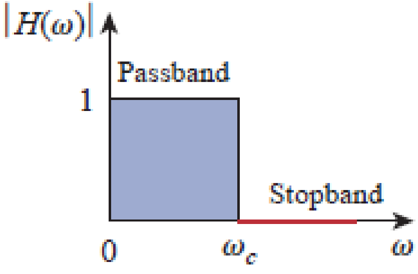
EEE/ETI 2204

CIRCUITS & NETWORK THEORY 11

Lecture 7

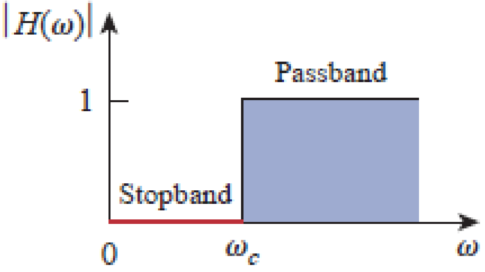
Filters

- A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.
- A filter is a *passive filter* if it consists of only passive elements *R*, *L*, and *C*. It is said to be an *active filter* if it consists of active elements (such as transistors and op amps) in addition to passive elements *R*, *L*, and *C*.
- There are four types of filters whether passive or active:



(a)

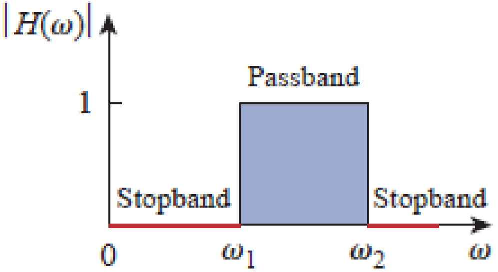
1. A *lowpass filter* passes low frequencies and stops high frequencies, as shown ideally in Fig. (a).



(b)

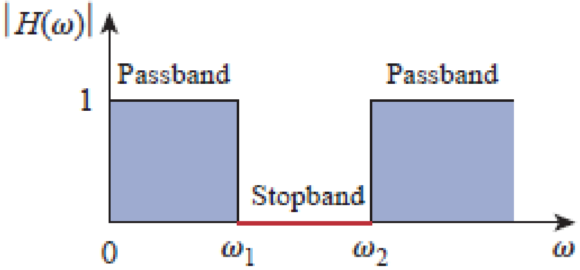
2. A *highpass filter* passes high frequencies and rejects low frequencies, as shown ideally in Fig. (b).

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.



(c)

3. A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. (c).



(d)

4. A *bandstop filter* passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. (d).

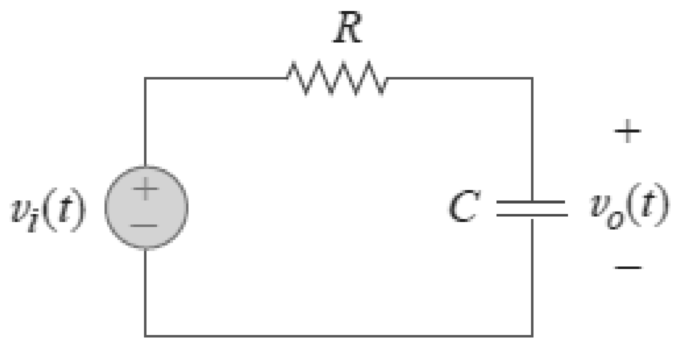
Lowpass Filter

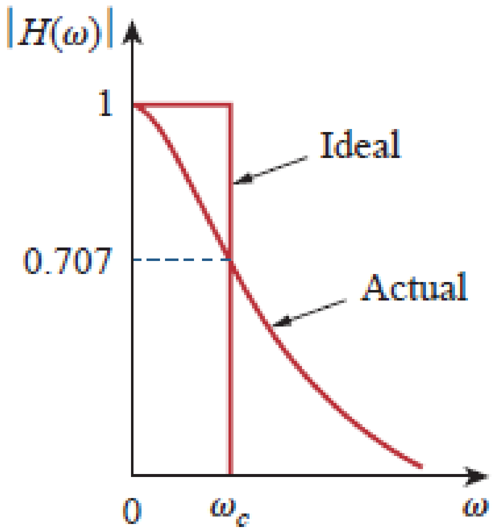
- A lowpass filter is designed to pass only frequencies from dc up to the cutoff frequency, ω_c.
- The cutoff frequency is the frequency at which the transfer function **H** drops in magnitude to 70.71% of its maximum value.
- It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.
- The cutoff frequency is also called the *rolloff frequency*.
- A lowpass filter can also be formed when the output of an *RL* circuit is taken off the resistor. Of course, there are many other circuits for lowpass filters.

