EEE/ETI:2204

CIRCUIT & NETWORK THEORY(CNT) II

Lecture 05

Introduction

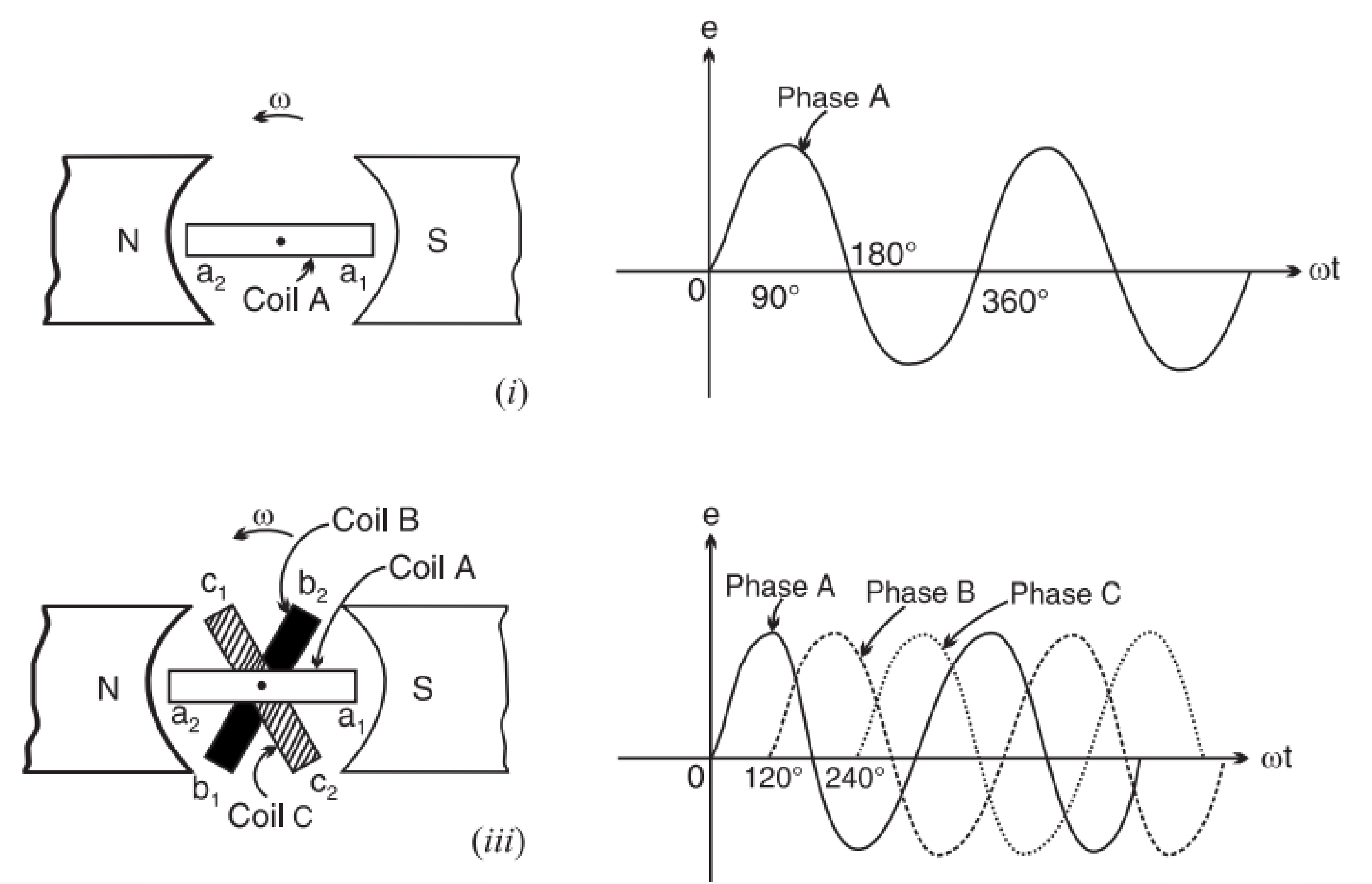
- The a.c. circuits discussed so far are termed as single phase circuits because they contain a single alternating current and voltage wave. The generator producing a single phase supply (called single-phase generator) has only one armature winding. But if the generator is arranged to have two or more separate windings displaced from each other by equal electrical angles, it is called a polyphase generator and will produce as many independent voltages as the number of windings or phases.
- The electrical displacement between the windings depends upon the number of windings or phases.
- Although several polyphase systems are possible, the 3-phase system is by far the most popular because it is the most efficient of all the supply systems.
- Generation, transmission and distribution of electricity via the National Grid system is accomplished by three phase alternating current.

