Tools and Techniques, Lecture 3

Performance

- Algorithmic complexity
- Memory access patterns
- Benchmarking

LUNDS UNIVERSITET

CERN School of Computing

CERN School of Computing 2025, Lund

Sten Åstrand, Lund University

Why?

Image credit: Giulio Eulisse, “Benchmarking and Profiling” lecture, CSC2024

How does it scale?

ALICE
Event Processing
Nodes Cluster

24640 CPU cores
2800 GPUs

Image credit: Giulio Eulisse, “Benchmarking and Profiling” lecture, CSC2024

How do you scale?

Man making Python plots, color photograph

Image from imgflip.com

Algorithmic complexity

Theoretical metric for estimating resource consumption (in our case runtime - time complexity).

“The time complexity of an algorithm represents the number of steps it has to take to complete.”

– Complexity Theory. Brilliant.org. Retrieved 13:58, June 18, 2025, from https://brilliant.org/wiki/complexity-theory/

Also useful: Devopedia. 2022. "Algorithmic Complexity." Version 8, February 19. Accessed 2024-06-25. https://devopedia.org/algorithmic-complexity

Big O notation

n items → ALGORITHM ⇢ *f*(*n*) steps of computation

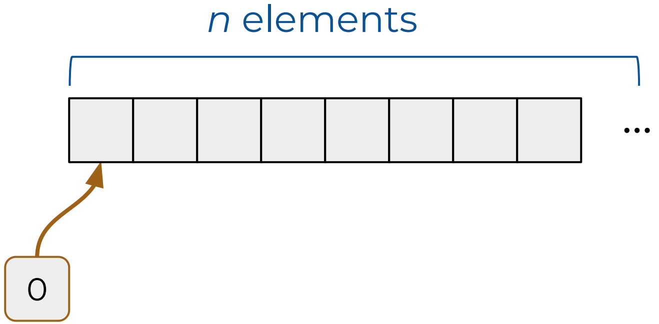
⇒ algorithm is on the order of *f*(*n*) or *O*(*f*(*n*))

Example: if an algorithm's runtime grows **linearly with the input size**, the **algorithm is *O*(*n*)**

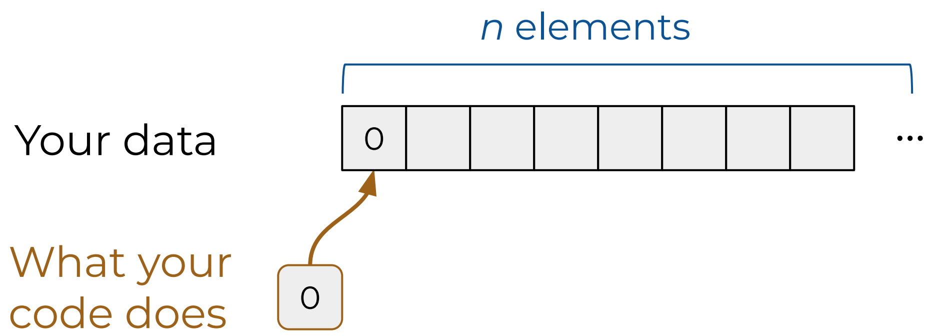
Strictly: *O*(*f*(*n*)) ⇔ "asymptotically bounded by *f*(*n*) up to some constant" (see Bachmann-Landau notation)

Big O Notation. Brilliant.org. Retrieved 13:54, June 18, 2025, from https://brilliant.org/wiki/big-o-notation/

Common Big O's and examples thereof

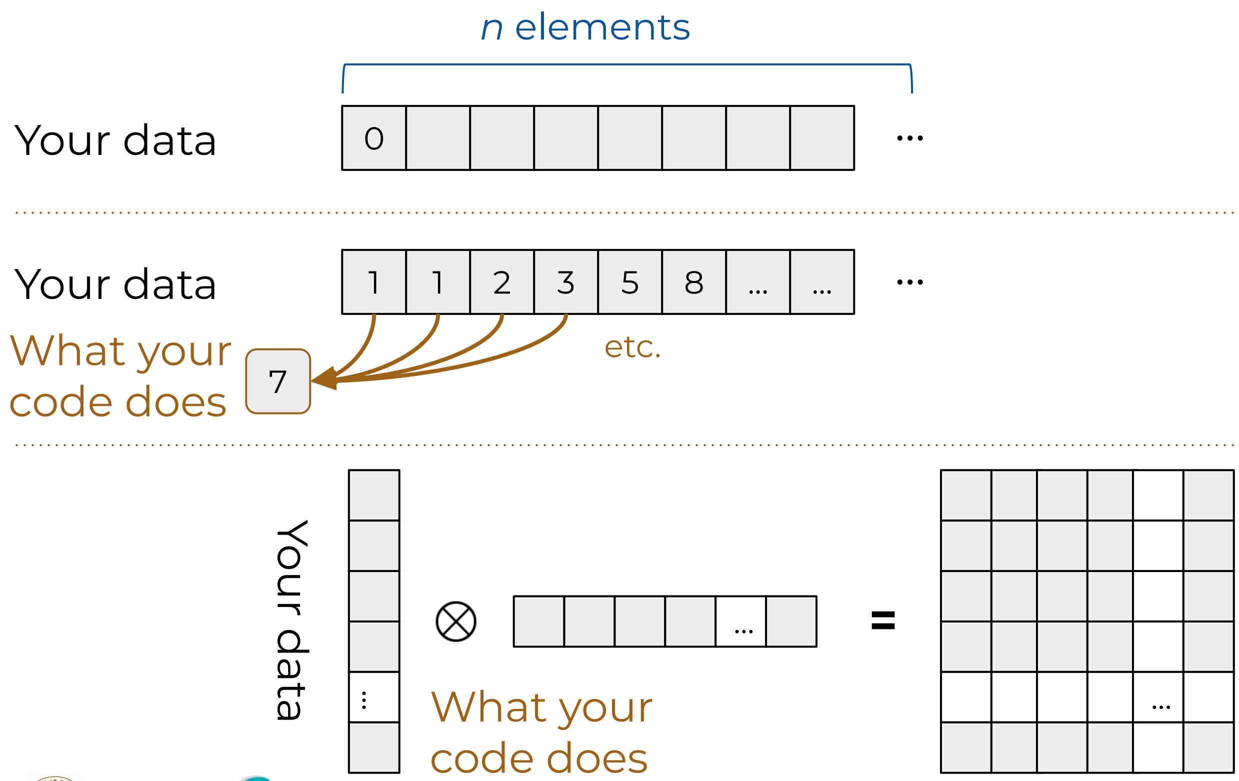
		Complexity	Time
Your data		?	?
What your code does			

Common Big O's and examples thereof



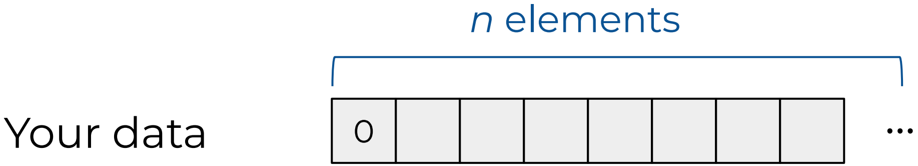
Complexity	Time
$O(1)$	Constant

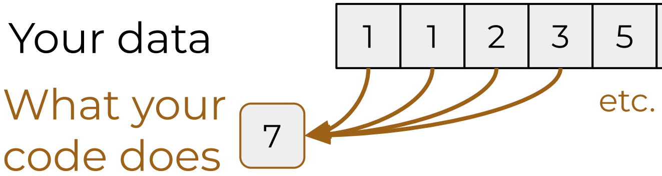
Common Big O's and examples thereof

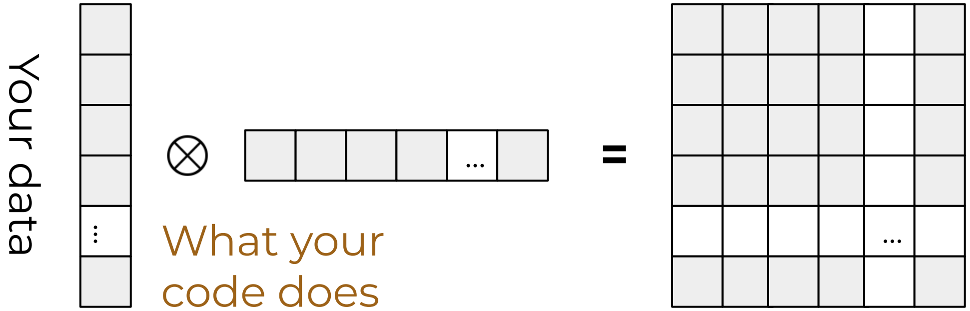


	Complexity	Time
	$O(1)$	Constant
	?	?
	?	?

Common Big O's and examples thereof

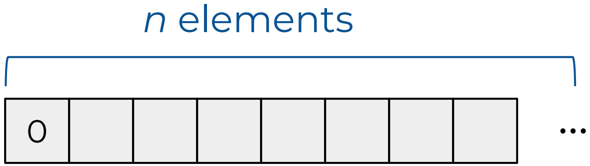
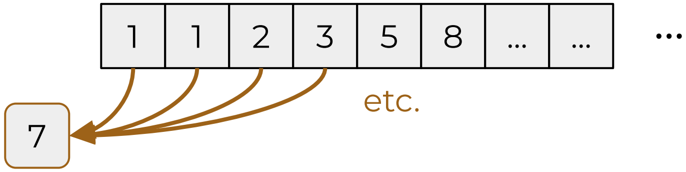
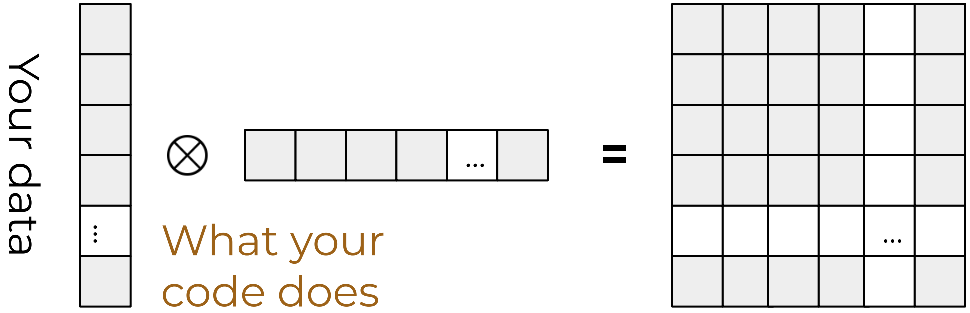






Complexity	Time
$O(1)$	Constant
$O(n)$	Linear
?	?