A typical lowpass filter is formed when the output of an RC circuit is taken off the capacitor as shown in Fig. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$





Note that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 0$. Figure *(b)* shows the plot of $|H(\omega)|$, along with the ideal characteristic. The half-power frequency, which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as the *cutoff frequency* ω_c, is obtained by setting the magnitude of $\mathbf{H}(\omega)$ equal to $1/\sqrt{2}$, thus,

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{RC}$$

Highpass Filter

A highpass filter is formed when the output of an RC circuit is taken off the resistor as shown in Fig. *(i)*. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

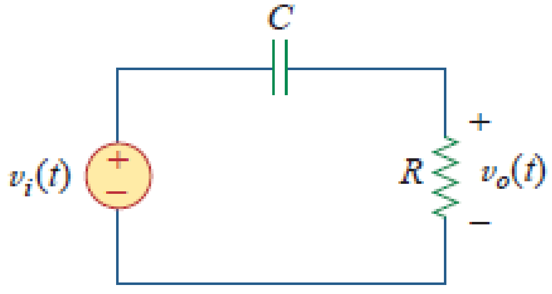
Note that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 1$. Figure *(ii)* shows the plot of $|H(\omega)|$.

Again, the corner or cutoff frequency is

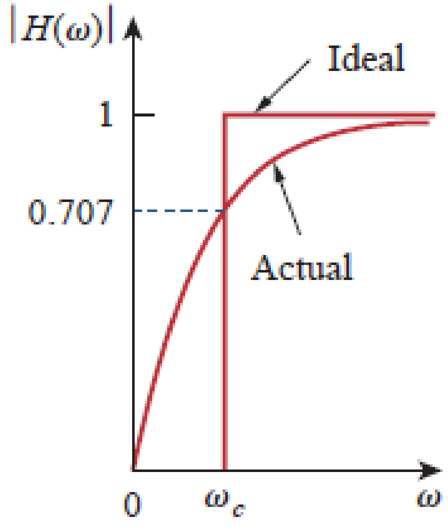
$$\omega_c = \frac{1}{RC}$$

A **highpass filter** is designed to pass all frequencies above its cutoff frequency ω_c.

A highpass filter can also be formed when the output of an RL circuit is taken off the inductor.



(i)

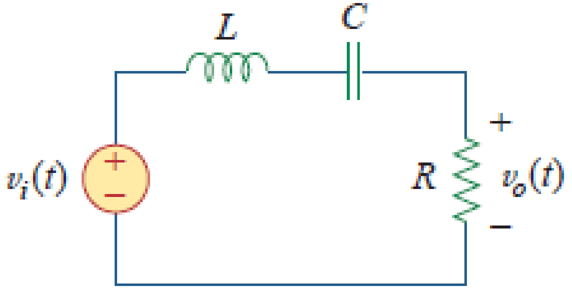


(ii)

Bandpass Filter

The *RLC* series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig. *(i)*. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

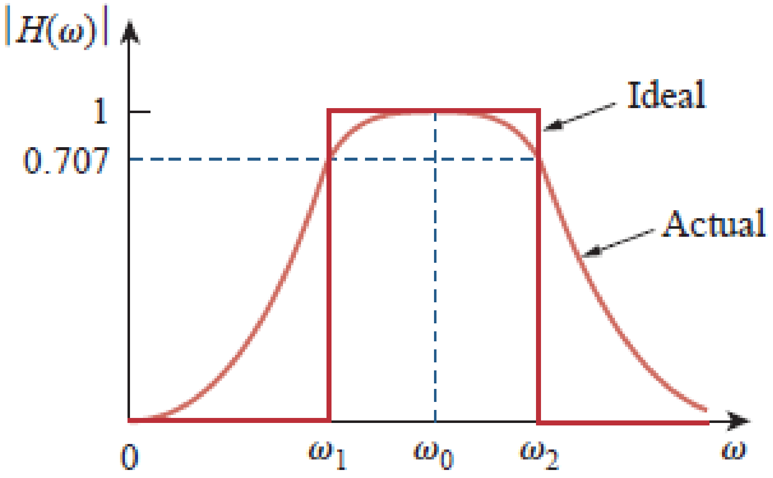


(i)

We observe that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 0$. The bandpass filter passes a band of frequencies $(\omega_1 < \omega < \omega_2)$ centered on ω_0, the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

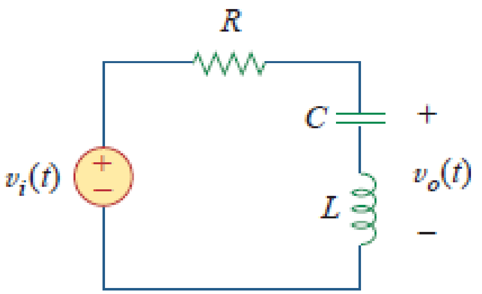
A **bandpass filter** is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.



