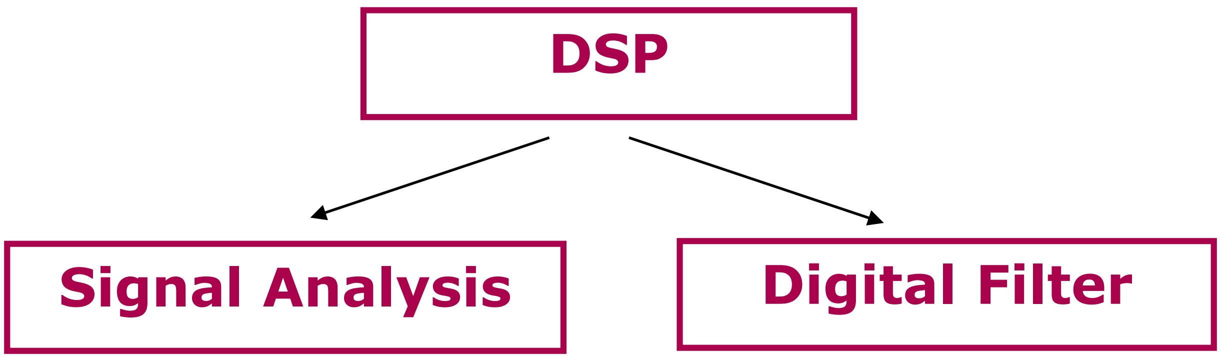
DSP Review

DSP Overview

DSP could be classified into two parts:

——Signal Analysis and Digital Filter



DSP Overview

Chapter 8: Infinite Impulse Response Filters

Analog Filter design

Design Low-pass IIR Digital Filter

Impulse Invariance IIR Filter Design Method

Bilinear Transform IIR Filter Design Method

Chapter 7: Design of FIR Digital Filter

Properties of FIR Filters

FIR filter design based on Windows

Chapter 6: Filter Structures

Block Structure

Mason and Transpose Theorem

IIR and FIR Structures

Chapter 8

8.1 An Introduction to Infinite Impulse Response Filters

8.2 The Laplace Transform

8.3 Analog Low-Pass Filters

8.4 Impulse Invariance IIR Filter Design Method

8.5 Bilinear Transform IIR Filter Design Method

8.6 Low-Pass IIR Filter Design

8.7 Other Types IIR Filter Design

8.3 Analog Filter design

Contents:

Filter Specifications

Butterworth Approximation

Chebyshev Approximation

Cauer Approximation

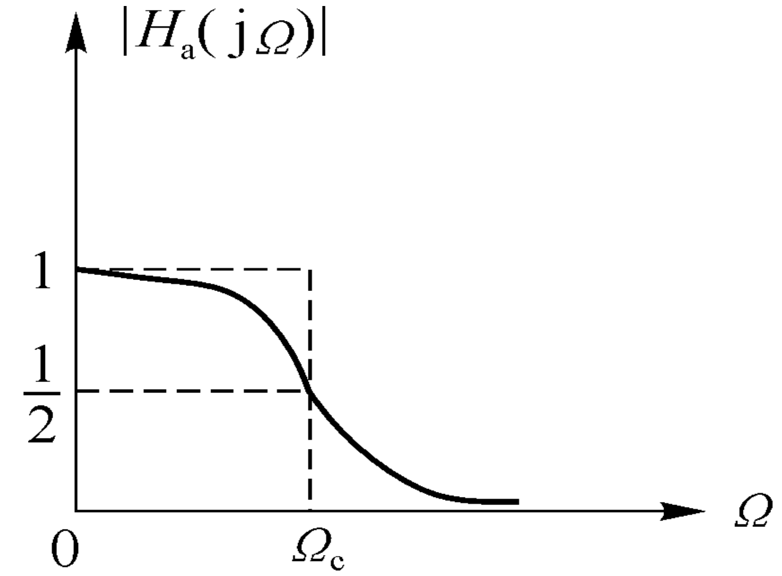
Comparison of above Analog Filters

(1) Butterworth Approximation

Magnitude Frequency Character:

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

N is a positive integer, the order of Filter. Ω_c is the cut-off frequency.



Features:

- ***Maximally Flat Magnitude***
- ***3dB Cutoff Frequency***

Summary for Calculation Method

(1) Requirements

(2) Calculate N

(3) Calculate Ω_c

(4) Find H(s)

Useful Formula

N=1:

$$H(s) = \frac{\Omega_c}{s + \Omega_c}$$

N=2:

$$H(s) = \frac{(\Omega_c)^2}{s^2 + \sqrt{2}\Omega_c s + \Omega_c^2}$$

N=3:

$$H(s) = \frac{(\Omega_c)^3}{s^3 + 2\Omega_c s^2 + 2\Omega_c^2 s + \Omega_c^3}$$

8.6 Low-Pass IIR Filter Design

Procedures of Designing IIR DF with AF design:

- **Required Targets for DF**
- **Transfer Function for Analogy Filter** ***$H_a(S)$***
- **Filter transition** ***(s plane -> z plane)*** **to obtain Transfer Function for digital filter** ***$H(z)$***
- **Digital frequency transition, to obtain other digital filters according to the digital LP filter**

Two Methods for $H_a(s)$ to H(z):

- ***Impulse Response Invariance method - IRI***
- ***Bilinear Transformation method - BLT***

Design IIR LP DF with BLT Method

Steps:

1) **Given ω_s, ω_p, α_p and α_s of DF.**

2) **According to:**

$$\boxed{\Omega = \frac{2}{T} tg \frac{\omega}{2}}$$

to calculate pre-warped critical frequency: Ω_s, Ω_p

3) **According to Ω_p, Ω_s, α_p and α_s, design the prototype of LP AF and get $H_a(s)$**

4) **Using BLT, we can get the Transfer Function of DF.**

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}}$$

Chapter 7

7.1 An Introduction to Finite Impulse Response Filters (FIR)

7.2 Properties of FIR Filters

7.3 Low-Pass FIR Filter Design

7.4 Examples to Design other Types Linear Phase FIR Filter

FIR Filter

$$H(z) = \sum_{r=0}^{N-1} h(r) z^{-r}$$

Characteristic of FIR DF:

- **Always Stable: Poles at origin point**
- **Zeros of Linear Phase FIR:** $z_i, \quad z_i^*, \quad \frac{1}{z_i}, \quad \frac{1}{z_i^*}$
- **Linear Phase DF**

Phase Delay & Group Delay:

$$\tau_p(\omega) = -\frac{\theta(\omega)}{\omega}$$

$$\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$$

- **Both are constant**
  - **Even Symmetry:**

$$\begin{cases} \theta_0 = 0 \text{ and } \tau = \frac{N-1}{2} \\ h(n) = h(N-1-n) \end{cases}$$

- **Only Group Delay**
  - **Odd Symmetry:**

$$\begin{cases} \theta_0 = \pm\frac{\pi}{2} \text{ with } \tau = \frac{N-1}{2} \\ h(n) = -h(N-1-n) \end{cases}$$

+ Order N: odd or even

The Phenomenon of Gibbs

Summary:

The phenomenon of Gibbs results in the convolution of windowed frequency function and $H_d(e^{j\omega})$.

(1) Transition band: the band between positive and negative acromion.

(2) The width of transition band is the mainlobe width of windows spectrum. For rectangular $w_R(e^{j\omega})$, the width is *$4\pi/N$*.

The width of transition band is decided by the selected windows.

For one certain type of windows, increasing N can make transition band more steep.

Summary of Windows

Window Functions	Transition band-width	Peak Sidelobe/dB	Minimum stopband/dB
Rectangular	$4\pi/N$ | $1.8\pi/N$	-13	21
Hanning	$8\pi/N$ | $6.2\pi/N$	-32	44
Hamming	$8\pi/N$ | $6.6\pi/N$	-43	53
Blackman	$12\pi/N$ | $11\pi/N$	-58	74

FIR Design with Windows

Steps:

Performance requirements -> $H_d(e^{j\omega})$

(1) Expand $H_d(e^{j\omega})$ to Fourier Series, get $h_d(n)$;

(2) Truncate $h_d(n)$ to N=2M+1 *(windows)*;

(3) Shift the truncated $h_d(n)$ right with M points, get h(n);

(4) Multiply h(n) by the choosing windows function;

(5) Realize h(n) or H(z) by hardware or software.

Chapter 6

6.1 Block Structure

6.2 Mason and Transpose Theorem

6.3 Example of Filter Structures

Contents

Filters:

Described by H(z) or h(n)

Described by diagram (*Mason's Rule*)

Filter Structure:

•FIR Filter:

- ***Direct Form,*** ***Cascade Form***

•IIR Filter:

- ***Direct Form, Canonical,*** ***Cascade, Parallel Form***

Transpose Theorem:

- ***Every structure has two realizations at least.***

Mason's Rule

Mason's Rule:

If g_i denotes the route gain from the *Source Node* to *Destination Node,* and Δ_i is the cofactor of g_i, then, the Transfer Function *H* from the source to destination is:

$$\boxed{H = \frac{1}{\Delta} \sum_i g_i \Delta_i}$$

***Mason's Rule* provides a step by step method to obtain the Transfer Function from a block diagram or signal flow graph.**

Derived by Samuel Jefferson Mason.

Equivalent Structures

Target: various realization of a given Transfer Function.

Equivalent Structure: the same *Transfer Function*.

Transpose Theorem:

(Proved at section 4.72 of A. V. Oppenheim's Book)

- ***Reverse the direction of all paths;***
- ***Maintain the path gain;***
- ***Exchange the positions of input and output.***

The Transfer Function is the same to original one, when there are only *one input and one output*.