Common Big O's and examples thereof

		Complexity	Time
Your data		$O(1)$	Constant
Your data What your code does		$O(n)$	Linear
		$O(n^2)$	Quadratic

Common Big O's and examples thereof

O(n) - linear time (i.e. input size doubles = runtime doubles)

- Radix sort

O(n²) - quadratic time (input size doubles = runtimes quadruples)

- "For loop in a for loop"
- Outer product of two *n*-length vectors ⇒ *n*-by-*n* matrix
- Selection sort, insertion sort

Common Big O's and examples thereof

O(n) - linear time (i.e. input size doubles = runtime doubles)

- Radix sort

aktshually $O(k \cdot n)$

O(n²) - quadratic time (input size doubles = runtimes quadruples)

- "For loop in a for loop"
- Outer product of two *n*-length vectors ⇒ *n*-by-*n* matrix
- Selection sort, insertion sort

Common Big O's and examples thereof, cont.

$O(\log_2 n)$ - logarithmic time

- Binary search of a sorted array

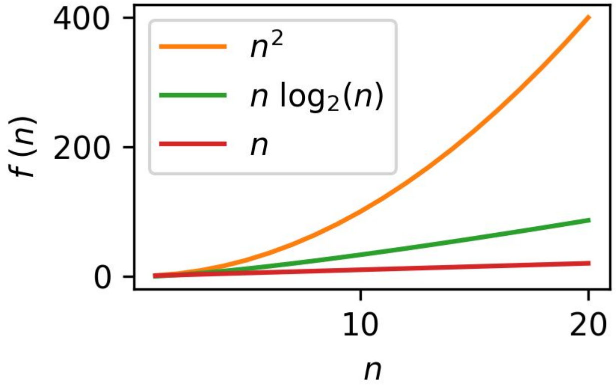
$O(n \log_2 n)$ - log-linear or "linearithmic" time
(close to *$O(n)$* for "reasonably sized" *n*)

- Many sorting algorithms, e.g. merge sort, heap sort

$O(n^3)$ - cubic time

- "For loop in a for loop in a for loop"
- "Schoolbook" matrix multiplication

Common Big O's and examples thereof, cont.

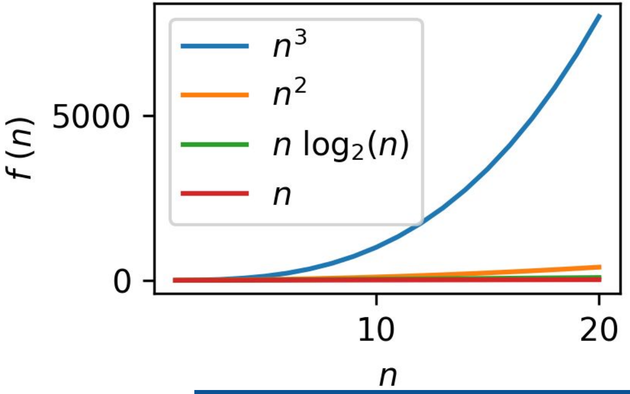


O(log$_2$ n) - logarithmic time

- Binary search of a sorted array

O(n log$_2$ n) - log-linear or "linearithmic" time (close to *O(n)* for "reasonably sized" *n*)

- Many sorting algorithms, e.g. merge sort, heap sort



O(n³) - cubic time

- "For loop in a for loop in a for loop"
- "Schoolbook" matrix multiplication

Two really Big O's (to avoid if possible)

O(2^n) and ***O(n!)*** - exponential time and factorial time

- Brute-force search, combinatorics, NP-hard problems
- Finding prime numbers

Matrix multiplication

- “Schoolbook” matrix multiplication $O(n^3)$
- The Strassen algorithm (1969) $O(n^{2.805})$
- State of the art (2023) $O(n^{2.373})$

For 4 x 4 matrices:

Strassen algorithm: 49 multiplications

Google Deepmind AlphaEvolve algorithm: 48 multiplications

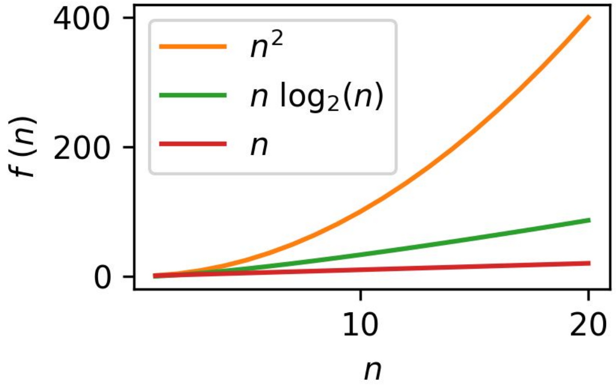
AlphaEvolve: A Gemini-powered coding agent for designing advanced algorithms
https://deepmind.google/discover/blog/alphaevolve-a-gemini-powered-coding-agent-for-designing-advanced-algorithms/

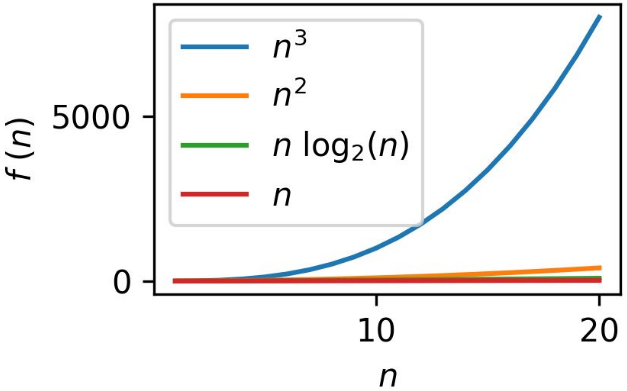
Algorithmic complexity - conclusion

```cpp
int binary_search(const std::vector<int>& v, int target) {
    int left = 0, right = v.size() - 1;

    while (left <= right) {
        int mid = left + (right - left) / 2;
        if (v[mid] == target)
            return mid;
        else if (v[mid] < target)
            left = mid + 1;
        else
            right = mid - 1;
    }

    return -1; // not found
}
```

Common code patterns

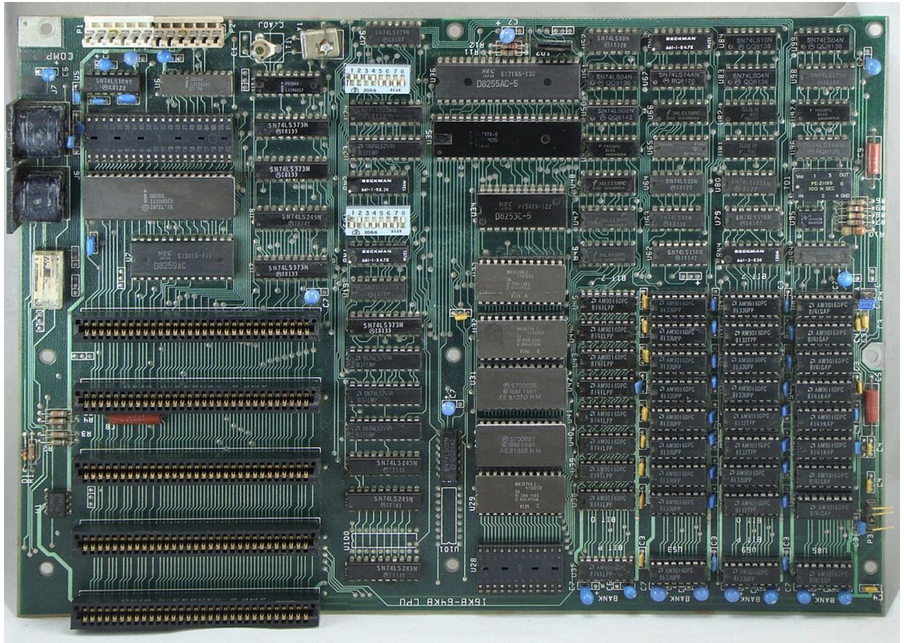
- For loop in a for loop
  - Solve differently?
  - Combine loops?
- Searching through data
  - Consider sorting first?
  - Use look-up table?
- Recomputing values
  - Compute once, keep in a table?





Pit stop

Memory access patterns



An IBM Personal Computer Model 5150 motherboard, 1981 - the first “PC”. Image credit: user GermanX on Wikimedia Commons, under license Attribution-ShareAlike 2.5 Generic

Memory access patterns

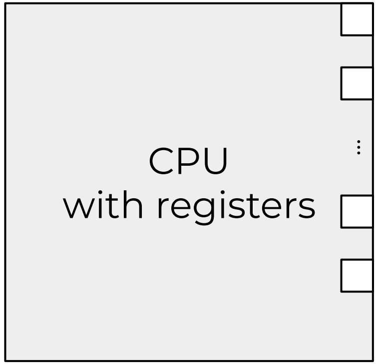
Image credit: user Gravislizard on Wikimedia Commons, under license Attribution-ShareAlike 2.0 France

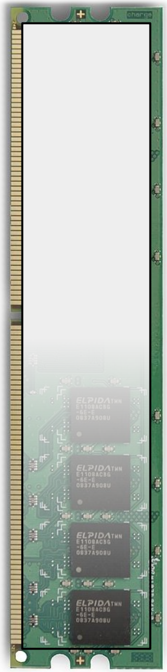
An IBM Personal Computer Model 5150 motherboard, 1981 - the first “PC”. Image credit: user GermanX on Wikimedia Commons, under license Attribution-ShareAlike 2.5 Generic