Polyphase System

- A polyphase alternator has two or more separate but identical windings (called phases) displaced from each other by equal electrical angle and acted upon by the common uniform magnetic field. Each winding or phase produces a single alternating voltage of the same magnitude and frequency. However, these voltages are displaced form one another by equal electrical angle.
- (i) Fig. (i) shows an elementary single-phase alternator. It has one winding or coil *A* rotating in anticlockwise direction with an angular velocity *w* in the 2-pole field. The equation of the e.m.f. induced in the coil is given by ;

$$e_{a1a2} = E_m \, Sin \, \omega t$$

Single phase & 3-phase



- Fig.(iii) shows an elementary 3-phase alternator. It has three identical windings or coils A, B and C displaced 120 electrical degrees from each other and rotating in anticlockwise direction with an angular velocity ω in the 2-pole field. Note that the corresponding terminals a_1, b_1 and c_1 are 120^o apart. Likewise the terminals a_2, b_2 and c_2 are 120 electrical degrees apart. Since the three coils are identical and have the same angular velocity, the e.m.f.s induced in them will be of the same magnitude and frequency.

- However, the three e.m.f.s will be displaced from one another by 120^o. Note that e.m.f. in coil B will be 120^o behind that of coil A and the e.m.f. in coil C will be 240^o behind that of coil A.

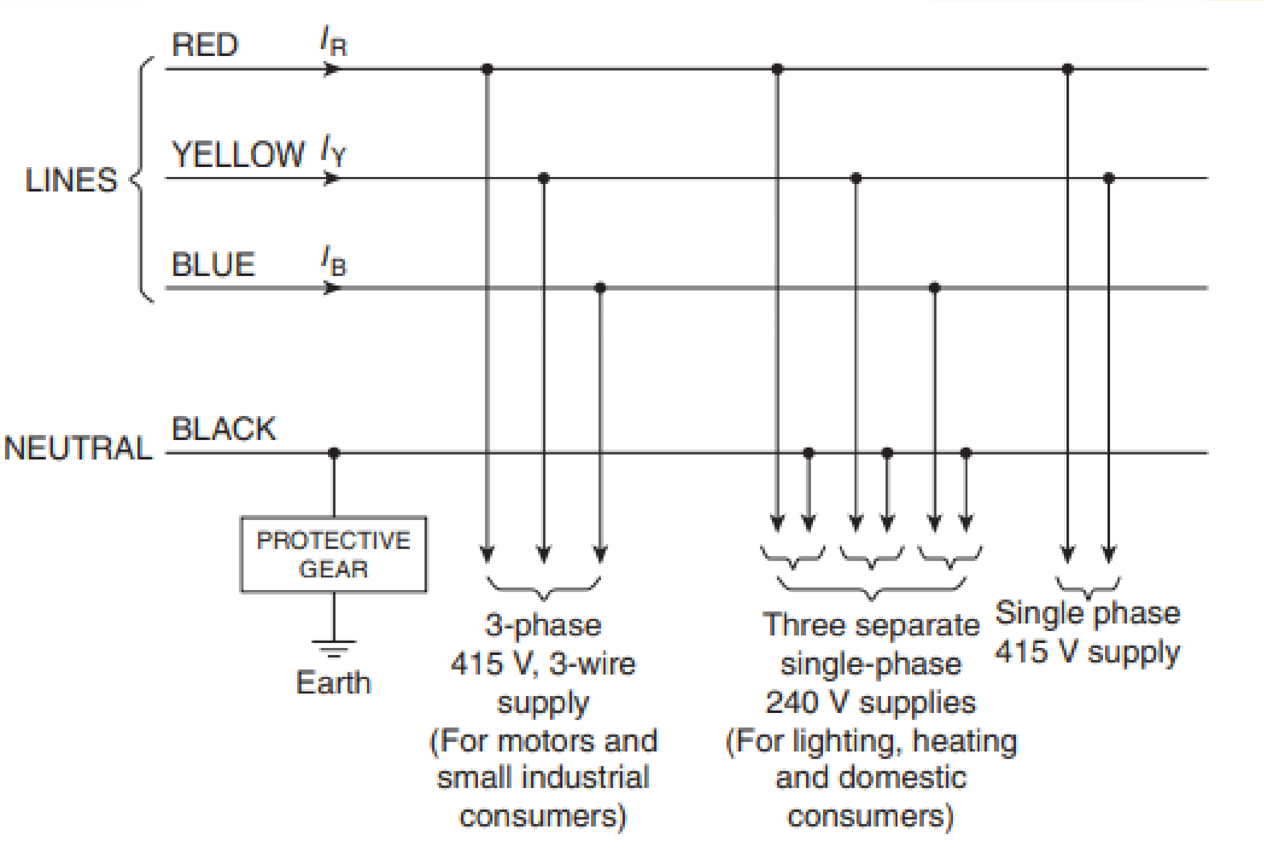
- The equations of the three e.m.f.s can be represented as :

