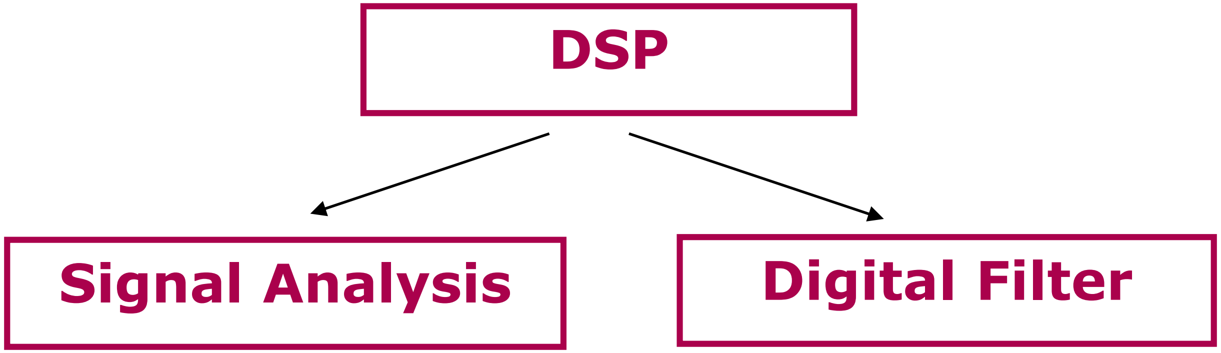
DSP Review

DSP Overview

DSP could be classified into two parts:

——Signal Analysis and Digital Filter



DSP Overview

Chapter 8: Infinite Impulse Response Filters

Analog Filter design

Design Low-pass IIR Digital Filter

Impulse Invariance IIR Filter Design Method

Bilinear Transform IIR Filter Design Method

Chapter 7: Design of FIR Digital Filter

Properties of FIR Filters

FIR filter design based on Windows

Chapter 6: Filter Structures

Block Structure

Mason and Transpose Theorem

IIR and FIR Structures

Chapter 8

8.1 An Introduction to Infinite Impulse Response Filters

8.2 The Laplace Transform

8.3 Analog Low-Pass Filters

8.4 Impulse Invariance IIR Filter Design Method

8.5 Bilinear Transform IIR Filter Design Method

8.6 Low-Pass IIR Filter Design

8.7 Other Types IIR Filter Design

8.3 Analog Filter design

Contents:

Filter Specifications

Butterworth Approximation

Chebyshev Approximation

Cauer Approximation

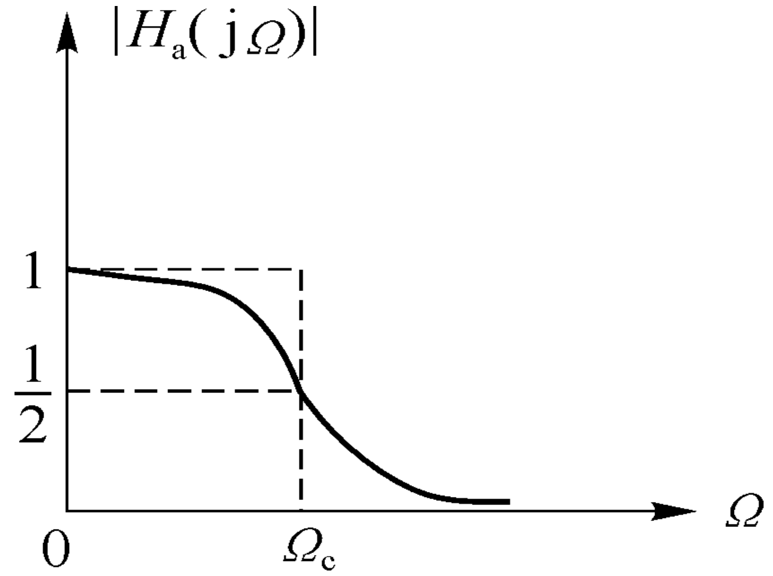
Comparison of above Analog Filters

(1) Butterworth Approximation

Magnitude Frequency Character:

$$\left|H(j\Omega)\right|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

N is a positive integer, the order of Filter. Ω_c is the cut-off frequency.