CPU-to-memory layout

CPU registers

Memory





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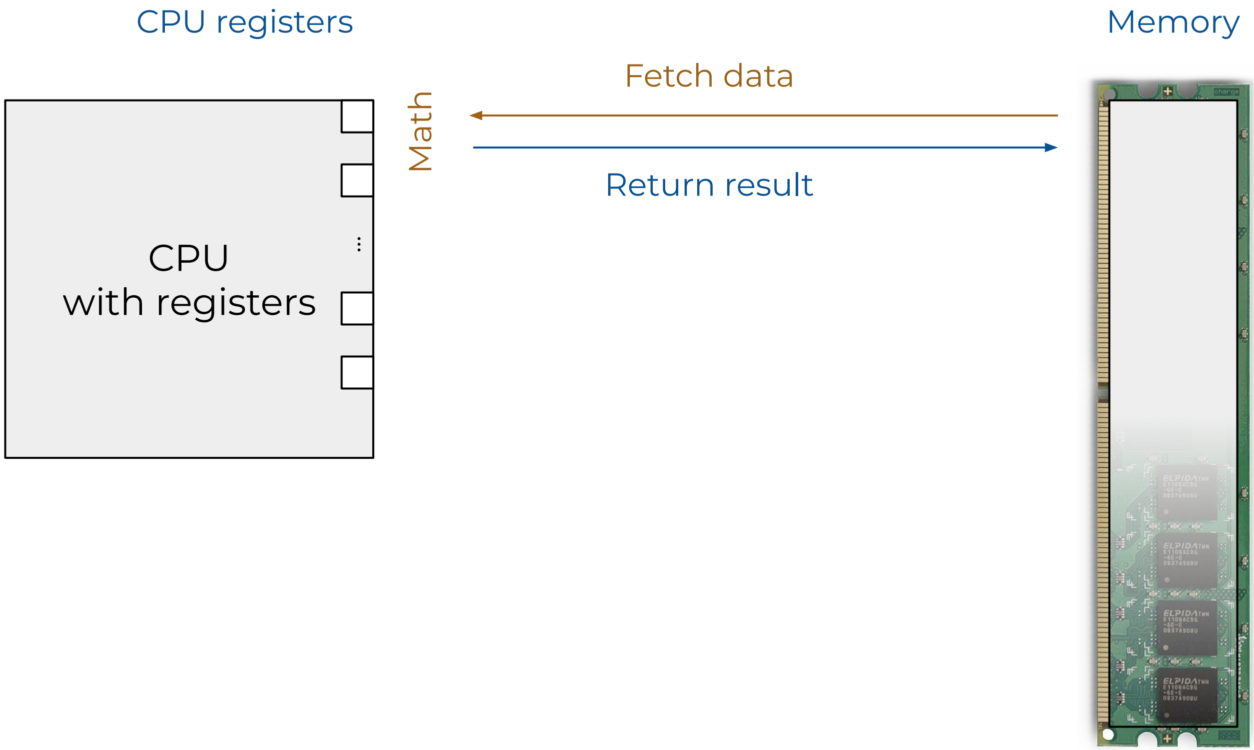

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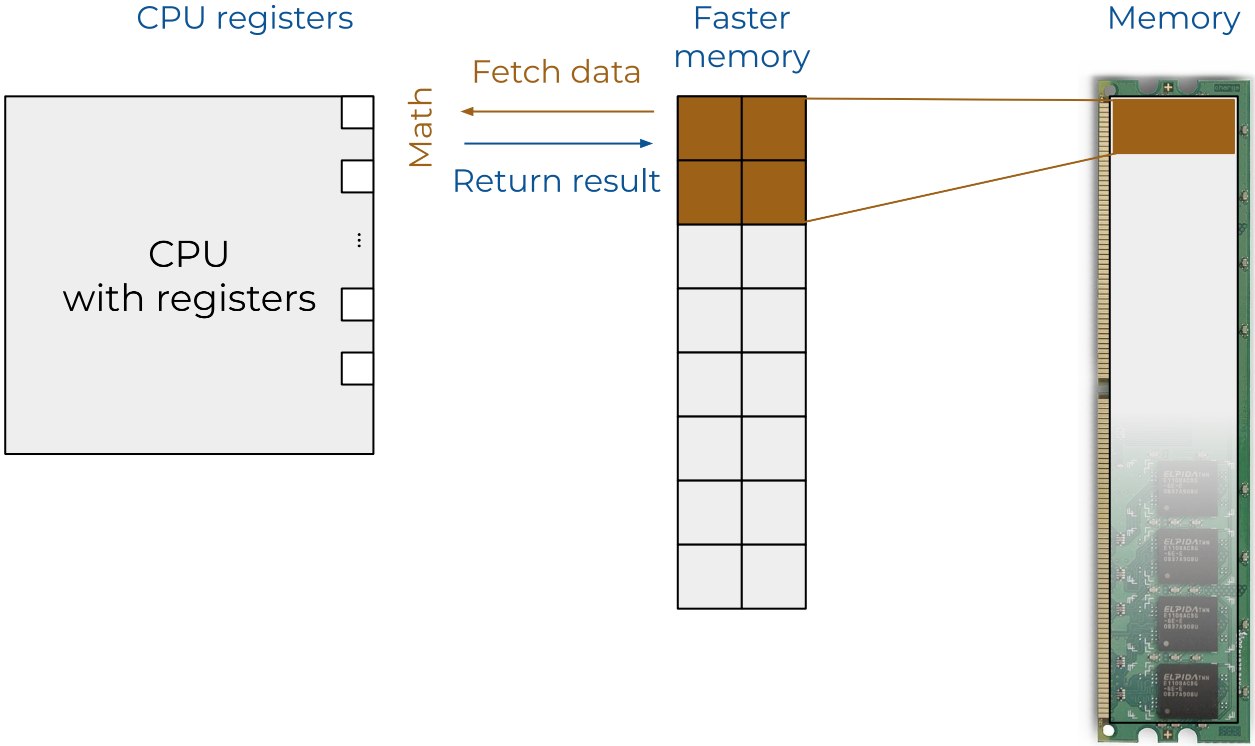

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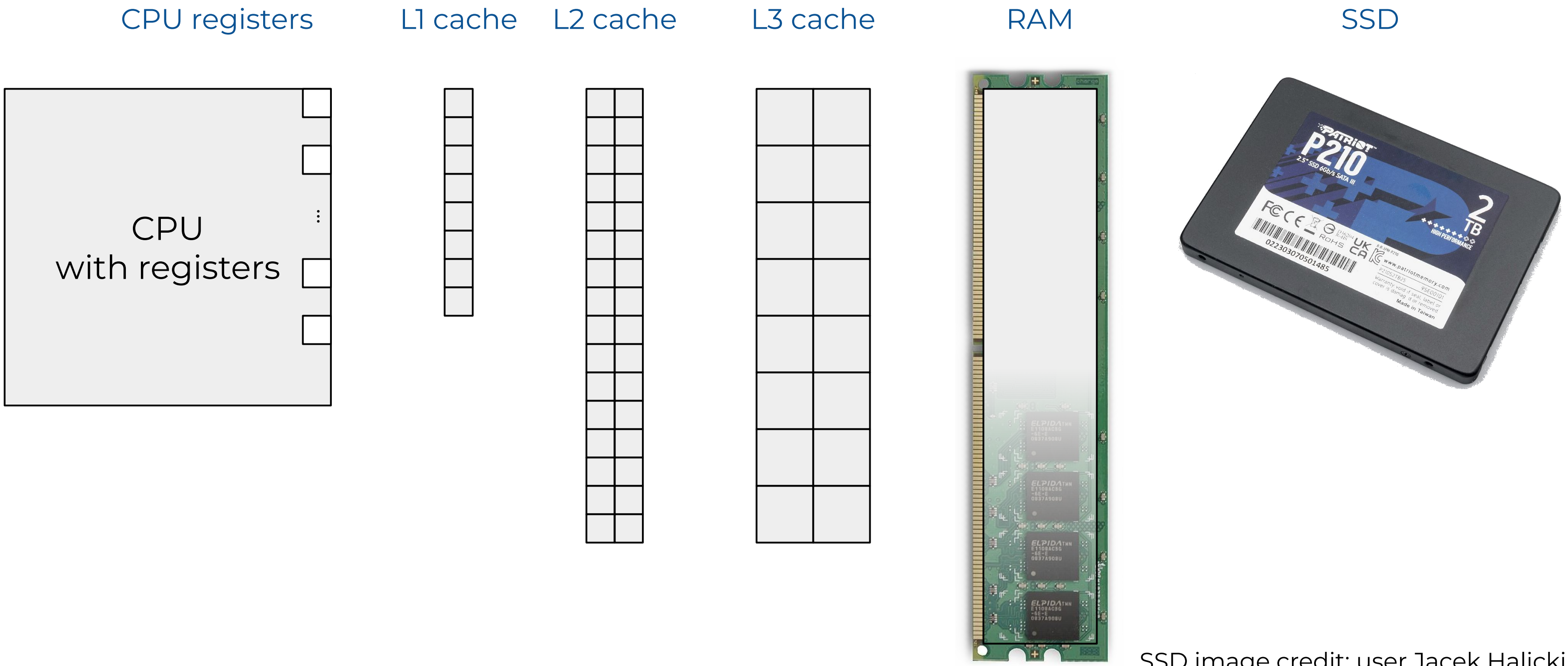

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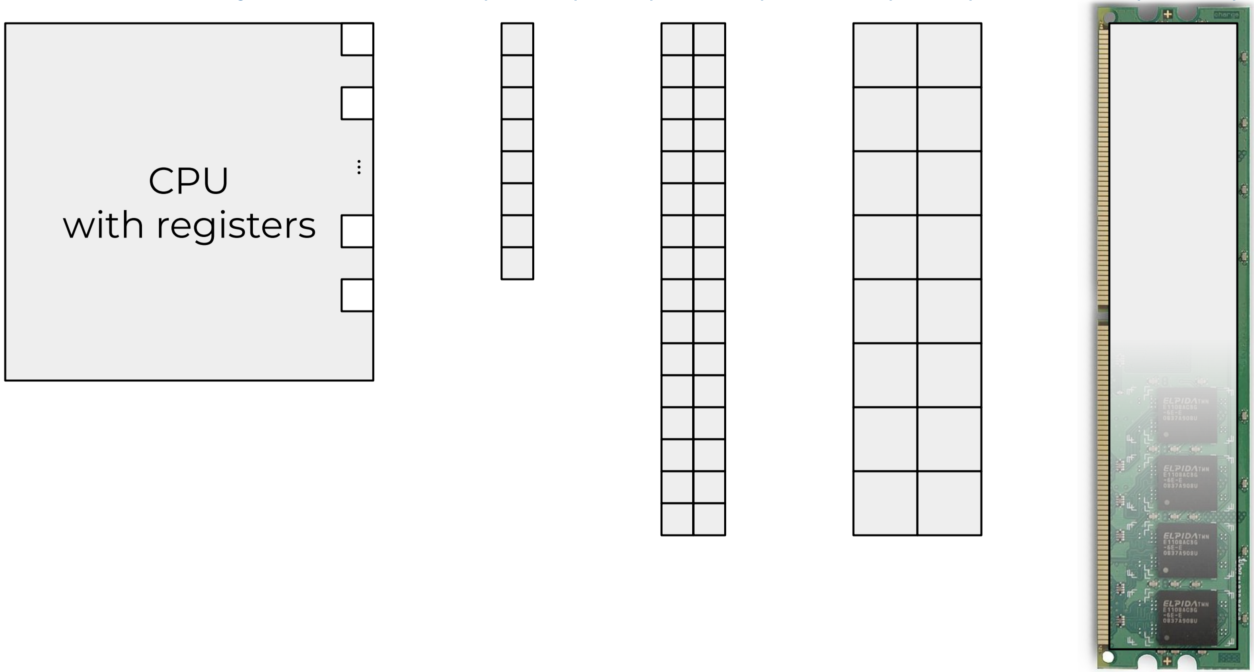
CPU-to-memory layout