$$e_{a1a2} = E_m \, Sin\, \omega t$$

$$e_{b1b2} = E_m \, Sin(\omega t - 120°)$$

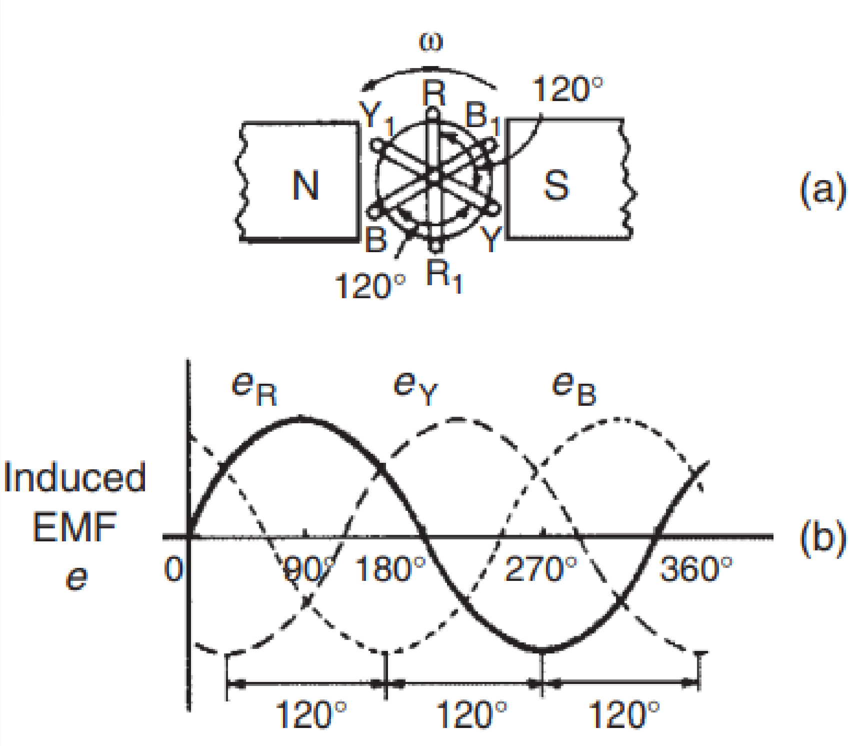
$$e_{c1c2} = E_m \, Sin(\omega t - 240°)$$

- Most consumers are fed by means of a single-phase a.c. supply. Two wires are used, one called the live conductor (usually coloured red) and the other is called the neutral conductor (usually coloured black).
- The neutral is usually connected via protective gear to earth, the earth wire being coloured green. The standard voltage for a single phase a.c. supply is 230V or 240V.
- The majority of single-phase supplies are obtained by connection to a three-phase supply.



3-phase system

- A three-phase supply is generated when three coils are placed 120^o apart and the whole rotated in a uniform magnetic field as shown in Fig. The result is three independent supplies of equal voltages which are each displaced by 120^o from each other.
- The convention adopted to identify each of the phase voltages is: R-red, Y-yellow, and B-blue.
- The phase-sequence is given by the sequence in which the conductors pass the point initially taken by the red conductor. The national standard phase sequence is R, Y, B.



- A three-phase a.c. supply is carried by three conductors, called ‘lines’ which are coloured red, yellow and blue.

- The currents in these conductors are known as line currents (I_L) and the p.d.’s between them are known as line voltages (V_L).

- A fourth conductor, called the neutral (coloured black, and connected through protective devices to earth) is often used with a three-phase supply.

