

EEE/ETI3105

ELECTRICAL MACHINES I

LECTURE 3: PRACTICAL TRANSFORMERS

Practical Transformer

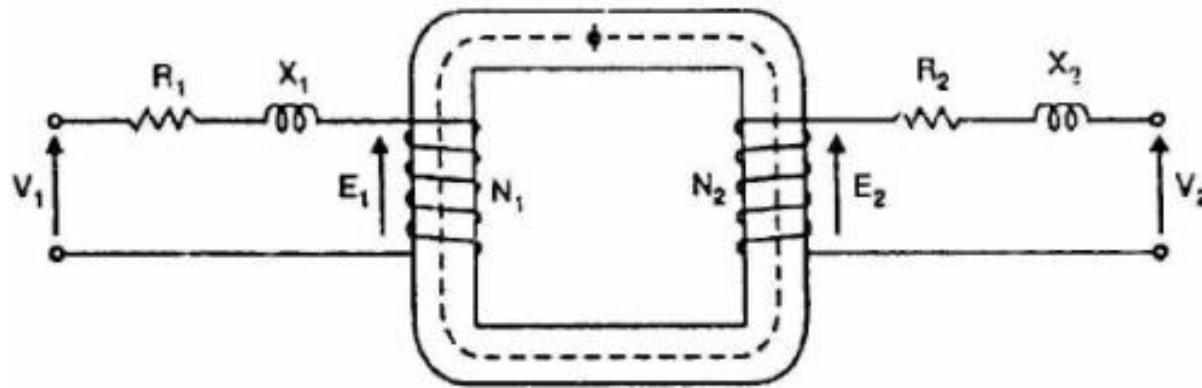
- A practical transformer differs from the ideal transformer in many respects.
- The practical transformer has
 - iron losses
 - winding resistances
 - magnetic leakage, giving rise to leakage reactance.

Iron losses.

- Since the iron core is subjected to alternating flux, there occurs *eddy current* and *hysteresis loss* in it.
- These two losses together are known as iron losses or core losses .
- The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc.
- It may be noted that magnitude of iron losses is quite small in a practical transformer.

Winding resistance

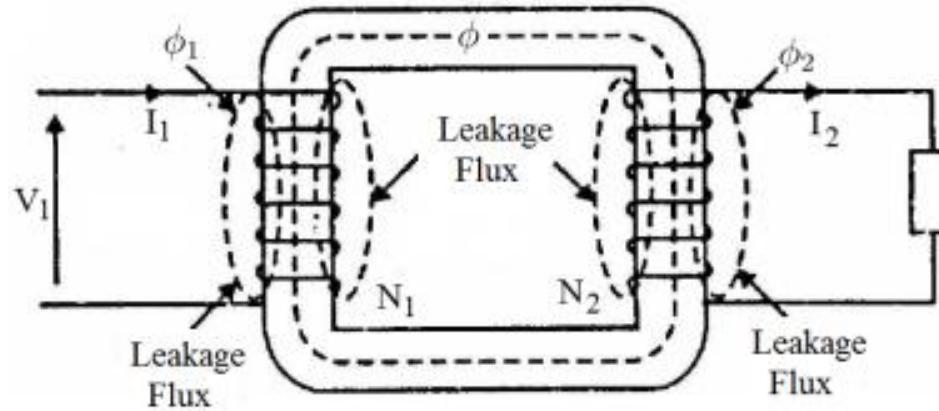
- Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance.
- The primary resistance R_1 and secondary resistance R_2 act in series with the respective windings as shown below.



- When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop.
- This will affect the power factor and E_1 will be less than V_1 while V_2 will be less than E_2

Leakage Reactance

- Both primary and secondary currents produce flux.
- The flux (Φ) which links both the windings is the useful flux and is called mutual flux.
- However, primary current would produce some flux Φ_1 which would not link the secondary winding (See Figure below).



- Similarly, secondary current would produce some flux Φ_2 that would not link the primary winding.
- The flux such as Φ_1 or Φ_2 which links only one winding is called leakage flux.
- The leakage flux paths are mainly through the air.

- The effect of these leakage fluxes would be the same as though inductive reactance were connected in series with each winding of transformer that had no leakage flux as shown in the Figure [previous slide].
- The effect of primary leakage flux (Φ_1) is to introduce an inductive reactance X_1 in series with the primary winding as previously shown.
- Similarly, the secondary leakage flux (Φ_2) introduces an inductive reactance X_2 in series with the secondary winding.
- There will be no power loss due to leakage reactance.
- However, the presence of leakage reactance in the windings changes the power factor as well as there is voltage loss due to IX drop.

Note.

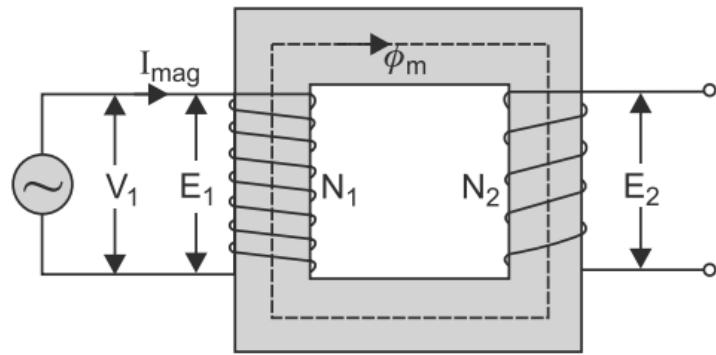
- Although leakage flux in a transformer is quite small (about 5% of Φ) compared to the mutual flux, yet it cannot be ignored.
- It is because leakage flux paths are through air of high reluctance and hence require considerable e.m.f.
- It may be noted that energy is conveyed from the primary winding to the secondary winding by mutual flux (Φ) which links both the windings.