Typical sizes of different memory hardware

CPU registers	L1 cache	L2 cache	L3 cache	RAM	SSD
8 bytes	O(10 kB)	O(100 kB)	O(1 MB)	O(10 GB)	O(1 TB)



CPU-to-memory layout

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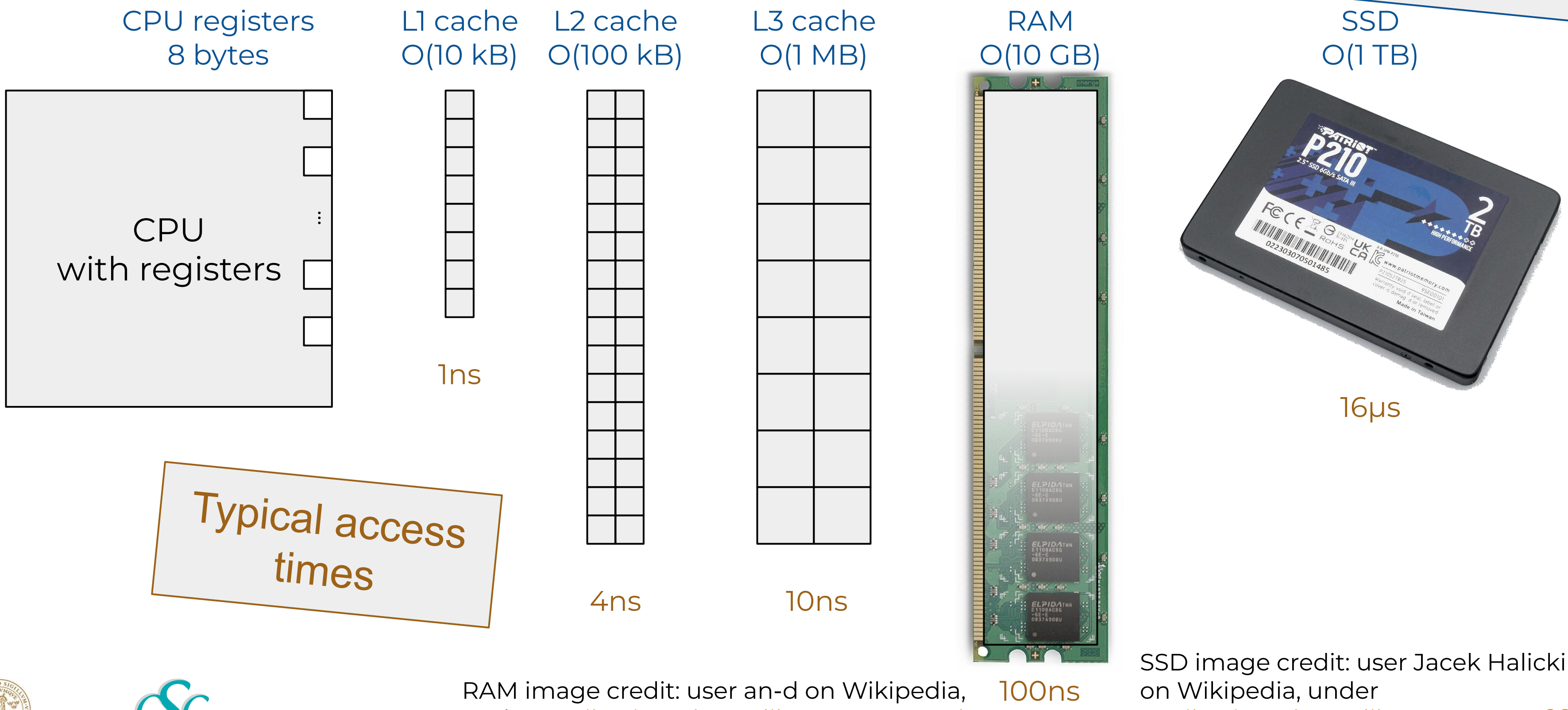

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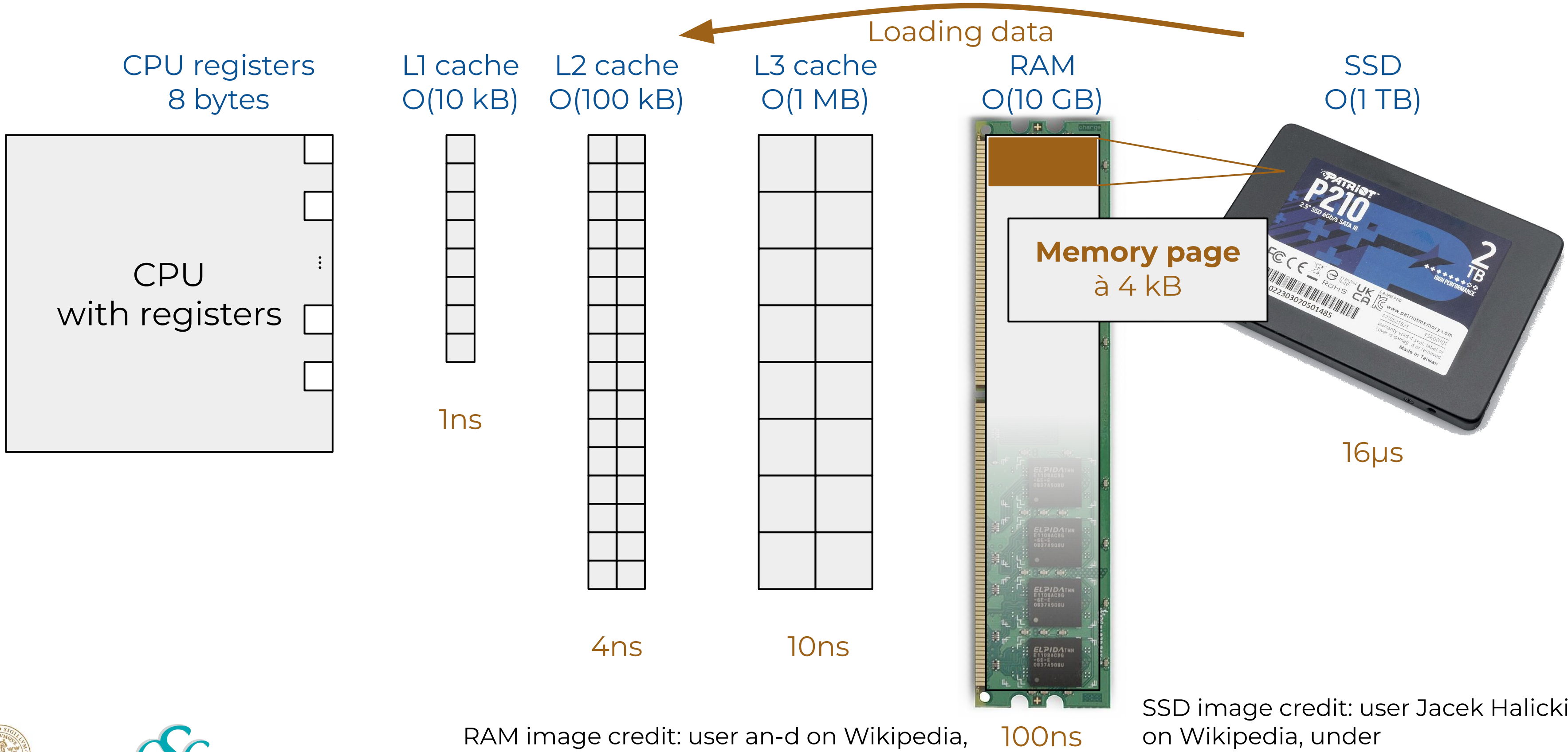
Block-based memory access, the “how”



