EML EEE 2205

ELECTROMAGNETICS 1

Course Outline

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⇒ Permittivity : ε :

⇒ Relative Permittivity ε_o :

Units for Permittivity [Farad / m]

Coulombs Law

$$F = \frac{1}{4\pi\varepsilon} \frac{Q_1 \cdot Q_2}{r^2} \bar{a}_r \quad N$$

$$\bar{a}_r = \frac{\bar{a}}{|a|}$$

⇒ Electric Field Intensity: Force per unit charge

$$\boxed{\bar{E} = \frac{F}{Q}\,\bar{a}_r \ N/C \text{ or } Volt/m}$$

⇒ Capacitance:

⇒ Voltage:

Coulombs Law Questions.

1. $Q_1 = 2.0 \times 10^{-19}\ C$

$Q_2 = -0.5 \times 10^{-19}\ C$

At 4cm apart, Find: $= 4 \times 10^{-2}\ m$

a) Force

b) Force if they are brought together and separated

a) $F = \frac{1}{4\pi\varepsilon} \cdot \frac{Q_1 \cdot Q_2}{r^2}\,\bar{a}_r\ N$

-ve ⇒ Attraction force

+ve ⇒ Repulsive force

$$F = \frac{1}{4 \times \frac{22}{7} \times 8.854 \times 10^{-12}} \cdot \frac{2.0 \times 10^{-19} \cdot (-0.5 \times 10^{-19})}{(4 \times 10^{-2})^2}$$

$$F = \frac{2 \times (-0.5)}{4 \times \frac{22}{7} \times 8.854 \times 16} \times \frac{10^{-19} \times 10^{-19}}{10^{-4} \times 10^{-12}}$$

$$\underline{F = -5.6173 \times 10^{-6}\ \bar{a}_r\ N}$$

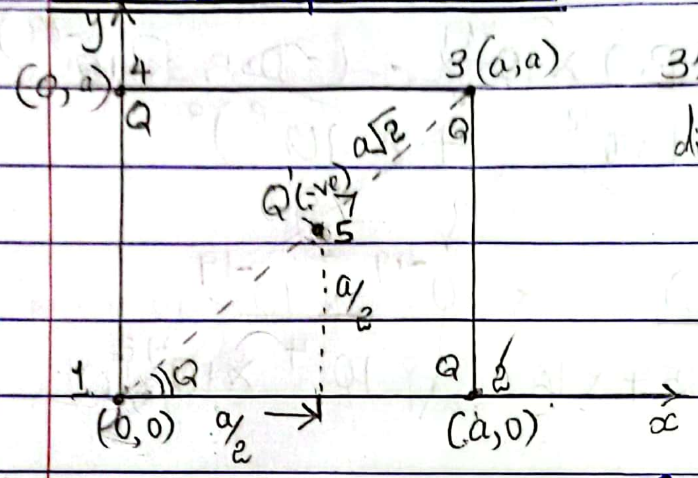
b) When brought together the values of charge in Q_1 and Q_2 will change i.e $Q_1' = Q_2' = \frac{Q_1 + Q_2}{2}$

$$= \frac{2.0 \times 10^{-19} + (-0.5 \times 10^{-9})}{2}$$

$$= 2 \times 10^{-} \quad \underline{0.75\ nC}$$

$$F' = \frac{Q_1' Q_2'}{4 \pi \varepsilon r^2}\ \bar{a}_r\ N$$

Practice Question 1



31 ⇒ Neither in y nor in x direction.

It is required to hold ~~from~~ four equal point charges to each in equilibrium at the corners of a square. Find the point charge which will do this if placed at the centre of the square.

forcing acting on 1 coz of 2

$$F_{21} = \frac{Q^2}{4\pi\varepsilon a^2}(-\bar{a}_x)\ N \qquad F_{51} = \frac{1 Q^2}{4\pi\varepsilon}$$

$$F_{41} = \frac{Q^2}{4\pi\varepsilon a^2}(-\bar{a}_y)\ N$$

$$F_{31} = \frac{Q^2}{4\pi\varepsilon(\sqrt{2}a)^2}(\bar{a}_{31})\ N$$

$$\boxed{= \frac{Q^2}{8\pi\varepsilon a^2}\left[\underbrace{\cos\alpha(-a_x) + \sin\alpha(-\bar{a}_y)}_{\text{Direction}}\right]}$$

$\alpha = 45°$

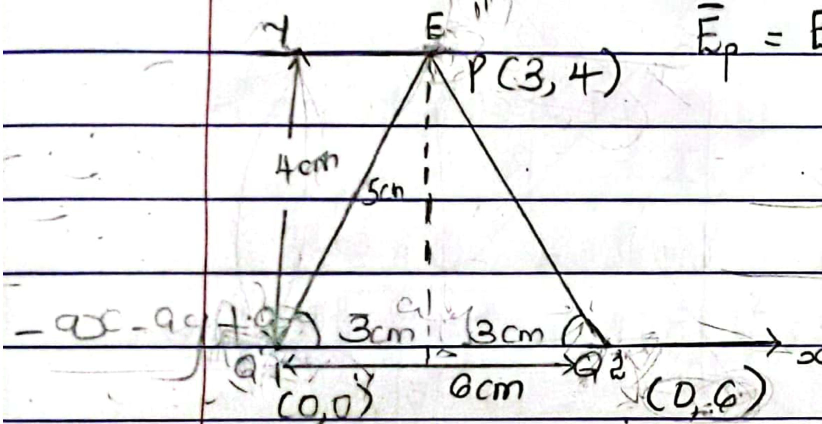
$$F_{51} = \frac{Q\,Q'\,\bar{a}_{51}}{4\pi\varepsilon(|R_{51}|)^2} = \frac{QQ'}{4\pi\varepsilon r^2}\,\bar{a}_r = \frac{QQ'}{2\sqrt{2}\pi\varepsilon a^2}\,(\bar{a}_x+\bar{a}_y)$$

For equilibrium $\Sigma F = 0 = F_{21} + F_{41} + F_{31} + F_{51}$

$Q' = -0.9569\,Q$

Practice Question 2.

$\bar{E}_p = E_{(3,4)} = E_{P_1} + E_{P_2}$



$V = -Ed$

Find E at the vertex of an isosceles triangle if equal charge = 10 mC are placed at the base of the given triangle.

Electric Field

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\,\bar{a}_r \ (V/m) \qquad \bar{E} = \frac{F}{Q}}$$

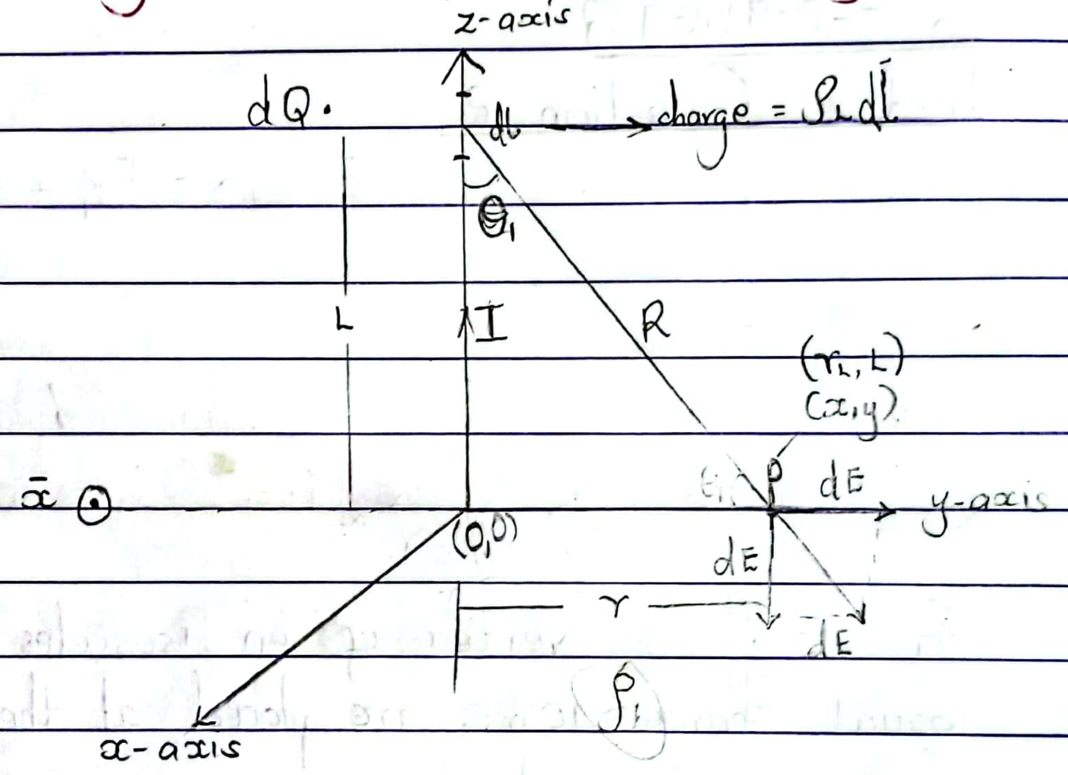
$$E_1 = \frac{10\times10^{-3}}{4\pi\varepsilon(5\times10^{-2})^2}\,\bar{a}_r \qquad \bar{E} = \bar{E}_1 + \bar{E}_2 = 2\left[\frac{10\times10^{-3}}{\frac{1}{9}\times 2}\cdot 4a\right.$$

Line charge: Charge distribution along a one-dimensional curve or line L in space.

Line charge density: the quantity of charge per unit length, measured in Coulombs per meter ($C \cdot m^{-1}$) at any point on a line charge distribution.

$$\bar{E} = \bar{E}_1 + \bar{E}_2 = 2\left[\frac{10 \times 10^{-3}}{\frac{1}{9} \times 10^{-9} \times 25}\right] \cdot (+a_y)$$

Finding Field Of A Line Charge



Charge density * Length = Charge

* Let ρ_L is the charge density (c/m)

1. Point
2. Line
3. Surface ρ_s (C/m^2)
4. Volume ρ_v (C/m^3)

electric field intensity due to each of the charge distributions ρ_L, ρ_s, ρ_v may be regarded as the summation of the field contributed by the numerous point charges making up the charge distribution.

i.e $E = \int \frac{\rho_L dl}{4\pi\epsilon_0 R^2} a_R$ (line charge) $E = \int \frac{\rho_s ds}{4\pi\epsilon_0 R^2} a_R$ (surface) $E = \int \frac{\rho_v dv}{4\pi\epsilon_0 R^2} a_R$ (volume)

$$\frac{1}{4\pi\varepsilon} = 9\times10^{+9}\ \mathrm{M/F}$$ $[\rho, \phi, L]$

ρ without subscript = radial distance in cylindrical coordinate

$$E = \frac{Q}{4\pi\varepsilon r^2}\,\bar{a}_r\ (N/L) = \frac{\rho_l\, d\bar{l}}{R\sqrt{L^2+\rho^2}} \Big/ \frac{\rho_l\, dl}{R\sqrt{x^2+y^2}}$$

vector

$$\boxed{\sin\theta_1 = y/R} \qquad \boxed{\frac{\rho_l\, dl \sin\theta}{4\pi\varepsilon\sqrt{x^2+y^2}}} \Rightarrow \boxed{\frac{\rho_l\, dl}{4\pi\varepsilon\sqrt{x^2+y^2}}\cdot\frac{y}{R}}$$

$$R^2 = L^2 + \rho^2 = x^2 + y^2$$

$$\frac{\rho_l\, dl \sin\theta}{4\pi\varepsilon\sqrt{x^2+y^2}} = \frac{\rho_L \cdot \rho}{4\pi\varepsilon(L^2+\rho^2)^{3/2}}\,dl$$ (ρ, x)

let $\rho = L\cot\theta$

$L^2 + L^2\cot\theta$

$\bar{a}_x$

$$\boxed{E_\rho = \frac{\rho_L}{2\pi\varepsilon_0\rho}\cdot\bar{a}_\rho}$$

$\Rightarrow$ Incase of point charge, E varies inversely with the square of distance. $\frac{Q}{4\pi\varepsilon r^2}$

$\Rightarrow$ Incase of line charge, E varies inversely with distance.

