

EEE/ETI3105

ELECTRICAL MACHINES I

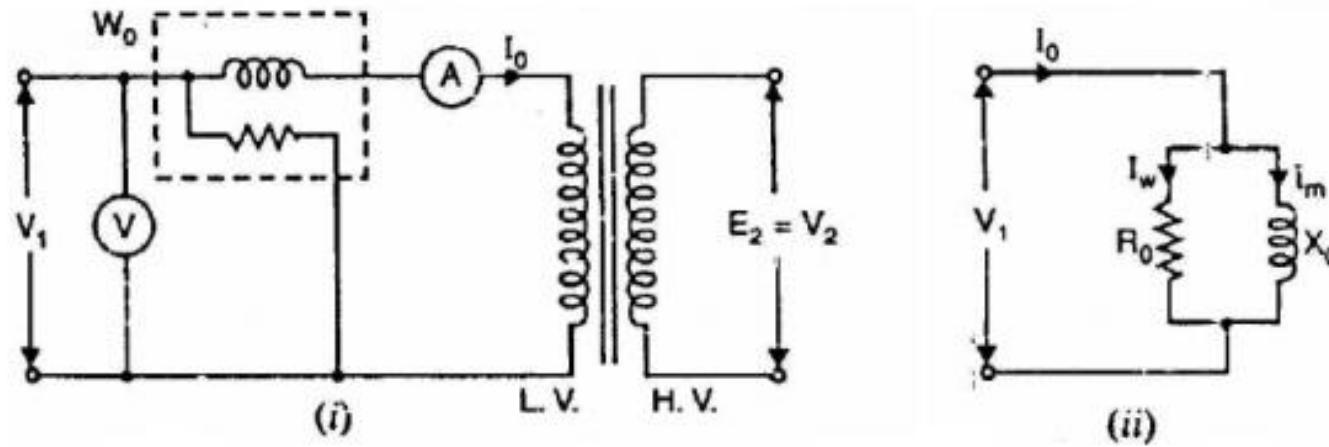
LECTURE 5: TRANSFORMER TESTS

TRANSFORMER TESTS

- The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests i.e.:
 - ✓ open-circuit test
 - ✓ short-circuit test.
- These tests are very convenient as they provide the required information without actually loading the transformer.
- Further, the power required to carry out these tests is very small as compared with full-load output of the transformer.
- These tests consist of measuring the input voltage, current and power to the primary first with secondary open-circuited (open-circuit test) and then with the secondary short-circuited (short circuit test).

Open-Circuit or No-Load Test

- This test is conducted to determine the iron losses (or core losses) and parameters R_0 and X_0 of the transformer.
- In this test, the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open-circuited.



- The applied primary voltage V_1 is measured by the voltmeter, the no-load current I_0 by ammeter and no-load input power W_0 by wattmeter as shown in Figure (i).
- As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core.
- Hence wattmeter will record the iron losses and small copper loss in the primary.