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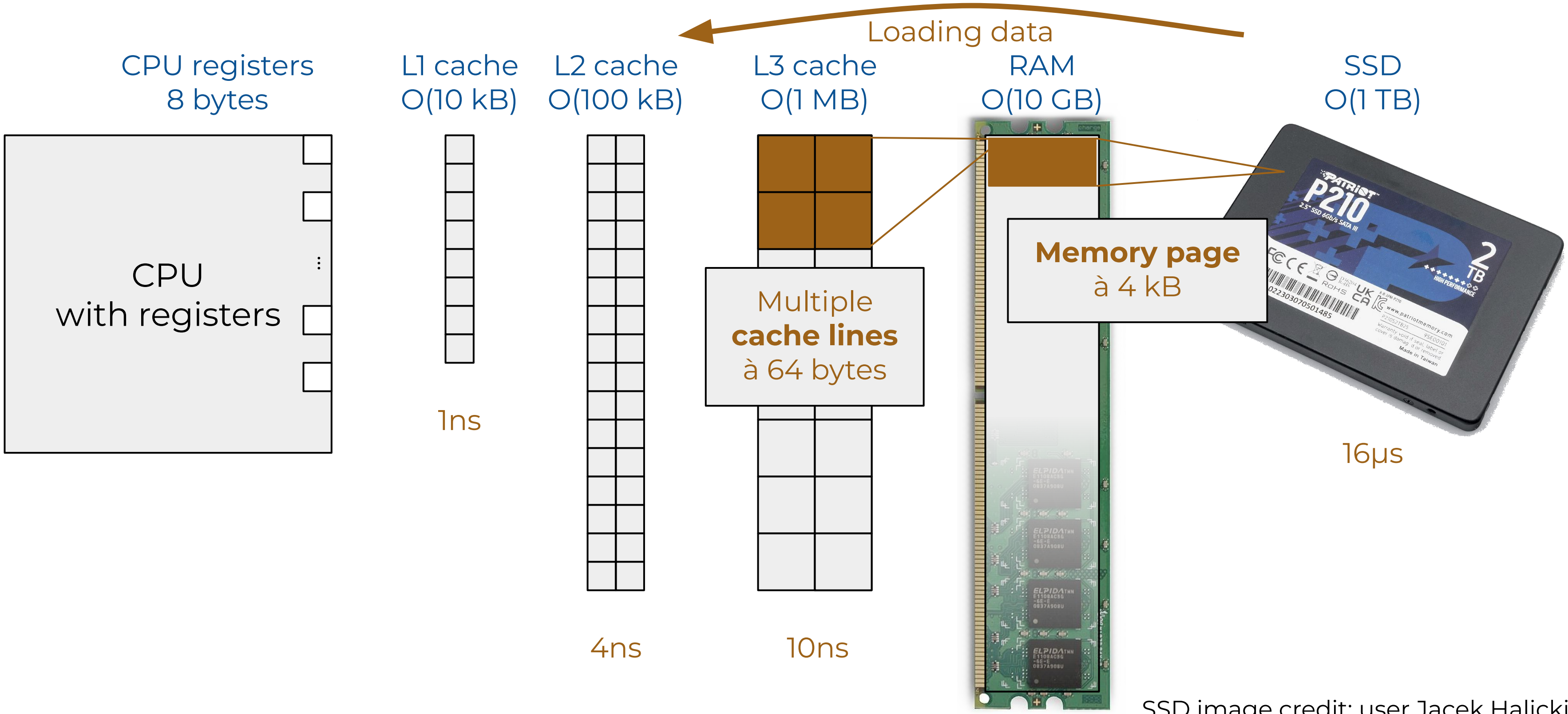

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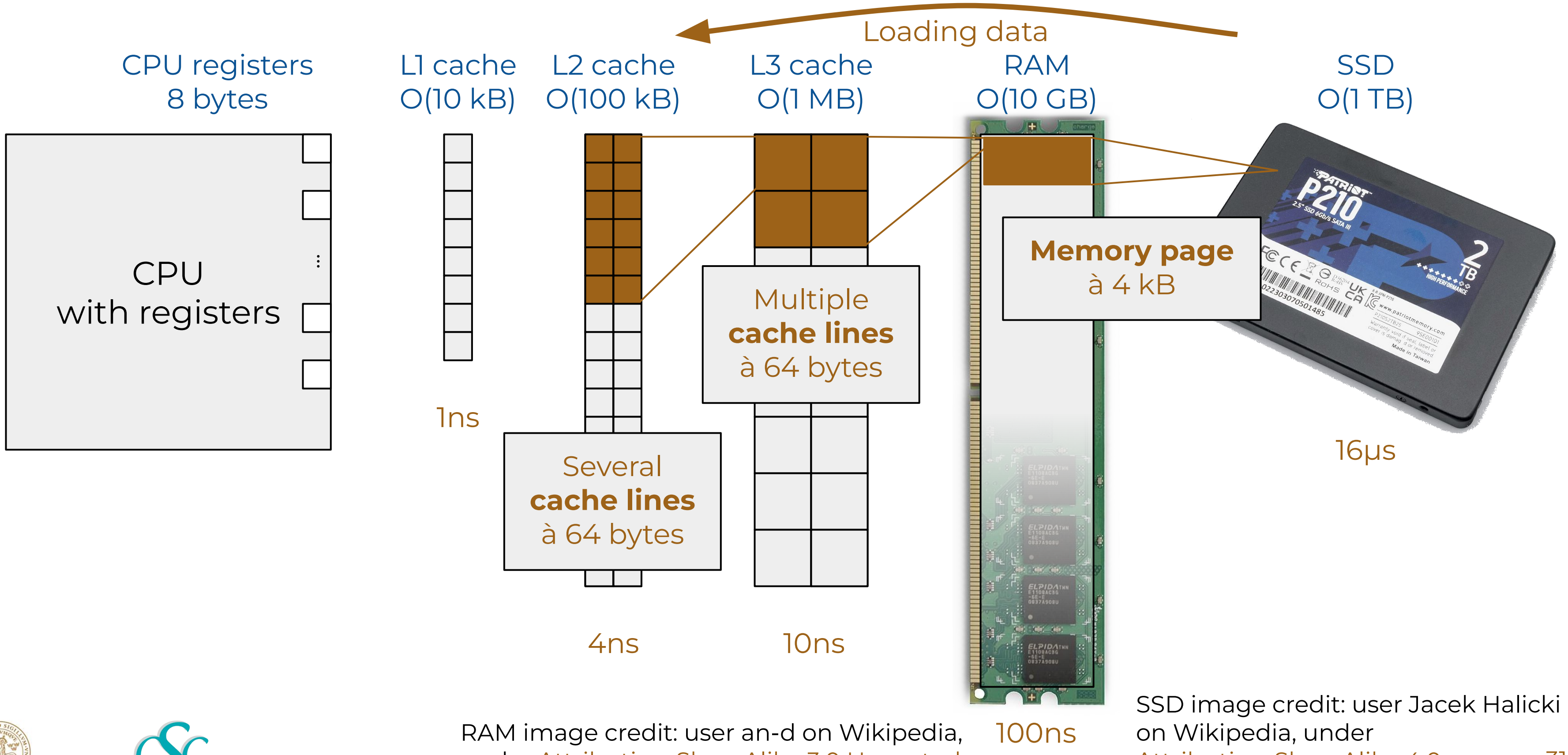

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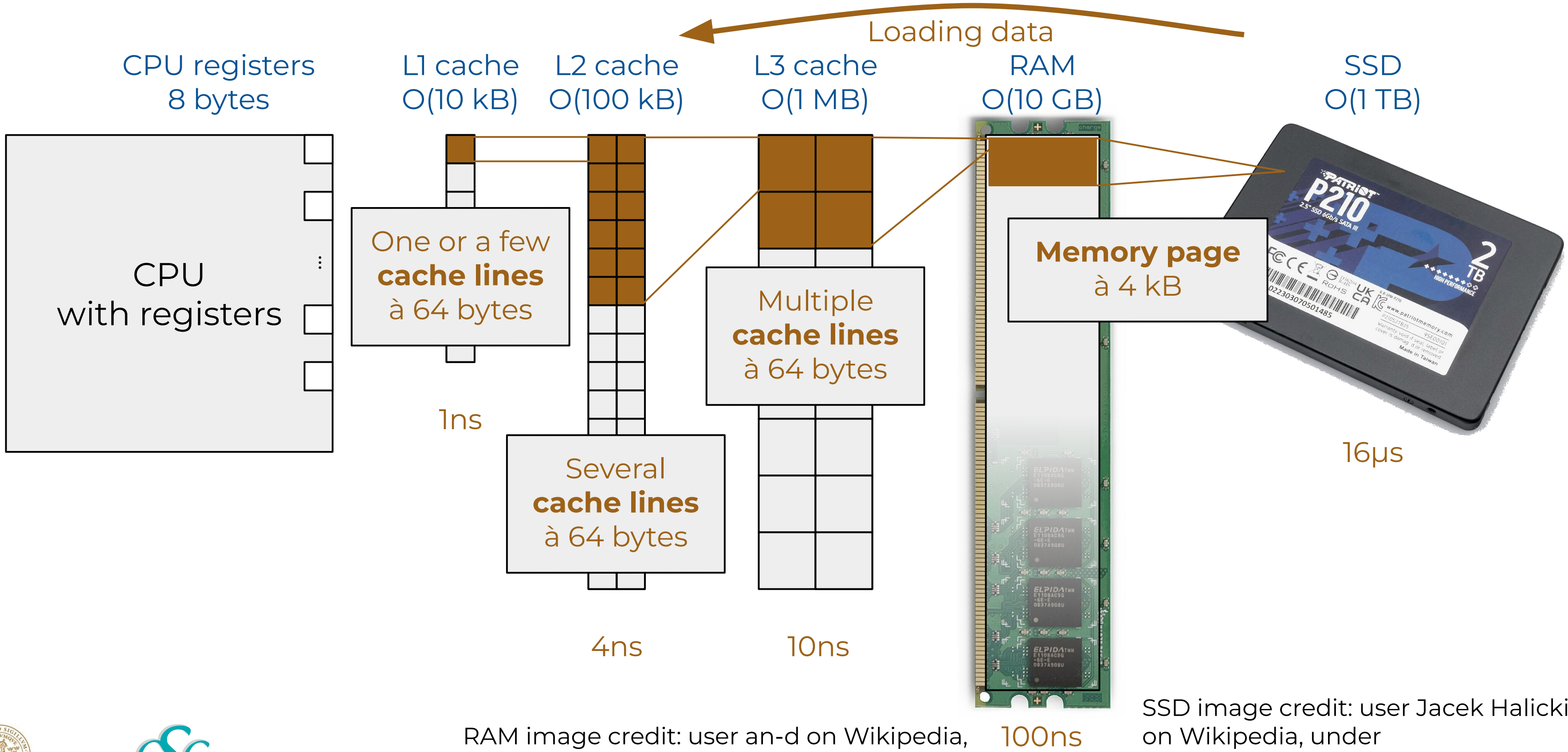

