15/1/2019

09.30 - 11.30am

Basement 1, Kevin Street

W228/282/2

DUBLIN INSTITUTE OF TECHNOLOGY

DT228 B. Sc. (Hons.) Degree in Computer Science
&
DT282 B. Sc. (Hons.) Degree in Computer Science (International)

Stage 2

WINTER EXAMINATIONS 2018/2019

CMPU 2012
MATHEMATICS 2

MR. PAUL SMYTH
DR. CHRIS HILLS
DR. MARTIN CRANE

15TH. JANUARY, 2019 09:30 – 11:30 AM

TIME: 2 HOURS

INSTRUCTIONS:
ANSWER QUESTION 1 AND TWO OF THE OTHER THREE QUESTIONS.
APPROVED CALCULATORS MAY BE USED.

MATHEMATICAL TABLES ARE PROVIDED.
NEW CAMBRIDGE STATISTICAL TABLES ARE NOT PERMITTED.

1. (a) State Fermat's Little Theorem. Using the fact that 2801 is prime, calculate the residue of

$$31^{7221204} \pmod{2801}$$

modulo 2801.

(8 marks)

(b) Using the Chinese Remainder Theorem solve the simultaneous congruence equations:

$$x \equiv 1 \pmod 5$$
$$x \equiv 2 \pmod 6$$
$$x \equiv 3 \pmod 7.$$

(8 marks)

(c) Write down the steps of Kruskal's algorithm for constructing a minimal weight spanning tree for a graph and use it to construct a minimal weight spanning tree for the weighted graph H shown in Figure 1. What is the weight of this minimal spanning tree?

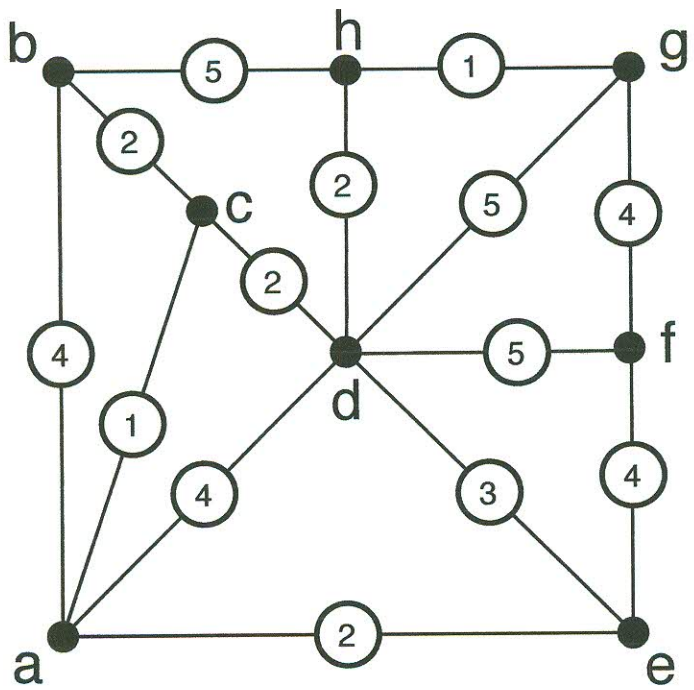

Figure 1: The weighted graph H.

(8 marks)

(d) Let the domain of discourse be the set of integers $\mathbb{Z}$. Given the predicates

$$\begin{aligned} E(x) &= x \text{ is even} \\ O(x) &= x \text{ is odd} \\ D(x,y) &= x|y \\ P(x) &= x \text{ is prime} \end{aligned}$$

then write the following statements as English sentences and state whether they are true or false:

i) $\forall x(E(x) \rightarrow D(2,x))$
ii) $\exists x(O(x) \wedge D(3,x))$
iii) $\forall x(P(x) \rightarrow \neg D(2,x))$
iv) $\forall x(\neg(E(x) \wedge O(x)))$

(8 marks)

(e) Using proof by induction prove that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-1) \cdot n \cdot (n+1)}{3}$$

for all $n \geq 2$, where $\cdot$ indicates multiplication. Show all the steps in the proof.

(8 marks)

2. (a) Ten people are lined up in a row. Five are engineers, three are scientists, and two are mathematicians. Calculate the probabilities of the following events:

i) The first four lined up are all engineers.
ii) The first position in the row is occupied by one of the scientists and the last two positions in the row are occupied by the remaining two scientists.
iii) The first three in the row are engineers and the last two are scientists.

(6 marks)

(b) A random experiment consists of rolling two fair dice once. Define the events

A : the numbers rolled are each greater than 2
B : the sum of the two dice is 8
C : the sum of the two dice is 5
D : both numbers rolled are odd
E : one number rolled is even and the other is odd

and calculate $P(B|A), P(C|A), P(D|A)$ and $P(E|A)$ giving your answer as a fraction in each case. Is event D independent of A? Justify your answer.