Open-Circuit or No-Load Test

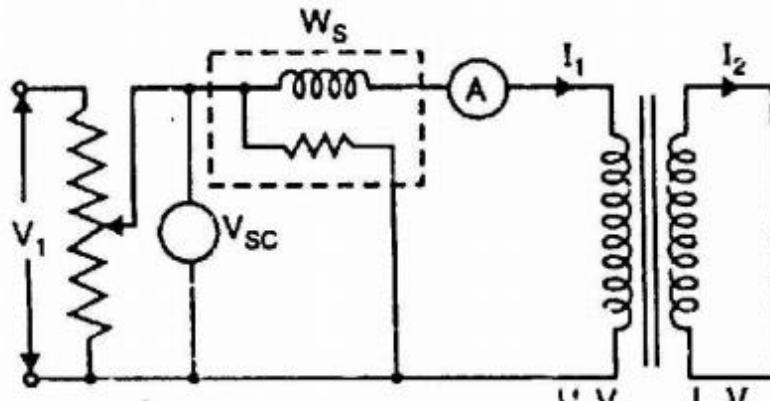
- ✳ Since no-load current I_0 is very small (usually 2-10 % of rated current), Cu losses in the primary under no-load condition are negligible as compared with iron losses.
- ✳ Hence, wattmeter reading practically gives the iron losses in the transformer. The iron losses are the same at all loads.
- ✳ Figure (ii) shows the equivalent circuit of transformer on no-load.
- ✳ Iron losses, $P_i = \text{Wattmeter reading} = W_0$
- ✳ No load current = Ammeter reading = I_0
- ✳ Applied voltage = Voltmeter reading = V_1
- ✳ Input power, $W_0 = V_1 I_0 \cos \phi_0$
 \therefore No - load p.f., $\cos \phi_0 = W_0 / V_1 I_0$
 $I_W = I_0 \cos \phi_0$; $I_m = I_0 \sin \phi_0$
 $R_0 = V_1 / I_w$ and $X_0 = V_1 / I_m$
- ✳ Thus open-circuit test enables us to determine iron losses and parameters R_0 and X_0 of the transformer.

Short-Circuit or Impedance Test

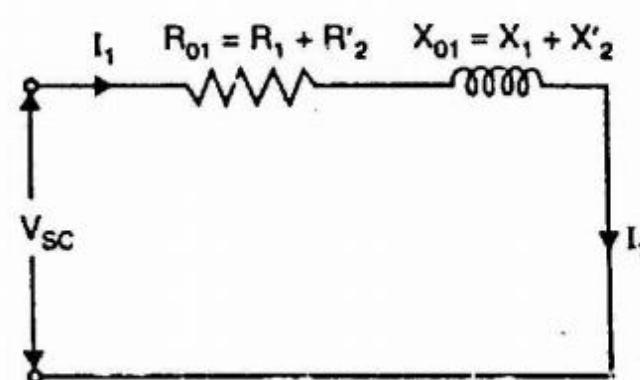
- This test is conducted to determine R_{01} (or R_{02}), X_{01} (or X_{02}) and full-load copper losses of the transformer.
- In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in Fig. (i).
- The low input voltage is gradually raised till at voltage V_{SC} , full-load current I_1 flows in the primary.
- Then I_2 in the secondary also has full-load value since $I_1/I_2 = N_2/N_1$. Under such conditions, the copper loss in the windings is the same as that on full load.

Short-Circuit or Impedance Test

- There is no output from the transformer under short-circuit conditions.
- Therefore, input power is all loss and this loss is almost entirely copper loss.
- It is because iron loss in the core is negligibly small since the voltage V_{SC} is very small.
- Fig. (ii) shows the equivalent circuit of a transformer on short circuit as referred to primary; the no-load current being neglected.



(i)



(ii)

Short-Circuit or Impedance Test

- ✖ Hence, the wattmeter will practically register the full-load copper losses.
- ✖ The no-load current being neglected due to its smallness.
- ✖ Full load Cu loss, P_C = Wattmeter reading = W_S
- ✖ Applied voltage = Voltmeter reading = V_{SC}
- ✖ F.L. primary current = Ammeter reading = I_1

$$P_C = I_1^2 R_1 + I_1^2 R'_2 = I_1^2 R_{01} \Rightarrow R_{01} = P_C / I_1^2$$

where R_{01} is the total resistance of transformer referred to primary.

- ✖ Total impedance referred to primary, $Z_{01} = V_{SC} / I_1$
 - ✖ Total leakage reactance referred to primary, $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$
 - ✖ Short-circuit p.f, $\cos \phi_2 = P_C / V_{SC} I_1$
-
- *Note: The SC test will give full-load Cu loss only if the applied voltage V_{SC} is such so as to circulate full-load currents in the windings.*

Advantages of Transformer Tests

- The two simple transformer tests offer the following advantages:
 1. The power required to carry out these tests is very small as compared to the full-load output of the transformer. In case of open-circuit test, power required is equal to the iron loss whereas for a short-circuit test, power required is equal to full-load copper loss.
 2. These tests enable us to determine the efficiency of the transformer accurately at any load and p.f. without actually loading the transformer.
 3. The short-circuit test enables us to determine R_{01} and X_{01} (or R_{02} and X_{02}). We can thus find the total voltage drop in the transformer as referred to primary or secondary. This permits us to calculate voltage regulation of the transformer.

