

String Manipulation

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KMP Algorithm

Hash

Trie

Others

KMP Algorithm

Sample 1

Sample 1

Given two strings T and P, find the first position where string P occurs in string T.

A sample

Sample Input:

$T = \text{"abcdefg"}$

$P = \text{"cde"}$

Sample Output:

2

Another sample

Sample Input:

T = "abcdefghijk"

P = "abce"

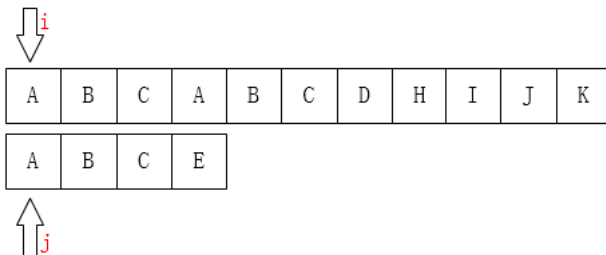
Sample Output:

-1

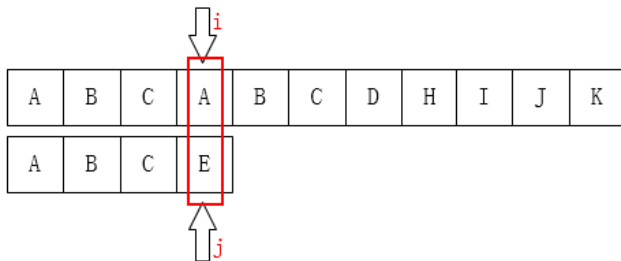
Naive Solution

A	B	C	D	E	F	G	H	I	J	K
A	B	C	E							

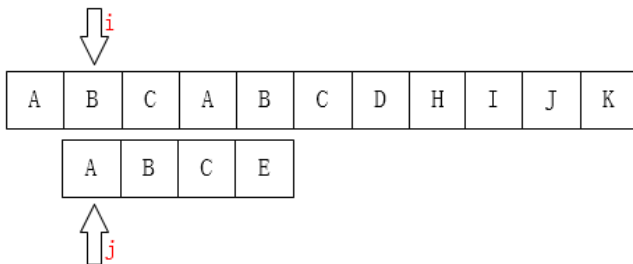
Naive Solution



Naive Solution



Naive Solution



Code 1: Naive Solution

```
1 def func(T, P):  
2     for i in xrange(len(T) - len(P) + 1):  
3         if T[i:i + len(P)] == P:  
4             return i  
5     return -1
```

Naive Solution

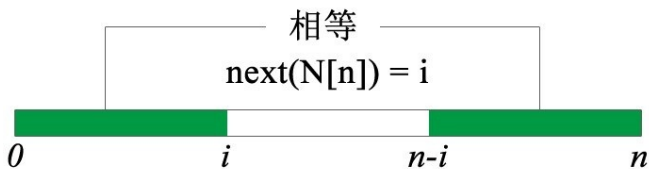
Time complexity is $O(n \times m)$

What if n and m are very large? for example $n = 100000$
and $m = 100000$

How to improve?



寻找最长首尾匹配位置



Sample

考虑下面几个字符串的最长首尾匹配长度:

$S1 = \text{"abcdefgabc"} \implies 3$

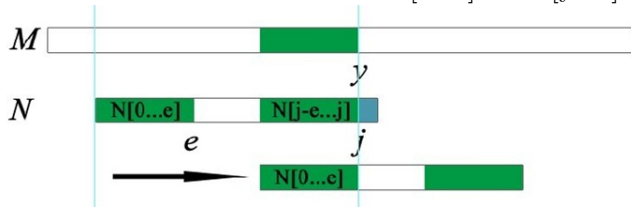
$S2 = \text{"abcdefgab"} \implies 2$

$S3 = \text{"aaaaa"} \implies 4$

$S4 = \text{"abcd"} \implies 0$

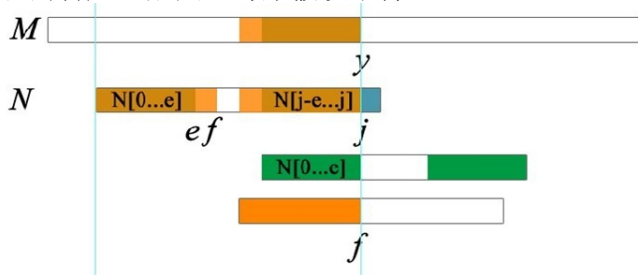
证明匹配

如果位置 $j+1$ 不匹配，将 N 后移至首尾相等位置，仍然可以满足匹配，接下来只需查看 $N[e+1]$ 与 $M[y+1]$ 是否相等即可



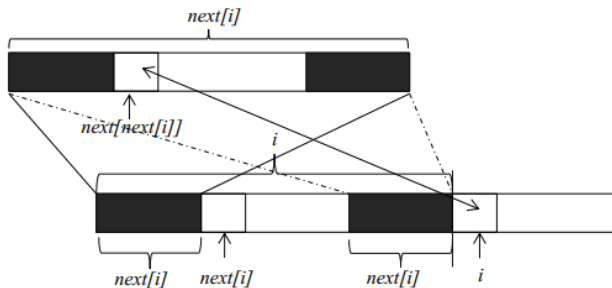
证明最优

用反证法，假设存在 f , $f > e$, 满足 $N[0...f] = M[y-f...y]$, 即其匹配位置出现在更早的位置，由于 $M[y-j...y] = N[0...j]$, $M[y-f...y] = N[j-f...j]$, 则满足 $N[j-f...j] = N[0...f]$, 则 e 就不是最长的首尾匹配点，与原假设不符



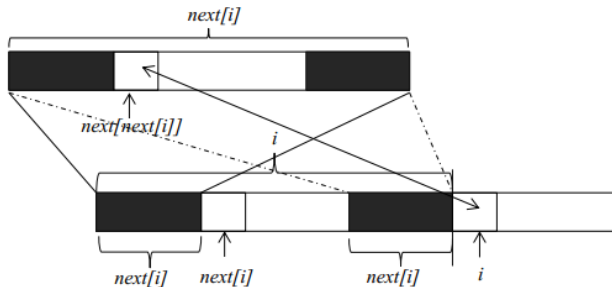
next 数组计算

假设我们现在已经求得 $\text{next}[1]$ 、 $\text{next}[2]$ 、…… $\text{next}[i]$ ，分别表示长度为 1 到 i 的字符串的前缀和后缀最大公共长度，现在要求 $\text{next}[i+1]$



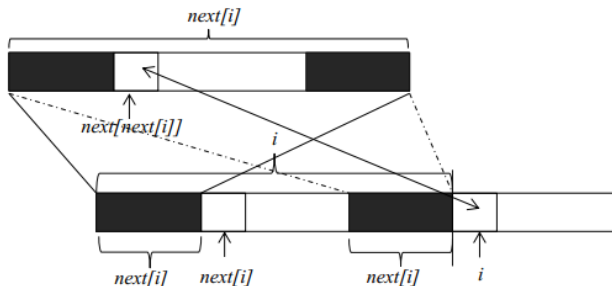
next 数组计算

如果位置 i 和位置 $\text{next}[i]$ 处的两个字符相同，则 $\text{next}[i+1]$ 等于 $\text{next}[i]$ 加 1。



next 数组计算

如果两个位置的字符不相同，我们可以将长度为 $\text{next}[i]$ 的字符串继续分割，获得其最大公共长度 $\text{next}[\text{next}[i]]$ ，然后再和位置 i 的字符比较。直到字符串长度为 0 为止



next 数组计算

$P = \text{"ABRACADABRA"}$

i	0	1	2	3	4	5	6	7	8	9	10	11
P[i]	A	B	R	A	C	A	D	A	B	R	A	无
next[i]	0	0	0	0	1	0	1	0	1	2	3	4

Table : example of calculating next array

Code 2: Golden Solution

```
1 void getNext(const string &P) {  
2     int m = P.length();  
3     next[0] = next[1] = 0;  
4     int j = 0;  
5     for (int i = 1; i < m; ++ i) {  
6         while (j && P[j] != P[i]) j = next[j];  
7         if (P[j] == P[i]) ++ j;  
8         next[i + 1] = j;  
9     }  
10 }
```

确保你真正理解了 next 数组

请思考该字符串的 next 数组的值:

$P = \text{"ABCABC"}$

问: 现在有了 next 数组, 如何对两个字符串进行匹配呢?

答: 匹配的过程和计算 next 数组的过程很相似

Code 3: Golden Solution

```
1 int find(const string &T, const string &P) {  
2     int n = T.length(), m = P.length();  
3     getNext(P);  
4     int j = 0;  
5     for (int i = 0; i < n; ++ i) {  
6         while (j && P[j] != T[i]) j = next[j];  
7         if (P[j] == T[i]) ++ j;  
8         if (j == m) return i - m + 1;  
9     }  
10    return -1;  
11 }
```

Golden Solution

Time complexity is $O(n + m)$ (提示: 使用摊还分析)

Hash (字符串哈希)

Rabin-Karp Algorithm

1. $S = s_1 s_2 s_3 \dots s_m$
2. $H(S) = (s_1 b^{m-1} + s_2 b^{m-2} + s_3 b^{m-3} + \dots + s_m b^0) \bmod M$
(可以将 S 看作 b 进制的数，另外实际使用中常用自然溢出来代替取模)
3. $H(S[k+1..k+m]) = (H(S[k..k+m-1]) \times b - s_k b^m + s_{k+m}) \bmod M$

Sample

$S = \text{"abcde"}$, 求 S 的所有后缀的 Hash 值。

$$\begin{cases} H(4) &= s[4] \\ H(3) &= s[4]x + s[3] \\ H(2) &= s[4]x^2 + s[3]x + s[2] \\ H(1) &= s[4]x^3 + s[3]x^2 + s[1]x + s[0] \\ H(0) &= s[4]x^4 + s[3]x^3 + s[2]x^2 + s[1]x + s[0] \end{cases}$$

Sample 2

Given two strings T and P, find the first position where string P occurs in string T.