(12 marks)

(c) A computer program contains a loop which outputs the results of a numerical calculation. Unfortunately the loop contains a bug, which means that 25% of the time the loop is executed, the bug outputs an incorrect value. If each execution of the loop can be considered an independent Bernoulli trial and the loop is executed 6 times in a particular run of the program, then calculate the probability that:

i) The program outputs no incorrect values.
ii) The program outputs one incorrect value.
iii) The program outputs exactly three incorrect values.
iv) The program outputs two or less incorrect values.
v) The program outputs at least three incorrect values.

In each case give your answer as a fraction.

(12 marks)

3. (a) If $9454 \equiv 1 \mod 137$ then solve the linear congruence equation:

$$4727x \equiv 61 \mod 137.$$

(4 marks)

(b) Show that the congruence equation

$$54x \equiv 51 \,(\mathrm{mod}\ 87),$$

has solutions, and find all the incongruent solutions. No marks will be awarded for brute force approaches.

(14 marks)

(c) The ciphertext

LTWMSG

was encrypted by means of a Hill digraph cipher, using the matrix

$$A = \begin{pmatrix} 17 & 11 \\ 4 & 19 \end{pmatrix}$$

modulo 26, where $A = 0, B = 1, \ldots, Z = 25$. Find the inverse of A modulo 26 and hence retrieve the message plaintext.

(12 marks)

4. (a) For the graph G shown in Figure 2, answer each of the following questions. Be careful to justify your answers.

i) Is G complete?

ii) Is G bipartite? If it is bipartite then draw the graph in partitioned form.

iii) Is G complete bipartite?

iv) Does G have an Euler path?

v) Does G have an Euler cycle?

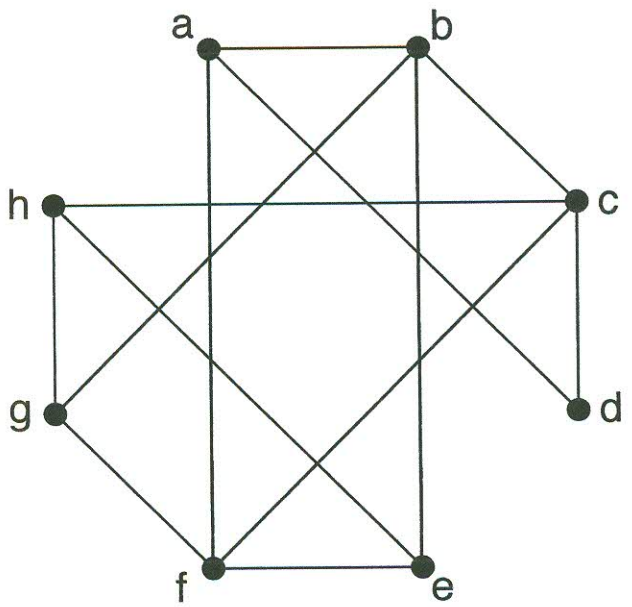

Figure 2: The graph G.

(12 marks)

(b) Using Dijkstra's Algorithm calculate a minimum weight path, from a to c, for the weighted graph K, shown in Figure 3.

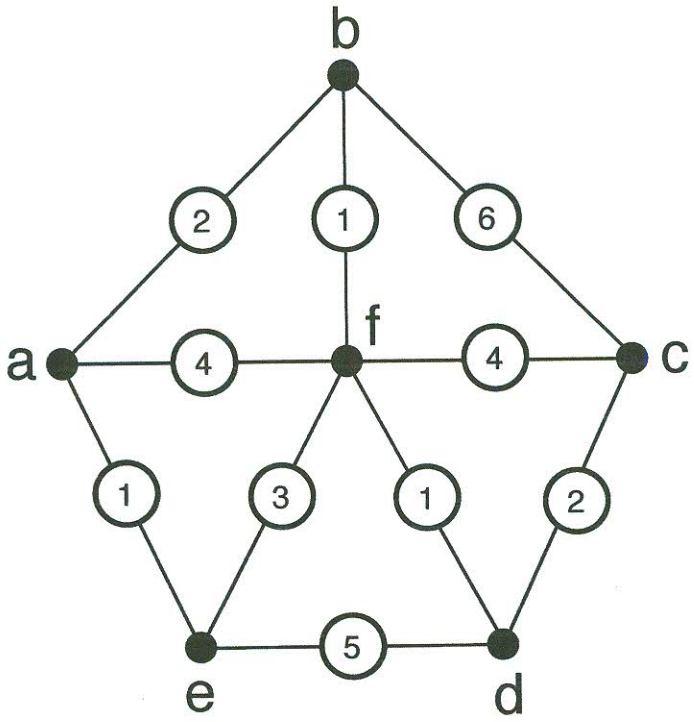

Figure 3: The weighted graph K.

(18 marks)