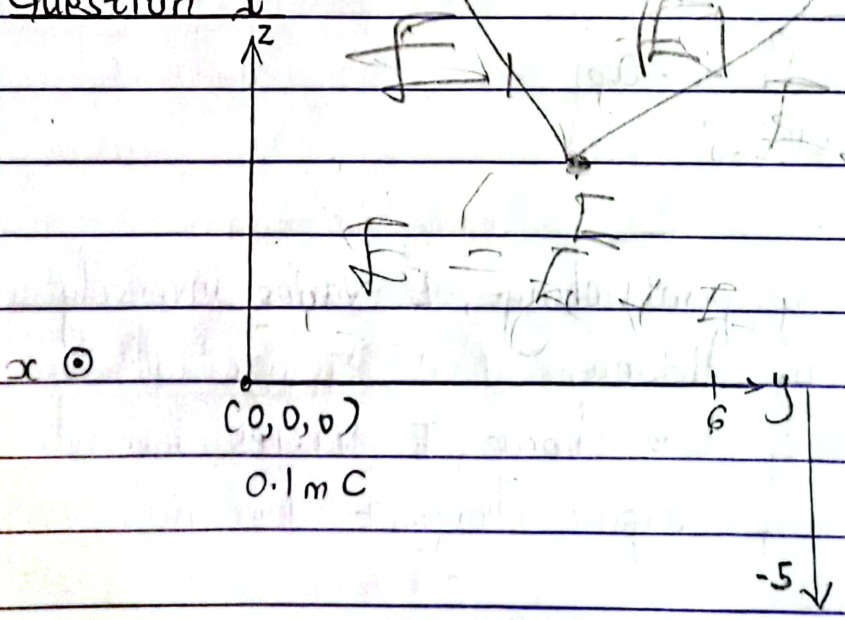
$\Rightarrow$ Incase of surface charge, E does not vary i.e $E_\rho = \frac{\rho_L}{2\pi\varepsilon}$

Examples

1. Find the electric field intensity at $P(-4, 6, -5)$ in free space caused by a charge at $0.1\,mC$ at
a) Origin
b) $(2, -1, -3)$

2. A point $Q_1 = 2\mu C$ is located at $P_1(-3, 7, -4)$ in free space, while $Q_2 = -5\mu C$ is at $P_2(2, 4, -1)$. At the point $(12, 15, 18)$. Find:
a) $\bar{E}$
b) $|E|$
c) $\bar{a}_E$

Question 1



a) $\bar{E} = \frac{Q}{4\pi\varepsilon r^2} \bar{a}_r$ $\qquad \bar{E} = \frac{0.1 \times 10^{-3}}{4\pi\varepsilon \times (8.775)^2}$

$r = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{(-4)^2 + 6^2 + (-5)^2} = \underline{8.775}$

$\bar{a}_r = \bar{r}/|r|$ $\qquad \bar{E} = \frac{0.1 \times 10^{-3}}{4\pi\varepsilon \times (77)}$

Vector = $-4 + 6 - 5$ $\qquad \bar{a}_r = \frac{-3}{8.775} = -0.34188$

Absolute magnitude = 8.775

$\mathcal{E}$ 8. Ans =

a) $5.32\,\bar{a}_x + 7.98\,\bar{a}_y - 6.659\,\bar{a}_z$ N/C

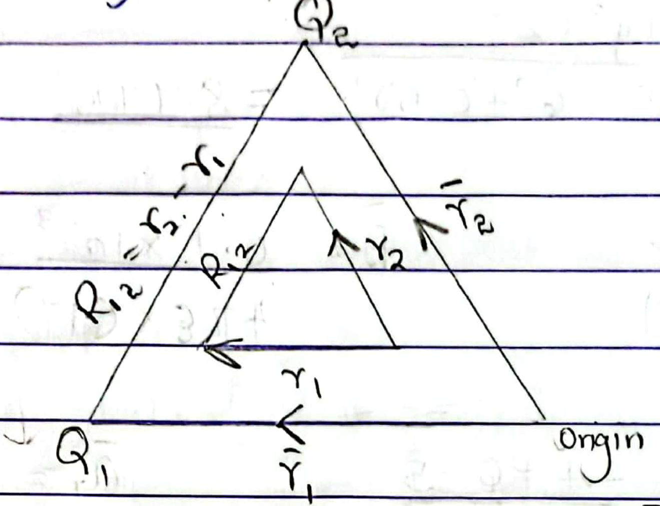
b) Ans = $(-6.42\,\bar{a}_x + 7.98\,\bar{a}_y - 2.14\,\bar{a}_z$ KV/m)

1.341×10^{12}

Position vector: Distance of the vector from the origin directed from the origin to the vector.

3. $Q_1 = 3\times10^{-4}$ $(1,2,3)$ C, $Q_2 = -10^{-4}$ C $(2,0,5)$

Find the force between them.



$F_{21} = \dfrac{Q_1 Q_2}{4\pi\varepsilon R_{12}^2}\,\bar{a}_{12}$ where $\bar{a}_{12} = \dfrac{\bar{R}_{12}}{|R_{12}|}$ $\Rightarrow$ Position vector, $\Rightarrow$ Magnitude

$= \dfrac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|} \longrightarrow (2-1)\bar{a}_x + (0-2)\bar{a}_y + (5-3)\bar{a}_z$

$\bar{r}_2 - \bar{r}_1 = \bar{a}_x - 2\bar{a}_y + 2\bar{a}_z$

$|\bar{r}_2| = \sqrt{2^2 + 0^2 + 5^2} = 5.3852$

$|\bar{r}_1| = \sqrt{1^2 + 2^2 + 3^2} = \cancel{3.6056}\ 3.7417$

$|\bar{r}_2 - \bar{r}_1| = \sqrt{1^2 + (-2^2) + (2)^2} = \underline{\underline{3}}$

Dot product $A \cdot B = AB \cos\theta_{AB}$

Cross product $A \times B = AB \sin\theta_{AB}\, a_n$

$|\ | =$ Magnitude / Ab...

$$\bar{a}_{12} = \frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}$$

$$F_2 = \frac{3\times10^{-4} \times (-10^{-4})}{4\pi \cdot \frac{1}{36\pi} \times 10^{-9} \times (3)^2} \cdot \frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}$$

(i)

$$F_2 = \frac{3 \times 1}{\cancel{\frac{1}{9}} \times \cancel{9}} \cdot \frac{10^{-4} \times 10^{-4}}{10^{-9}} \cdot \frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}$$

$$F_2 = [3 \times 10] \cdot \frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}$$

$$\text{Ans} = 30\left(\frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}\ N\right) \qquad |F_2| = 30N$$

$$\text{Or} = 10\,\bar{a}_x - 20\,\bar{a}_y + 20\,\bar{a}_z\ N$$

$$\boxed{F_1 = -F_2}$$

Position vector ... value

4 $Q_1 = 2mC$ at $(-3, 7, -4)$ and $Q_2 = 5mC$ at $(2, 4, -1)$. Find the force on Q_2

Ans $= -159\,\bar{a}_x + 0.956\,\bar{a}_y - 0.956\,\bar{a}_z$ N

ii) Find force on Q_1

Ans $= -ve\ (\hat{i})$

Short Notes:

Electromagnetics: it is a branch of physics or electrical engineering which is used to study the electric and magnetic phenomena.

Field: it is a function that specifies a quantity everywhere in a region or a space.

Magnetic field: It is the field produced due to magnetic effects.

Electric field: it is the field produced due to an electric charge.

Scalar quantity: A quantity which is wholly characterized by its magnitude.

Vector quantity: A quantity which is characterized by both, magnitude and direction.

Scalar field e.g: temperature of atmosphere

Vector field eg: velocity of particles in a moving fluid

Laws: Cummulative → $A + B = B + A$

Associative → $\bar{A} + (\bar{B} + \bar{C}) = (\bar{A} + \bar{B}) + \bar{C}$

Distributive → $\alpha(\bar{A} + \bar{B}) = \alpha\bar{A} + \alpha\bar{B}$

NB: Letter $\bar{a}$ is used to indicate the unit vector and its suffix indicates the direction of the unit vector. Thus $\bar{a}_x$ indicates the unit vector along x axis direction

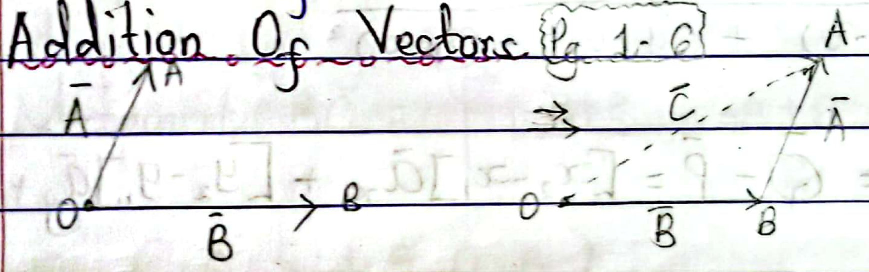
$$\boxed{\text{Unit Vector } \bar{a}_{OA} = \frac{\overline{OA}}{|OA|}}$$

Multiplication By a Scalar

Multiplication of a vector by $-$ve, changes the direction of the vector.

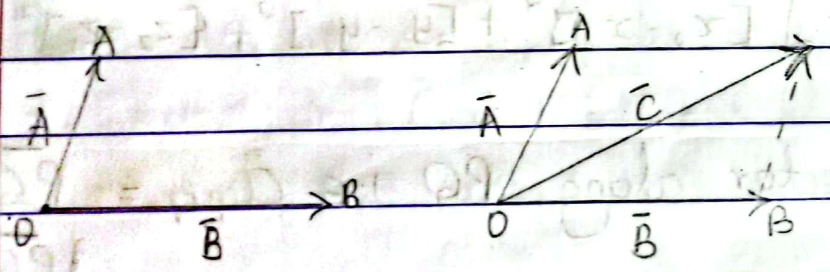
Addition Of Vectors (Pg. 1-6)

Method 1



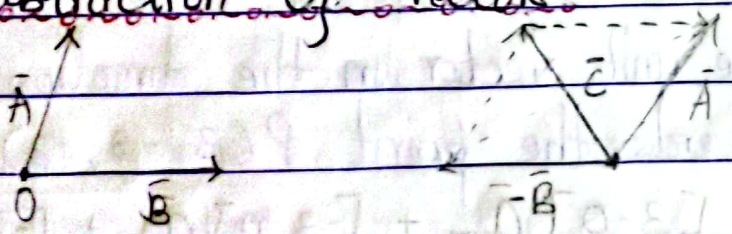
$\bar{C} = \bar{A} + \bar{B}$

Method 2 Parallelogram rule



$\bar{C} = \bar{A} + \bar{B}$

Subtraction Of Vectors:



$\bar{C} = \bar{A} - \bar{B}$

Co-ordinate Systems

(right handed system, left handed system)

- ✓ Cartesian or rectangular co-ordinate system
- ✓ Cylindrical co-ordinate system.
- ✓ Spherical co-ordinate system.

i) Cartesian Co-ordinate System

Position Vector: example $\bar{r}_{op} = x_1 \bar{a}_x + y_1 \bar{a}_y + z_1 \bar{a}_z$.

i.e $\Rightarrow (x_1 - 0)\bar{a}_x + (y_1 - 0)\bar{a}_y + (z_1 - 0)\bar{a}_z$

Magnitude of Vector: example $|\bar{r}_{op}| = \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2 + (z_1 - 0)^2}$

$$\bar{PQ} = \bar{Q} - \bar{P} = [x_2 - x_1]\bar{a}_x + [y_2 - y_1]\bar{a}_y + [z_2 - z_1]\bar{a}_z$$

$$|\bar{PQ}| = \sqrt{[x_2 - x_1]^2 + [y_2 - y_1]^2 + [z_2 - z_1]^2}$$

Unit Vector along PQ i.e $\bar{a}_{PQ} = \dfrac{\bar{PQ}}{|\bar{PQ}|}$

Example:

1. Obtain the unit vector in the direction from the origin towards the point $P(3, -3, -2)$

$$\bar{OP} = [3-0]\bar{a}_x + [-3-0]\bar{a}_y + [-2-0]\bar{a}_z$$

$$\bar{OP} = 3\bar{a}_x - 3\bar{a}_y - 2\bar{a}_z$$

$$|\bar{OP}| = \sqrt{3^2 + (-3)^2 + (-2)^2} = 4.69042$$

$\therefore$

$$\bar{a}_{OP} = \frac{3\bar{a}_x - 3\bar{a}_y - 2\bar{a}_z}{4.69042}$$

$$= 0.6396\,\bar{a}_x - 0.6396\,\bar{a}_y - 0.4264\,\bar{a}_z$$

2. Given three points in cartesian co-ordinate system as $A(3,-2,1)$, $B(-3,-3,5)$, $C(2,6,-4)$. Find the vector from A to the midpoint of the straight line joining B to C.

$$\text{Midpoint of } BC = \left[\frac{-3+2}{2}, \frac{-3+6}{2}, \frac{5-4}{2}\right]$$

$$= (-0.5, 1.5, 0.5)$$

Hence vector from A to midpoint of BC

$$= [-0.5-3]\,\bar{a}_x + [1.5-(-2)]\,\bar{a}_y + [0.5-1]\,\bar{a}_z$$

$$= -3.5\,\bar{a}_x + 3.5\,\bar{a}_y - 0.5\,\bar{a}_z$$

Cylindrical Co-ordinate System (Pg. 1-15)

Has axes / points

$0 \le r < \infty$ — ϕ is expressed in radians

$0 \le \phi \le 2\pi$ — $P(r_1, \phi_1, z_1)$

$-\infty < z \le \infty$ — clockwise $\phi = -ve$

Anticlockwise $\phi = +ve$

$dl = dr\,\bar{a}_r + r\,d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$

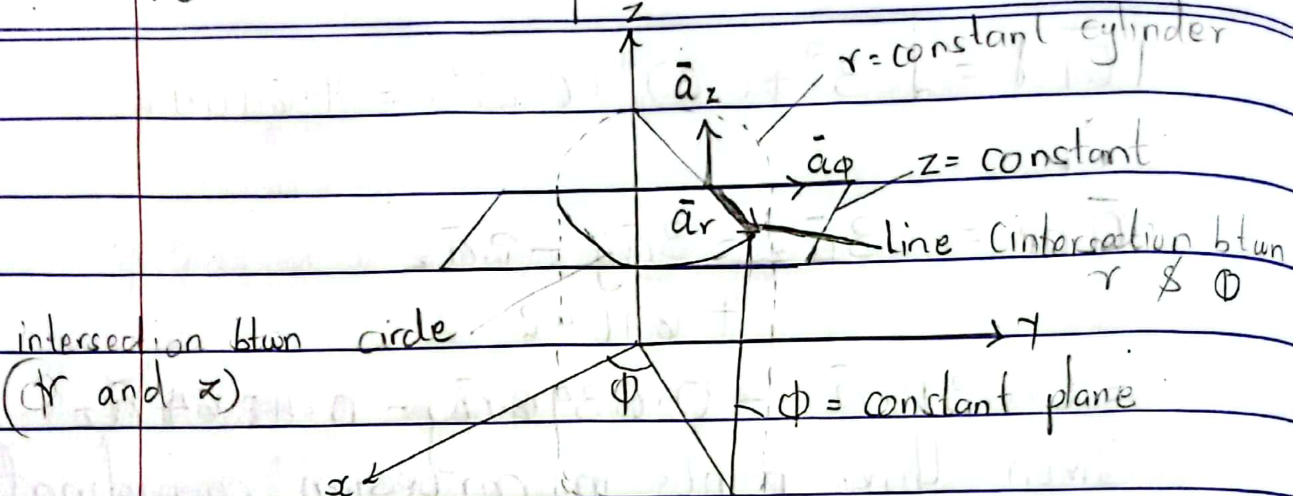
$|\bar{dl}| = \sqrt{(dr)^2 + (r\,d\phi)^2 + (dz)^2}$

$dv = r\,dr\,d\phi\,dz$

$d\bar{S}_r = r\,d\phi\,dz\,a_r$

$d\bar{S}_\phi = dr\,dz\,\bar{a}_\phi$

$d\bar{S}_z = r\,dr\,d\phi\,\bar{a}_z$



(Base Vectors) $\bar{a}_r, \bar{a}_\phi, \bar{a}_z$ $\quad \bar{P} = P_r\bar{a}_r + P_\phi\bar{a}_\phi + P_z\bar{a}_z$

Position vector and magnitude is found using the same procedure as in cartesian plane.