- If the three-phase windings are kept independent then six wires are needed to connect a supply source (such as a generator) to a load (such as motor). To reduce the number of wires it is usual to interconnect the three phases.

- There are two ways in which this can be done, these being: (a) **a star connection**, and (b) **a delta, or mesh, connection.**

- Sources of three-phase supplies, i.e. alternators, are usually connected in star, whereas three-phase transformer windings, motors and other loads may be connected either in star or delta.

Reasons for the Use of 3-phase System

- Constant power. In a single-phase circuit, the instantaneous power varies sinusoidally from zero to a peak value at twice the supply frequency. This pulsating nature of power is objectionable for many applications. However, in a balanced 3-phase system, the power supplied at all instants of time is constant. Because of this, the operating characteristics of 3-phase apparatus, in general, are superior to those of similar single-phase apparatus.
- Greater output. The output of a 3-phase machine is greater than that of a single-phase machine for a given volume and weight of the machine. In other words, a 3-phase machine is smaller than a single-phase machine of the same rating.
- Cheaper. The three-phase motors are much smaller and less expensive than single-phase motors because less material (copper, iron, insulation) is required. Moreover, 3-phase motors are self-starting i.e. they do not require any special provision to get them started. However, single phase motors require internal starting device.

- Power transmission economics. Transmission of electric power by 3-phase system is cheaper than that of single-phase system, even though three conductors are required instead of two. For example, to transmit the same amount of power over a fixed distance at a given voltage, the 3-phase system requires only 3/4th the weight of copper than that required by the single-phase system. This means a saving in the number and strength of transmission towers.
- Three-phase rectifier service. Rectified 3-phase voltage is smoother than rectified single phase voltage. As a result, it is easier to filter out the ripple component of 3-phase voltage than that of a single-phase voltage. This is especially useful where large a.c. power is to be converted into steady d.c. power e.g. radio and television transmitters.
- Miscellaneous advantages. Other advantages of three-phase system over the single-phase system are : (i) A 3-phase system can set-up a rotating uniform magnetic field in stationary windings. This cannot be done with a single-phase current. (ii) The 3-phase motors are more efficient and have a higher power factor than single phase motors of the same capacity.