Code 4: Hash Function

```
1 typedef unsigned long long ll;  const ull B = 131;
2 int contain(const string &a, const string &b) {
3     int n = S.length(), m = b.length();
4     if (n > m) return false;
5     ull t = 1;
6     for (int i = 0; i < n; ++i) t *= B;
7     ull ah = 0, bh = 0;
8     for (int i = 0; i < n; ++i) ah = ah * B + a[i];
9     for (int i = 0; i < n; ++i) bh = bh * B + b[i];
10    for (int i = 0; i + n <= m; ++i) {
11        if (ah == bh) return i;
12        if (i + n < m) bh = bh * B + b[i + n] - b[i] * t;
13    }
14    return -1;
15 }
```

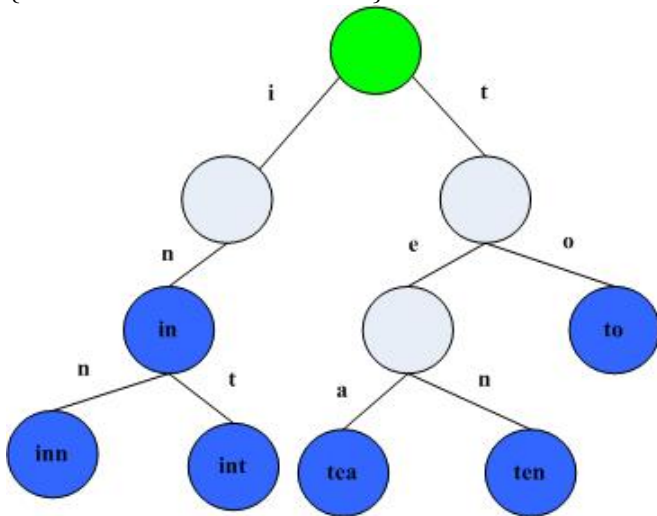
Trie (字典树, 前缀树)

How to read?

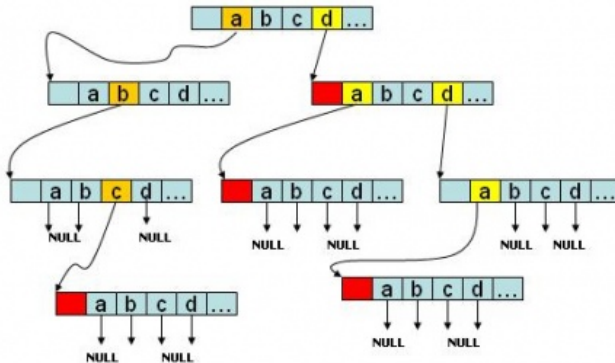
Trie: /tra /

Its pronunciation is same with word “try” .

{tea, ten, to, in, inn, int}



Another picture



Function of Trie

1. 字符串检索：检索/查询功能是 Trie 树最原始的功能。思路就是从根节点开始一个一个字符进行比较
2. 词频统计：Trie 树常被搜索引擎系统用于文本词频统计。
3. 前缀匹配：trie 树前缀匹配常用于搜索提示。如当输入一个网址，可以自动搜索出可能的选择。当没有完全匹配的搜索结果，可以返回前缀最相似的可能。

Code 5: Trie

```
1 struct Trie {
2     int ch[maxNode][sigma_size], val[maxNode], sz;
3     Trie() { sz = 1; memset(ch[0], 0, sizeof(ch[0])); }
4     void insert(char *s) {
5         int u = 0, n = strlen(s);
6         for (int i = 0; i < n; ++i) {
7             int c = s[i] - 'a';
8             if (!ch[u][c]) {
9                 memset(ch[sz], 0, sizeof(ch[sz]));
10                val[sz] = 0; ch[u][c] = sz ++;
11            } u = ch[u][c];
12        }
13        val[u] = 1;
14    }
15 };
```

Others (其他)

Several more string algorithms

1. Manacher Algorithm
2. Suffix Array
3. Extended KMP Algorithm
4. Aho-Corasick Automachine
5. Suffix Automachine

The end
Thank you!