Practical Transformer on No Load

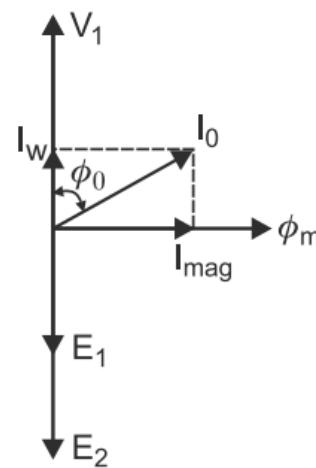
- A transformer is said to be on no-load when its secondary winding is kept open and no-load is connected across it. As such, no current flows through the secondary i.e., $I_2 = 0$.
- Hence, the secondary winding is not causing any effect on the magnetic flux set-up in the core or on the current drawn by the primary. But the losses cannot be ignored.
- At no-load, a transformer draws a small current I_o (usually 2 to 10% of the rated value). This current has to supply the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper loss in the primary (the primary copper losses are so small as compared to core losses that they are generally neglected moreover secondary copper losses are zero as I_2 is zero)
- Therefore, current I_o lags behind the voltage vector V_1 by an angle I_o which is less than 90° , as shown in Fig. (b). The angle of lag depends upon the losses in the transformer.

The no-load current I_o has two components;

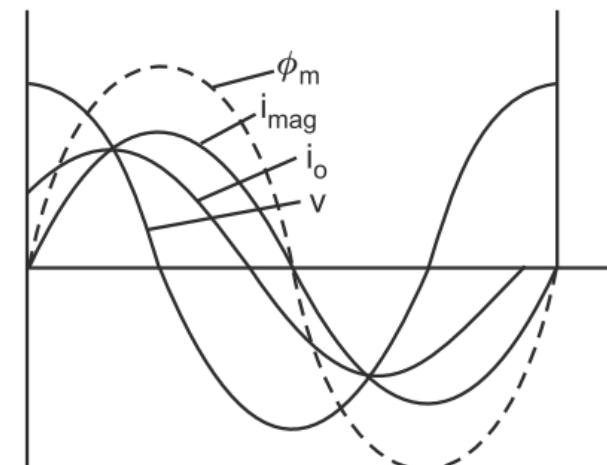
- One, I_w in phase with the applied voltage V_1 , called active or working component. It supplies the iron losses and a small primary copper losses.
- The other, I_{mag} in quadrature with the applied voltage V_1 , called reactive or magnetizing component. It produces flux in the core and does not consume any power.



(a) Circuit diagram



(b) Phasor diagram



(c) Wave diagram

Working component,

$$I_w = I_0 \cos \phi_0$$

Magnetising component,

$$I_{mag} = I_0 \sin \phi_0$$

No-load current,

$$I_0 = \sqrt{I_w^2 + I_{mag}^2}$$

Primary p.f. at no-load,

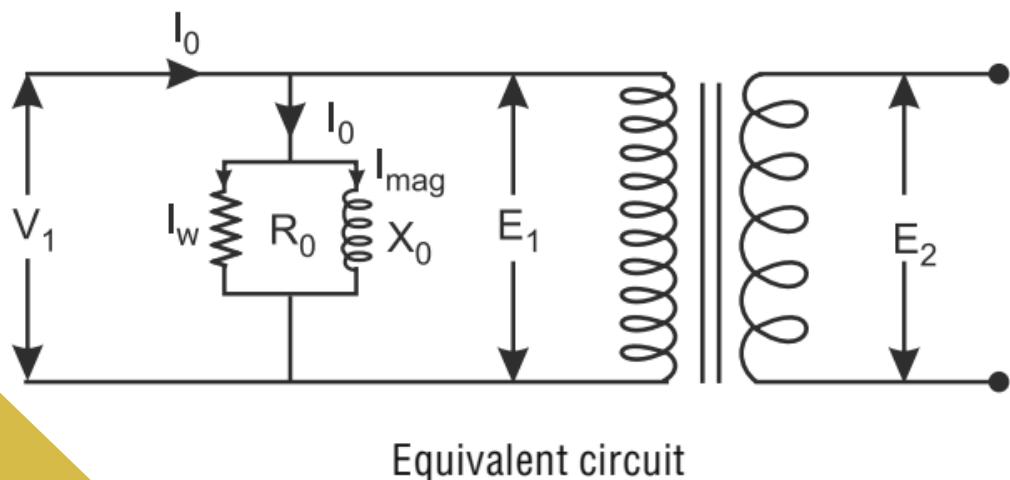
$$\cos \phi_0 = \frac{I_w}{I_0}$$

No-load power input,

$$P_0 = V_1 I_0 \cos \phi_0$$

Exciting resistance, $R_0 = \frac{V_1}{I_w}$

Exciting reactance, $X_0 = \frac{V_1}{I_{mag}}$



- The equivalent circuit of a transformer at no-load is shown in Fig..
- Here, R_0 represents the exciting resistance of the transformer which carries power loss component of no-load current, i.e., I_w used to meet with the no-load losses in the transformer, whereas X_0 represents the exciting reactance of the transformer which carries wattless component of no-load current, i.e., I_{mag} used to set-up magnetic field in the core.

