DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

PRIVATE BAG 10143, DEDAN KIMATHI, NYERI KENYA

ELECTRICAL & ELECTRONIC ENGINEERING DEPARTMENT

EEE 3208 ELECTROMAGNETICS III

Prerequisites

EEE 3103 ELECTROMAGNETICS II

Purpose of the Course
The aim of this course is to enable the student to understand advanced analytical techniques for formulating and solving problems in applied electromagnetics.

Expected Learning Outcomes
By the end of this course, the learner should be able to;
1. Understand Time Varying Electromagnetics Fields.
2. Understand boundary value problems, with applications to electrostatics and magnetostatics, time varying fields , and radiating systems.
3. Solve Spectral domain field representations - complex multi-valued functions.
4. Understand Radio wave propagation in stratified media.

Course Content
Introduction to Time Varying Electromagnetic Fields: The electromagnetic spectrum. Maxwell's equations - differential and integral forms.

Applications to wave propagation in dielectrics and good conductors.

Uniform plane waves: magnitude and direction; in vacuum, conducting and non-conducting media.

Waves propagation, Wave Propagation in Lossy Dielectrics, Plane Waves in Lossless Dielectrics, Plane Waves in Free Space, Plane Waves in Good Conductors,

Power and the Poynting Vector, Reflection of a Plane Wave at Normal Incidence, and at Oblique Incidence.

Laboratory/Practical Exercises
i) Radio wave propagation
ii) Wave propagation in Dielectric materials
iii) Planes in free space

Course Assessment

Cats	10%
Assignments	5%
Labs	15%
Exam	70%
Total	**100%**

Reference Text Books

Bhag S. G., & Hüseyin R. H. (2004) Electromagnetic field theory fundamentals, Cambridge University Press, 2nd Ed.

Zahn, M. (2003). Electromagnetic Field Theory: A Problem Solving Approach, Krieger Publishing Company, reprint Ed.

Edminister, J.A. (1994). Schaum's Outline of Theory and Problems of Electromagnetics, McGraw-Hill, 2nd Ed.

Dipak L. S, & Valdis V. L. (2006). Applied Electromagnetics and Electromagnetic Compatibility, John Wiley & Sons

Introduction: to Time Varying Electromagnetic Fields:

In Electromagnetics I & II, focus was mainly on **static electric and magnetic fields** (electrostatics and magnetostatics).

Now, we extend to **time-varying fields**, which are the foundation for understanding **wave propagation, radiation, and radio communication systems**.

Key **concept**: When fields vary with time, **electric and magnetic fields are coupled** → a changing electric field induces a magnetic field, and vice versa.

The electromagnetic spectrum.

Electromagnetic (EM) waves are classified based on frequency f or wavelength λ.