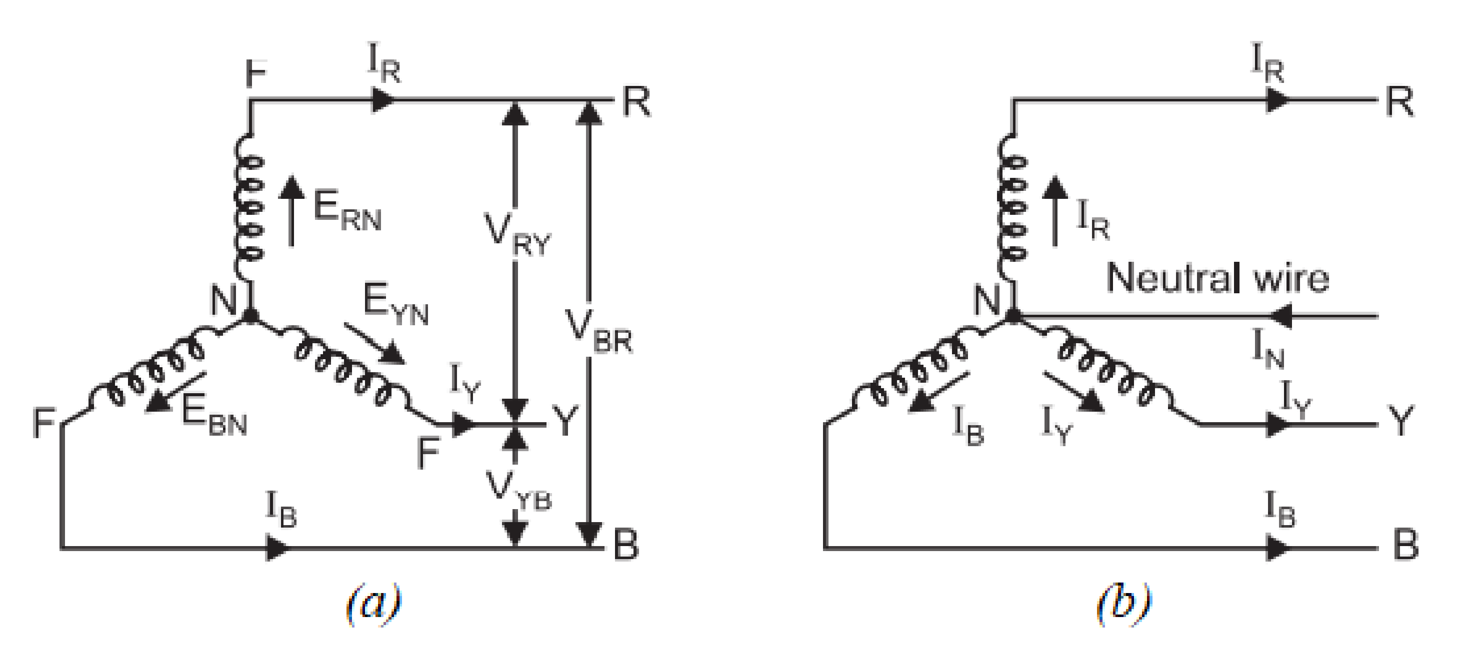
Three phase terms

- Phase voltage: these are voltages between the lines and neutral line (voltage in each phase).
- Line voltage: these are voltages between two lines in a three phase system.
- Phase current: the current flowing through each phase in a three-phase load.
- Line current: the current flowing from the generator to the load in each transmission line in a three phase system.
- Balanced phase voltages: these are phase voltages equal in magnitude and are out of phase with each other by 120^o.
- Balanced load: a balanced load is one in which the phase impedances are equal in magnitude and in phase.
- Phase sequence: The order in which the voltages in the three phases (or coils) of an alternator reach their maximum positive values is called phase sequence or phase order.

Star or Wye (Y) Connected System

- In this method, similar ends (start or finish) of the three phases of the alternator are joined together to form a common junction N as shown in Fig. (a)
- The common junction N is called the star point or neutral point. The three line conductors are run from the three ends (finish ends F in this case) and are designated as R, Y and B.
- This constitutes a 3-phase, 3-wire star-connected system.
- The voltage between any line and the neutral point (i.e., voltage across each winding) is called the phase voltage while the voltage between any two lines is called the line voltage.
- The currents flowing in the phases are called the phase currents while those flowing in the lines are called the line currents. Note that the phase sequence is RYB.
- Sometimes, a 4th wire, called neutral wire, is run from the neutral point as shown in Fig. (b). This gives 3-phase, 4-wire star-connected system.

- The 3-wire star-connected system is used for *balanced loads* (i.e. load in each phase of the alternator has the same impedance and power factor) because then current in the neutral conductor is zero and no neutral conductor is required. However, 4-wire star-connected system is used *for unbalanced loads* because in that case, neutral current exists and the neutral conductor provides the return path as shown in Fig. (b).



Points to note

(*i*) The three phase voltages (*i.e.* E_{RN}, E_{YN}, and E_{BN}) are equal in magnitude but displaced 120° from each other. The same is true for line voltages (*i.e.* V_{RY}, V_{YB} and V_{BR}). *Such a supply system is called* **balanced supply system.**

(*ii*) It can be shown that in case of balanced star-connected supply system, the magnitude of line voltage is $\sqrt{3}$ times the magnitude of phase voltage *i.e.*

Line voltage = $\sqrt{3}$ × Phase voltage ... *in magnitude* $V_L = \sqrt{3}\, V_p$

Thus Y-connected balanced supply system enables us to use two voltages viz. phase voltage and line voltage.

(*iii*) In star connection, the lines are in series with their respective phases. Therefore, magnitude of line current is equal to the magnitude of phase current *i.e.*

Line current = Phase current ... *in magnitude* $I_L = I_p$

(*iv*) For balanced loads, all line currents (or phase currents) are equal in magnitude but displaced 120° from each other.

(*v*) For 3-phase, 4-wire star-connected system, the current I_N in the neutral wire is the phasor sum of the three line currents. For a balanced load, $I_N = 0$. If the load is not balanced, the neutral wire will carry current equal to the phasor sum of the three line currents.