- It is emphasized here that no load primary copper loss, ($I_0^2 R_1$) is very small and may be neglected.
- Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,
- No load input power, W_0 = Iron loss
- **Note.**
- At no load, there is no current in the secondary so that $V_2 = E_2$.
- On the primary side, the drops in R_1 and X_1 , due to I_o are also very small because of the smallness of I_o .
- Hence, we can say that at no load, $V_1 = E_1$

Practice Question

- **Qn 1:** A 230/110 V single-phase transformer has a core loss of 100 W. If the input under no-load condition is 400 VA, find core loss current, magnetizing current and no-load power factor angle.
- **Qn 2:** A 230V, 50 Hz transformer has 200 primary turns. It draws 5 A at 0.25 p.f lagging at no-load.

Determine:

- (i) Maximum value of flux in the core;
- (ii) Core loss;
- (iii) Magnetizing current
- (iv) Exciting resistance and reactance of the transformer.
- (v) Draw its equivalent circuit.

Solution Q1

Here, $V_1 = 230$ V; $V_2 = 110$ V; $P_i = 100$ W

Input at no-load = 400 VA

i.e.,

$$V_1 I_0 = 400$$

or

$$\text{No-load current, } I_0 = \frac{400}{230} = 1.739 \text{ A}$$

$$\text{Core loss current, } I_w = \frac{P_i}{V_1} = \frac{100}{230} = 0.4348 \text{ A (Ans)}$$

$$\text{Magnetising current, } I_{mag} = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.739)^2 - (0.4348)^2} = 1.684 \text{ A (Ans)}$$

$$\text{No-load power factor, } \cos \phi_0 = \frac{I_w}{I_0} = \frac{0.4348}{1.739} = 0.25 \text{ lag (Ans)}$$

$$\text{No-load power factor angle, } \phi_0 = \cos^{-1} 0.25 = 75.52^\circ \text{ (Ans)}$$

Solution Q2

(i) Using the relation,

$$E_1 = 4.44 N_1 f \phi_m$$

$$230 = 4.44 \times 220 \times 50 \times \phi_m$$

\therefore Maximum value of flux $\phi_m = 5.18 \text{ m Wb}$ (Ans)

(ii) Core loss,

$$P_0 = V_1 l_0 \cos \phi_0 = 230 \times 5 \times 0.25 = 287.5 \text{ W} \text{ (Ans)}$$

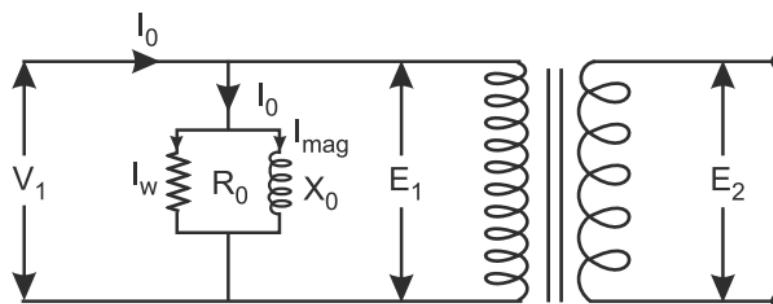
(iii) No-load p.f., $\cos \phi_0 = 0.25$; $\sin \phi_0 = \sin \cos^{-1} 0.25 = 0.9682$

Magnetising current component, $I_m = I_0 \sin \phi_0 = 5 \times 0.9682 = 4.84 \text{ A}$ (Ans)

$$\text{Exciting resistance, } R_0 = \frac{V_1}{I_w} = \frac{230}{I_0 \cos \phi_0} = \frac{230}{5 \times 0.25} = 184 \Omega \text{ (Ans.)}$$

$$\text{Exciting reactance, } X_0 = \frac{V_1}{I_{mag}} = \frac{230}{4.84} = 47.52 \Omega \text{ (Ans.)}$$

The values of different quantities are mentioned in the solution itself.



Equivalent circuit of transformer at no-load.

Practice Question

- The no load current of a transformer is 10A at power factor of 0.25 lagging, when connected to 400 V, 50 Hz supply. Calculate,
 - a) Magnetizing component of no load current
 - b) Iron loss
 - c) Maximum value of flux in core. Assume primary winding turns as 500.