Table 1; The Electromagnetic Spectrum

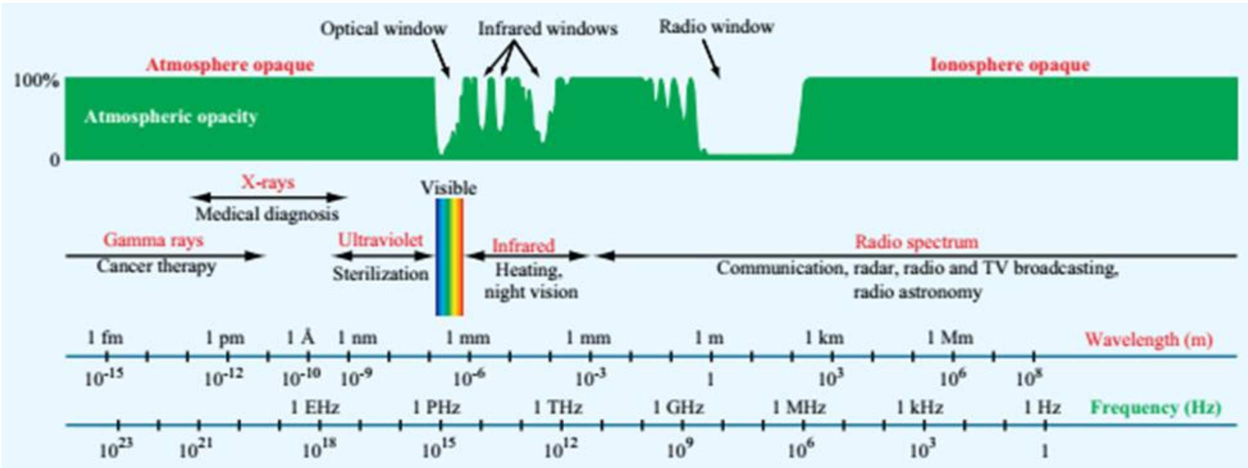
Band Designation	Frequency	Wavelength	Example Uses
ELF (Extremely Low Frequency)	3 to 30 Hz	100 to 10 Mm	
SLF (Super Low Frequency)	30 to 300 Hz	10 to 1 Mm	Power lines
ULF (Ultra Low Frequency)	300 to 3 kHz	1 Mm to 100 km	
VLF (Very Low Frequency)	3 to 30 kHz	100 to 10 km	Submarine comm.
LF (Low Frequency)	30 to 300 kHz	10 to 1 km	RFID
MF (Medium Frequency)	300 kHz to 3 MHz	1 km to 100 m	AM broadcast
HF (High Frequency)	3 to 30 MHz	100 to 10 m	Shortwave broadcast
VHF (Very High Frequency)	30 to 300 MHz	10 to 1 m	FM and TV broadcast
UHF (Ultra High Frequency)	300 MHz to 3 GHz	1 m to 10 cm	TV, WLAN, GPS, Microwave ovens
SHF (Super High Frequency)	3 to 30 GHz	10 to 1 cm	Radar, WLAN, Satellite comm.
EHF (Extremely High Frequency)	30 to 300 GHz	10 to 1 mm	Radar, Radio astronomy, Point-to-point high rate data links, Satellite comm.
Microwaves	1 to 300 GHz	30 cm to 1 mm	
Millimeter waves	30 to 300 GHz	10 to 1 mm	
Submillimeter waves	>300 GHz	<1 mm	

EM waves because they share the following fundamental properties:

- A ***monochromatic*** (single frequency) EM wave consists of electric and magnetic fields that oscillate at the same frequency f.

- The phase velocity of an EM wave propagating in vacuum is a universal constant given by the velocity of light.
- In vacuum, the wavelength λ of an EM wave is related to its oscillation frequency f by $\lambda = \frac{c}{f}$

Region	**Frequency Range**	**Wavelength Range**	**Applications**
Radio (RF)	3 Hz – 300 GHz	100 km – 1 mm	Broadcasting, communication
Microwave	300 MHz – 300 GHz	1 m – 1 mm	Radar, satellite, Wi-Fi
Infrared (IR)	300 GHz – 400 THz	1 mm – 0.7 µm	Remote control, thermal imaging
Visible Light	400 – 790 THz	700 – 380 nm	Human vision
Ultraviolet	790 THz – 30 PHz	380 – 10 nm	Sterilization
X-Rays	30 PHz – 30 EHz	10 nm – 0.01 nm	Medical imaging
Gamma Rays	>30 EHz	<0.01 nm	Nuclear processes



Electromagnetic spectrum

We shall examine situations in which electric and magnetic fields are dynamic, or time varying. It should be mentioned first that in static EM fields, electric and magnetic fields are independent of each other, whereas in dynamic EM fields, the two fields are interdependent. In other words, a time-varying electric field necessarily involves a corresponding time-varying magnetic field. Second, time-varying EM fields, represented by $\boldsymbol{E}(x, y, z, t)$ and $\boldsymbol{H}(x, y, z, t)$, are of more practical value than static EM fields.

In summary:

Stationary charges → electrostatic fields

Steady currents → magnetostatic fields

Time-varying currents → electromagnetic fields (or waves). Time-varying EM fields are governed by **Maxwell's Equations**.

FARADAY'S LAW

Induced emf, *V*emf equal to the time rate of change of the magnetic flux linkage by the circuit.