Relationship Between Cartesian And Cylindrical Systems.

$$\boxed{x = r\cos\phi,\quad y = r\sin\phi,\quad z = z}$$

Whereby $r = \sqrt{x^2+y^2}$ $\quad \tan\phi = y/x$

Transformation from cartesian to cylindrical

$r = \sqrt{x^2+y^2}$, $\phi = \tan^{-1} y/x$ & $z = z$

r is alway either +ve or 0

NB: For ϕ if x and y are +ve then ϕ is in 1st quadrant, if x is -ve and y +ve then ϕ is in 2nd quadrant $\Rightarrow$ $\phi = +90/+180$ i.e $<180° \Rightarrow -270°$

e.g $x = -2$ $y = 1$ $\quad \phi = \tan^{-1}\left[\frac{1}{-2}\right] = -26.56°$ but $\Rightarrow$ $-26.56° + 180$

when y is -ve and x is +ve, ϕ is in 4^{th} quadrant i.e within $0^\circ \to 90^\circ$ i.e 270° to 360°.

when x & y are -ve, ϕ is in 3^{rd} quadrant i.e ϕ is within -90° to -180° i.e $180 \to 270$

so 180° must be subtracted from ϕ

e.g $x = y = -3$ then $\phi = \tan^{-1}\left[\frac{-3}{-3}\right] = +45$

$45 - 180^\circ = -135$ i.e $-135 + 360^\circ = 225^\circ$

Example 1 (Pg 1-20)

1. Consider a cylinder of length L and radius R. Obtain its volume by integration.

$dv = r\,dr\,d\phi\,dz \qquad r \to R,\ z \to L,\ \phi \to 2\pi$

$$\int_0^1 \int_0^{2\pi} \int_0^R r \qquad \int_0^1 \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^R d\phi\, dz$$

$$= \frac{R^2}{2} \int_0^1 [\phi]_0^{2\pi} dz \qquad 2\pi \frac{R^2}{2} [z]_0^L = \underline{\underline{\pi R^2 L}}$$

$$= \pi R^2 L$$

Spherical Co-ordinate System

Ranges of Variables

$0 \leq r < \infty$

$0 \leq \phi \leq 2\pi$

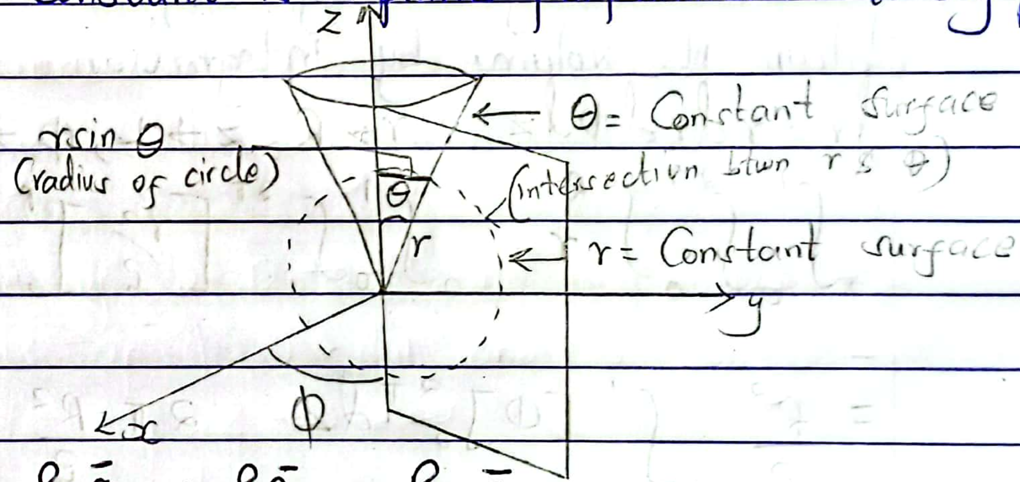
$0 \leq \theta \leq \pi$ as half angle

$P(r, \theta, \phi)$

r = Constant which is a sphere with center as origin.

θ = Constant which is right circular zocone with apex as origin & axis as z-axis.

ϕ = Constant is a plane perpendicular to xy plane



Base Vectors $\bar{P} = P_r \bar{a}_r + P_\theta \bar{a}_\theta + P_\phi \bar{a}_\phi$

$$\bar{dl} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi$$

$$|\bar{dl}| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta\,d\phi)^2}$$

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$d\bar{S}_r = r^2 \sin\theta \, d\theta \, d\phi$

$d\bar{S}_\theta = r \sin\theta \, dr \, d\phi$

$d\bar{S}_\phi = r \, dr \, d\theta$

relationship btwn Cartesian And Spherical Systems

$x = r\sin\theta\cos\phi$ $\quad z = r\cos\theta$

$y = r\sin\theta\cos\phi$

$r = \sqrt{x^2+y^2+z^2}$ $\quad \tan\phi = y/x$ $\quad \cos\theta = z/r$

Example: Pg 1-27

Calculate the volume of a sphere of radius R using integration.

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$r \to R \quad \theta \to \pi \quad \phi \to 2\pi$

$$v = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\int_0^{2\pi}\int_0^{\pi}\left[\frac{r^3}{3}\right]_0^R \sin\theta = \frac{R^3}{3}\int_0^{2\pi}[-\cos\theta]_0^{\pi} \, d\phi$$

$$= \frac{R^3}{3}\left[-\cos\pi - (-\cos 0)\right]\int_0^{2\pi} d\phi = \frac{R^3}{3}\left[(-(-1)) - (-1)\right]$$

$$[\phi]_0^{2\pi}$$

$$= \frac{R^3}{3} \times 2 \times 2\pi = \frac{4}{3}\pi R^3$$

Orthogonal system: system in which the co-ordinates are mutually perpendicular.

st P·1

Co-ordinate Systems

Cartesian co-ordinate system (orthogonal system): A co-ordinate system in which the three axes are mutually perpendicular to each other.

Unit Vector:

Circular Cylindrical Co-ordinates (ρ, ϕ, z)

$z \rightarrow$ units (meters)

$\rho \rightarrow$ Perpendicular distance from z axis to the point in consideration. (Measured in meters)

$\phi \rightarrow$ The angle between a plane from ρ and perpendicular to the x-axis. / angle of projection of ρ in the x-y plane.

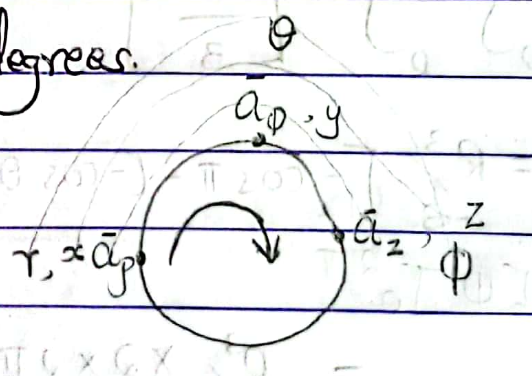
NB $\Rightarrow$ Limits of ρ $0 \rightarrow \infty$ and is always +ve

Limits of ϕ $0 \rightarrow 2\pi$ and is always +ve

Limits of z $-\infty \rightarrow \infty$

$\bar{a}_\phi \rightarrow$ is not in degrees.

NB: $a_\rho \cdot a_\rho = 1$



Stokes theorem

Transformation From Rect to Cylindrical system

$$\boxed{\rho = \sqrt{x^2+y^2}, \quad \phi = \tan^{-1} y/x \quad z = z}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Spherical Co-ordinate System (r, θ, ϕ)

r - line from origin to the point in consideration/ radius of the sphere.

θ - angle between the z - axis and the line r

NB:

ϕ - The angle between a plane from ρ and perpendicular to the x - axis / angle of projection of ρ in the x-y plane.

DEL Operator $\bar{\nabla}$

Operator with 4 - operations

$$\bar{\nabla} = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

In 3-D:

1. Gradient of a scalar V, written as $\bar{\nabla} V$
2. The divergence of a vector A, written as $\bar{\nabla} \cdot A$
3. The Curl of a vector A, written as $\bar{\nabla} \times A$
4. The Laplacian of a scalar V, written as $\bar{\nabla}^2 V$

⇒ The gradient of a scalar field V is a vector that represents both the magnitude and the direction

1. The magnitude of $\bar{\nabla} V$ equals the maximum rate of change in V per unit distance.

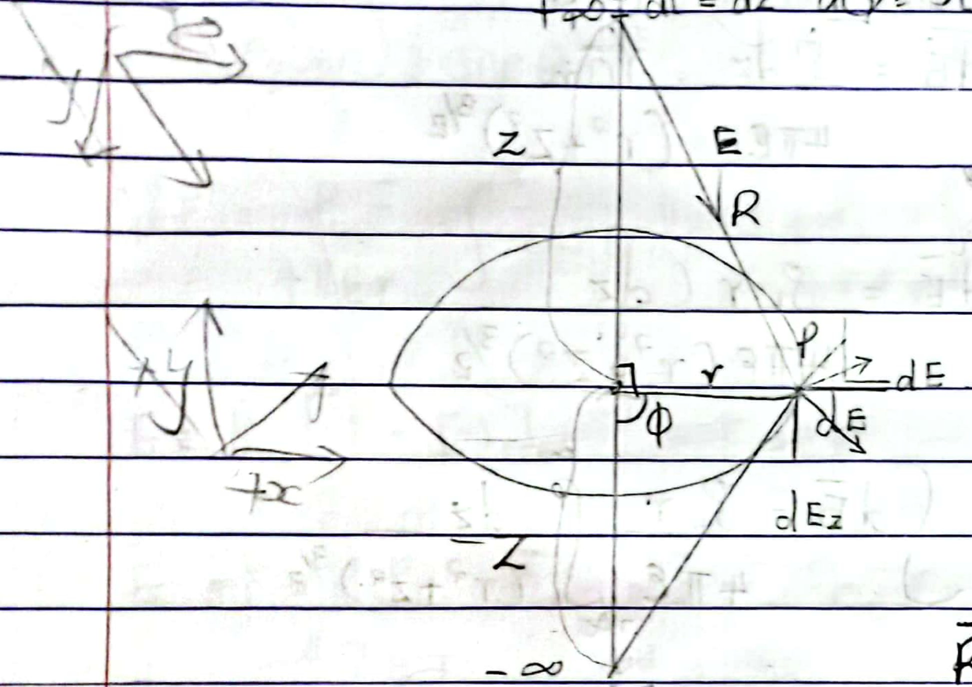
2.