Solution

The given values are, $I_o = 10 \text{ A}$, $\cos \phi_b = 0.25$, $V_1 = 400 \text{ V}$ and $f = 50 \text{ Hz}$

a) $I_m = I_o \sin \phi_b = \text{magnetising component}$

$$\phi_b = \cos^{-1}(0.25) = 75.522^\circ$$

$$\therefore I_m = 10 \times \sin(75.522^\circ) = 9.6824 \text{ A}$$

b) $P_i = \text{iron loss} = \text{power input on no load}$

$$= W_o = V_1 I_o \cos \phi_b = 400 \times 10 \times 0.25$$

$$= 1000 \text{ W}$$

c) On no load, $E_1 = V_1 = 400 \text{ V}$ and $N_1 = 500$

$$E_1 = 4.44 f \phi_m N_1$$

$$\therefore 400 = 4.44 \times 50 \times \phi_m \times 500$$

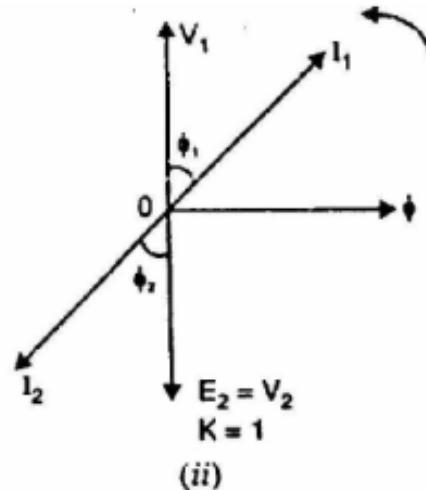
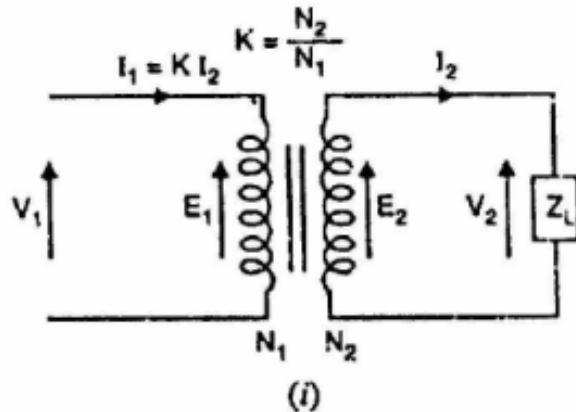
$$\therefore \phi_m = 3.6036 \text{ mWb}$$

Practice Questions

- **Qn 1.:** At open circuit, transformer of 10 kVA, 500/250 V, 50 Hz draws a power of 167 watt at 0.745 A, 500V. Determine the magnetising current, wattful current, no-load power factor, hysteresis angle of advance, equivalent resistance and reactance of exciting circuit referred to primary side.
- **Qn 2.:** A transformer working on 2200 V, 50 Hz has 220 primary turns. The core has a mean length of magnetic path of 100 cm and cross-sectional area 1000 sq. cm. the iron having a relative permeability of 100. The iron loss is 400 watt. Calculate primary no-load current
- **Qn 3.:** A transformer takes a current of 0.6 A and absorbs 64W when primary is connected to its normal supply of 200V, 50 Hz; the secondary being an open circuit. Find the magnetizing and iron loss currents.

Ideal Transformer on Load

- Let us connect a load Z_L across the secondary of an ideal transformer as shown in Fig. (i).
- The secondary e.m.f. E_2 will cause a current I_2 to flow through the load.



$$I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$$

- The angle at which I_2 leads or lags V_2 (or E_2) depends upon the resistance and reactance of the load.

- ✳ In the present case, we have considered inductive load so that current I_2 lags behind V_2 (or E_2) by ϕ_2 .
- ✳ The secondary current I_2 sets up an m.m.f. $N_2 I_2$ which produces a flux in the opposite direction to the flux ϕ originally set up in the primary by the magnetizing current (Lenz law).
- ✳ This will change the flux in the core from the original value.
- ✳ However, the flux in the core should not change from the original value.
- ✳ In order to fulfill this condition, the primary must develop an m.m.f. which exactly counterbalances the secondary m.m.f.
- ✳ $N_2 I_2$. Hence a primary current I_1 must flow such that:

$$N_1 I_1 = N_2 I_2$$

$$\text{or } I_1 = \frac{N_2}{N_1} I_2 = K I_2$$

*Thus when a transformer is loaded and carries a secondary current I_2 , then a current $I_1 (= K I_2)$ must flow in the primary to maintain the m.m.f. balance.

- ✳ In other words, the primary must draw enough current to neutralize the demagnetizing effect of secondary current so that mutual flux ϕ remains constant.
- ✳ Thus as the secondary current increases, the primary current $I_1 (= KI_2)$ increases in unison and keeps the mutual flux ϕ constant.
- ✳ The power input, therefore, automatically increases with the output. For example if $K = 2$ and $I_2 = 2A$, then primary will draw a current $I_1 = KI_2 = 2 \times 2 = 4A$.
- ✳ If secondary current is increased to $4A$, then primary current will become $I_1 = KI_2 = 2 \times 4 = 8A$.

Phasor diagram:

- ✳ Figure (ii) shows the phasor diagram of an ideal transformer on load.
- ✳ Note that in drawing the phasor diagram, the value of K has been assumed unity so that primary phasors are equal to secondary phasors.

- The secondary current I_2 lags behind V_2 (or E_2) by ϕ_2 . It causes a primary current $I_1 = KI_2 = 1 \times I_2$ which is in *antiphase* with it.