(ii)

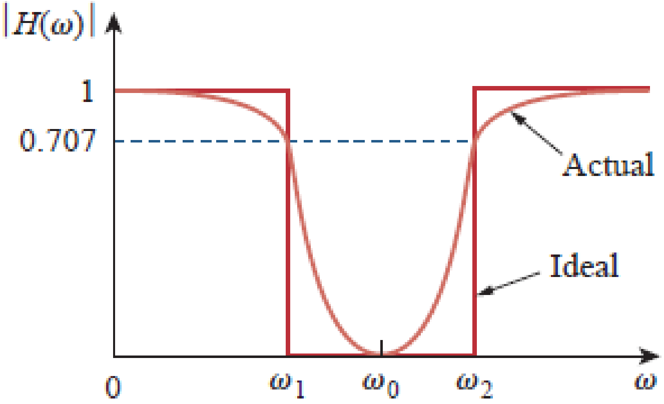
Since the bandpass filter in Fig. *(i)* is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor are determined as shown before.

Bandstop filter



A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a *bandstop, band-reject*, or *notch* filter. A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig. *(i)* . The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$



Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

while the half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas in Section for a series resonant circuit. Here, ω_0 is called the *frequency of rejection*, while the corresponding bandwidth ($B = \omega_2 - \omega_1$) is known as the *bandwidth of rejection*. Thus,

A **bandstop filter** is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

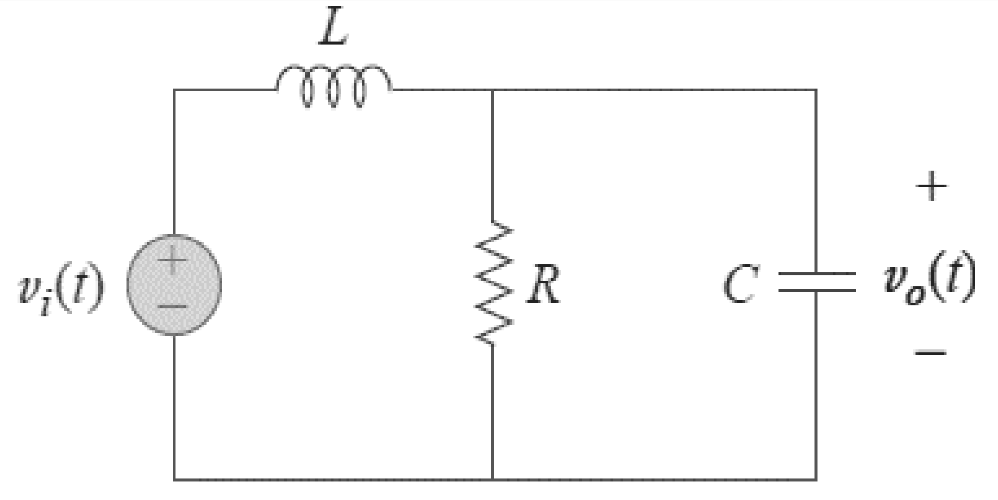
Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

Practice Questions

a) Given the circuit shown in **Fig below.** Determine the type of filter. Calculate the corner or cutoff frequency given R = 2 kΩ, L = 2 H and C = 2 μF.



Solution

The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \qquad s = j\omega$$

$$R \left\| \frac{1}{sC} \right. = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \qquad s = j\omega$$

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R}$$

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, we conclude is a second-order lowpass filter.