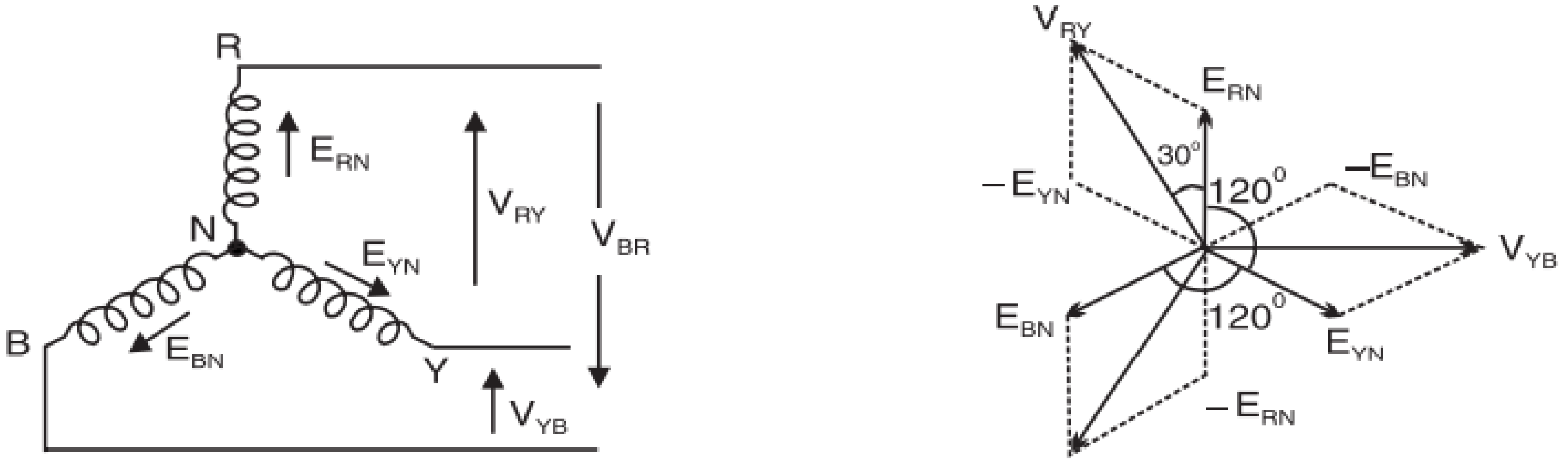
Voltages and Currents in Balanced Y-Connected Supply System

- Fig.(a) shows a balanced 3-phase, Y-connected supply system in which the r.m.s. values of the e.m.f.s generated in the three phases are E_{RN}, E_{YN} and E_{BN}. Since the supply system is balanced, these e.m.f.s will be equal in magnitude (say each equal to E_{ph}, the phase voltage) but displaced 120^o from one another as shown in the phasor diagram in Fig.(a). It is clear from the circuit diagram (Fig. (b) that p.d. between any two line terminals (i.e. line voltage) is the phasor difference between the potentials of these two terminals w.r.t. the neutral i.e.

P.D. between lines R and Y, $V_{RY} = E_{RN} - E_{YN}$ ... *phasor difference*

P.D. between lines Y and B, $V_{YB} = E_{YN} - E_{BN}$ ... --do--

P.D. between lines B and R, $V_{BR} = E_{BN} - E_{RN}$ ... --do--



(a) (b)

Relation between line voltage and phase voltage. Considering the lines R and Y, the line voltage V_{RY} is equal to the phasor difference of E_{RN} and E_{YN}. To subtract E_{YN} from E_{RN}, reverse the phasor E_{YN} and find its phasor sum with E_{RN} as shown in the phasor diagram

The two phasors E_{RN} and $-E_{YN}$ are equal in magnitude ($= E_{ph}$) and are 60° apart.

$$V_{RY} = 2\,E_{ph} \cos(60° / 2) = 2\,E_{ph} \cos 30° = \sqrt{3}\,E_{ph}$$

$$V_{YB} = E_{YN} - E_{BN} \text{ ...phasor difference } = \sqrt{3}\,E_{ph}$$

$$V_{BR} = E_{BN} - E_{RN} \text{ ...phasor difference } = \sqrt{3}\,E_{ph}$$

By phasor algebra. The above relation can be easily established by phasor algebra. Suppose the magnitude of each phase voltage is E_{ph}. Then,

$$\boldsymbol{E_{RN}} = E_{ph} \angle 0° = E_{ph}(1 + j\,0)$$

$$\boldsymbol{E_{YN}} = E_{ph} \angle -120° = E_{ph}(-0.5 - j\,0.866)$$

$$\boldsymbol{E_{BN}} = E_{ph} \angle -240° = E_{ph}(-0.5 + j\,0.866)$$

Now,

$$\boldsymbol{V_{RY}} = \boldsymbol{E_{RN}} - \boldsymbol{E_{YN}} = E_{ph}(1 + j\,0) - E_{ph}(-0.5 - j\,0.866)$$

$$= E_{ph}(1.5 + j\,0.866) = \sqrt{3}\;E_{ph} \angle 30°$$

$\therefore$

$$\boldsymbol{V_{RY}} = \sqrt{3}E_{ph} \angle 30°$$

Similarly,

$$V_{YB} = \sqrt{3}\,E_{ph} \text{ and } V_{BR} = \sqrt{3}\,E_{ph}$$

Power

The total power in the circuit is quite logically, the sum of powers in the three phases.