Practice Question

- **Qn.:** A 15 kVA, 440/230 V, 50 Hz, single phase transformer gave the following test results:

Open Circuit (LV side) 250 V, 1.8A, 95 W.

Short Circuit Test (HV side) 80 V, 12.0 A, 380 W.

Compute the parameters of the equivalent circuit referred to LV side

Solution

Transformer rating = 15 kVA; $E_1 = 440$ V; $E_2 = 230$ V; $f = 50$ Hz

Open circuit test (LV side); $V_2 = 250$ V; $I_0 = 1.8$ A; $W_0 = 95$ W

Short circuit test (HV side); $V_{1(sc)} = 80$ V; $I_{1(sc)} = 12$ A; $W_c = 380$ W

From open circuit test performed on *LV* side;

$$I_w = \frac{W_0}{V_2} = \frac{95}{250} = 0.38 \text{ A} \quad I_{mag} = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.8)^2 - (0.38)^2} = 1.75943 \text{ A}$$

$$\text{Exciting resistance, } R_0 = \frac{V_2}{I_w} = \frac{250}{0.38} = 658 \Omega \quad \text{Exciting reactance, } X_0 = \frac{V_2}{I_{mag}} = \frac{250}{1.75943} = 142 \Omega$$

From short circuit test performed on *HV* side;

$$Z_{ep} = \frac{V_{1(sc)}}{I_{1(sc)}} = \frac{80}{12} = 6.667 \Omega \quad R_{ep} = \frac{W_c}{(I_{1(sc)})^2} = \frac{380}{(12)^2} = 2.639 \Omega$$

$$X_{ep} = \sqrt{Z_{ep}^2 - R_{ep}^2} = \sqrt{(6.667)^2 - (2.639)^2} = 6.122 \Omega$$

$$\text{Transformation ratio, } K = \frac{E_2}{E_1} = \frac{230}{440} = 0.5227$$

Transformer resistance and reactance referred to *LV* (secondary) side;

$$R_{es} = R_{ep} \times K^2 = 2.639 \times (0.5227)^2 = 0.7211 \Omega$$

$$X_{es} = X_{ep} \times K^2 = 6.122 \times (0.5227)^2 = 2.673 \Omega$$

Efficiency of a Transformer

- Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW)
i.e., Efficiency = Output power/Input power
- It may appear that efficiency can be determined by directly loading the transformer and measuring the input power and output power.
- However, this method has the following drawbacks:
 1. Since the efficiency of a transformer is very high, even 1% error in each wattmeter (output and input) may give ridiculous results. This test, for instance, may give efficiency higher than 100%.
 2. Since the test is performed with transformer on load, considerable amount of power is wasted. For large transformers, the cost of power alone would be considerable.

Efficiency of a Transformer

- 3) It is generally difficult to have a device that is capable of absorbing all of the output power.
- 4) The test gives no information about the proportion of various losses.
- Due to these drawbacks, direct loading method is seldom used to determine the efficiency of a transformer. In practice, open-circuit and short-circuit tests are carried out to find the efficiency.

Efficiency from Transformer Tests

Efficiency = Output / input = Output / (Output + Losses)

The losses can be determined by transformer tests.

F.L. Iron loss = P_i ...from open-circuit test

F.L. Cu loss = P_C ...from short-circuit test

Total F.L. losses = $P_i + P_C$

We can now find the full-load efficiency of the transformer at and p.f. without actually loading the transformer.

F.L. efficiency $\eta_{FL} = \{ \text{Full - load VA} \times \text{p.f.} \} / \{ (\text{Full - load VA} \times \text{p.f.}) + P_i + P_C \}$

Note that iron loss remains the same at all loads.

