

3. Computing Spectrum of Audio Signals

- (a) Determine DFT coefficients of signal $x[n]$ represented by vector $[1 \ -1 \ 0 \ 2]$ and compare it to the DTFT transform $X(e^{j\omega})$ of $x[n]$ assuming $x[n] = 0$ for $n < 0$, $n > 3$.
- (b) State and prove the linearity and time delay properties for DFT.
- (c) Evaluate circular convolution for $x[n] = [1 \ -1 \ 0 \ 2]$ and $h[n] = [1 \ 0 \ 1 \ 0]$.
- (d) State and prove the circular convolution property for DFT.
- (e) If $X(k)$ denotes DFT coefficients for length N signal $x[n]$, show that the inverse DFT of $X(0), X(1), \dots, X(N-1)$ yields the original signal $x[0], x[1], \dots, x[N-1]$.
- (f) Write your own Matlab function *my_DFT* to implement DFT for a specific input signal in vector **x**. Use the summation expression from the basic DFT definition.
- (g) Determine the number of FLOPs to get all DFT coefficients in part (f).

4. Fast Fourier Transform Algorithm

- (a) Show how through decimation in time, DFT of a length N signal can be determined through calculation of two $N/2$ DFT's and the use of appropriate 'twiddle' factors. (Hint: N is assumed to be even, then separate the original DFT sum into two sums for odd and even time indexes n .)
- (b) If N is a power of 2, repeat the approach from (a) to further reduce length $N/2$ DFT into two length $N/4$ DFT's, then a length $N/4$ DFT into two length $N/8$ DFT's, etc. until only size 2 DFT's and appropriate twiddle factors are needed.
- (c) Determine the reduced computational complexity of calculating DFT through the approach in (b). Plot the computational complexity as a function of signal length N and compare it to computational complexity for the original approach from Question 3 (g). Use $N=2^M$ for $M=1,2,\dots,30$.
- (d) Implement your own Matlab function '*my_FFT*' that uses recursive calls to implement FFT for length $N=2^M$ signals. Test your routine and compare your transform results to the ones obtained by the built in FFT.
- (e) Implement your own '*FFT_16*' routine to calculate length 16 FFT using appropriate butterfly structure. Test your implementation against built-in FFT.
- (f) Use the FPGA board to implement and demo the algorithm from (e) in real time.