(i)

$$\phi_1 = \phi_2$$

or $\cos \phi_1 = \cos \phi_2$

Thus, power factor on the primary side is equal to the power factor on the secondary side

- (ii) Since there are no losses in an ideal transformer, input primary power is equal to the secondary output power i.e.,

$$V_1 I_1 \cos \phi_1 = V_2 I_2 \cos \phi_2$$

Practice Questions

- ① An Ideal transformer has 1000 turns on its primary and 500 turns on its secondary. The driving voltage on the primary side is 100 V and the load resistance is 5Ω ; Calculate V_2 , I_1 and I_2 .

- ② An ideal transformer having 90 turns on the primary side and 2250 turns on the secondary is connected to 200 V, 50Hz supply. The load across the secondary draws a current of 2 A at a p.f. of 0.8 lagging. Calculate;
 - (i) the value of primary current
 - (ii) the peak value of flux linked with the secondary

Also draw the phasor diagram

Solution Qn 1&2

①

$$K = N_2/N_1 = 500/100 = 1/2$$

$$V_2 = KV_1 = (1/2) \times 100 = \mathbf{50 \text{ V}}$$

$$I_2 = V_2/R_2 = (50/5) = \mathbf{10 \text{ A}}$$

$$I_1 = KI_2 = (1/2) \times 10 = \mathbf{5 \text{ A}}$$

②

$$K = N_2/N_1 = 2250/90 = 25$$

(i)

$$I_1 = KI_2 = 25 \times 2 = \mathbf{50 \text{ A}}$$

(ii)

$$E_1 = 4.44fN_1\phi_m \leftrightarrow 200 = 4.44 \times 50 \times 90 \times \phi_m$$

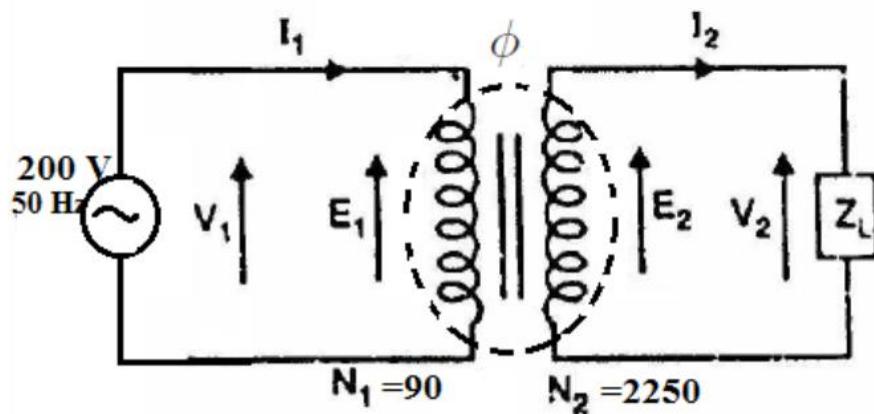
$$\phi_m = \frac{200}{4.44 \times 50 \times 90} = \mathbf{0.01 \text{ Wb}}$$

Conti....Solution Qn 2

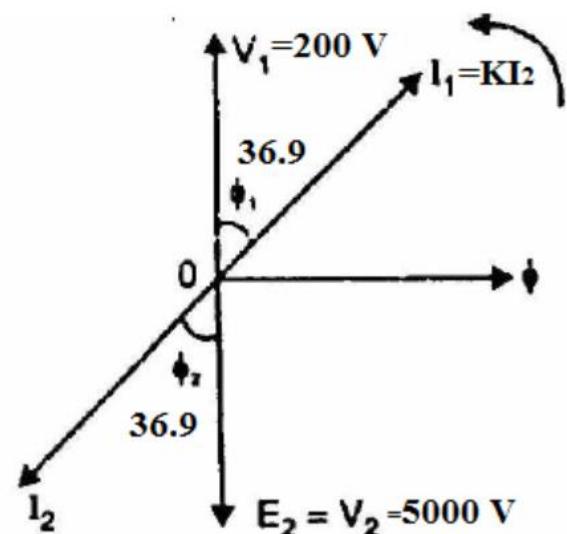
The conditions of the problem are represented in Figure (i)

Figure (ii) shows the phasor diagram (not to scale) of the transformer.

$$E_2 = KE_1 = 5000 \text{ V}$$



(i)