E Due To Line Charge

Example:

Find E due to uniform ρ_L along a filamentary wire of infinite length

$dl = dz \quad dQ = \rho_L \, dl$



$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2} \bar{a}_r \qquad \bar{a}_r = \frac{\bar{R}}{|R|} \quad \frac{\bar{a}_r \Rightarrow \text{Vector}}{|a_r| \Rightarrow \text{Magnitude}}$

$dE = \frac{dQ}{4\pi\epsilon R^2} \frac{\bar{R}}{|R|}$ where $\bar{R} = r\bar{a}_r - Z\bar{a}_z$

$|R| = \sqrt{r^2 + z^2}$

Because of the symmetry, the z component cancels since each $\rho_L dl$ will have $-\rho_L dl$ charge on the opposite sides.

$$d\bar{E} = \rho_L dl \Rightarrow \frac{\rho_L dz}{4\pi\epsilon} \cdot \frac{r\bar{a}_r}{(r^2+z^2)\sqrt{r^2+z^2}}$$

$$\Rightarrow d\bar{E} = \frac{\rho_L dz}{4\pi\epsilon} \cdot \frac{r\bar{a}_r}{(r^2+z^2)^{3/2}}$$

$$\Rightarrow d\bar{E} = \frac{\rho_L r\, dz}{4\pi\epsilon(r^2+z^2)^{3/2}}$$

$$E \Rightarrow \int d\bar{E} = \frac{\rho_L r}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{dz}{(r^2+z^2)^{3/2}}$$

$$\bar{E} = \frac{\rho_L r}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{dz}{(r^2+z^2)^{3/2}}$$

Put $z = r\tan\theta$ $\qquad r^2+z^2 = r^2\tan^2\theta + r^2$

$dz = r\sec^2\theta\, d\theta$ $\qquad r^2(1+\tan^2\theta) = r^2\sec^2\theta$

$$d\bar{E} = K'\int_{-\pi/2}^{\pi/2} \frac{r\sec^2\theta\, d\theta}{(r^2\sec^2\theta)^{3/2}} \qquad d\bar{E} = K'\int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta\, d\theta}{r^2(\sec\theta)^3}$$

dx dy dz

$$dE = k' \int_{-\pi/2}^{\pi/2} \frac{1}{r^2} \cdot \frac{1}{\sec\theta}\, d\theta$$

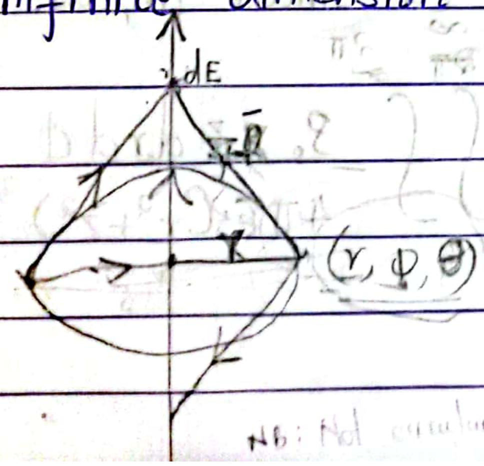
$$dE = \frac{k'}{r^2} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta}\, d\theta \Rightarrow dE = \frac{k'}{r^2} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$dE = \frac{\rho_L}{4\pi\epsilon r} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta = \frac{+\rho_L}{4\pi\epsilon r} \Big[\sin\theta\Big]_{-\pi/2}^{\pi/2}$$

$$E = k\,[1 - [-1]] \qquad E = 2k$$

$$\bar{E} = \frac{2\rho_L}{4\pi\epsilon r}\,\bar{a}_r \qquad \boxed{\bar{E} = \frac{\rho_L}{2\cancel{4}\pi\epsilon r}\,\bar{a}_r}$$

2. Find 'E' due to uniform ρ_s along on the surface of infinite dimension



NB: Not circular but extended

$$dQ = \rho_s \, dS \, \bar{a}_z = \rho_s \overset{dS}{(dr\, d\phi \cdot r)} \, \bar{a}_z$$

$$\bar{R} = -r\bar{a}_r + z\bar{a}_z$$

$$d\bar{E} = \frac{\rho_s (r\,dr\,d\phi)\bar{a}_z}{4\pi\epsilon R^2} \bar{a}_r \qquad \bar{a}_r = \frac{(\cancel{-r\bar{a}_r} + z\bar{a}_z)}{|R|}$$

Because of symmetry, the R component gets cancelled i.e the radial component (along the radius).

$$d\bar{E} = \frac{\rho_s (r\,dr\,d\phi)}{4\pi\epsilon R^2} \cdot \frac{z\bar{a}_z}{(\sqrt{r^2+z^2})} \Rightarrow d\bar{E} = \frac{\rho_s (r\,dr\,d\phi) \cdot z\bar{a}_z}{4\pi\epsilon (r^2+z^2)^{3/2}}$$

$$\Rightarrow d\bar{E} = \frac{\rho_s \, rz \, dr\, d\phi}{4\pi\epsilon (r^2+z^2)^{3/2}}$$

Consider a small $d\bar{A}$ on the plane

$$\bar{E} = \int_0^{\infty}\int_0^{2\pi} \frac{\rho_s \, rz \, dr\, d\phi}{4\pi\epsilon (r^2+z^2)^{3/2}}$$

$$E = \int_0^\infty \frac{\rho_s r z\, dr}{4\pi\varepsilon (r^2+z^2)} [\phi]_0^{2\pi} = \frac{z\rho_s}{2\pi\varepsilon}\int \frac{r\, dr}{(r^2+z^2)^{3/2}}$$

$$r^2+z^2 = m \qquad 2r\,dr + 0 = dm$$

$$\frac{z\rho_s}{2\varepsilon}\int \frac{dm/2}{m^3} = \frac{z\rho_s}{4\varepsilon}\int \frac{dm}{m^{3/2}}$$

$$E = \frac{z\rho_s}{4\varepsilon}\left[\frac{m^{-1/2}}{-1/2}\right] \qquad E = \frac{z\rho_s}{4\pi\varepsilon}\left[-2m^{-1/2}\right]_0^\infty$$

$$\frac{z\rho_s}{2\pi\varepsilon}\left[-\frac{1}{(r^2+z^2)^{1/2}}\right]_0^\infty \quad \frac{z\rho_s}{2\pi\varepsilon}\left[-0-\left(-\frac{1}{z}\right)\right]$$

$$\frac{\cancel{z}\rho_s}{2\pi\varepsilon}\cdot\frac{1}{\cancel{z}} = \frac{\rho_s}{2\pi\varepsilon}\cdot a_z$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\pi\varepsilon}\cdot\bar{a}_z}$$

1. Two parallel infinite sheets having uniform ρ_s placed at $x=0$ and $x=a$

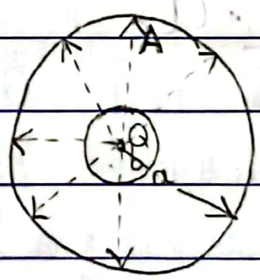
i) Find E in the space

ii) If one of the plates has opposite ρ_s (i.e $-\rho_s$ at $x=a$) repeat (i)

Assignment — 2. Three uniform sheets of charge are located in free space as follows $2\mu C/m^2$ at $x=-3$; $5\mu C/m^2$ at $x=1$ and $4\mu C/m^2$ at $x=5$. Determine E at $(0,0,0)$

Ans. $169.4\,\bar{a}_x$ N/C or V/m

D = Displacement flux density



$$D_a = \frac{Q}{4\pi a^2} \qquad D_b = \frac{Q}{4\pi b^2}$$

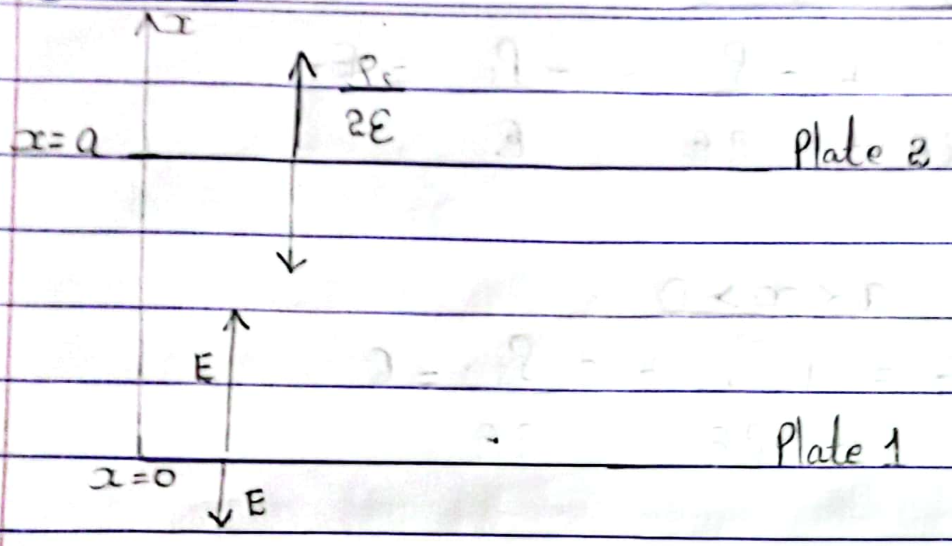
if $b=0$, then point charge

$$D_r = \frac{Q}{4\pi r^2}\cdot\bar{a}_r$$

But $E_r = \dfrac{Q}{4\pi \varepsilon r^2}\cdot\bar{a}_r$

$$\boxed{\bar{D} = \varepsilon E}$$

Solution:



$$E = \frac{\rho_s}{2\varepsilon} \cdot \bar{a}_n$$

i) $x > a$ $\quad E_T = E_0 + E_a$

$0 < x < a \quad E_T = \frac{\rho_s}{2\varepsilon} + \frac{-\rho_s}{2\varepsilon} = 0$

$\underbrace{\qquad\qquad\qquad}_{\text{between the plates}}$

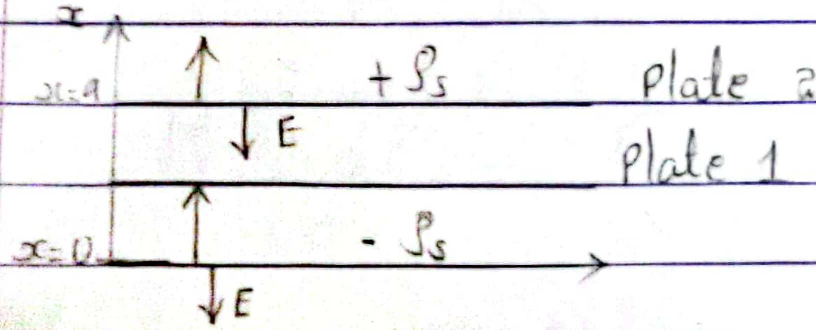
for $0 \cancel{>} x < a$

$$-\frac{\rho}{2\varepsilon} + \frac{-\rho}{2\varepsilon} = \frac{-\rho}{\varepsilon} \cdot \bar{a}_x$$

x $a < x > 0$

$$E_T = \frac{\rho_s}{2\varepsilon} + \frac{\rho_s}{2\varepsilon} = \frac{\rho_s}{\varepsilon} \cdot \bar{a}_n$$

ii)



$\iint \cdot d.$

For $a > x > 0$

$$\frac{-\rho_s}{2\varepsilon} + \frac{-\rho_s}{2\varepsilon} = \frac{-\rho_s}{\varepsilon} = E_T$$

For $a < x > 0$

$$E_T = +\frac{\rho_s}{2\varepsilon} + \frac{-\rho_s}{2\varepsilon} = 0$$

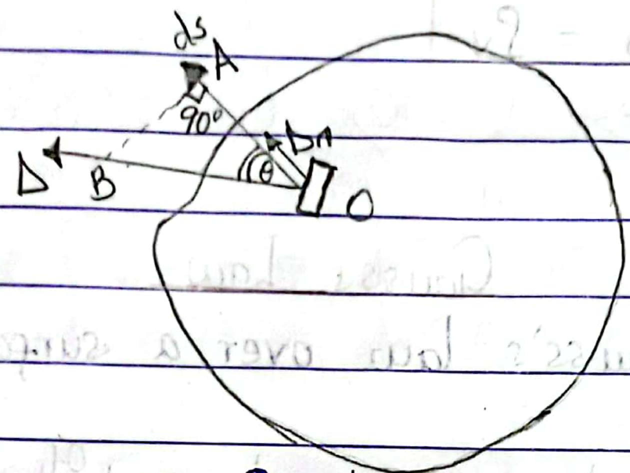
For $0 > x < a$

$$E_T = +\frac{\rho_s}{2\varepsilon} + \frac{-\rho_s}{2\varepsilon} = 0$$

Gauss's Law: The electric flux crossing any closed surface is equal to the total charge enclosed by the surface.

$$d\overline{\psi} = \overline{D} \cdot d\overline{s}$$

$$\underset{\text{flux}}{\psi} = \oiint \underbrace{\overline{D} \cdot d\overline{s}}_{d\psi} = \underset{\text{charge}}{Q}$$



$\frac{OA}{OB} = \cos\theta \quad ds \qquad D_n = D\cos\theta$

$d\psi = \bar{D}\cdot d\bar{s} \Rightarrow D\cdot ds\cos\theta \qquad D = \epsilon\bar{E}$

$ds \Rightarrow dy\,dz\,\bar{a}_r,\ dx\,dy\,\bar{a}_z,\ dx\,dz\,\bar{a}_y$

$$\frac{\oint\!\!\oint \bar{D}\cdot d\bar{s}}{\Delta V} = \frac{Q}{\Delta V}$$

Derivation Of Maxwell's Equation.

Step 1 ⇒ $$\frac{\oint\!\!\oint \bar{D}\cdot d\bar{s}}{\Delta V} = \frac{Q}{\Delta V}$$

$$\lim_{\Delta V\to 0} \frac{\oint\!\!\oint \bar{D}\cdot d\bar{s}}{\Delta V} = \lim_{\Delta V\to 0} \frac{Q}{\Delta V} = \rho_v \ c/m^3$$

$\nabla \Rightarrow$ divergence

$$\boxed{\nabla \cdot D = \rho_v}$$

$$\nabla = \frac{1}{m} \Rightarrow \frac{1}{m} \cdot \frac{C}{m^2} = C/m^3$$

Gauss's Law

Proof: $\Rightarrow$ Verify Gauss's law over a surface of a sphere.