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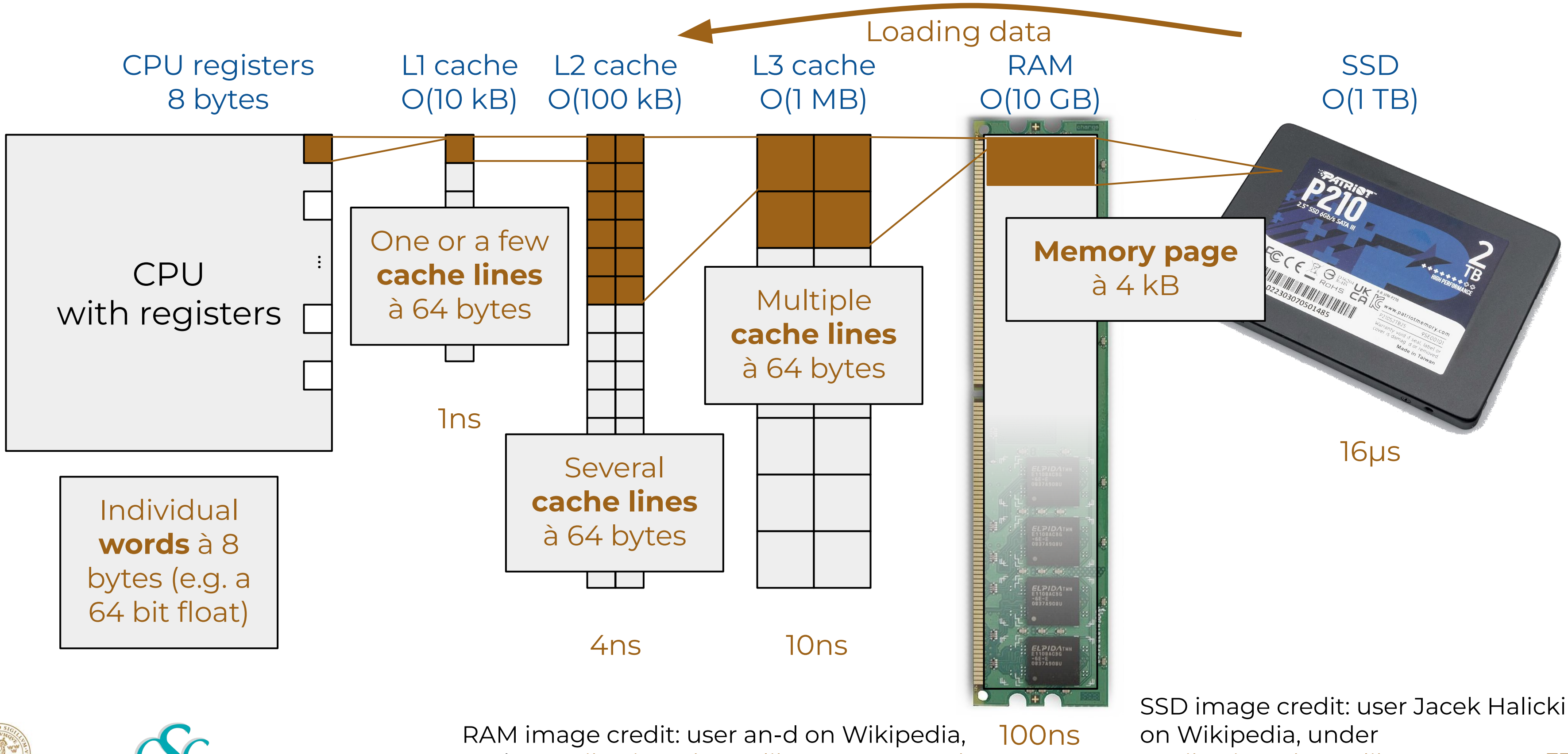

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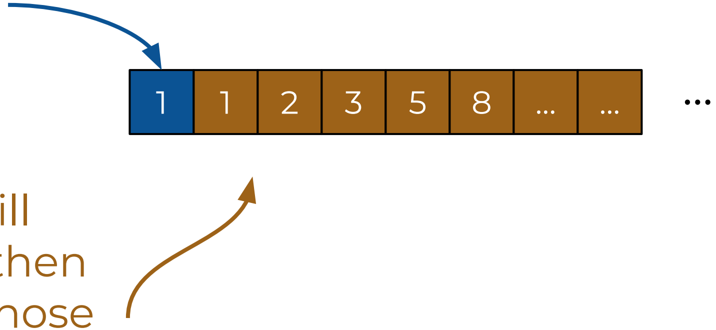
SSD image credit: user Jacek Halicki on Wikipedia, under Attribution-ShareAlike 4.0

Block-based memory access, the “why”

“**Principle of locality**” or “**data locality**” - data access often happens on many elements close to each other.

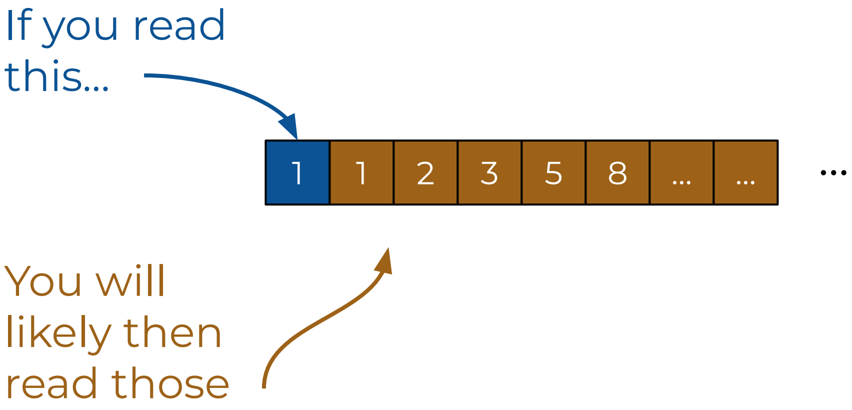
If you read this…



You will likely then read those

Block-based memory access, the "why"

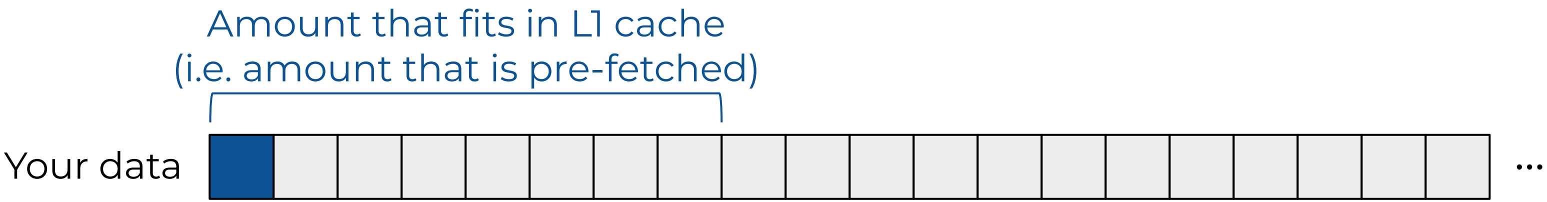
"**Principle of locality**" or "**data locality**" - data access often happens on many elements close to each other.



- **space locality**
- **time locality**

Pre-fetching: loading data that is likely to be needed soon

Memory access patterns



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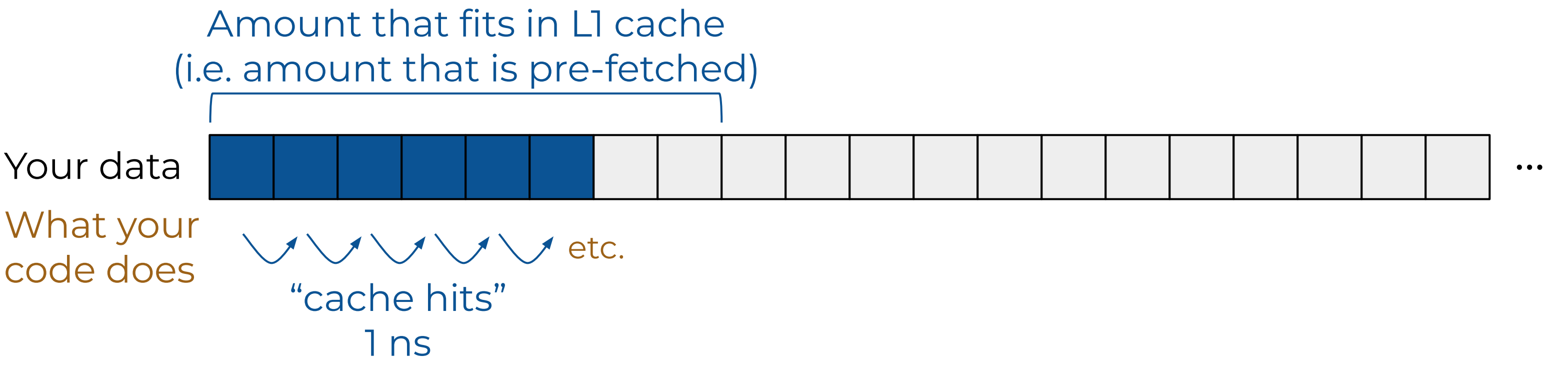

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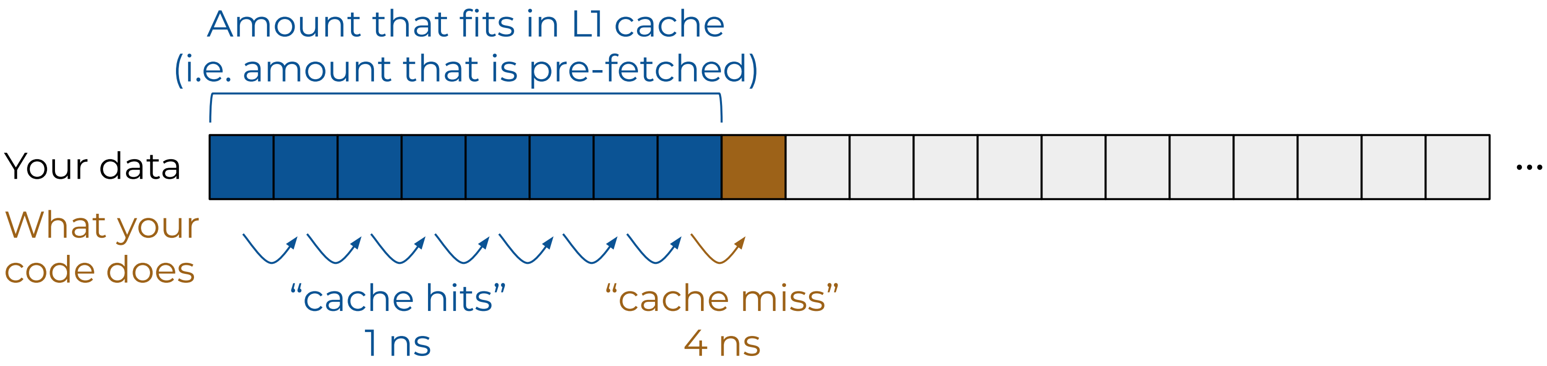