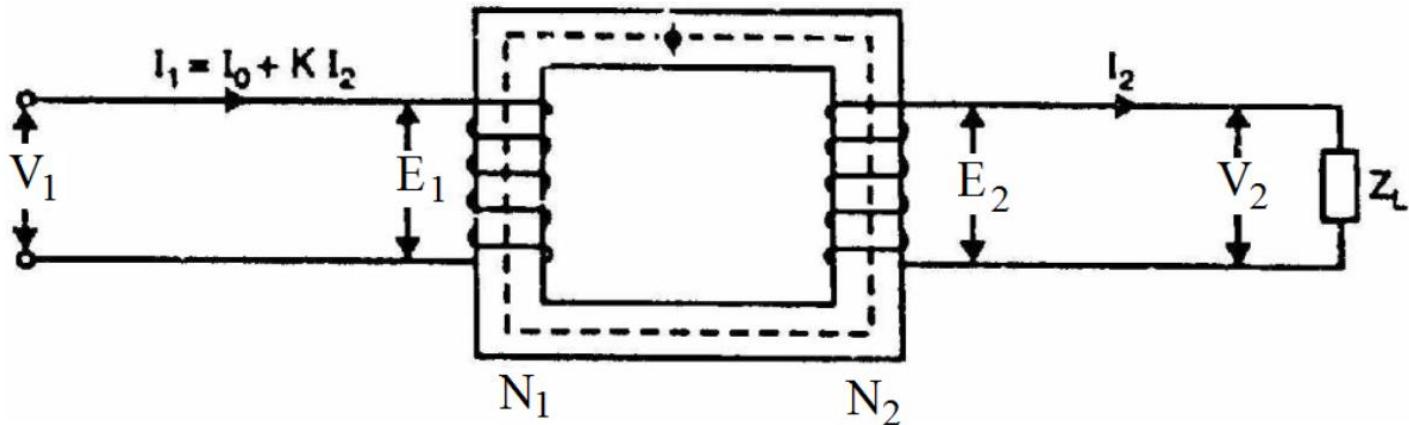
(ii)

Practical Transformer on Load

- ✳ We shall consider two cases:
 - (i) when such a transformer is assumed to have no winding resistance and leakage flux
 - (ii) when the transformer has winding resistance and leakage flux.

Practical Transformer with no winding resistance and leakage flux

- ✳ The Figure shows a practical transformer with the assumption that resistances and leakage reactances of the windings are negligible.



- ✳ With this assumption, V₂ = E₂ and V₁ = E₁.

Practical Transformer with no winding resistance and leakage flux

- ✳ Let us take the usual case of inductive load which causes the secondary current I_2 to lag the secondary voltage V_2 by ϕ .
- ✳ The total primary current I_1 must meet two requirements which are:
 - (a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.
 - (b) It must supply a current I'_2 to counteract the demagnetizing effect of secondary current I_2 . The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$
$$\text{or } I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

- ✳ The total primary current I_1 is the phasor sum of I'_2 and I_0 i.e.,

$$I_1 = I'_2 + I_0$$

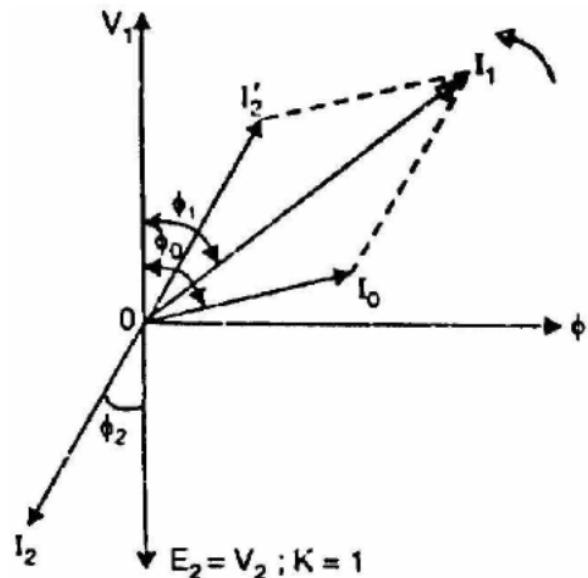
where $I'_2 = -K I_2$

Note that I'_2 is 180° out of phase with I_2 .

Practical Transformer with no winding resistance and leakage flux

- ➊ The phasor diagram for the usual case of inductive load is as shown.
- ➋ Both E_1 and E_2 lag behind the mutual flux ϕ by 90° .
- ➌ The current I'_2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2 .
- ➍ Now $I'_2 = K I_2$ and is antiphase with I_2 .
- ➎ I_0 is the no-load current of the transformer.
- ➏ The phasor sum of I'_2 and I_0 gives the total primary current I_1 .

Note that in drawing the phasor diagram, the value of K is assumed to be unity so that primary phasors are equal to secondary phasors.



$$\text{Primary p.f.} = \cos \phi_1 \quad \& \quad \text{Secondary p.f.} = \cos \phi_2$$

$$\text{Input power to transformer, } P_1 = V_1 I_1 \cos \phi_1$$

$$\text{Output power of transformer, } P_2 = V_2 I_2 \cos \phi_2$$

Practice Questions

- **Qn1.** A single-phase transformer with a ratio of 440/110 V takes no-load current of 5 A at 0.2 p.f. lagging. If the secondary supplies a current of 120 A at a p.f. 0.8 lagging, find the current taken by the primary.
- **Qn2.** A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no-load current of 3 A at 0.2 p.f. lagging. Calculate the primary and power factor when the secondary current is 280 A at a p.f. 0.8 lagging
- **Qn 3.** A single phase transformer with a ratio of 6600/400 V (primary to secondary voltage) takes to no-load current of 0.7 A at 0.24 power factor lagging. If a current of 120 A at a power factor of 0.8 lagging is supplied by its secondary. Estimate the current drawn by the primary winding. [7.83 A]