From L.H.S

Let the center of the sphere of radius r ~~to~~ be at the is at origin if it has a point charge 'Q'

Then $E = \frac{Q}{4\pi\epsilon r^2} \cdot \bar{a}_r \Rightarrow D = \frac{Q}{4\pi r^2} \cdot \bar{a}_r$ at surface

$\Delta s = d\bar{s}$ at surface $= r^2 \sin\theta \, d\theta \, d\phi \cdot \bar{a}_r$

$$D \cdot ds = \frac{Q}{4\pi r^2} \cdot \bar{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \cancel{dr} \, \bar{a}_r$$

$$\iint \bar{D} \cdot d\bar{s} = \iint \frac{Q}{4\pi r^2} r^2 \sin\theta \, d\theta \, d\phi$$

$$\frac{Q\cancel{r}}{4\pi} \int_0^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$\frac{Q\cancel{r}}{4\pi\cancel{r}} \cdot 2\pi \int_0^{\pi} \sin\theta \, d\theta$$

$$\frac{Q\cancel{r}}{2}\Big[-\cos\theta\Big]_0^{\pi} \qquad \frac{Q}{2}\left[\frac{1}{-0.99985} - [-1\right]$$

$$-\frac{Q}{2}\left[\frac{-0.99985 + 1}{(1)}\right] \qquad -\frac{Q}{\cancel{2}} \times -\cancel{2}$$

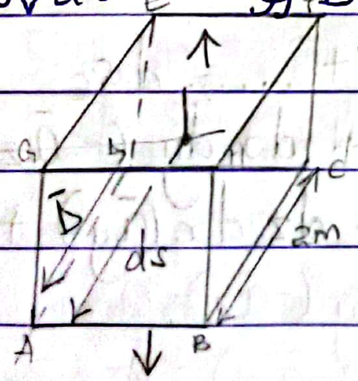
$$\boxed{= Q}$$

2. Verify divergence theorem for the volume of a cube 2m on an edge, centered of the origin and with sides parallel to the axes

Given $D = 10 x^3 / 3 \ \bar{a}_x \ c/m^2$

$$Q = \int \rho_v \, dv = \oint \bar{D} \cdot d\bar{s} = \iiint \nabla \cdot D \, dv$$

L.H.S. R.H.S



Solution

$$\left(a_x^{i}\frac{\partial}{\partial x} + a_y^{j}\frac{\partial}{\partial y} + a_z^{k}\frac{\partial}{\partial z}\right) \cdot \frac{10x^3}{3} \cdot \bar{a}_x$$

$$a_x \cdot \frac{\partial}{\partial x}\left(\frac{10x^3}{3}\right) + 0 + 0$$

$$10\,\frac{\cancel{3}x^2}{\cancel{3}} \qquad \iiint 10x^2\, dx\,dy\,dz$$

$$[y]_{-1}^{1}[z]_{-1}^{1}\int_{-1}^{1} 10x^2 \cdot dx$$

$$(2)(2)\left[\frac{10x^3}{3}\right]_{-1}^{+1} = \frac{80}{3}\, C \Rightarrow R.H.S$$

L.H.S

$ds = ds_1 + ds_2 + \ldots\ldots ds_6$

$ds = dx\,dy(\bar{a}_z) + dx\,dy(-\bar{a}_z) + dy\,dz(\bar{a}_x) + dy\,dz(-\bar{a}_x) + dz\,dx(\bar{a}_y) + dz\,dx(-\bar{a}_y)$

$dx\,dy(\bar{a}_z) + dx\,dy(-\bar{a}_z) = 0$

$dy\,dz(\bar{a}_x) + dy\,dz(-\bar{a}_x) = 0$ since D has $a_x \neq 0$

$dz\,dx(\bar{a}_y) + dz\,dx(-\bar{a}_y) = 0$

$$\bar{D}\cdot d\bar{s} = \frac{10x^3}{3}\cancel{a_x}\cdot[dydz]\cancel{\bar{a}_x}$$

$$\iint \bar{D}\cdot d\bar{s} = \frac{10x^3}{3}\int_{-1}^{1} dy \int_{-1}^{+1} dz$$

$$\iint \bar{D}\cdot d\bar{s} = \frac{10x^3}{3}[y]_{-1}^{1}\,[z]_{-1}^{1} \quad \text{for } +\bar{a}_x$$

$$\iint \bar{D}\cdot d\bar{s} = -\frac{10x^3}{3}[y]_{-1}^{1}\,[z]_{-1}^{1}$$

$$\frac{40}{3}x^3 - \frac{40x^3}{3}x^3$$
$$\frac{40}{3}(1)^3 - \frac{40}{3}(-1)^3$$

$$= \frac{80}{3}\ C.$$

3. Given $\bar{D} = 50y^3\cdot\bar{a}_y$ ~~cm~~ C/m^3. Evaluate both sides of divergence theorem for the volume of a cube of side 3m on an edge centered at the origin and with edge parallel to the axes.

$$Q = \int \rho_v \, dv = \oiint \bar{D} \cdot d\bar{s} = \iiint \nabla \cdot D \, dv$$

POTENTIAL

Potential: Work done by a unit charge from ∞ to the point under consideration in the given electric field.

0 potential = infinity.

-ve field doing the work.

Work done = Force x distance. = $Q\vec{E} \cdot dl$

$$\Delta w = Q\bar{E} \cdot \bar{dl} \qquad 'U' \Rightarrow \Phi$$

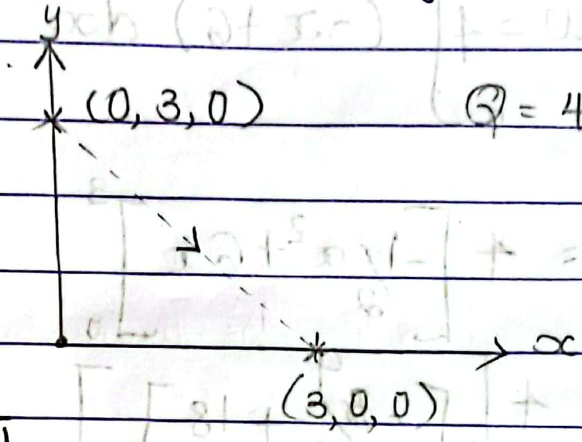
$$d\Phi_1 - d\Phi_2 = -\int_{P_2}^{P_1} E \cdot dl \Rightarrow \text{Work done.}$$

$$\oint_{P_1 = P_2}^{P_1} \bar{E} \cdot \bar{dl} = 0 \quad \text{Electric Static field / Conservative field}$$

$$dl = \bar{a}_x dx + \bar{a}_y dy + \bar{a}_z \cdot dz$$

$a_y \cdot a_y = 1$
$\bar{a}_x \cdot \bar{a}_y = 0$

1. Find the work done by 4C of charge towards (3,0,0)m from (0,3,0)m. If the electric field is $(x \bar{a}_x - 2y\, a_y)$ V/m



$Q = 4C$

$\rho_v = \bar{\nabla} \cdot D$

$dv = \bar{E} \cdot \bar{dl}$ $\quad (x\bar{a}_x - 2y\, a_y) \cdot (a_x\, dx + a_y\, dy)$

$dw = Q\, dv$

Find equation of the line.

i.e $x + y = 3$ $\qquad dx + dy = 0 \qquad dx = -dy$

C/m^2 $\quad [x \bar{a}_x - 2(3-x)\bar{a}_y] \cdot [a_x dx + dy]$

$x\,dx - 2(3-x)\,dy \;,\; x\,dx - 2(3-x)(-dx)$

$= -x\,dx + 6\,dx$

$dv = -x\,dx + 6\,dx$

$dw = Q\,dv$

$= 4(-x\,dx + 6\,dx)$

$$\int dw = \int 4\,(-x\,dx + 6\,dx)$$

$$\int dw = 4\int_0^3 (-x+6)\,dx$$

$$W = 4\left[-\frac{1}{2}x^2 + 6x\right]_0^3$$

$$W = 4\left[\left[-\frac{\cancel{3}^{\,9}}{2} + 18\right]\right] = 54\,J$$

2. If the same charge is moved first along the y-axis and then the x-axis. Find the workdone

Short notes:

Position vector: Position vector of a point is the directed distance from a particular point.

Distance vector: It is the displacement from one point to another.

Exercise:

1. If $A = 10a_x - 4a_y + 6a_z$ and $B = 2a_x + a_y$, fin

a) The component of A long a_y

b) The magnitude of 3A-B

c) A unit vector along A+2B

Sol:

a) $= -4$

b) $3A - B = 3(10a_x - 4a_y + 6a_z) - (2a_x + a_y)$

$= 30a_x - 12a_y + 18a_z - 2a_x - a_y$

$= 28a_x - 13a_y + 18a_z$

$\sqrt{28^2 + (-13)^2 + 18^2} = \underline{35.74}$

c) $A + 2B = (10a_x - 4a_y + 6a_z) + 2(2a_x + a_y)$

$A + 2B = 10a_x - 4a_y + 6a_z + 4a_x + 2a_y$

$= 14a_x - 2a_y + 6a_z$

unit vector $= \dfrac{\text{Position Vector}}{\text{Magnitude}}$

$M = \sqrt{14^2 + (-2)^2 + 6^2} = 15.362$

$= \dfrac{14a_x - 2a_y + 6a_z}{15.362}$

$= \underline{0.9113a_x - 0.1302a_y + 0.3906a_z}$

2. Given vectors $A = a_x + 3a_z$ and $B = 5a_x + 2a_y - 6a_z$ determine:

a) $|A+B|$, b) $5A - B$; c) The component of A long a_y ; d) A unit vector parallel to $3A+B$.

Sol:

a) $a_x + 3a_z + 5a_x + 2a_y - 6a_z = A + B$

$= 6a_x - 3a_z + 2a_y$

$\sqrt{6^2 + (-3)^2 + 2^2} = \underline{7}$

b) $5A - B = 5(a_x + 3a_z) - (5a_x + 2a_y - 6a_z)$

$5A - B = \cancel{5a_x} + 15a_z - \cancel{5a_x} - 2a_y + 6a_z$

$5A - B = -2a_y + 21a_z$

$= \underline{(0, -2, 21)}$

c) $= \underline{0}$

d) $3A + B = 3(a_x + 3a_z) + 5a_x + 2a_y - 6a_z$

$3A + B = 3a_x + 9a_z + 5a_x + 2a_y - 6a_z$

$= 8a_x + 2a_y + 3a_z$

$$\text{Magnitude} = \sqrt{8^2 + 2^2 + 3^2} = 8.775$$

$$= \frac{8a_x + 2a_y + 3a_z}{8.775}$$

$$= 0.9117a_x + 0.2279a_y + 0.3419a_z$$

3. Points P and Q are located at (0, 2, 4) and (−3, 1, 5). Calculate:
a) The position vector P, b) The distance vector from P to Q, c) The distance between P and Q, d) A vector parallel to PQ with magnitude of 10.

Sol:

a) $r_P = 2a_y + 4a_z$

b) $P \to Q$ $(-3-0)a_x + (1-2)a_y + (5-4)a_z$
$= \underline{-3a_x - a_y + a_z}$

c) distance $= \sqrt{(-3)^2 + (-1)^2 + (1)^2} = 3.317$ units

d) $A = Aa_A$

$$\pm 10\frac{(-3a_x - a_y + a_z)}{3.317} \qquad \pm 10(0.9040a_x - 0.3015a_y + 0.3015a_z)$$

4. Given points P(1, −3, 5), Q(2, 4, 6) and R(0, 3, 8)
Find a) the position vectors of P and R
b) The distance vector r_{QR} c) distance between

Q and R.

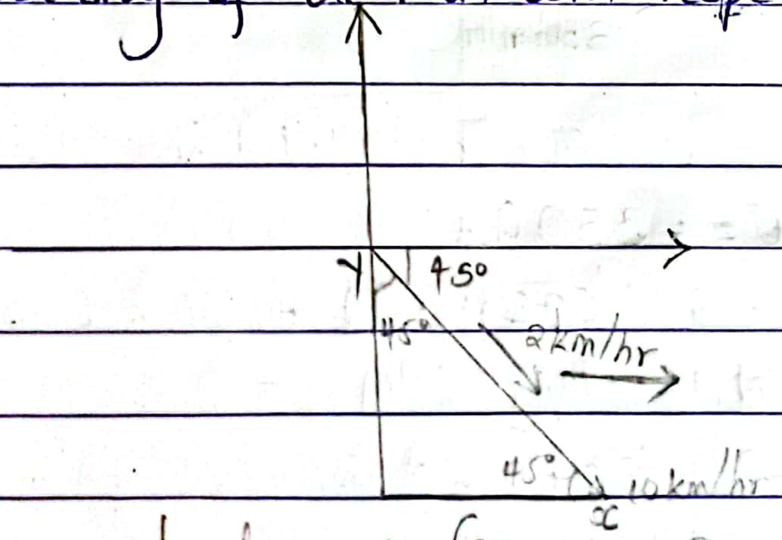
a) $r_p = a_x - 3a_y + 5a_z \quad r_r = 3a_y + 8a_z$

b) $(0-1)a_x + (3-(-3))a_y + (8-5)a_z$

$= -a_x + 6a_y + 3a_z$

c) $\sqrt{(-1)^2 + (6)^2 + (3)^2} = 6.78$

5. A river flows south east at 10km/hr and a boat flows upon it with its bow pointed in the direction of travel. A man walks upon the deck at 2km/hr in a direction to the right and perpendicular to the direction of the boat's movement. Find the velocity of the man with respect to the earth.



Velocity of boat $= 10(\cos 45\, a_x - \sin 45\, a_y)$

Man's Velocity $= 2(-\cos 45\, a_x - \sin 45\, a_y)$

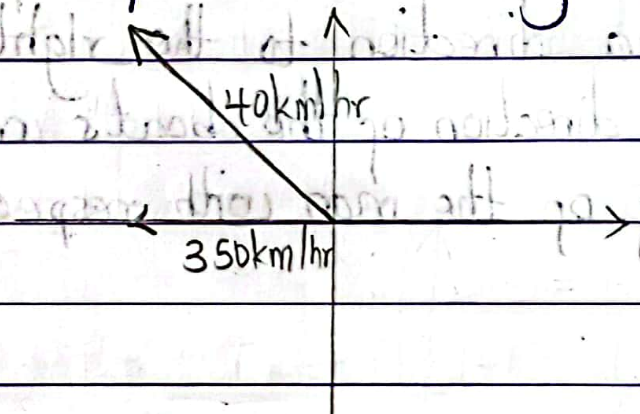
Boat $= 7.071\, a_x - 7.071\, a_y$

Man's Velocity $= -1.414\, a_x - 1.414\, a_y$

$U_{ab} = U_m + U_b = 5.657 a_x - 8.485 a_y$

$|U_{ab}| = 10.2 \angle -56.3^\circ$ 10.2 km/hr at 56.3° south east.