Features:

- ***Maximally Flat Magnitude***
- ***3dB Cutoff Frequency***

Summary for Calculation Method

(1) Requirements

(2) Calculate N

(3) Calculate Ω_c

(4) Find H(s)

Useful Formula

N=1:

$$H(s) = \frac{\Omega_c}{s + \Omega_c}$$

N=2:

$$H(s) = \frac{(\Omega_c)^2}{s^2 + \sqrt{2}\Omega_c s + \Omega_c^2}$$

N=3:

$$H(s) = \frac{(\Omega_c)^3}{s^3 + 2\Omega_c s^2 + 2\Omega_c^2 s + \Omega_c^3}$$

8.6 Low-Pass IIR Filter Design

Procedures of Designing IIR DF with AF design:

- **Required Targets for DF**
- **Transfer Function for Analogy Filter *$H_a(S)$***
- **Filter transition *(s plane -> z plane)* to obtain Transfer Function for digital filter *H(z)***
- **Digital frequency transition, to obtain other digital filters according to the digital LP filter**

Two Methods for $H_a(s)$ to H(z):

- ***Impulse Response Invariance method - IRI***
- ***Bilinear Transformation method - BLT***

Design IIR LP DF with BLT Method

Steps:

1) **Given ω_s, ω_p, α_p and α_s of DF.**

2) **According to:**

$$\boxed{\Omega = \frac{2}{T} tg \frac{\omega}{2}}$$

to calculate pre-warped critical frequency: Ω_s, Ω_p

3) **According to Ω_p, Ω_s, α_p and α_s, design the prototype of LP AF and get $H_a(s)$**

4) **Using BLT, we can get the Transfer Function of DF.**

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}}$$

Chapter 7

FIR Filter

$$H(z) = \sum_{r=0}^{N-1} h(r) z^{-r}$$

Characteristic of FIR DF:

- Always Stable: Poles at origin point
- Zeros of Linear Phase FIR: $z_i, \quad z_i^*, \quad \frac{1}{z_i}, \quad \frac{1}{z_i^*}$
- Linear Phase DF

Phase Delay & Group Delay:

$$\boxed{\tau_p(\omega) = -\frac{\theta(\omega)}{\omega}}$$

$$\boxed{\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}}$$

- Both are constant
  - Even Symmetry: $\begin{cases} \theta_0 = 0 \text{ and } \tau = \dfrac{N-1}{2} \\ h(n) = h(N-1-n) \end{cases}$
- Only Group Delay
  - Odd Symmetry: $\begin{cases} \theta_0 = \pm\dfrac{\pi}{2} \; with \; \tau = \dfrac{N-1}{2} \\ h(n) = -h(N-1-n) \end{cases}$

+ Order N: odd or even

The Phenomenon of Gibbs

Summary:

The phenomenon of Gibbs results in the convolution of windowed frequency function and $H_d(e^{j\omega})$.

(1) Transition band: the band between positive and negative acromion.

(2) The width of transition band is the mainlobe width of windows spectrum. For rectangular $w_R(e^{j\omega})$, the width is $4\pi/N$.

The width of transition band is decided by the selected windows.

For one certain type of windows, increasing N can make transition band more steep.

Summary of Windows

Window Functions	Transition band-width	Peak Sidelobe/dB	Minimum stopband/dB
Rectangular	$4\pi/N$ | $1.8\pi/N$	-13	21
Hanning	$8\pi/N$ | $6.2\pi/N$	-32	44
Hamming	$8\pi/N$ | $6.6\pi/N$	-43	53
Blackman	$12\pi/N$ | $11\pi/N$	-58	74

FIR Design with Windows

Steps:

Performance requirements -> $H_d(e^{j\omega})$

(1) Expand $H_d(e^{j\omega})$ to Fourier Series, get $h_d(n)$;

(2) Truncate $h_d(n)$ to N=2M+1 *(windows)*;

(3) Shift the truncated $h_d(n)$ right with M points, get h(n);

(4) Multiply h(n) by the choosing windows function;

(5) Realize h(n) or H(z) by hardware or software.