Solution Qn 1

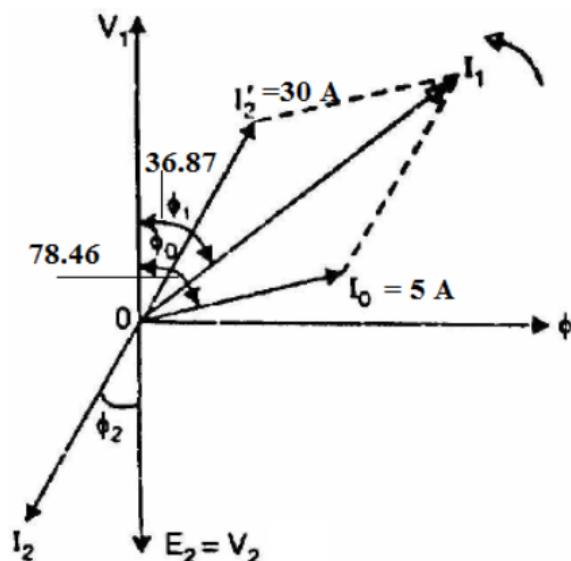
$$\cos \phi_2 = 0.8 \therefore \phi_2 = 36.87^\circ$$

$$\cos \phi_0 = 0.2 \therefore \phi_0 = 78.46^\circ$$

Now $K = V_2/V_1 = 110/440 = 1/4$

$$I'_2 = K I_2 = (1/4) \times 120 = 30A$$

$$I_0 = 5 A$$



Angle between I'_2 and I_1 = $78.46^\circ - 36.87^\circ = 41.59^\circ$

Using Parallelogram method, we have,

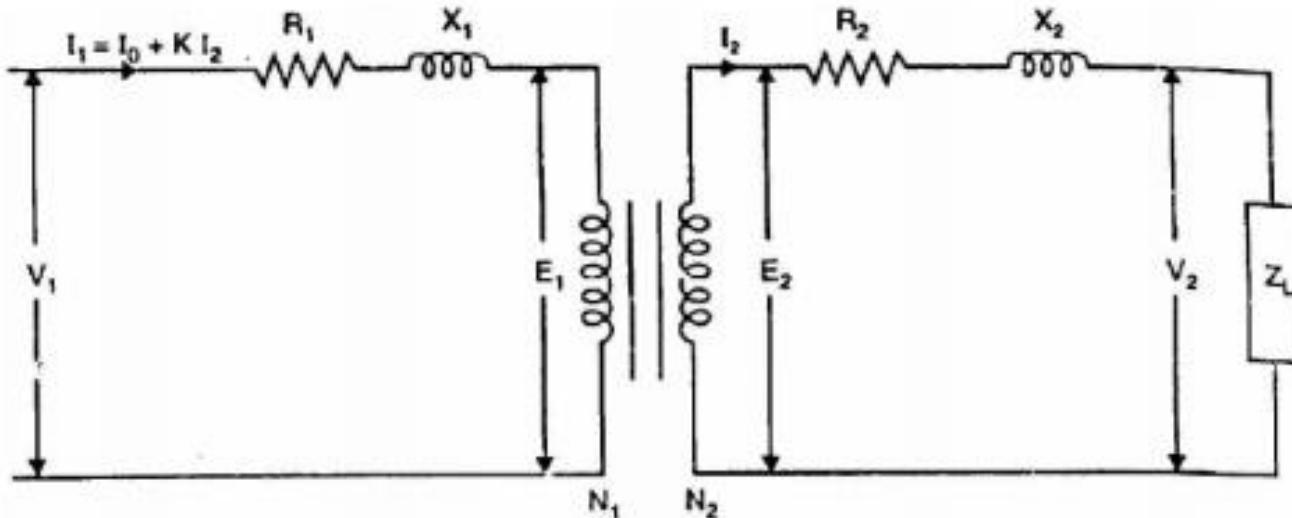
$$I_1 = \sqrt{(I_0)^2 + (I'_2)^2 + 2I_0 I'_2 \cos \theta}$$

$$I_1 = \sqrt{(I_0)^2 + (I'_2)^2 + 2I_0 I'_2 \cos 41.59^\circ}$$

$$= 33.9 A$$

Practical Transformer with resistance and leakage reactance

- These are the actual conditions that exist in a transformer.



- There is voltage drop in R_1 and X_1 so that primary e.m.f. E_1 is less than the applied voltage V_1 .
- Similarly, there is voltage drop in R_2 and X_2 so that secondary terminal voltage V_2 is less than the secondary e.m.f. E_2 .
- Let us take the usual case of inductive load which causes the secondary current I_2 to lag behind the secondary voltage V_2 by ϕ_2

Practical Transformer with resistance and leakage reactance

- ✳ The total primary current I_1 must meet two requirements which are:
 - (a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.
 - (b) It must supply a current I'_2 to counteract the demagnetizing effect of secondary current I_2 . The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$
$$\text{or } I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