The magnitude of $\mathbf{H}$ is

$$H = \frac{R}{\sqrt{(R - \omega^2RLC)^2 + \omega^2L^2}}$$

Solution

The corner frequency is the same as the half-power frequency, i.e., where **H** is reduced by a factor of $1/\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency,

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = (1 - \omega_c^2\, 4 \times 10^{-6})^2 + (\omega_c\, 10^{-3})^2$$

Assuming that ω_c is in krad/s,

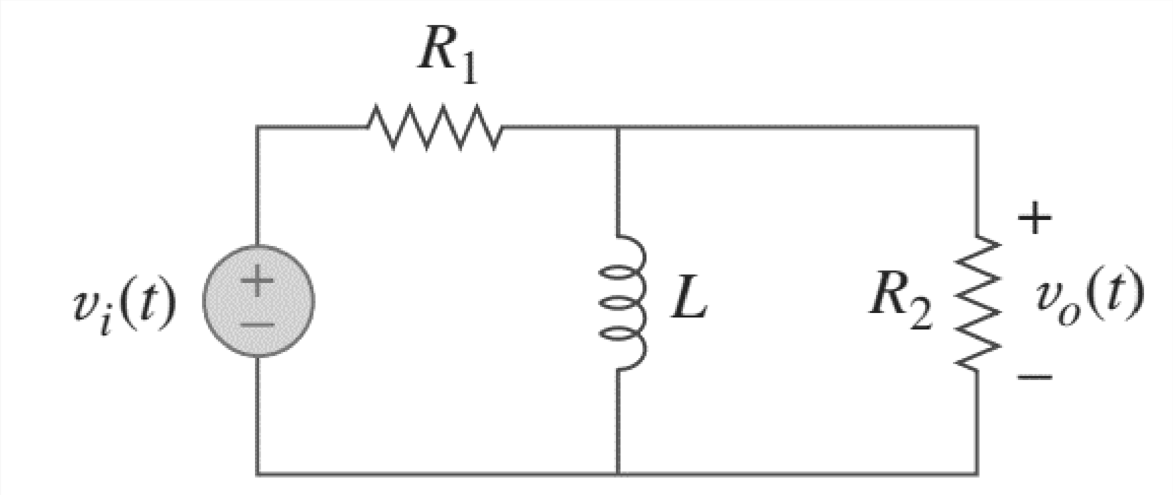
$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$ and -0.1134. Since ω_c is real,

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

Practice Question

- For the circuit in Fig. below, obtain the transfer function in terms of voltage gain. Identify the type of filter the circuit represents and determine the corner frequency. Take R_1 = R_2 100Ω and L=2mH



Answer: $\dfrac{R_2}{R_1 + R_2}\left(\dfrac{j\omega}{j\omega + \omega_c}\right)$, highpass filter $\quad \omega_c = \dfrac{R_1 R_2}{(R_1 + R_2)L} = 25 \text{ krad/s}.$

Practice

If the bandstop filter in Fig. is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of L and C. Take $R = 150\ \Omega$ and the bandwidth as 100 Hz.

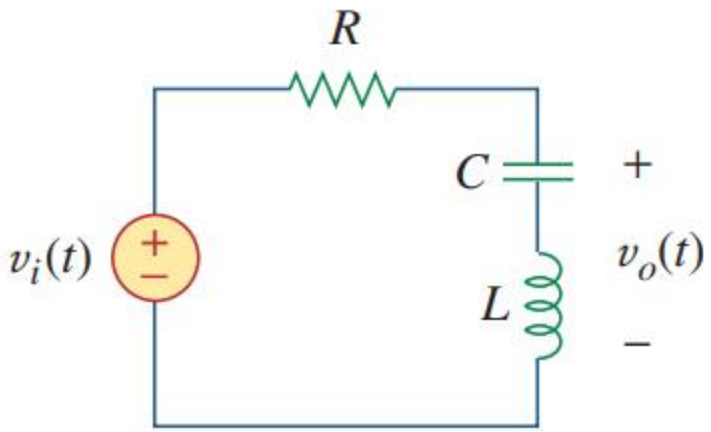
Solution:

use the formulas for a series resonant circuit

$$B = 2\pi(100) = 200\pi \text{ rad/s}$$

But

$$B = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{B} = \frac{150}{200\pi} = 0.2387 \text{ H}$$



Rejection of the 200-Hz sinusoid means that f_0 is 200 Hz, so that ω_0

$$\omega_0 = 2\pi f_0 = 2\pi(200) = 400\pi$$

Since $\omega_0 = 1/\sqrt{LC}$,

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2(0.2387)} = 2.653\ \mu\text{F}$$

Active Filters

- There are three major limitations to the passive filters:
  - Passive elements cannot add energy to the network.
  - They may require bulky and expensive inductors.
  - They perform poorly at frequencies below the audio frequency range
- Nevertheless, passive filters are useful at high frequencies.
- Active filters consist of combinations of resistors, capacitors, and op amps.
- They offer some advantages over passive *RLC* filters.
  - They are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters.
  - They can provide amplifier gain in addition to providing the same frequency response as *RLC* filters
  - Active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects. This isolation allows designing the stages independently and then cascading them to realize the desired transfer function.

