0.87103398051	2.47283651262

Table 1: Numerical method iteration table

$$|y(x_0) - y_0| = 2 - 2 = 0$$

$$|y(x_1) - y_1| = 2.08575 - 2.05637221015 = 0.02937778985$$

$$|y(x_2) - y_2| = 2.47283651262 - 2.41440123702 = 0.0584352756$$