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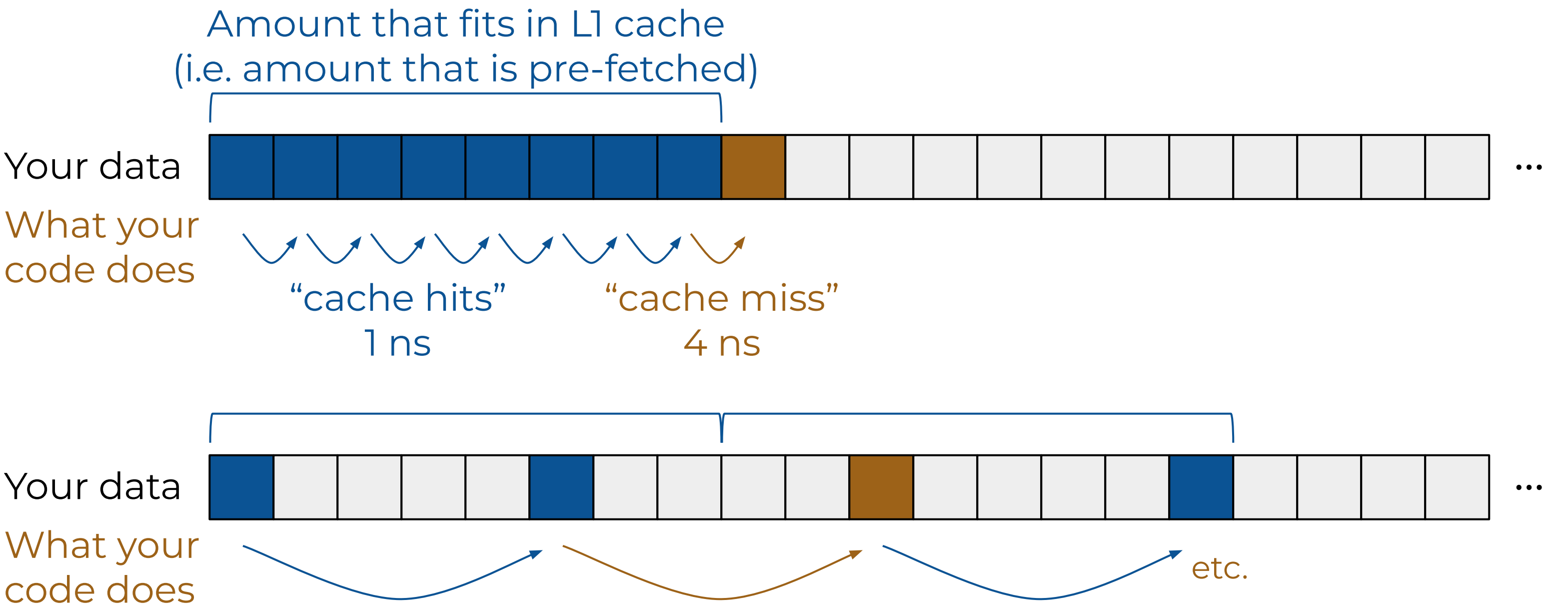

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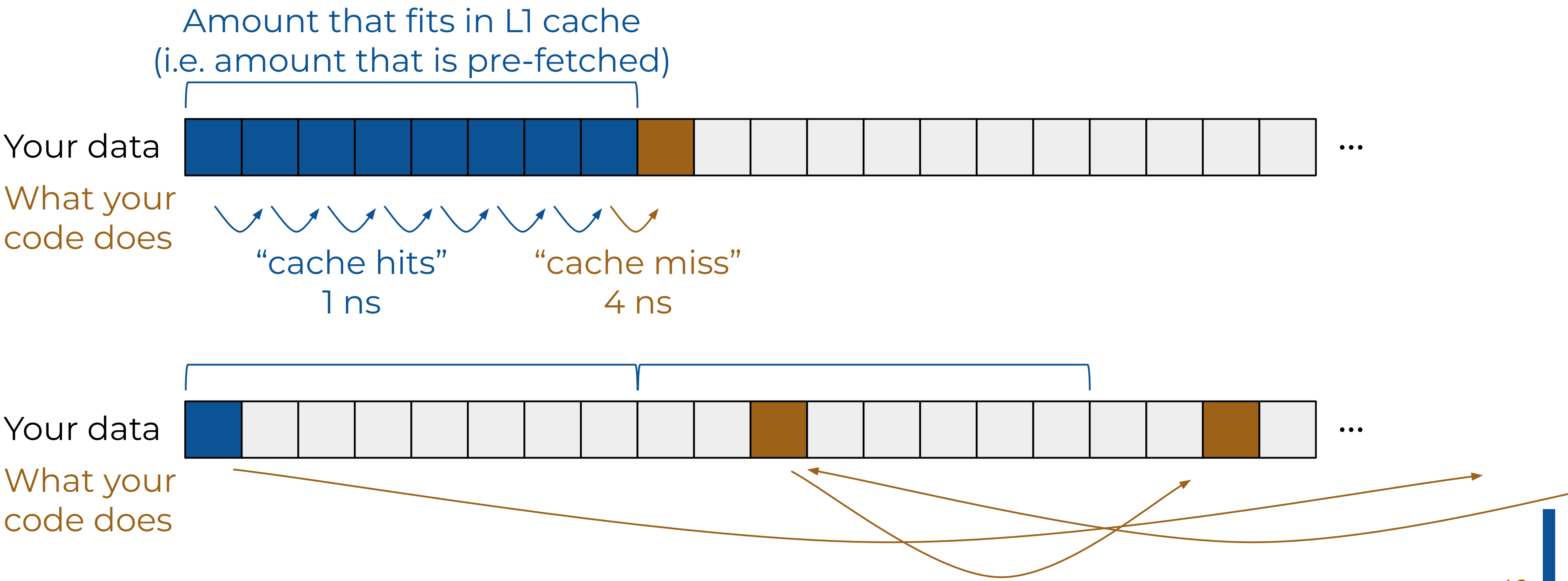
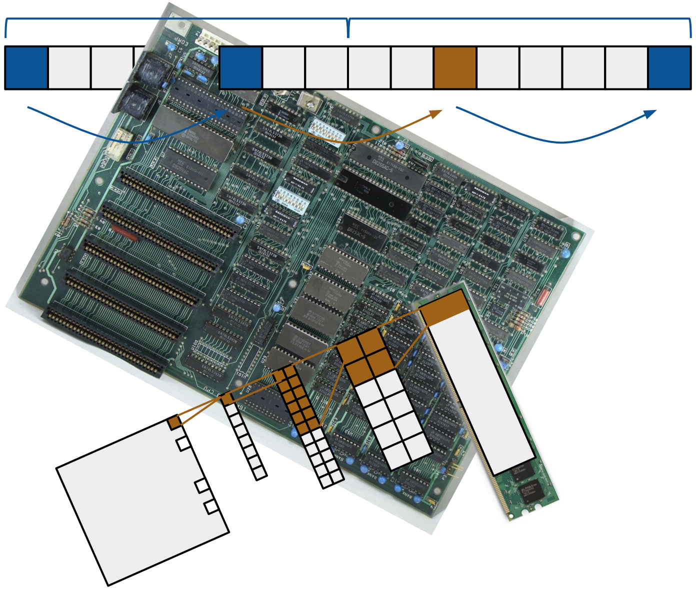
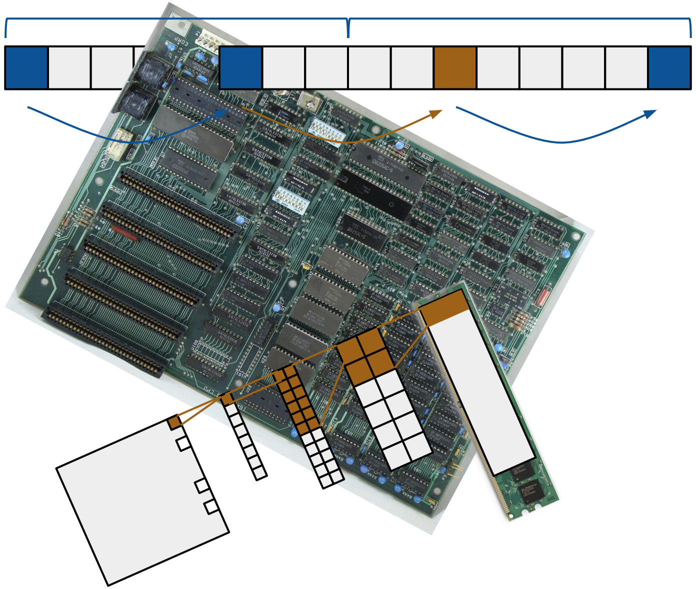


Memory access patterns - conclusions



Pit stop 2

Benchmarking



Benchmarking, pitfalls of

...but measuring is hard.

- Compiler optimizations
- Programs running in parallel ⇒ variance
- CPUs boost clock frequencies
- ...

Takeaway: healthy scepticism and reasonable expectations

Linux **time** command

```
int main() {
        int temp;
        for (int i = 0; i < 1000000000; i++) {
                temp = i;
        }
        return temp;
}
```

```
$ g++ -O3 time_example.cpp -o time_example
$ time ./time_example

real    0m0.009s
user    0m0.002s
sys     0m0.002s
```

- **real:** total time until your program finished
- **user:** time spent executing your program
- **sys:** time the system spent on behalf of your program

Note: blunt tool

Linux **time** command

```
int main() {
        int temp;
        for (int i = 0; i < 1000000000; i++) {
                temp = i;
        }
        return temp;
}
```

godbolt.org Compiler Explorer:

```
main:
        mov     eax, 999999999
        ret
```

```
$ g++ -O3 time_example.cpp -o time_example
$ time ./time_example

real    0m0.009s
user    0m0.002s
sys     0m0.002s
```

- **real:** total time until your program finished
- **user:** time spent executing your program
- **sys:** time the system spent on behalf of your program

Note: blunt tool

Compiler pitfalls

```cpp
int main() {
    int size = 1000;
    int index = 0;
    for (int i = 0; i < 1000000000; i++) {
        index = i % size;
        // … vector operations
    }
}
```

standard string-to-int conversion

```cpp
int main() {
    int size = std::stoi(string_from_user);
    int index = 0;
    for (int i = 0; i < 1000000000; i++) {
        index = i % size;
        // … vector operations
    }
}
```