Efficiency of a Transformer

The efficiency of a transformer is defined as the ratio of output to the input power, the two being measured in same units (either in watts or in kW).

$$\text{Transformer efficiency, } \eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

or

$$\begin{aligned}\eta &= \frac{\text{output power}}{\text{output power} + \text{iron losses} + \text{copper losses}} \\ &= \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + P_c}\end{aligned}$$

where, V_2 = Secondary terminal voltage

I_2 = Full load secondary current

$\cos \phi_2$ = p.f. of the load

P_i = Iron losses = Hysteresis losses + eddy current losses (constant losses)

P_c = Full load copper losses = $I_2^2 R_{es}$ (variable losses)

If x is the fraction of the full load, the efficiency of the transformer at this fraction is given by the relation;

$$\eta_x = \frac{x \times \text{output at full load}}{x \times \text{output at full load} + P_i + x^2 P_c} = \frac{x V_2 I_2 \cos \phi_2}{x V_2 I_2 \cos \phi_2 + P_i + x^2 I_2^2 R_{es}}$$

The copper losses vary as the square of the fraction of the load.

Conditions for Maximum Efficiency of a Transformer

- Output power = $V_2 I_2 \cos \phi_2$
- If R_{02} is the total resistance of the transformer referred to secondary, then, the total Cu loss, $P_C = I_2^2 R_{02}$
Total losses = $P_i + P_C$

$$\therefore \eta_{tx} = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{02}} = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + P_i/I_2 + I_2 R_{02}} \dots \text{(i)}$$

- For a normal transformer, V_2 is approximately constant.
- Hence for a load of given p.f., efficiency depends upon load current I_2 .
- It is clear from exp (i) above that numerator is constant and for the efficiency to be maximum, the denominator should be minimum i.e.,

$$\frac{d}{dI_2} (\text{denominator}) = 0$$

$$\frac{d}{dI_2} (V_2 \cos \phi_2 + P_i/I_2 + I_2 R_{02}) = 0$$

$$-P_i/I_2^2 + R_{02} = 0$$

$$\Rightarrow P_i = I_2^2 R_{02} \dots \text{(ii)}$$

- Hence efficiency of a transformer will be maximum when copper losses are equal to constant or iron losses.
- From (ii) above, the load current I_2 corresponding to maximum efficiency is given by:

$$I_2 = \sqrt{\frac{P_i}{R_{02}}}$$

- The relative value of these losses is in the control of the designer of the transformer according to the relative amount of copper and iron used.
- A transformer which is to operate continuously on full-load would, therefore, be designed to have-maximum efficiency at full-load.
- However, distribution transformers operate for long periods on light load.
- Therefore, their point of maximum efficiency is usually arranged to be between three-quarter and half full-load.
- **Note.** In a transformer, iron losses are constant whereas copper losses are variable. In order to obtain maximum efficiency, the load current should be such that total Cu losses become equal to iron losses.

Output kVA Corresponding to Maximum Efficiency

- Let P_C = Copper losses at full-load kVA

P_i = Iron losses

x = Fraction of full-load kVA at which efficiency is maximum

Total Cu losses = $x^2 P_C$

$$\therefore x^2 P_C = P_i \quad \text{or } x = \sqrt{\frac{P_i}{P_C}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu Loss}}} \\ \therefore$$

- Output kVA corresponding to maximum efficiency

$$= x \times \text{Full - load kVA} = \text{Full - load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu Loss}}}$$

- It may be noted that the value of kVA at which the efficiency is maximum is independent of p.f. of the load.