- ✳ The total primary current I_1 is the phasor sum of I'_2 and I_0 i.e.,

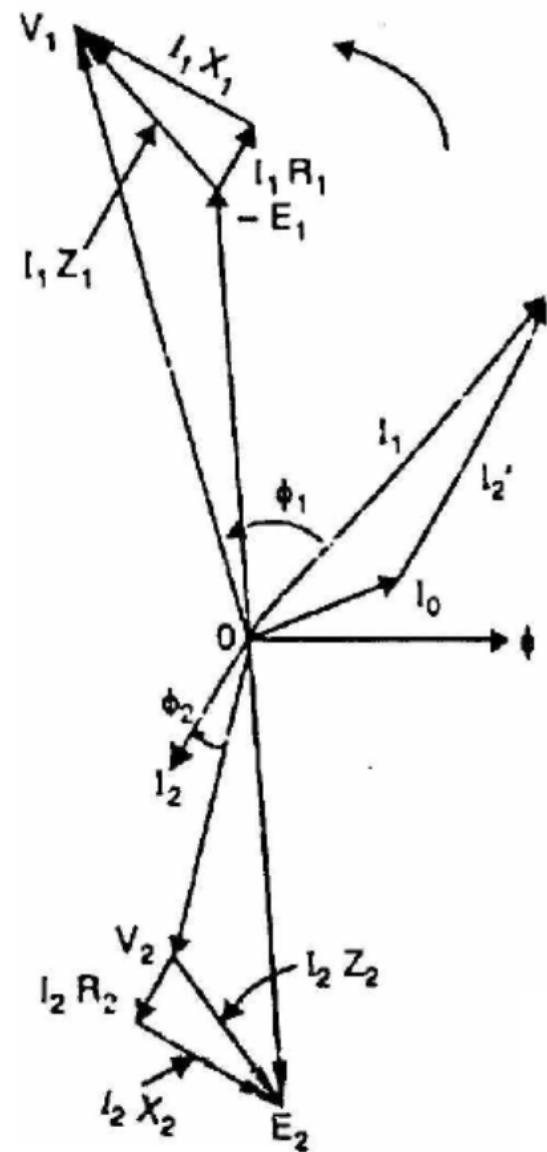
$$I_1 = I'_2 + I_0 \quad \text{where} \quad I'_2 = -K I_2$$
$$V_1 = -E_1 + I_1(R_1 + jX_1) \quad \text{where} \quad I_1 = I_0 + (-K I_2)$$
$$= -E_1 + I_1 Z_1$$

The magnitude $V_1 = E_1 + I_1 Z_1$

$$V_2 = E_2 - I_2(R_2 + jX_2) = E_2 - I_2 Z_2$$

PT with resistance and leakage reactance: Phasor diagram

- ✖ Consider the usual case of inductive load.
- ✖ Both E_1 and E_2 lag the mutual flux ϕ by 90° .
- ✖ The current I'_2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2 .
- ✖ Now $I'_2 = K I_1$ and is opposite to I_1 . Also I_0 is the no-load current of the transformer.
- ✖ The phasor sum of I'_2 and I_0 gives the total primary current I_1 .
- ✖ Note that counter e.m.f. that opposes the applied voltage V_1 is $-E_1$.
- ✖ Therefore, if we add $I_1 R_1$ (in phase with I_1) and $I_1 X_1$ (90° ahead of I_1) to $-E_1$, we get the applied primary voltage V_1 .



- The phasor E_2 represents the induced e.m.f. in the secondary by the mutual flux ϕ .
- The secondary terminal voltage V_2 will be what is left over after subtracting I_2R_2 and I_2X_2 from E_2 .
- Load power factor = $\cos \phi_2$
- Primary power factor = $\cos \phi_1$
- Input power to transformer, $P_1 = V_1 I_1 \cos \phi_1$
- Output power of transformer, $P_2 = V_2 I_2 \cos \phi_2$

Practice Questions

- **Qn1.** Qn.1 The primary of a 1000/250V transformer has a resistance of 0.15Ω and a leakage reactance of 0.8Ω . Find the primary induced e.m.f. when the primary current is 60A at a 0.8 p.f. lagging.
- **Qn. 2.** The voltage on the secondary side of a single phase transformer is 200V when supplying a load of 8 kW at a pf of 0.8 lagging. The secondary resistance is 0.04Ω and secondary leakage reactance is 0.8Ω . Calculate the induced e.m.f. in the secondary.

Solution Qn 1

Primary Impedance, $Z_1 = 0.15 + j0.8 = 0.814\angle79.6^0$

Power factor angle, $\phi_1 = \cos^{-1} 0.8 = 36.9^0$

Taking the applied voltage as the reference phasor, we have, $V_1 = 1000\angle0^0$ V.

$$\begin{aligned}\text{Now, } \quad -E_1 &= V_1 - I_1 Z_1 \\ &= 1000\angle0^0 - (60\angle-36.9^0 \times 0.814\angle79.6^0) \\ &= 1000\angle0^0 - (48\angle42.7^0) \\ &= 1000\angle0^0 - (36 + j33) \\ &= 964 - j33 = 964.5\angle-2^0\end{aligned}$$

\therefore Primary e.m.f, $E_1 = -964 + j33 = 964.5\angle178^0$

Self Assessment

- Even at no-load, a transformer draws current from the mains. Why?
- What do you know about reactance in a transformer?
- When load current of a transformer increases, how does the input current adjust to meet the new conditions?
- How does leakage flux occur in a transformer?
- A transformer working on 4200 V, 50 Hz has 200 primary turns. The core has a mean length of magnetic path of 100 cm and cross-sectional area 1000cm^2 . the iron having a relative permeability of 100. The iron loss is 600W. Calculate primary no-load current.