6. An airplane has a ground speed of 350 km/hr in the direction due west. If there is a wind blowing northwest at 40 km/hr, calculate the true air speed and heading of the airplane



Airplane $= -350 a_x$

Air $= 40(-\cos 45 a_x + \sin 45 a_y)$

Air $= 40(-0.7071 a_x + 0.7071 a_y)$

Air $= (-28.284 a_x + 28.284 a_y) + (-350 a_x)$

Air $= -378.284 a_x + 28.9911 a_y$

$= 379.4$ km/hr $\angle -4.370^\circ$

Coulomb's Law

Statement: The force F between two point charges Q_1 and Q_2 is:

1. Along the line joining them
2. Directly proportional to the product $Q_1 Q_2$ of the charges
3. Inversely proportional to the square of the distance R between them i.e

$$F = \frac{k Q_1 Q_2}{R^2} \qquad k = \frac{1}{4\pi \varepsilon_0}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} = \frac{10^{-9}}{36\pi} \qquad k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9} \, m/F$$

$$\frac{1}{4\pi \cdot \frac{10^{-9}}{36\pi}} = \underline{9 \times 10^{9}}.$$

Force F_{12} i.e force on Q_2 due to Q_1

$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} a_{R_{12}}$$

Exercise:

1. Find the electric field intensity at $P(-4, 6, -5)$ in free space caused by a charge at $0.1 m C$ at

origin

b) $(2,-1,-3)$

a) $E = \frac{Q}{4\pi\epsilon_0 r^2}$ $\quad P(-4,6,-5)\quad (0,0,0)$

$a_r = -4a_x + 6a_y - 5a_z$

$\sqrt{(-4)^2 + 6^2 + (-5)^2} = 8.775$

$E = \frac{0.1\times10^{-3}}{\cancel{4\pi}\times10^{-9}\times(8.775)^2}$

$\cancel{36}\pi \; 9$

$E = \frac{0.1\times9\times10^{-3}\times10^{9}}{(8.775)^2} \quad \frac{-4a_x+6a_y-5a_z}{8.775}$

$E = 0.01169\times10^{6}\cdot\left(-0.4558a_x + 0.6838a_y - 0.5698a_z\right)$

$E = (-5.328a_x + 7.993a_y - 6.66a_z)\ kV/m$

b) $(2,-1,-3)\quad P(-4,6,-5)$

$-6a_x + 7a_y - 2a_z \quad$ Magnitude $= 9.434$

$E = \frac{Q}{4\pi\epsilon_0 R^2} \quad \frac{0.1\times10^{-3}\times9\times10^{9}}{89}$

$\frac{0.1\times9\times10^{-3}\times10^{9}}{89} = 0.0101\times10^{6}$

Maverick

$E = 0.0101 \times 10^{6} \cdot (-0.6366\, a_x + 0.7142\, a_y - 0.2112\, a_z)$

$\bar{E} = (-6.423\, a_x + 7.494\, a_y - 2.133\, a_z \ kv/m)$

Definitions:

i) Electric field intensity: This is the force per unit charge.

ii) Coulomb's law: The force between two point charges acts along the line joining the two charges, it is directly proportional to the product of the two charges and is inversely proportional to the square of the distance between the two charges.

iv) Conditions for a vector to be solenoidal:

If the vector lacks divergence and Curl

Angles between two

$\cos^{-1}. \frac{A \cdot B \cdot C}{|A| |B| |C|} = \theta$ $\quad \sin^{-1}. \frac{A \times B \times C}{|A| |B| |C|} = \theta$

i) Find the angle between $D = 4i - 2j - k$ and $B = i + 4j - 4k$

$$\cos^{-1} = \frac{B \cdot D}{|B|\,|D|}$$

$(4i - 2j - k) \cdot (i + 4j - 4k)$

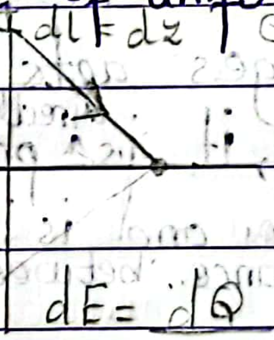
$(4 \times 1)i + (-2 \times 4)j + (-1 \times -4)k$

$4 - 8 + 4$

$= 90$

ii) Obtaining the 'E' in free space due to an infinite line charge of uniform linear charge.

$dl = dz \quad Q = \rho_L dl$



$$dE = \frac{dQ}{4\pi\varepsilon r^2}\,\bar{a}_r \qquad dE = \frac{dQ}{4\pi\varepsilon r^2}\,\frac{\bar{R}}{|\bar{R}|}$$

$\bar{R} = +r a_r - z a_z$

$|R| = \sqrt{r^2 + z^2}$

$$dE = \frac{\rho_L dz}{4\pi\varepsilon (r^2+z^2)(r^2+z^2)^{1/2}}\,+r a_r$$

$$dE = \frac{r \rho_L dz}{4\pi\varepsilon (r^2+z^2)^{3/2}}\,a_r$$

$$\int dE = \frac{r\rho_L}{4\pi\epsilon_0}\int \frac{dz}{(r^2+z^2)^{3/2}}$$

$z = r\tan\theta$ $\quad$ $r^2 + r^2\tan^2\theta$

$dz = r\sec^2\theta$ $\quad$ $r^2(1+\tan^2\theta) = r^2\sec^2\theta$

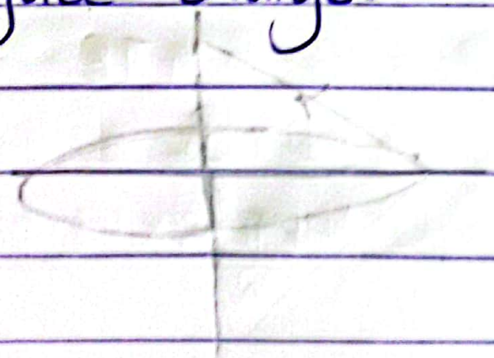
$$\frac{r\rho_L}{4\pi\epsilon}\int_{-\pi/2}^{\pi/2}\frac{r\sec^2\theta}{(r^2\sec^2\theta)^{3/2}}\,d\theta$$

$$\frac{r\rho_L}{4\pi\epsilon}\int_{-\pi/2}^{\pi/2}\frac{\cancel{r}\sec^2\theta}{\cancel{r^3}_{r^2}\sec^3\theta}\,d\theta$$

$$\frac{\cancel{r}\rho_L}{r^{\cancel{2}}4\pi\epsilon}\int_{-\pi/2}^{\pi/2}\frac{1}{\sec\theta}\,d\theta$$

$$\frac{\rho_L}{4\pi\epsilon r}\Big[\sin\theta\Big]_{-\pi/2}^{\pi/2} = \frac{2\rho_L}{4\pi\epsilon r}$$

For Surface charge.



$$dE = \frac{\rho_s\, ds}{4\pi\varepsilon R^2} \cdot \bar{a}_r \qquad ds = r\,dr\,d\phi$$

$$dE = \frac{r\rho_s\, dr\, d\phi}{4\pi\varepsilon R^2} \frac{\bar{R}}{|R|} \qquad \bar{R} = -r a_r + z a_z$$

$$|R| = \sqrt{r^2 + z^2}$$

$$dE = \frac{z r \rho_s\, dr\, d\phi\, a_z}{4\pi\varepsilon (r^2+z^2)(r^2+z^2)^{1/2}}$$

$$dE = \frac{z r \rho_L\, dr\, d\phi\, a_z}{4\pi\varepsilon (r^2+z^2)^{3/2}} \qquad \int dE = \int_0^{2\pi}\int_0^{\infty} \frac{z r \rho_L\, dr\, d\phi}{4\pi\varepsilon (r^2+z^2)}$$

$$\int_0^{2\pi} \frac{z\rho_L}{4\pi\varepsilon} \int_0^{\infty} \frac{r\, dr\, d\phi}{(r^2+z^2)^{3/2}} \qquad m = r^2 + z^2$$

$$dm = 2r\,dr$$

$$\int_0^{2\pi} \frac{z\rho_L}{4\pi\varepsilon} \int_0^{\infty} \frac{1}{2} \cdot m^{-3/2+1}\, dm \qquad r\,dr = \frac{dm}{2}$$

$$\int_0^{2\pi} \frac{z\rho_L}{4\pi\varepsilon} \left[-\frac{m}{(r^2+z^2)^{1/2}} \right]_0^{\infty} \qquad \int_0^{2\pi} \frac{z\rho_L}{4\pi\varepsilon} \left(- \left(-\frac{1}{z} \right) \right)$$

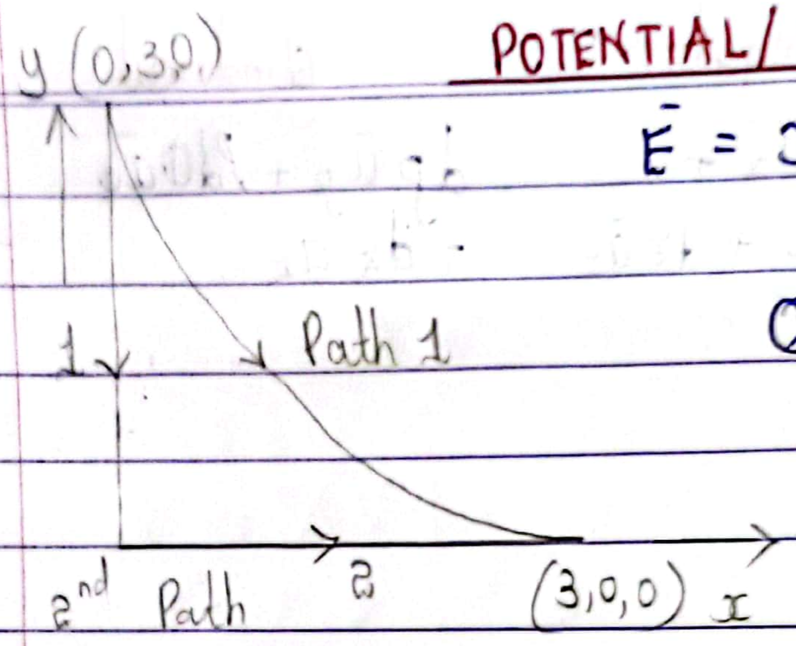
$$\int_0^{2\pi} \frac{\rho_L}{4\pi\varepsilon}\, d\phi = \frac{\rho_L}{4\pi\varepsilon} [\phi]_0^{2\pi}$$

$$\frac{2\pi \rho_L}{4\pi\varepsilon} = \frac{\rho_L}{2\varepsilon}$$

Differential elements	Rectangular (Cartesian)	Cylindrical	Spherical
length dl:	$dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$	$d\rho\,\bar{a}_\rho + \rho\, d\phi\,\bar{a}_\phi + dz\,a_z$.	

limit of $y \to 0 \to 3$ $\quad \bar{a}_x \cdot \bar{a}_y = 0$

$\bar{a}_x \cdot \bar{a}_x = 1$



POTENTIAL / WORK DONE

$\bar{E} = x\bar{a}_x - 2y\bar{a}_y$

$Q = 4C$

moved as shown find work done

$dV_1 = \bar{E} \cdot d\bar{l}$

$= (x\bar{a}_x - 2y\bar{a}_y)(-\bar{a}_y dy)$

$= +2y\,dy$

$V_1 = 2\int y\,dy = \left[2\frac{y^2}{2}\right]_3^0 = \frac{2(-3)^2}{2}$

$= 9$ volts.

$dV_2 = (x\bar{a}_x - 2y\bar{a}_y)(dx\,\bar{a}_x)$

$= x\,dx$

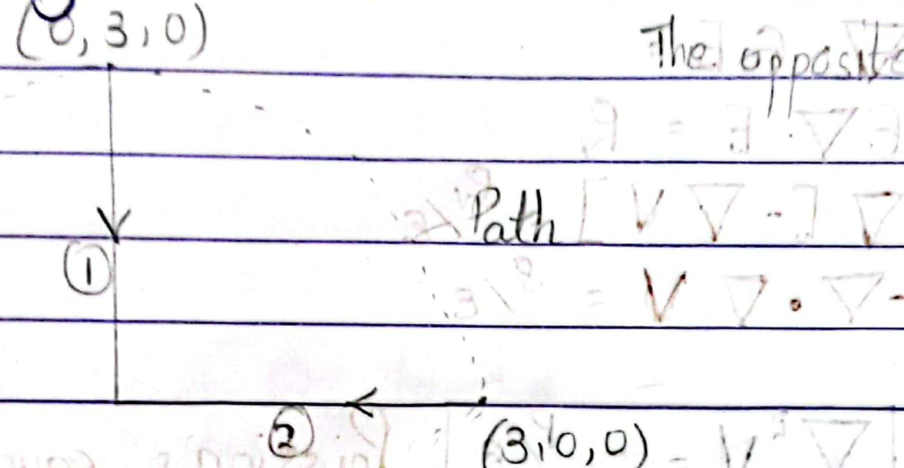
$V_2 = \left[\frac{x^2}{2}\right]_0^3 = 9/2$ Volts or J/C

$V = V_1 + V_2 = 13.5\,V$

Workdone $= QV = 4 \times 13.5 = 54\,J$

Ans = 54 J

ii) Find the work done when moving along $x^2+y^2 = (3)^2$ circle.