Calculation of Efficiency from O.C. and S.C. Tests

From O.C. test, $W_o = P_i$

From S.C. test, $W_{sc} = (P_{cu}) F.L.$

$$\therefore \% \eta \text{ on full load} = \frac{V_2 (I_2) F.L. \cos \phi}{V_2 (I_2) F.L. \cos \phi + W_o + W_{sc}} \times 100$$

Thus for any p.f. $\cos \phi_2$ the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos \phi}{n \times (\text{VA rating}) \times \cos \phi + W_o + n^2 W_{sc}} \times 100$$

where $n = \text{fraction of full load}$

or
$$\% \eta = \frac{n V_2 I_2 \cos \phi}{n V_2 I_2 \cos \phi + W_o + n^2 W_{sc}} \times 100$$

where $I_2 = n (I_2) F.L.$

Practice Question

- **Qn.:** A 50 MVA, 60 Hz single-phase transformer indicates that it has a voltage rating of 8 kV: 78 kV. Open circuit test and short circuit test gave the following results:

Open Circuit Test: 8 kV, 61.9 A and 136 kW

Short Circuit Test: 650 V, 6.25 kA and 103 kW.

Determine the efficiency and voltage regulation if the transformer is operating at rated voltage and a load of 0.9 p.f. lagging.

Solution

Here, rating of transformer = 50 MVA = 50×10^6 VA

$$V_1 = 8 \text{ kV}; V_2 = 78 \text{ kV}; \text{Load p.f., } \cos \phi = 0.9 \text{ lag}$$

Open circuit (*LV* side): $V_1 = 8 \text{ kV}; I_0 = 61.9 \text{ A}; W_0 = 136 \text{ kW}$

Short circuit test (*LV* side): $V_{1(sc)} = 650 \text{ V}; I_{1(sc)} = 6.25 \text{ kA}; W_c = 103 \text{ kW}$

From open circuit test, iron losses of the transformer,

$$P_0 = W_0 = 136 \text{ kW}$$

At rated capacity, full-load current of the transformer on *LV* side,

$$I_{1(fl)} = \frac{\text{Rated capacity}}{V_1} = \frac{50 \times 10^6}{8 \times 10^3} = 6.25 \text{ kA}$$

Since $I_{1(sc)} = I_{1(fl)} = 6.25 \text{ kA}$, the short circuit test is performed at full load.

\therefore Full load copper losses, $P_c = W_c = 103 \text{ kW}$

Full-load efficiency,

$$\begin{aligned}\eta &= \frac{\text{Rated kVA} \times \cos \phi}{\text{Rated kVA} \times \cos \phi + P_i \text{ in kW} + P_c \text{ in kW}} \\ &= \frac{50 \times 10^3 \times 0.9}{50 \times 10^3 \times 0.9 + 136 + 103} = 0.9947 = \mathbf{99.47\%} \quad (\text{Ans.})\end{aligned}$$

Conti... Solution

Considering the data of short circuit test;

Transformer impedance referred to primary,

$$Z_{ep} = \frac{V_{1(sc)}}{I_{1(sc)}} = \frac{650}{6.25 \times 10^3} = 0.104 \Omega$$

Transformer resistance referred to primary,

$$R_{ep} = \frac{W_c}{(I_{1(sc)})^2} = \frac{103 \times 10^3}{(6.25 \times 10^3)^2} = 0.0165 \Omega$$

Transformer reactance referred primary,

$$X_{ep} = \sqrt{Z_{cp}^2 - R_{cp}^2} = \sqrt{(0.104)^2 - (0.0165)^2} = 0.10317 \Omega$$

Load p.f., $\cos \phi = 0.9$; $\sin \phi = \sin \cos^{-1} 0.9 = 0.4359$

$$\begin{aligned} E_1 &= V_1 - I_1 R_{ep} \cos \phi - I_1 X_{ep} \sin \phi \\ &= 8 \text{ kV} - 6.25 \text{ kA} \times 0.0165 \times 0.9 - 6.25 \text{ kA} \times 0.10317 \times 0.4359 \\ &= 8 - 0.0928 - 0.28107 = 7.626 \text{ kV} \end{aligned}$$

$$\text{Voltage regulation, \% } Reg = \frac{V_1 - E_1}{V_1} = \frac{8 - 7.626}{8} \times 100 = \mathbf{4.675\%} \text{ (Ans.)}$$