Chapter 6

6.1 Block Structure

6.2 Mason and Transpose Theorem

6.3 Example of Filter Structures

Contents

Filters:

Described by H(z) or h(n)

Described by diagram (*Mason's Rule*)

Filter Structure:

•FIR Filter:

- ***Direct Form, Cascade Form***

•IIR Filter:

- ***Direct Form, Canonical, Cascade, Parallel Form***

Transpose Theorem:

- ***Every structure has two realizations at least.***

Mason's Rule

Mason's Rule:

If g_i denotes the route gain from the *Source Node* to *Destination Node,* and Δ_i is the cofactor of g_i, then, the Transfer Function *H* from the source to destination is:

$$\boxed{H = \frac{1}{\Delta}\sum_i g_i \Delta_i}$$

***Mason's Rule* provides a step by step method to obtain the Transfer Function from a block diagram or signal flow graph.**

Derived by Samuel Jefferson Mason.

Equivalent Structures

Target: various realization of a given Transfer Function.

Equivalent Structure: the same *Transfer Function*.

Transpose Theorem:

(Proved at section 4.72 of A. V. Oppenheim's Book)

- ***Reverse the direction of all paths;***
- ***Maintain the path gain;***
- ***Exchange the positions of input and output.***

The Transfer Function is the same to original one, when there are only *one input and one output*.

DSP Overview

Chapter 0: Introduction of signal and signal processing

- Characterization and Classification of Signals
- Digital Signal Processing
- DSP Processes & Features

Chapter 1: Discrete Sequences and Systems

- Discrete Sequences
- Discrete Linear Time-Invariant Systems
- Causality and Stability Properties of LTI systems

Chapter 2: Periodic Sampling

- Sampling Low-Pass Signals
- Discrete Convolution

DSP Overview

Chapter 3: z-Transform

z-Transform & ROC

Inverse z-Transform

Chapter 4: The Discrete Fourier Transform

DTFT, DFS, DFT

DFT Leakage, Resolution, Properties

Circular Convolution, Frequency Response

Chapter 5: The Fast Fourier Transform

FFT (*Butterfly algorithm*) & Inverse FFT

Linear convolution with DFT

Piecewise Convolution for Long Sequence

Chapter 1

Chapter 1

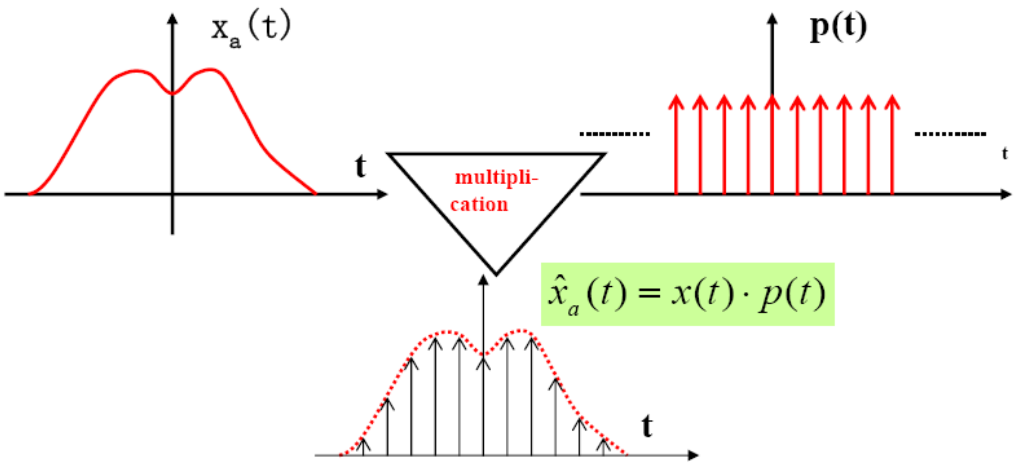
1. <u>Basic Sequences</u>
   a. Left-side, Right-side and Two-side
   b. Unit sample, Unit step, Rectangular sequence
   c. Real exponent, Sinusoidal sequence
2. <u>Signal Processing Operational Symbols</u>
   a. Basic operation of sequences
   b. Multiplication, Addition, Unit Delay
3. <u>Discrete LTI System</u>
   a. Linear
   b. Time-shift Invariant
   c. Causality
   d. Stability
4. <u>For LTI systems, y(n)=x(n)* x(n)</u>