The opposite path = -54 J

$dV_1 = \bar{E} \cdot d\bar{l} = (x a_x - 2y \bar{a}_y)(\bar{a}_x dx + \bar{a}_y dy + \cancel{\bar{a}_z dz})$ (x-y plane)

Differentiate $x^2+y^2 = (3)^2$

$2x\,dx + 2y\,dy = 0$ $\qquad x\,dx + y\,dy = 0$

$x\,dx = -y\,dy$ $\qquad y = \frac{x\,dx}{dy}$ $\qquad x = \frac{-x\,dx}{dy}$

replacing in the equation.

$x\bar{a}_x - 2\left[-\frac{x\,dx}{dy}\right]\bar{a}_y]$

$\nabla \cdot D = \rho_v$ $\bar{D} \cdot d\bar{s} = dq$ $Q = \oint\oint \bar{D} \cdot$

Derivation

$\nabla \cdot \epsilon E =$

$\epsilon \nabla \cdot E = \rho_v$

$\nabla [-\nabla V] = \rho_v / \epsilon$

$-\nabla \cdot \nabla V = \rho / \epsilon$

$\boxed{\nabla^2 V = -\rho/\epsilon}$ Poission's equations

At region where charge is free

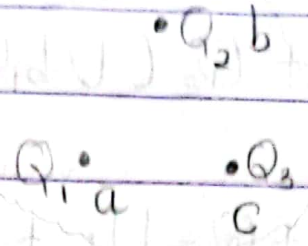
$\boxed{\nabla^2 V = 0}$ Laplace equation

Potential $V = \phi$

$\nabla \phi = \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z}$

$= \left(a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z}\right)\left(\phi_x \bar{a}_x + \phi_y \bar{a}_y + \phi_z \bar{a}_z\right)$

Finding Energy In Static Electric Field



Initially no charge then bring Q_1 to a. $W_1 = 0$
Since $E = 0$
Then bring Q_2 to b.
$W_2 = Q_2 V_{b1}$
Then bring Q_3 to C.
$W_3 = Q_3 (V_{c1} + V_{c2})$
Total Workdone: $W = W_1 + W_2 + W_3$
$= 0 + Q_2 V_{b1} + Q_3 (V_{c1} + V_{c2})$

Starting With Charge 3
$W_3 = 0$ since $E = 0$
Then bring Q_2 to b
$W_2 = Q_2 V_{b3}$
Then bring Q_1 to a
$W_1 = Q_1 (V_{a2} + V_{a3})$
Total Workdone: $W = 0 + Q_2 V_{b3} + Q(V_{a2} + V_{a3})$
NB: Total work done is the same.

$W + W' = 2W$

$\Rightarrow Q(U_{a2} + U_{a3}) + Q_2(U_{b_1} + U_{b_3} + Q_3(U_{c_1} + U_{c_2}$

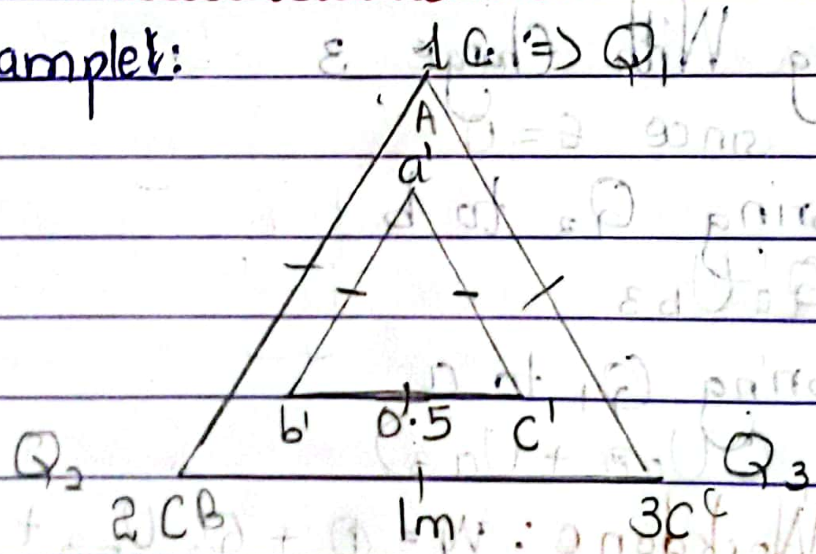
$$\boxed{W = \frac{1}{2}\left[Q_1 U_a + Q_2 U_b + Q_3 U_c + \dots\right]}$$

$$\boxed{= \frac{1}{2} \sum Q U} \; J$$

Energy in static electric field is equal to the (½) sum of the product of a charge Q and potential at its place due to other charges.

$$\boxed{= \frac{1}{2} \sum_{j=a}^{m} \sum_{i=1}^{n} Q_i U_j}$$

Example:



Concentric $\Delta \Rightarrow$ Center of 2 Δ is the same

1. Find work done if charges are moved from abc to a'b'c' in the given figure.

Solution:

Energy stored when charges are at outer triangle

$$E = \frac{1}{2} \sum QU \quad \frac{1}{2}\left[Q_1 U_a + Q_2 U_b + Q_3 U_c\right]$$

$$\frac{1}{2}\left[1(U_{a_2}+U_{a_3}) + 2(U_{b_1}+U_{b_3}) + 3(U_{c_1}+U_{c_3})\right]$$

Potential at $U_{a_2} = \frac{Q}{4\pi\varepsilon(1)}$ $\qquad U_p = \frac{Q}{4\pi\varepsilon r}$

$U_{a2} = 1.8 \times 10^{10}$ $\quad U_{b1} = 9\times10^9$ $\quad U_{c_1} = 9\times10^9$

$U_{a3} = 2.7\times10^{10}$ $\quad U_{b3} = 2.7\times10^{10}$ $\quad U_{c_2} = 1.8\times10^{10}$

$$2W = \frac{1}{4\pi\varepsilon}\left[\frac{1(2+3)}{Q_1} + \frac{2(3+1)}{Q_2} + \frac{3(1+2)}{Q_3}\right]$$

$$\frac{1}{2}\left[1((1.8\times10^{10}) + (2.7\times10^{10})) + 2((9\times10^9)+(2.7\times10^{9})) + 3((9\times10^9)+(1.8\times10^{10}))\right]$$

$$\frac{1}{2}\left[4.5\times10^{10} + \right.$$

Ans = 99×10^{9} J

NB: Energy stored when charges are at inner Δ = twice the previous one as the distance between vertices is halved.

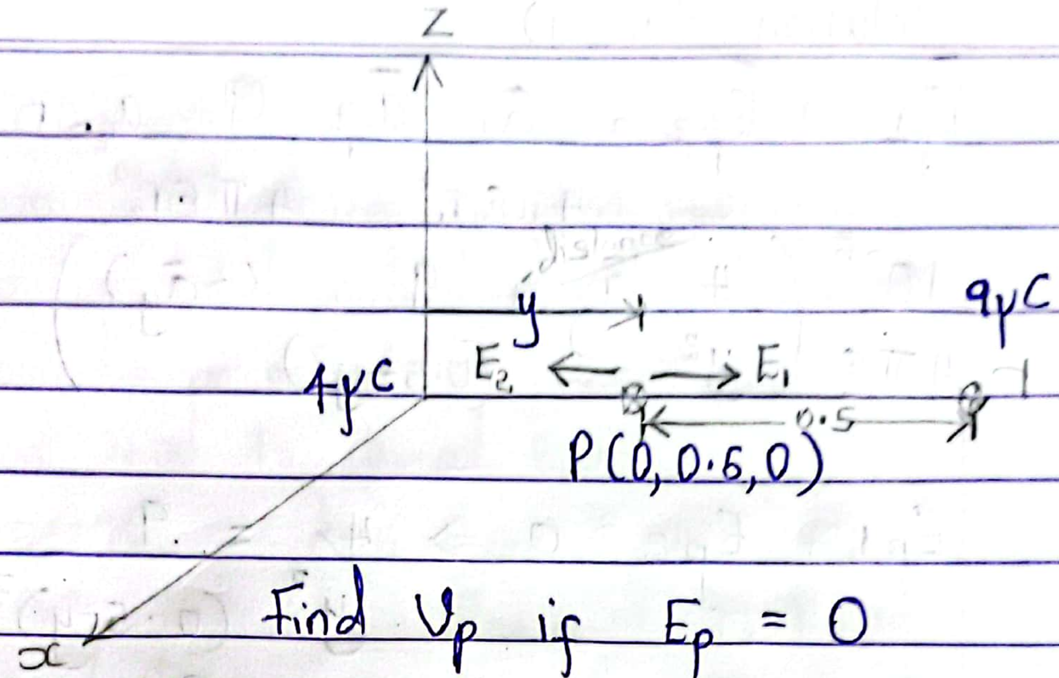
$$W_E = \frac{1}{2}\sum_{n=1}^{n} Q_n V_n = \frac{1}{2}\int \rho_v \, dv \, , \, \phi$$

Volume

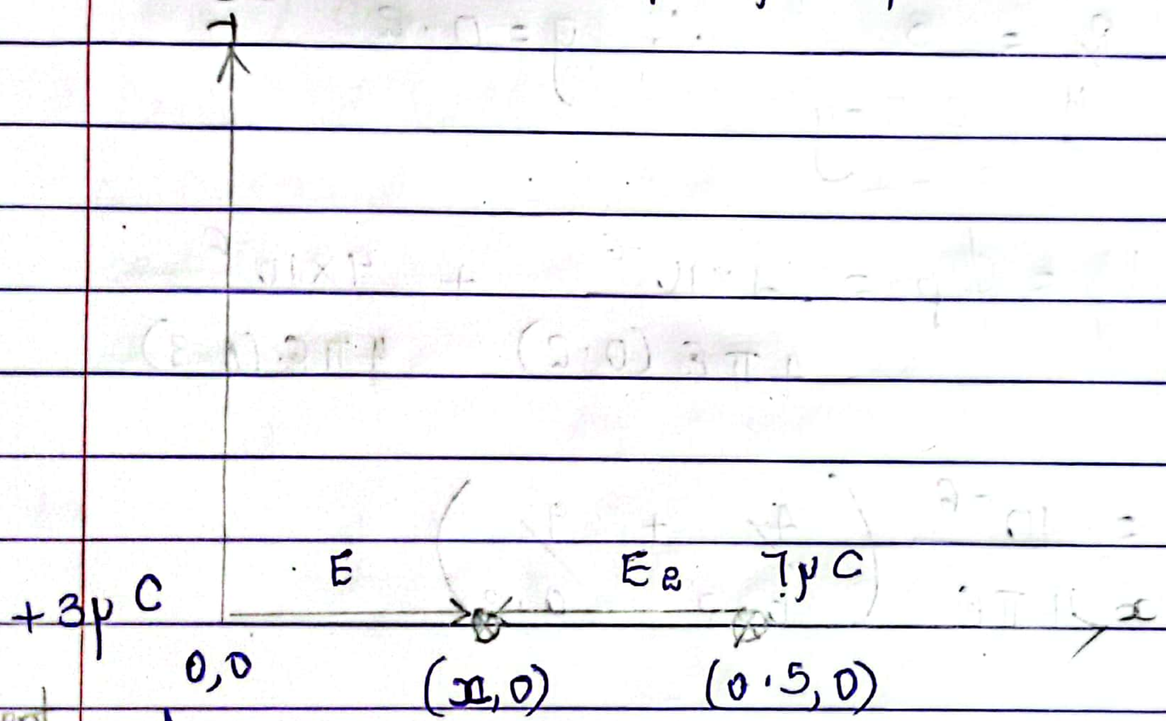
$$\frac{1}{2}\int \varepsilon E^2 dV = \frac{1}{2}\int \frac{D^2}{\varepsilon} dV = \frac{1}{2}\int \bar{D} \cdot \bar{E} \, dV$$

Energy Density $= \frac{1}{2}\bar{D}\cdot\bar{E} = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\frac{D^2}{\varepsilon}$ J/m³

Q1.



i) Find V_p if $E_p = 0$



Assignment

ii) Find V_p if $E_p = 0$

Ans: $x = 0.196$ Ans: $V = \Phi = 343.8$ KV

$x = 0.946$

Solution for i)

$$\bar{E}_{p1} + \bar{E}_{p2} = \frac{Q_1}{4\pi\varepsilon r_1^2}\bar{a}_y + \frac{Q_2}{4\pi\varepsilon r_2^2}\bar{a}_y = 0$$

$$= \frac{10^{-6}}{4\pi\varepsilon}\left(\frac{4}{y^2}\bar{a}_y + \frac{9}{(0.5-y^2)}(-\bar{a}_y)\right)$$

$$E_{p1} + E_{p2} = 0 \Rightarrow \frac{4}{y^2} = \frac{9}{(0.5-y)^2}$$

$$\frac{2}{y} = \frac{3}{0.5-y} \qquad \therefore\ y = 0.2$$

$$V_p = \phi_p = \frac{4\times10^{-6}}{4\pi\varepsilon(0.2)} + \frac{9\times10^{-6}}{4\pi\varepsilon(0.3)}$$

$$= \frac{10^{-6}}{4\pi\varepsilon}\left(\frac{4}{0.2} + \frac{9}{0.3}\right)$$

$$= 450\,\text{kV}$$

Point charges +Q 4, - Q are located on the x-axis at +1m & -1m respectively. Find the ratio of total potential at $x = +100m$ to Φ_{200} i.e potential at $x = +200m$