First-Order Lowpass Filter

One type of first-order filter is shown in Fig. The components selected for Z_i and Z_f determine whether the filter is lowpass or highpass, but one of the components must be reactive.

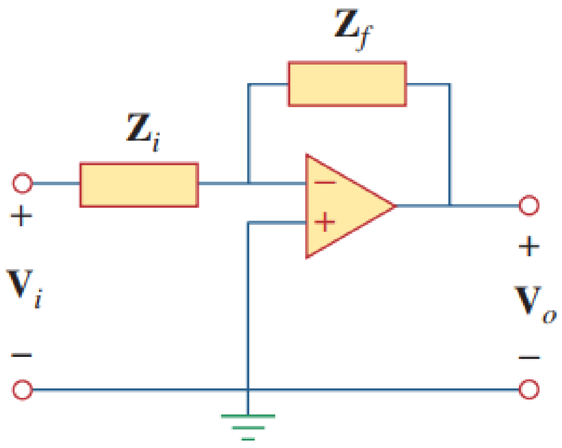


Figure shows a typical active lowpass filter. For this filter, the transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

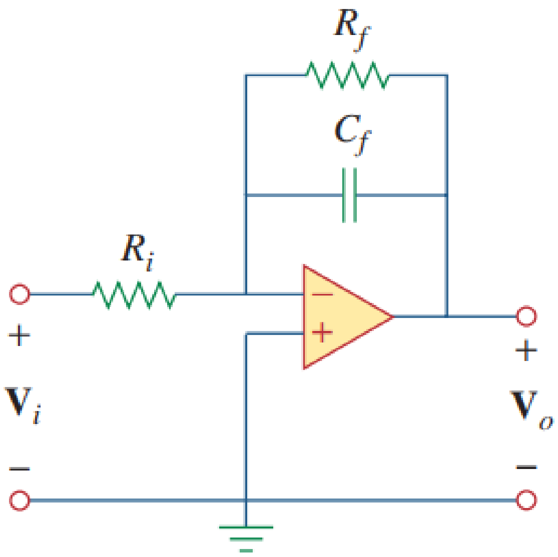
where $\mathbf{Z}_i = R_i$ and

$$\mathbf{Z}_f = R_f \left\| \frac{1}{j\omega C_f} = \frac{R_f/j\omega C_f}{R_f + 1/j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f} \right.$$

Therefore,

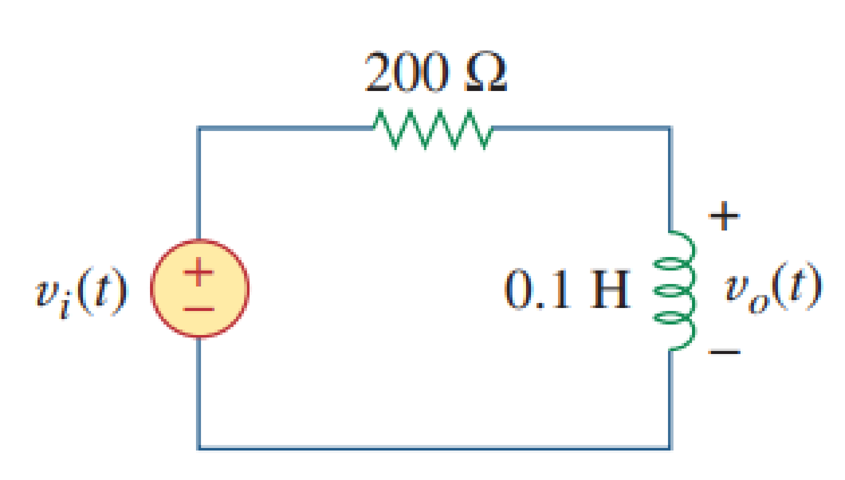
$$\mathbf{H}(\omega) = -\frac{R_f}{R_i} \frac{1}{1 + j\omega C_f R_f}$$

$$\omega_c = \frac{1}{R_f C_f}$$



Practice

- Determine what type of filter is in Fig. shown below. Calculate the corner frequency.



Practice Question

- Find the transfer function for each of the active filters in Fig. below.