Chapter 2

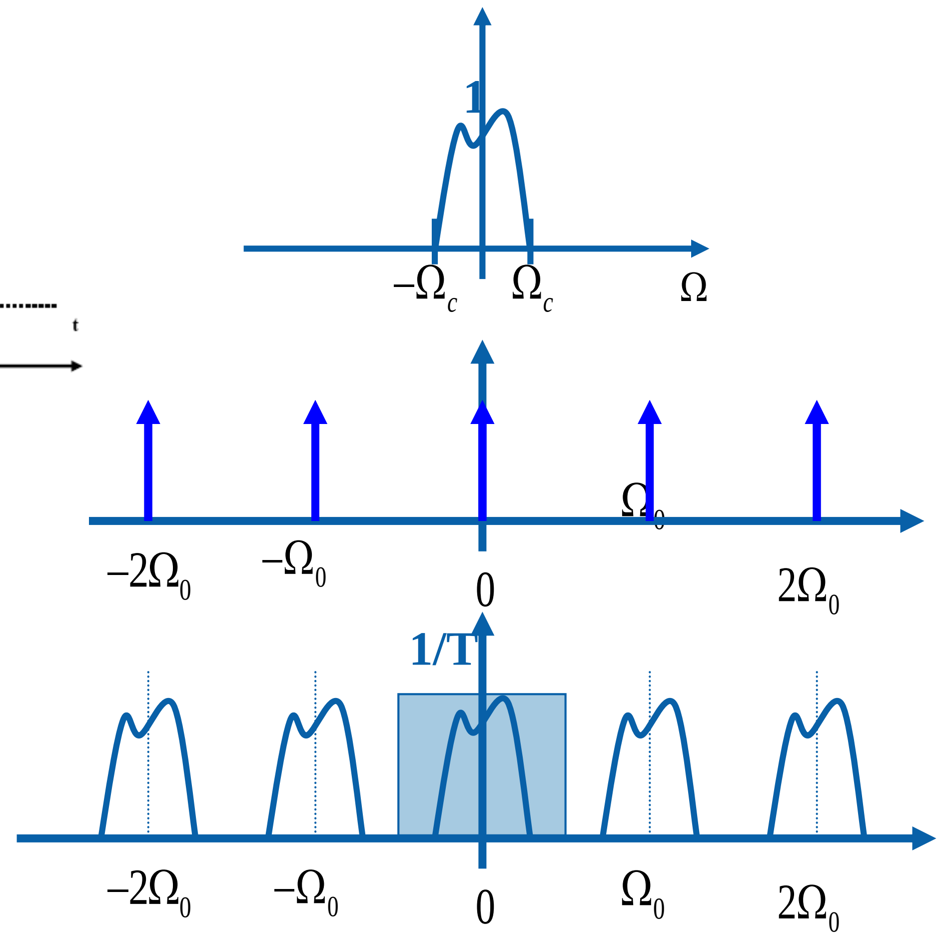
Sampling Theorem-Shannon Theorem

1. Band-limited signals

2. $f_s \geq 2 f_m$



$$\hat{X}_a(\Omega) = \frac{1}{2\pi} X_a(\Omega) * P(\Omega)$$
$$= \frac{1}{T} \sum_{m=-\infty}^{\infty} X_a(\Omega - m\Omega_0)$$



Discrete Convolution in the Time Domain

Calculation steps:

(1) *Time-reversed*

h(-k) ←h(k)

(2) *Right shift n*

h(n-k)←h(-k)