Practice Question

- **Qn.** A 5 kVA, 500/250V, 50Hz, single phase transformer gave the following readings,
 - O.C. Test: 500V, 1 A, 50 W (L.V. side open)
 - S.C. Test: 25 V, 10 A, 60 W (L.V. side shorted)
- Determine
 - The efficiency on full load, 0.8 lagging p.f
 - the voltage regulation on full load, 0.8 leading p.f
 - the efficiency on 60% of full load, 0.8 leading p.f
 - Draw the equivalent circuit referred to primary and insert all values in it.

All-Day (or Energy) Efficiency

- The ordinary or commercial efficiency of a transformer is defined as the ratio of output power to the input power i.e.,
Commercial efficiency = Power Output/Power Input
- There are certain types of transformers whose performance cannot be judged by this efficiency.
- For instance, distribution transformers used for supplying lighting loads have their primaries energized all the 24 hours in a day but the secondaries supply little or no load during the major portion of the day.
- Constant loss (i.e., iron loss) occurs during the whole day but copper loss occurs only when the transformer is loaded and would depend upon the magnitude of load.

All-Day (or Energy) Efficiency

- Consequently, the copper loss varies considerably during the day and the commercial efficiency of such transformers will vary from a low value (or even zero) to a high value when the load is high.
- The performance of such transformers is judged on the basis of energy consumption during the whole day (i.e., 24 hours).
- This is known as **all-day or energy efficiency**.
- The ratio of output in kWh to the input in kWh of a transformer over a 24-hour period is known as all-day efficiency i.e.,

$$\eta_{\text{all-day}} = \frac{\text{kWh output in 24 hours}}{\text{kWh input in 24 hours}}$$

All-Day (or Energy) Efficiency

$$\begin{aligned}\% \text{ All day } \eta &= \frac{\text{Output energy in kWh during a day}}{\text{Input energy in kWh during a day}} \times 100 \\ &= \frac{\text{Output energy in kWh during a day}}{\text{Output energy} + \text{Energy spent for total losses}} \times 100\end{aligned}$$

- All-day efficiency is of special importance for those transformers whose primaries are never open-circuited but the secondaries carry little or no load much of the time during the day.
- In the design of such transformers, efforts should be made to reduce the iron losses which continuously occur during the whole day.
- **Note.** *Efficiency of a transformer means commercial efficiency unless stated otherwise.*

Practice Question

- **Qn 1.:** A 500 kVA, 600/400V, one-phase transformer has primary and secondary winding resistance of 0.42Ω and 0.0011Ω , respectively. The primary and secondary voltages are 600 V and 400 V, respectively. The iron loss is 2.9 kW. Calculate the efficiency at half full load at a power factor of 0.8 lagging.
- **Qn 2.:** In a 25 kVA, 2000/200 V power transformer the iron and full load copper losses are 350 W and 400 W, respectively. Calculate the efficiency at unity power factor at (i) full load and (ii) half load.
- **Qn 3.:** A 220/400 V, 10 kVA, 50Hz, single-phase transformer has copper loss of 120 W at full load. If it has an efficiency of 98% at full load, unity power factor, determine the iron losses. What would be the efficiency of the transformer at half full-load at 0.8 p.f. lagging.

Solution Qn 1

Transformer rating, = 500 kVA

Primary voltage, E_1 = 6600 V

Primary resistance, R_1 = 0.42 Ω

Secondary voltage, E_2 = 400 V

Secondary resistance, R_2 = 0.0011 Ω

Iron losses, P_i = 2.9 kW

Fraction of the load, $x = \frac{1}{2} = 0.5$

Load p.f., $\cos \phi = 0.8$ lagging

$$\text{Transformation ratio, } K = \frac{E_2}{E_1} = \frac{400}{6600} = \frac{2}{33}$$

$$\text{Primary resistance referred to secondary, } R'_1 = K^2 R_1 = \frac{2}{33} \times \frac{2}{33} \times 0.42 = 0.00154 \Omega$$