Φ at 400 to ϕ at 800

$x = 0.196$ Ans $\frac{\Phi_{100}}{\Phi_{200}} = 4$ $\frac{\Phi_{100}}{\Phi_{400}} = 16$

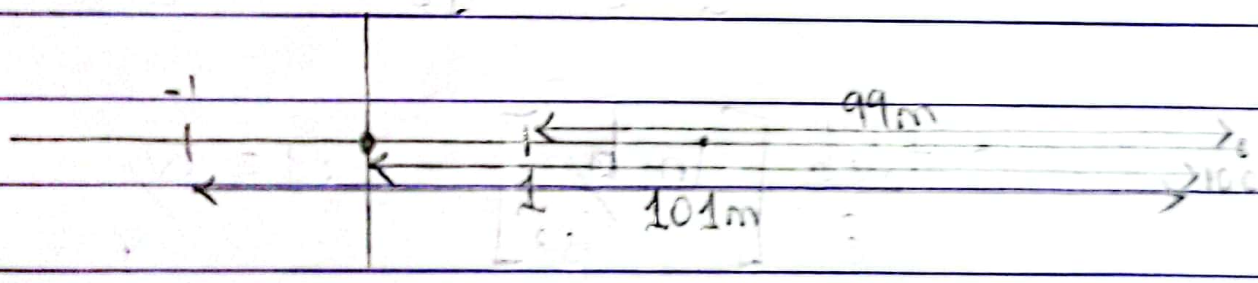
$x = 0.$

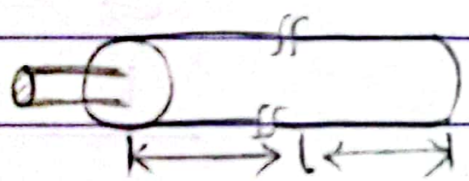
$\Phi_{100} / \Phi_{800} = 64$

Derive an expression for C per unit length of a co-axial cable.

~~$x = 0.196$~~ ~~Ans $V - \Phi = 343.8kV$~~

~~$x = 0.946$~~





$D = \varepsilon E$

$$\bar{D}_r = Q/A$$
$$= \frac{\rho_s 2\pi b l}{(2\pi r l)} a_r \, C/m^2$$
$$= \frac{\rho_s b}{r} a_r$$

$$E_r = \frac{\rho_s b}{\varepsilon r} a_r (N/C) \qquad V_{ab} = -\oint \bar{E} \cdot d\bar{l}$$

$d\bar{l} \to dr\,\bar{a}_r$

$$= -\oint \left(\frac{\rho_s b}{\varepsilon r}\right) \bar{a}_r \cdot dr\,\bar{a}_r$$

$$V_{ab} = -\frac{\rho_s b}{\varepsilon} \int_b^a \frac{1}{r} \, dr$$

$$V_{ab} = -\frac{\rho_s b}{\varepsilon} \Big[\ln r \Big]_b^a \Rightarrow -\frac{\rho_s b}{\varepsilon} \left[\ln a - \ln b\right]$$

$$= +\frac{\rho_s b}{\varepsilon} \left[\ln b/a \right] \qquad C = Q/V$$

$$Q = \frac{\cancel{\rho_s} 2\pi \cancel{b} l}{\frac{\cancel{\rho_s} \cancel{b}}{\varepsilon} \ln b/a}$$

$$\frac{C}{l} = \frac{2\pi \varepsilon}{\ln (b/a)} \quad f/m$$

$\varepsilon_0 \varepsilon_r$

$$\frac{D_{n_1}}{D_1} = \cos\theta_1 \qquad \frac{Et_1}{E_1} = \sin\theta_1$$

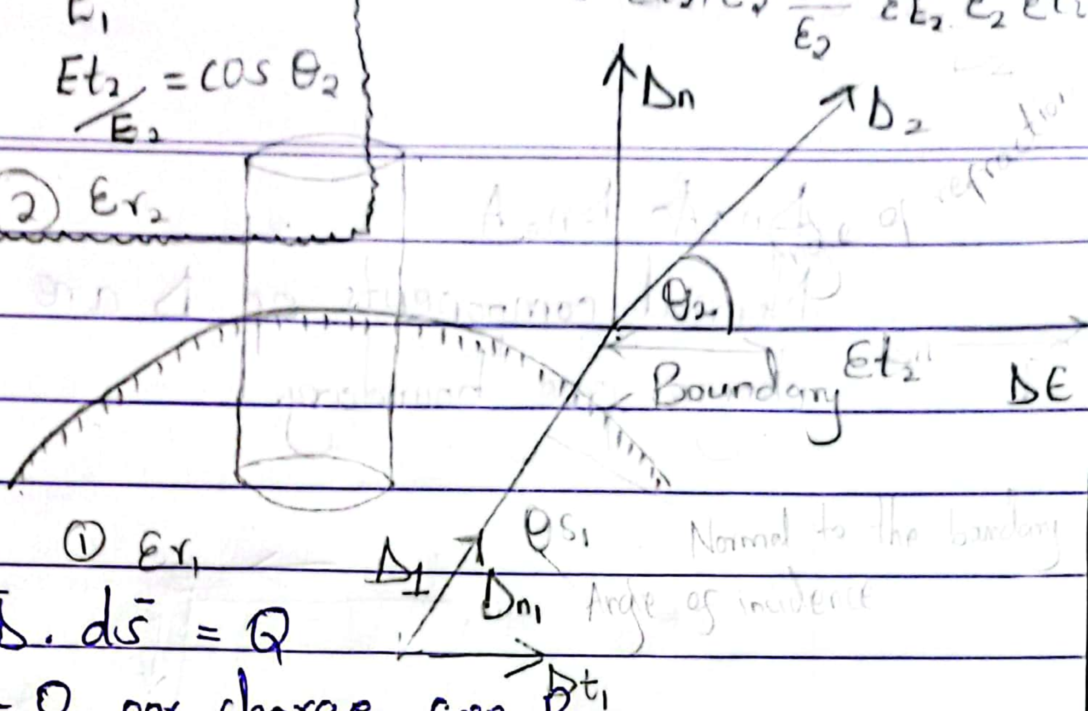
$$\frac{D_{n_2}}{D_2} = \sin\theta_2 \qquad \frac{Et_2}{E_2} = \cos\theta_2$$

$$\tan\theta_2 = \frac{\sin\theta_2}{\cos\theta_2} = \frac{D_{n_2}/D_2}{Et_2/\varepsilon_2} = \frac{D_{n_2}}{\frac{\varepsilon t_2}{\varepsilon_2}} \cdot \frac{1}{\varepsilon E_2} = \frac{1}{\varepsilon_2}\frac{D_{n_2}}{\varepsilon t_2}$$

$$\frac{\tan\theta_i}{\tan\theta_r} = \frac{\varepsilon r_1}{\varepsilon r_2}$$

$$\boxed{D_{1n} = D_{2n}}$$

$$\boxed{Et_1 = E_{2t}}$$



G. law $\bar{D}.d\bar{s} = Q$

$\bar{D}.d\bar{s} = 0$ for charge free B^{t_1}

Find boundary conditions for electrostatic field

Area : Bottom + Top + Curved = 0

At $\Delta h \Rightarrow 0 \Rightarrow$ Bottom + Top + 0

$\bar{D}.d\bar{s} \Rightarrow D_t \cdot \pi a^2 \uparrow$ $\Rightarrow$ for top surface

$\bar{D} = (D_n + D_t) \cdot \pi a^2)\, ds \uparrow$

$(\vec{D}_n \cdot A\vec{ds}) + (\vec{D}_t \cdot A d\vec{s})$ $\cos 0 \Rightarrow 1$

$(D_nA) + (D_t\, A\cos 90)$
$\cos 0$

$D_{n_2}A + 0$

Bottom Surface

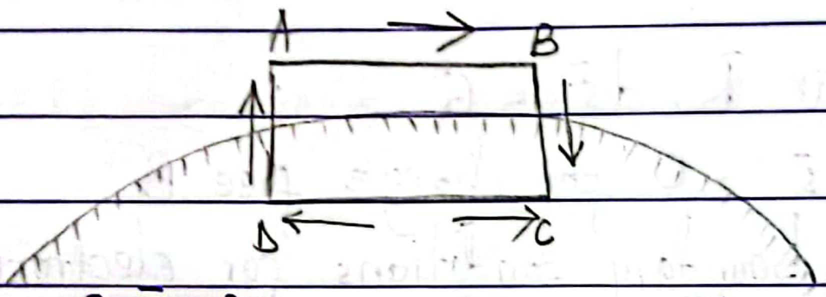
$(D_{n_2} + D_{t_2})\, A\, ds \downarrow$

$-D_{n_2}A$

Bottom + Top + curved = 0 $\quad -D_{n_1}A + D_{n_2}A + 0 = 0$

$D_{n_1} A = D_{n_2} A$

Normal components of D are continuous at the charge free boundary.



$$V = \oint \bar{E} \cdot d\bar{l} = 0 \qquad E(AB + \cancel{BC} + CD + \cancel{DA}) = 0$$

$$\Delta h \to 0 \quad = (\vec{E_n} + \vec{E_t})(\vec{AB} + \overleftrightarrow{CD})$$

$$E_n(\bar{AB} + \bar{CD}) + \vec{E_t}(\vec{AB} + \overleftrightarrow{CD})$$

$$0 + (E_{t_2} AB - E_{t_1} AB)$$

$$0 = E_{t_2} - E_{t_1} \qquad E_{t_1} = E_{t_2}$$

Tangential components of E are continuous at the charge free boundary.

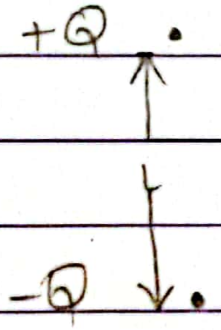
$\tan\theta_1 = \sin\theta_1 / \cos\theta_1 = E_{t1}/\varepsilon_1 \times \frac{D_1}{D_{n1}}$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}$$

Ratio of tangents equals to the ratio incident permittivity to the refracted permittivity of the medium.

Ratio of tangents equals of incidence angle to the refracted angle equals to the ratio of incident permittivity to the refracted permittivity of the medium.

Electric Dipole

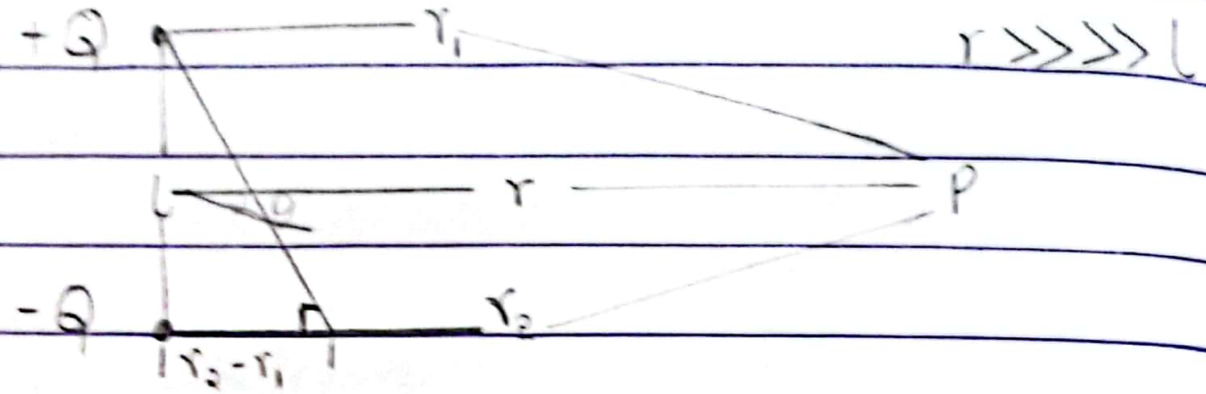


$\underline{\text{Dipole Moment}} = \bar{P} = Q \cdot d\bar{l}$

$r >>>> l$

P

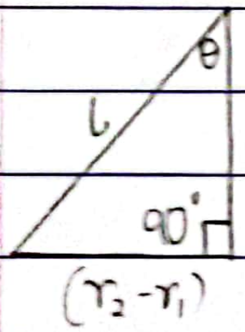
Finding Electric Field Of A Dipole



$r_1 \parallel r_2 \parallel r \quad |r_1| = |r_2|$

$$V_p = \frac{Q_1}{4\pi\varepsilon r_1} + \frac{-Q_2}{4\pi\varepsilon r_2} = \frac{Q}{4\pi\varepsilon}\cdot\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{r_2 - r_1}{r_1 r_2}\right) = \frac{Q}{4\pi\varepsilon}\left(\frac{r_2 - r_1}{r^2}\right)$$



$$V = \frac{Q\, l \cos\theta}{4\pi\varepsilon r^2}$$

$$\bar{E} = -\nabla V$$

$$\nabla = -\frac{\partial v}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial v}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial v}{\partial \theta}\bar{a}_\phi$$

Boundary Conditions
Dipole

$$\frac{Ql\cos\theta}{4\pi\epsilon} \cdot \frac{d}{dr}\frac{1}{r^2}\, a_r + \frac{1}{r}\frac{Ql}{4\pi\epsilon r^2}(-\sin\theta)\,\bar{a}_\theta + 0$$

Equation of Stream lines $\Rightarrow$ $E_\theta / E_r = \dfrac{\sin\theta}{2\cos\theta}$