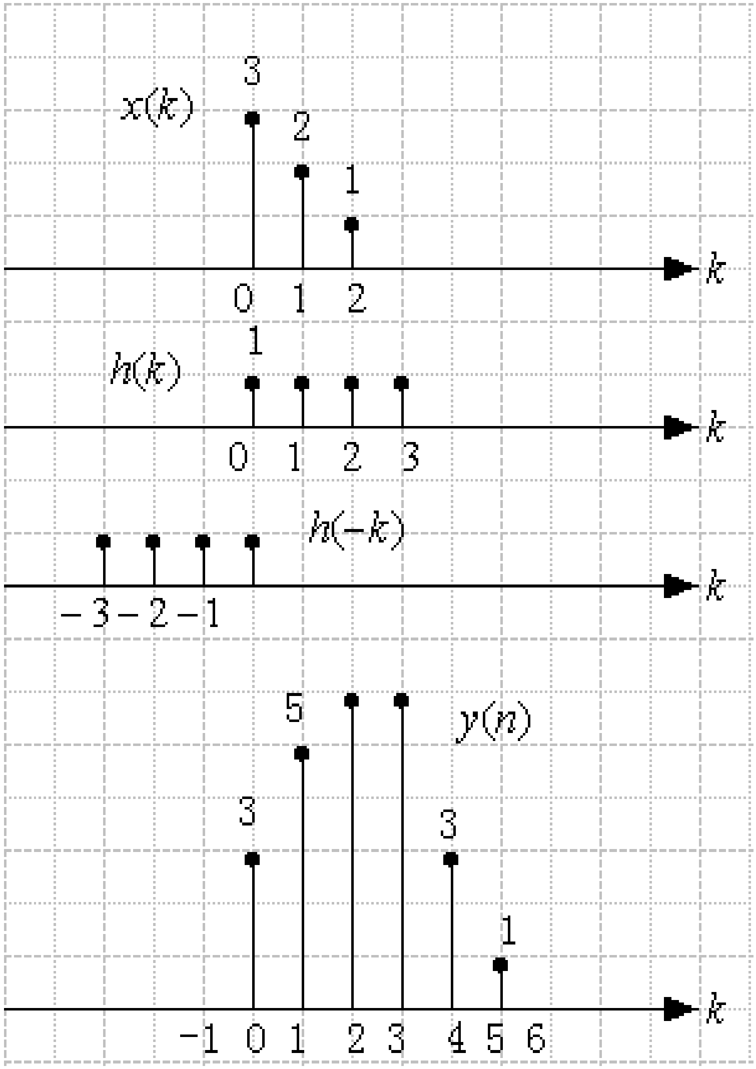
(3) *Multiplication*

x(n)·h(n-k)

(4) *Sum*

∑[x(n)·h(n-k)]

Note: the resulted length is *N+M-1.*



Chapter 3

3.1 The z-Transform

3.1.1 Poles and Zeros on the z-Plane and Stability

3.1.2 The ROC of z-Transform

3.1.3 The Properties of z-Transform

3.2 The Inverse z-Transform

3.2.1 General Expression of Inverse z-Transform

3.2.2 Inverse z-Transform by Partial-Fraction Expansion

Chapter 3

1. z-Transform

$$X(z) = Z[x(n)] = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

- ***ROC & Pole-zero plot***

(a) Right-side Sequence: ROC |z|>r1;

(b) Left-side Sequence: ROC |z|<r2;

(c) Two-side Sequence: ROC r1<|z|<r2.

Chapter 3

2. The Properties of ZT

(a) Linear

(b) Time shifting $Z[x(n+n_0)] = z^{n_0} X(z)$

(c) Frequency shifting (scaling in the z-domain) $Z[a^n x(n)] = X\left(\frac{z}{a}\right)$

(d) Differential $Z[nx(n)] = -z\frac{dX(z)}{dz} \qquad R_{x-} < |z| < R_{x+}$

(e) Conjugation

(f) Initial Value Theorem $\mathrm{x}(0) = \lim_{z\to\infty} X(z)$

(g) Convolution in z-domain

$$Z[x(n) * h(n)] = X(z) \cdot H(z)$$

Chapter 3

3. Inverse z-Transform

$$x(n) = \frac{1}{2\pi j} \oint_c X(z) z^{n-1} dz$$

(a) Part Fractional method

(b) General Expression - Residue method

- **Draw the zero-pole plot, find the ROC, and draw the closed curve C, containing the origin.**
- **Calculate the residue numbers in & out of C, get the value of x(n).**

Chapter 4

- DTFT
- Understanding the DFT Equation
- Inverse DFT
- DFT Leakage
- Windows
- DFT Resolution, Zero Padding, and Frequency-Domain Sampling
- DFT Properties
- Frequency Response

DTFT, DFS and DFT

1. DTFT:

$$\begin{cases} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega} \\ x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{jn\omega} d\omega \end{cases}$$

2. DFS:

$$\begin{cases} \widetilde{X}(k) = \sum_{n=0}^{N-1} \widetilde{x}(n) W_N{}^{kn} \\ \widetilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \widetilde{X}(k) W_N{}^{-kn} \end{cases}$$

3. DFT:

$$\begin{cases} X(k) = \sum_{n=0}^{N-1} x(n) W_N{}^{kn}, \quad k = 0, \cdots, N-1 \\ x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N{}^{-kn}, \quad n = 0, \cdots, N-1 \end{cases}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