$$\text{Total resistance referred to secondary, } R_{es} = R_2 + R'_1 = 0.0011 + 0.00154 = 0.00264 \Omega$$

$$\text{Full load secondary current, } I_2 = \frac{kVA \times 10^3}{E_2} = \frac{500 \times 10^3}{400} = 1250 \text{ A}$$

$$\begin{aligned}\text{Copper losses at full load, } P_c &= I_2^2 R_{es} = (1250)^2 \times 0.00264 \\ &= 4125 \text{ W} = 4.125 \text{ kW}\end{aligned}$$

Efficiency of transformer at any fraction (x) of the load,

$$\eta_x = \frac{xkVA \cos \phi}{xkVA \cos \phi + P_i + x^2 P_c} \times 100 = \frac{0.5 \times 500 \times 0.8}{0.5 \times 500 \times 0.8 + 2.9 + (0.5)^2 \times 4.125} \times 100 = 98.07\% \text{ (Ans.)}$$

Solution Qn 2

$$\eta_x = \frac{x \text{ kVA} \times 1000 \times \cos \phi}{x \text{ kVA} \times 1000 \times \cos \phi + P_i + x^2 P_c}$$

where, $\cos \phi = 1$; $P_i = 350 \text{ W}$; $P_c = 400 \text{ W}$

(i) At full-load $x = 1$

$$\therefore \eta = \frac{1 \times 25 \times 1000 \times 1}{1 \times 25 \times 1000 \times 1 + 350 + 1 \times 1 \times 400} \times 100 = 97\cdot087 \% \text{ (Ans)}$$

(ii) At half-load; $x = 0.5$

$$\therefore \eta = \frac{0.5 \times 25 \times 1000 \times 1}{0.5 \times 25 \times 1000 \times 1 + 350 + (0.5)^2 \times 400} \times 100 = 96\cdot525 \% \text{ (Ans)}$$

Practice Question

- A 400kVA, distribution transformer has full load iron loss of 2.5kW and copper loss of 3.5kW. During a day, its load cycle for 24hours is,
 - 6 hours 300kW at 0.8 pf
 - 10 hours 200kW at 0.7 pf
 - 4 hours 100kW at 0.9 p.f
 - 4 hours No load
- Determine its all day efficiency.

Practice Question

- **Qn.1** In a 50 kVA transformer has iron loss is 500 W and full load copper loss is 800W. Find the efficiency at full load and half full load at 0.8 p.f. lagging. [**full load = 96.85%; Half full load = 96.6%**]

- **Qn.2** A 40 kVA transformer has iron loss of 450W and full load copper loss of 850W. If the full load p.f. of the load is 0.8 lagging, calculate
 - (i)full-load efficiency
 - (ii) the kVA load at which maximum efficiency occurs
 - (iii)the maximum efficiency

Practice Question

- **Qn** A 5-kVA distribution transformer has a full-load efficiency at unity p.f of 95%, the copper and iron losses then being equal. Calculate its all-day efficiency if it is loaded throughout the 24 hours as follows:

No load for 10 hours;

Quarter load for 7 hours

Half load for 5 hours;

Full load for 2 hours

Assume load p.f of unity

Self Assessment

- What is voltage regulation of a transformer?
- Why does voltage drop in a transformer?
- Is the regulation at rated load of a transformer same at 0.8 p.f. lagging and 0.8 p.f. leading?
- Is the percentage impedance of a transformer same on primary and on secondary?
- Obtain the equivalent circuit of a 200/400V, 50Hz, single phase transformer from the following test data:

O.C test: 200V, 0.7A, 70W-on LV side

SC test: 15V, 10A, 85W-on HV side

Calculate the secondary voltage when delivering 5kW at 0.8 p.f. lagging.

Self Study

- Write short notes on
 - i. Transformer cooling methods eg, ONAN, ONAF, ODAF etc
 - ii. **Autotransformer** principle of operation and application