**Math & Consequences: Chain Rule in Context**
Math 133
6 March, 2024

| **Names:** |
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A company which works in transporting hazardous materials has tasked you with analyzing scenarios involving a chemical designated Chemical X. The exact nature of the chemical is proprietary information, but they tell you the following:

(i) Chemical X will be transported via an underwater pipeline, whose closest point to shore is 5km out.

(ii) Should a leak occur in the pipeline, chemical X will quickly rise to the surface, forming a film of a particular height $h$ cm, and spread from the point of origin in a circle.

(iii) The company plans to build a sensor 0.5km from shore in the direction of this closest point in order to detect chemical X before it reaches shore.

(iv) Chemical X is very dangerous to humans, and if it were to reach shore, an evacuation would be required.

The following formula for the volume of a cylinder will be helpful:

$$V = \pi r^2 h$$

Keeping in mind that $V$ and $r$ are changing over time, it might be more helpful to write this formula in the following way:

$$V(t) = \pi(r(t))^2 h$$

**Scenario One**

- The chemical forms a film of height 10cm.
- At the moment, the circular film has a radius of 1km and is spreading at a rate of 100m/hour.

Question 1: How quickly is chemical X leaking from the pipeline in $\mathrm{m}^3$/hour? *Hint: it would help to convert all the distances to the same unit.*

**Scenario Two**

- A leak has begun at the closest point to shore releasing at a constant rate of 150,000$\text{m}^3$/hour.
- The chemical forms a film of height 10cm.

Question 2: Using $\frac{dr}{dt}$ when $r = 4.5$km, estimate how long it would take the chemical X to reach the shore after being detected by the sensor.

Question 3: Is your estimate in the previous question an over- or under-estimate? How can you tell?

**Scenario Three**

- A leak has begun at the closest point to shore releasing at a constant rate of 150,000$\text{m}^3$/hour.
- The chemical initially forms a film of height 10cm, but the warmth at the surface is causing this height to shrink at a rate of 0.01cm/hour.

Question 4: Using $\frac{dr}{dt}$ when $r = 4.5$km and $h = 8$cm, estimate how long it would take the chemical X to reach the shore after being detected by the sensor.

Question 5: How does this compare to your answer in scenario two?

**Scenario Four**

- The chemical forms a film of height 10cm.
- It's estimated that it would take a full 24 hours to evacuate the area of shore closest to the pipeline.

Question 6: What is the maximum rate of leakage that would allow evacuation in time?

Question 7: How much chemical X would you advise is safe to transmit in this pipeline? What additional information or context would help you make this decision?