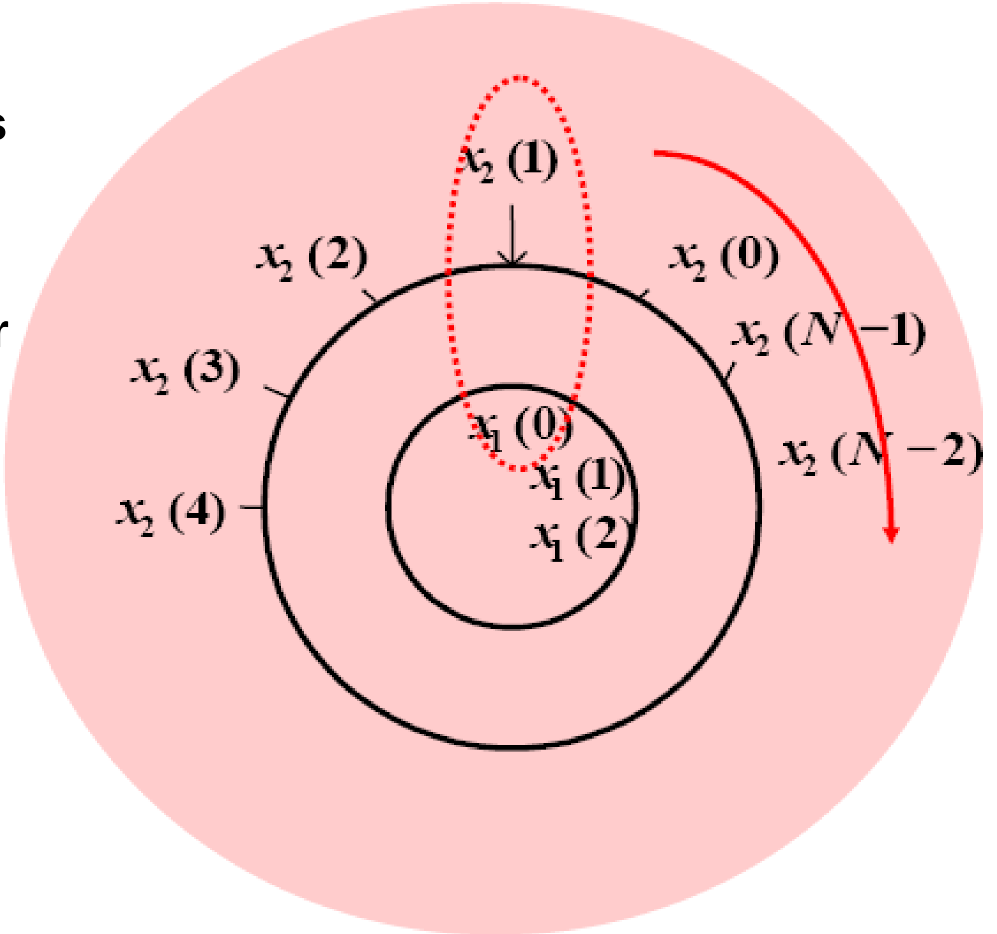
Circular Convolution

Concentric Method:

1) Multiply the corresponding values on the two circles and sum. We get $x_3(0)$;

2) Shift $x_2(n\text{-}m)$ 1 point, i.e., the outer circle rotate 1 point clockwise. Repeat (1) and we get $x_3(1)$;

3) In the same way, we get $x_3(n)$; $0 \le n \le N\text{-}1$.



Frequency Response

LTI System:

a. **Frequency Response:** $H(e^{jw})$

b. **Transfer Function:** $H(z)$

c. **Difference Equation**

Chapter 5

Fast Fourier Transform - FFT

FFT Reverse & Rearrangement and In-place Computation

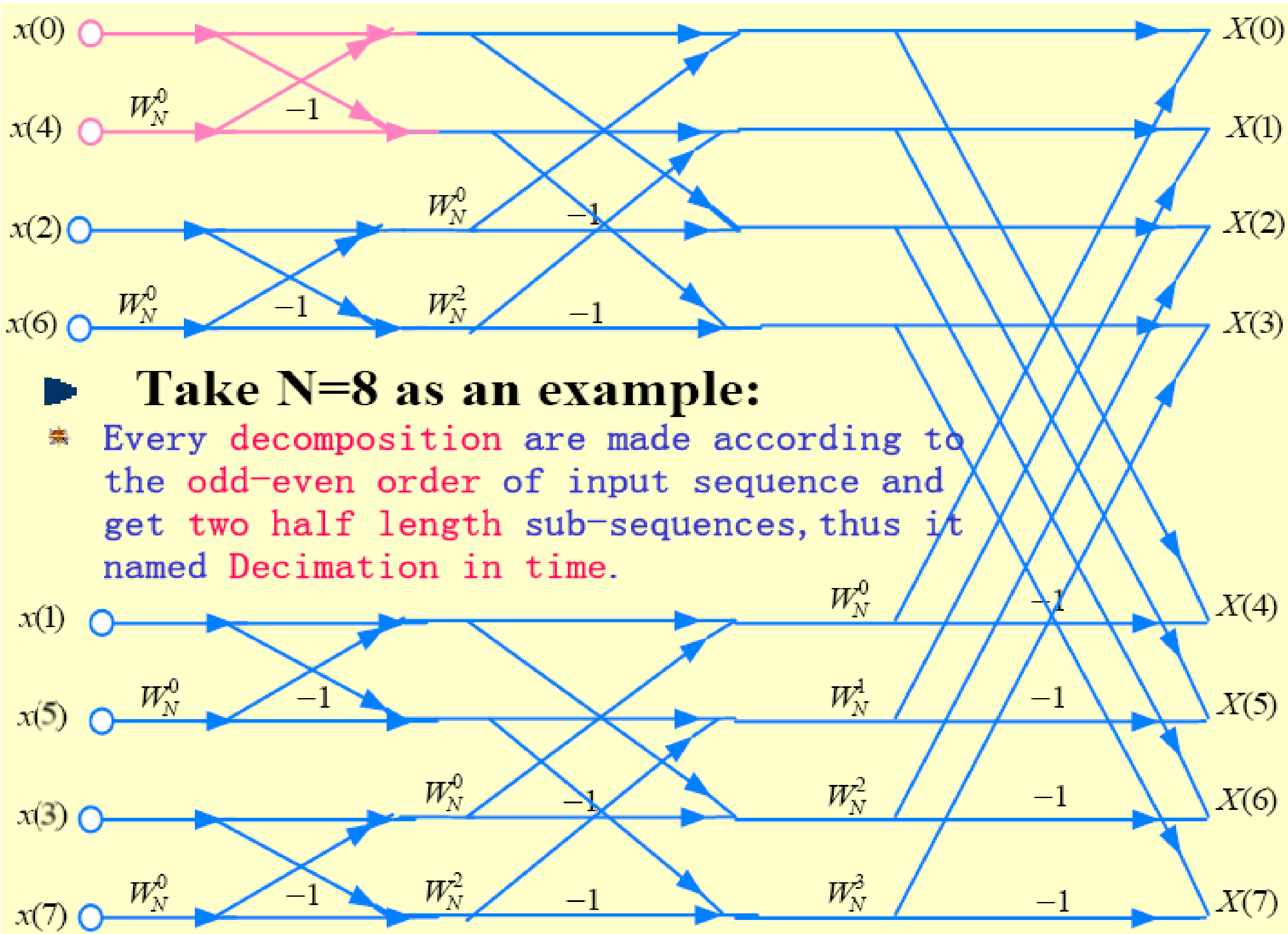
Inverse Fast Fourier Transform - IFFT

High-efficient FFT for Real Sequences

Discrete Convolution Using FFT

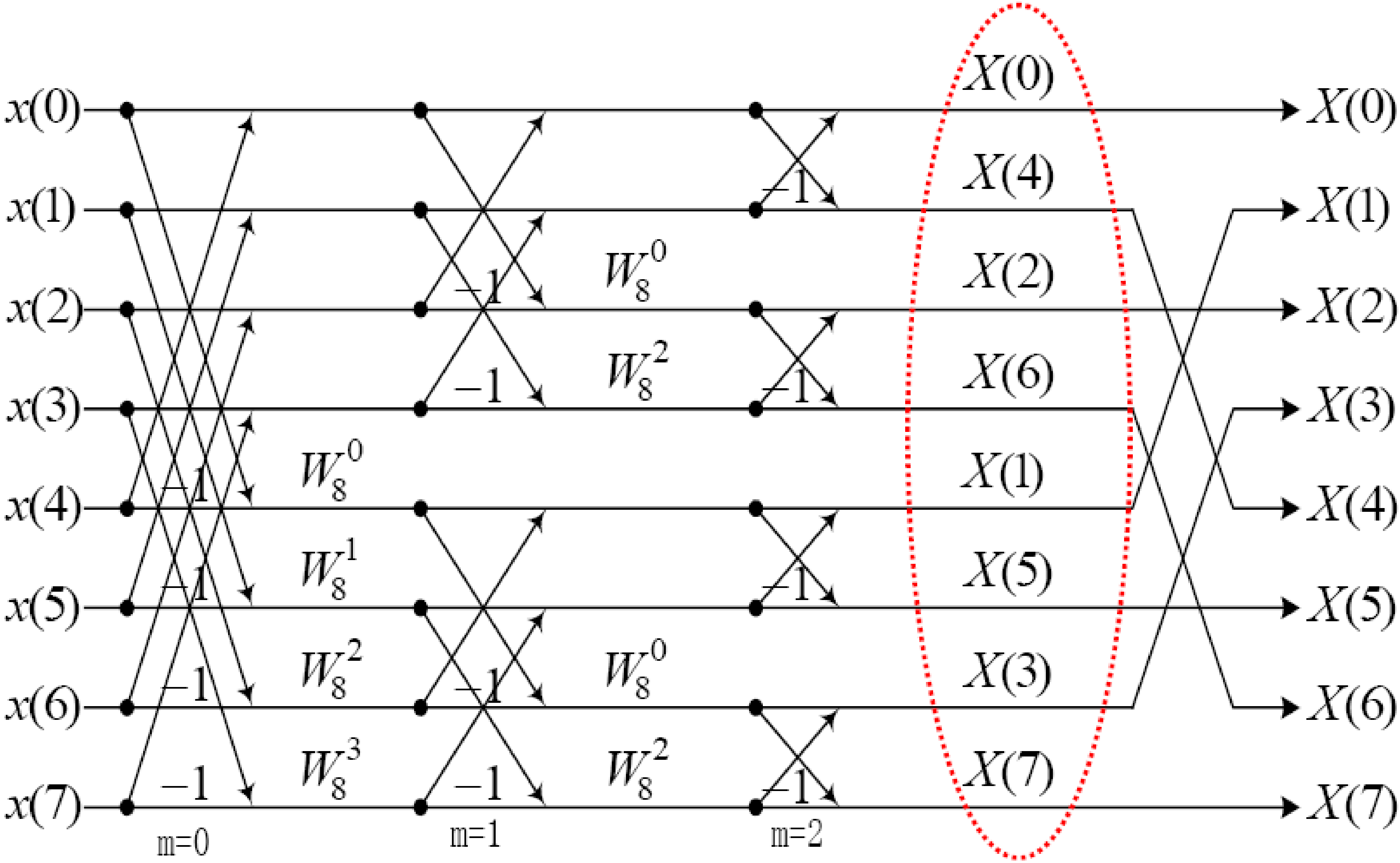
Piecewise Convolution for Long Sequences

Decimation in Time FFT (DIT-FFT)



- **Take N=8 as an example:**
  - Every decomposition are made according to the odd-even order of input sequence and get two half length sub-sequences, thus it named Decimation in time.

Decimation in Frequency FFT (DIF-FFT)



Discrete Conv. using DFT

1.Using DFT to do Discrete Convolution

The most important condition:

The Length of Circular Convolution must bigger than or equal to that of Linear Convolution:

N’>=N+M-1

2.Piecewise Convolution for Long Sequence:

a. Overlap-Add method

b. Overlap-Save method

Conclusion

***Chapter 1*: Discrete Sequences and Systems**

***Chapter 2*: Periodic Sampling**

***Chapter 3*: z-Transform and Inverse z-Transform**

***Chapter 4*: The Discrete Fourier Transform**

***Chapter 5*: The Fast Fourier Transform**

***Chapter 6*: Filter Structure**

***Chapter 7*: Finite Impulse Response Filters**

***Chapter 8*: Infinite Impulse Response Filters**

The End