It can be expressed as

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt} \quad 1$$

Where $\lambda = N\psi$ is the flux linkage, *N* is the number of turns in the circuit, and ψ is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This behavior is described as Lenz's law.

Lenz's law states the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current opposes change in the original magnetic field.

The variation of flux with time as in eq. (1) may be caused in three ways:

1. By having a stationary loop in a time-varying **B** field

2. By having a time-varying loop area in a static **B** field

3. By having a time-varying loop area in a time-varying **B** field

1. Stationary Loop in Time-Varying B Field (Transformer emf)

In Figure 1 a stationary conducting loop is in a time-varying magnetic **B** field

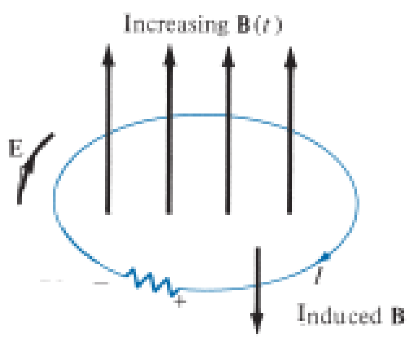


FIGURE 1 Induced emf due to a stationary loop in a timevarying **B** field. From equation

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

we have;

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \qquad 2$$

This emf induced by the time-varying current (producing the time-varying **B** field) in a stationary loop is often referred to as *transformer emf* in power analysis, since it is due to transformer action. By applying Stokes's theorem to the middle term in eq. (2), we obtain

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \qquad 3$$

For the two integrals to be equal, their integrands must be equal; that is,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad 4$$

This is one of the Maxwell's equations for time-varying fields. It shows that the timevarying **E** field is not conservative($\nabla \times E \neq 0$). This does not imply that the principles of energy conservation are violated. The work done in taking a charge about a closed path in a time-varying electric field, for example, is due to the energy from the time-varying magnetic field. Observe that Figure 1 obeys Lenz's law: the induced current I flows such as to produce a magnetic field that opposes the change in $\mathbf{B}(t)$.

2. Moving Loop in Static B Field (Motional emf)

When a conducting loop is moving in a static **B** field, an emf is induced in the loop. We recall) that the force on a charge moving with uniform velocity **u** in a magnetic field **B** is;

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

We define the *motional electric field* $\mathbf{E}m$ as

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

If we consider a conducting loop, moving with uniform velocity **u** as consisting of a large number of free electrons, the emf induced in the loop is;

$$V_{emf} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \qquad 5$$

This type of emf is called *motional emf* or *flux-cutting emf* because it is due to motional action. It is the kind of emf found in electrical machines such as motors, generators, and alternators.

By applying Stokes's theorem to eq. 5), we have

$$\int_S (\nabla \times \mathbf{E}_m) \cdot d\mathbf{S} = \int_S \nabla \times (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{S}$$

Or

$$\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B}) \qquad 6$$

3. Moving Loop in Time-Varying Field

In the general case, a moving conducting loop is in a time-varying magnetic field. Both transformer emf and motional emf are present. Combining eqs. (2) and (5) gives the total emf as;

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \qquad 7$$

Or

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B}) \qquad 8$$

DISPLACEMENT CURRENT

We have reconsidered Maxwell's curl equation for electrostatic fields and modified it for time-varying situations to satisfy Faraday's law. We shall now reconsider Maxwell's curl equation for magnetic fields (Ampère's circuit law) for timevarying conditions.