Compiler pitfalls

standard string-to-int conversion

```
int main() {
    int size = 1000;
    int index = 0;
    for (int i = 0; i < 1000000000; i++) {
        index = i % size;
        // … vector operations
    }
}
```

```
Array size: 1000
Steps taken: 1000000000
Time per step: 1.06009 ns
```

```
int main() {
    int size = std::stoi(string_from_user);
    int index = 0;
    for (int i = 0; i < 1000000000; i++) {
        index = i % size;
        // … vector operations
    }
}
```

```
Array size: 1000
Steps taken: 1000000000
Time per step: 2.02596 ns
```

⇒ **loop 2x slower**

Compiler pitfalls

```
int main() {
    int size = 1000;
    int index = 0;
    for (int i = 0; i < 1000000000; i++) {
        index = i % size;
        // … vector operations
    }
}
```

```
movabs rsi, 2361183241434822607

.L4:
mov rdx, rcx
shr rdx
mov rax, rdx
mul rsi
mov rax, rcx
shr rdx, 4
imul rdx, rdx, 250
sub rax, rdx
movsx rax, DWORD PTR [rbp+0+rax*4]
add ebx, DWORD PTR [rbp+0+rax*4]
```

standard string-to-int conversion

```
int main() {
    int size = std::stoi(string_from_user);
    int index = 0;
    for (int i = 0; i < 1000000000; i++) {
        index = i % size;
        // … vector operations
    }
```

```
mov rax, rcx
cqo
idiv rbp
movsx rax, DWORD PTR [rbx+rdx*4]
add esi, DWORD PTR [rbx+rax*4]
mov r12d, esi
```

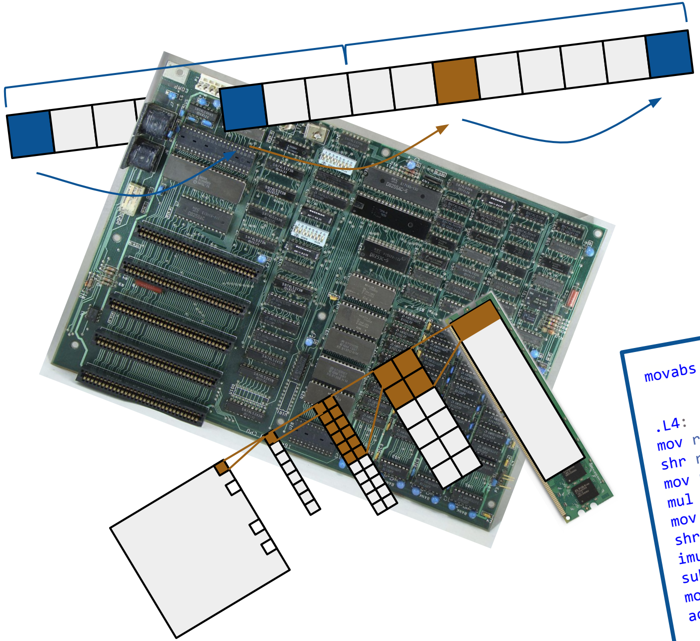
⇒ **loop 2x slower**

Benchmarking tools

Some more sophisticated benchmarking tools:

- C/C++
  - **Catch2**
  - Google Benchmark
- Rust
  - bench
- Python
  - pytest-benchmark
  - timeit
- General purpose
  - hyperfine
  - **perf** (Linux)

Benchmarking - conclusions



- Compiler optimizations
- Programs running in parallel ⇒ variance
- CPUs boost clock frequencies
- …

```
movabs rsi, 2361183241434

.L4:
mov rdx, rcx
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mov rax, rcx
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sub rax, rdx
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add ebx, DWORD PTR [rbp+0+rax*4]
```

```
mov rax, rcx
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```

<u>Healthy scepticism and reasonable expectations</u>

Conclusions - conclusions

Tools and Techniques, Lecture 3

Performance

- Algorithmic complexity
- Memory access patterns
- Benchmarking

https://github.com/Stoneandbeach/CSC2025

Backup

Catch2

```cpp
#include <catch2/catch_test_macros.hpp>
#include <catch2/benchmark/catch_benchmark.hpp>

int arithmetic_sum(int upper) {
    volatile int sum = 0;
    for (int i = 1; i <= upper; i++) {
        sum += i;
    }
    return sum;
}

TEST_CASE("Benchmarking Basics") {
    BENCHMARK("Arithmetic sum 1 to 100") {
        return arithmetic_sum(100);
    };
}
```

Catch2

```
#include <catch2/catch_test_macros.hpp>
#include <catch2/benchmark/catch_benchmark.hpp>

int arithmetic_sum(int upper) {
    volatile int sum = 0;
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    }
    return sum;
}

TEST_CASE("Benchmarking Basics") {
    BENCHMARK("Arithmetic sum 1 to 100") {
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    };
}
```

including Catch2 library

function to benchmark

defining a set of tests or benchmarks

configuring a benchmark

Catch2

```
#include <catch2/catch_test_macros.hpp>
#include <catch2/benchmark/catch_benchmark.hpp>

int arithmetic_sum(int upper) {
   volatile int sum = 0;
   for (int i = 1; i <= upper; i++) {
       sum += i;
   }
   return sum;
}

TEST_CASE("Benchmarking Basics") {
   BENCHMARK("Arithmetic sum 1 to 100") {
       return arithmetic_sum(100);
   };
}
```

including Catch2 library

function to benchmark

defining a set of tests or benchmarks

configuring a benchmark

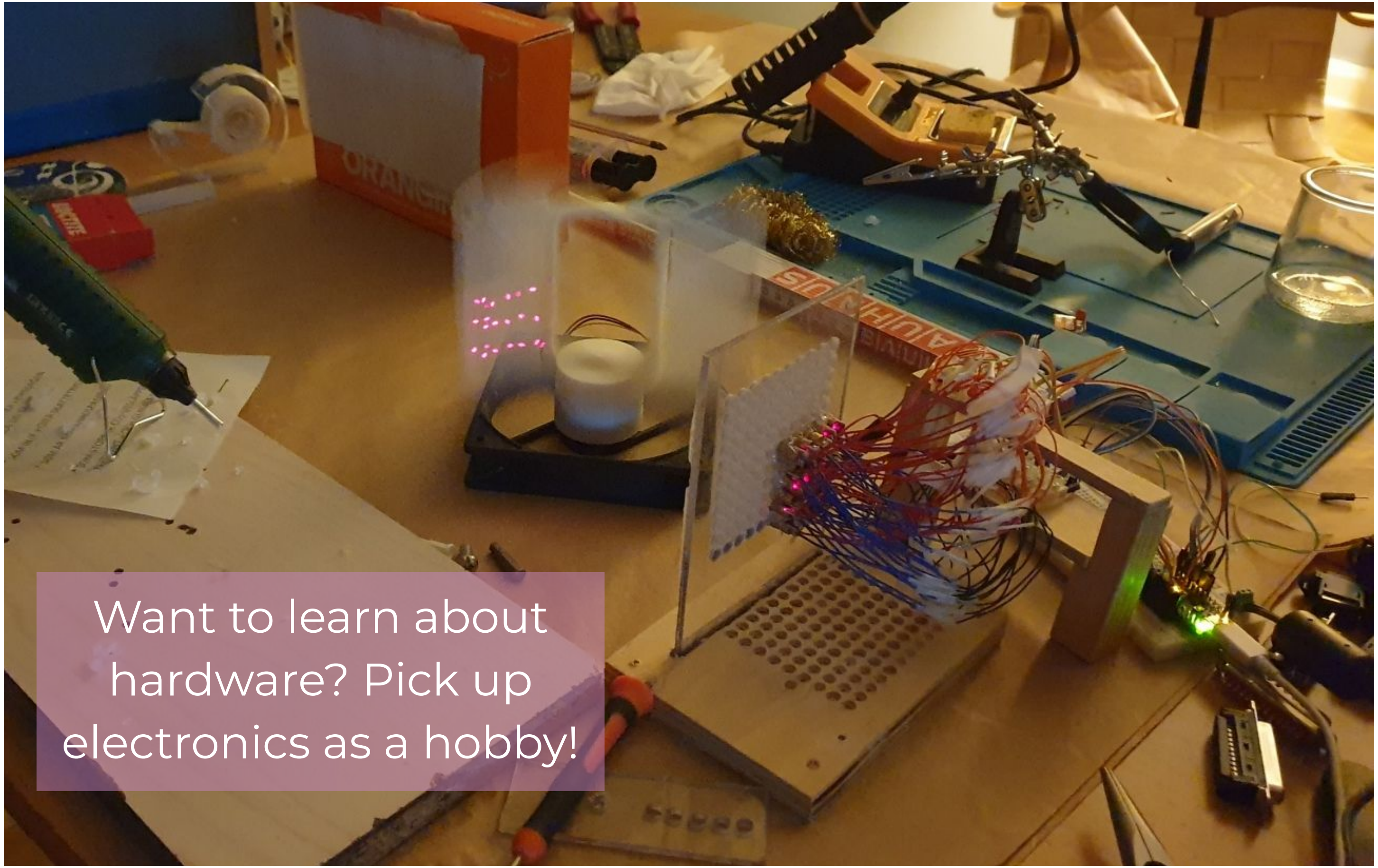
Catch2 output

```
-------------------------------------------------------------------------------
Benchmarking Basics
-------------------------------------------------------------------------------
/eos/user/k/kaastran/schools/CSC2025/prep/exercises/src/ex0.0_benchmarking_basics.cpp:15
...............................................................................

benchmark name                       samples       iterations    est run time
                                     mean          low mean      high mean
                                     std dev       low std dev   high std dev
-------------------------------------------------------------------------------
Arithmetic sum 1 to 100                        100           518     3.4706 ms
                                        38.5609 ns    36.9933 ns    40.8075 ns
                                        9.47688 ns    7.24546 ns     12.492 ns


===============================================================================
test cases: 1 | 1 passed
assertions: - none -
```

???


Want to learn about hardware? Pick up electronics as a hobby!

Further reading

Latency Numbers Every Programmer Should Know (originally from Jeff Dean):

https://people.eecs.berkeley.edu/~rcs/research/interactive_latency.html

A crazy breakdown of CPU instruction latency and throughput:

https://www.agner.org/optimize/instruction_tables.pdf

Nice discussion about CPU cycle and memory access costs:

http://ithare.com/infographics-operation-costs-in-cpu-clock-cycles/

Modern microprocessors in 90 minutes:

https://www.lighterra.com/papers/modernmicroprocessors/