For static EM fields, we recall that

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad 9$$

But the divergence of the curl of any vector field is identically zero. Hence,

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} \qquad 10$$

The continuity of current however, requires that

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \qquad 11$$

Thus eqs. (10) and (11) are obviously incompatible for time-varying conditions. We must modify eq. (9) to agree with eq. (11). To do this, we add a term to eq. (9) so that it becomes;

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d \qquad 12$$

where $\mathbf{J}_d$ is to be determined and defined. Again, the divergence of the curl of any vector is zero. Hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d \qquad 13$$

In order for eq. (13) to agree with eq. (11),

$$\nabla \cdot \mathbf{J}_d = -\nabla \cdot \mathbf{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

Or

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt} \qquad 14$$

Substituting eq. (14) into eq. (12) results in

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad 15$$

This is Maxwell's equation (based on Ampère's circuit law) for a time-varying field. The insertion of $\mathbf{J}_d$ into eq. (9) was one of the major contributions of Maxwell. Without the term $\mathbf{J}_d$, the propagation of electromagnetic waves (e.g., radio or TV waves) would be impossible. At low frequencies, $\mathbf{J}_d$ is usually neglected compared with $\mathbf{J}$. However, at radio frequencies, the two terms are comparable.

MAXWELL'S EQUATIONS IN FINAL FORMS

TABLE 2 Generalized Forms of Maxwell's Equations

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$	Ampère's circuit law

*This is also referred to as Gauss's law for magnetic fields.

Where;

$\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ (*magnetic field intensity*, A/m),

$\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ (*electric field intensity*, V/m),

$\mathbf{D} = \mathbf{D}(\mathbf{r}, t)$ (*electric flux density*, C/m^2),

$\mathbf{B} = \mathbf{B}(\mathbf{r}, t)$ (*magnetic flux density*, W/m^2),

$\rho = \rho(\mathbf{r}, t)$ (*volumetric charge density*, C/m^3),

$\mathbf{J} = \mathbf{J}(\mathbf{r}, t)$ (*current density*, A/m^2),

and r is the position vector for an ordinary point in the medium. Here ordinary point refers to a point wherein within its immediate neighborhood the physical properties of the medium are continuous. In other words, the small medium around r is considered to be homogeneous.

The concepts of linearity, isotropy, and homogeneity of a material medium still apply for time-varying fields; in a linear, homogeneous, and isotropic medium characterized by $\sigma, \varepsilon,$ and μ, the constitutive relations

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P} \quad 16$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_o (\mathbf{H} + \mathbf{M}) \quad 17$$

$$\mathbf{J} = \sigma \mathbf{E} + \rho_v \mathbf{u} \quad 18$$

Hold for time-varying fields. Consequently, the boundary conditions remain valid for timevarying fields, where $\mathbf{a}_n$ is the unit normal vector to the boundary.

$$E_{1t} - E_{2t} = 0 \quad \text{or} \quad (\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{a}_n = \mathbf{0}$$

$$H_{1t} - H_{2t} = K \quad \text{or} \quad (\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_n = \mathbf{K}$$

$$D_{1n} - D_{2n} = \rho_s \quad \text{or} \quad (\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{a}_n = \rho_s$$

$$B_{1n} - B_{2n} = 0 \quad \text{or} \quad (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{a}_n = 0$$

However, for a perfect conductor ($\sigma = \infty$) in a time-varying field,

$$\mathbf{E} = \mathbf{0}, \quad \mathbf{H} = \mathbf{0}, \quad \mathbf{J} = \mathbf{0}$$

And hence,

$$\mathbf{B}_n = \mathbf{0}, \quad \mathbf{E}_t = \mathbf{0}$$

TIME-HARMONIC FIELDS

A **time-harmonic field** is one that varies periodically or sinusoidally with time.

TABLE 3 Time-Harmonic Maxwell's Equations Assuming Time Factor $e^{j\omega t}$

Point Form	Integral Form
$\nabla \cdot \mathbf{D}_s = \rho_{vs}$	$\oint \mathbf{D}_s \cdot d\mathbf{S} = \int \rho_{vs}\, dv$
$\nabla \cdot \mathbf{B}_s = 0$	$\oint \mathbf{B}_s \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E}_s = -j\omega \mathbf{B}_s$	$\oint \mathbf{E}_s \cdot d\mathbf{l} = -j\omega \int \mathbf{B}_s \cdot d\mathbf{S}$
$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega \mathbf{D}_s$	$\oint \mathbf{H}_s \cdot d\mathbf{l} = \int (\mathbf{J}_s + j\omega \mathbf{D}_s) \cdot d\mathbf{S}$