For a balanced load, the power consumed in each load phase is the same.

$$\therefore \quad \text{Total power, } P = 3 \times \text{Power in each phase} = 3 \times V_{ph}\, I_{ph} \cos\phi \qquad ...(i)$$

$$\text{For a star connection, } V_{ph} = V_L/\sqrt{3} \quad ; \quad I_{ph} = I_L$$

$$\therefore \quad P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi \qquad ...(ii)$$

$$\text{Reactive power, } Q = \sqrt{3}\, V_L I_L \sin\phi$$

Either of relations (i) and (ii) can be used to determine the power. *It may be noted that ϕ is the phase difference between a phase voltage and the corresponding phase current and not between the line current and corresponding line voltage.*

$$\text{Apparent power, } S = \sqrt{3}\, V_L I_L$$

$$\therefore \quad S = \sqrt{P^2 + Q^2} \text{ and power factor } \cos\phi = \frac{P}{S}$$

Checking Correct Connections for Y-connected Alternator

In star-connected alternator, similar ends (start or finish) of the three coils of alternator are connected together to form the neutral point. If this is not done, the line voltages will not be 120° apart nor line voltages equal to $\sqrt{3}\,E_{ph}$. The proper connections can be checked with a voltmeter as under :

(*i*) Connect two coils in series and measure the voltage across the free ends. This voltage should be $\sqrt{3}\,E_{ph}$ where E_{ph} is the voltage of each coil. If it is less, reverse the connections of one coil.

(*ii*) Now connect one end of the third coil to the common junction (*i.e.*, neutral point). Again the voltage from the free end of this coil to the free end of each of the other two coils should be $\sqrt{3}\,E_{ph}$. If this value is not obtained, reverse the third coil connections.

Advantages of Star Connection

(*i*) In star connection, phase voltage $V_{ph} = V_L/\sqrt{3}$. Since the induced e.m.f. in the phase winding of an alternator is directly proportional to the number of turns, a star-connected alternator will require less number of turns than a Δ-connected alternator for the same line voltage.

(*ii*) For the same line voltage, a star-connected alternator requires less insulation than a delta-connected alternator.

Due to above two reasons, 3-phase alternators are generally star-connected. One can hardly find delta-connected alternators.

(*iii*) In star connection, we can get 3-phase, 4-wire system. This permits to use two voltages *viz*, phase voltages as well as line voltages. Remember that in star connection, $V_L = \sqrt{3}\, E_{ph}$. Single phase loads (*e.g.* lights etc) can be connected between any one line and the neutral wire while the 3-phase loads (*e.g.* 3-phase motors) can be put across the three lines. Such a flexibility is not available in Δ-connection.

(*iv*) In star connection, the neutral point can be earthed. Such a measure offers many advantages. For example, in case of line to earth fault, the insulators have to bear $1/\sqrt{3}$ (*i.e.* 57.7%) times the line voltage. Moreover, earthing of neutral permits to use protective devices (*e.g.* relays) to protect the system in case of ground faults.

Practice Questions

- **Qn. 1:** Three coils, each having a resistance of 20Ω and an inductive reactance of 15Ω, are connected in star to a 400 V, 3-phase, 50 Hz supply. Calculate (i) the line current (ii) power factor and (iii) power supplied.
- **Qn. 2:**A star-connected load consists of three identical coils each of resistance 30 Ω and inductance 127.3 mH. If the line current is 5.08A, calculate the line voltage if the supply frequency is 50 Hz.
- **Qn. 3:**Three similar coils, connected in star, take a total power of 1.5 kW at a p.f. of 0.2 lagging from a 3-phase 400 V, 50 Hz supply. Calculate (i) the resistance and inductance of each coil and (ii) the line currents if one of the coils is short-circuited.
- **Qn. 4:** A star-connected balanced system with a line voltage of 300 V is supplying a balanced Y-connected load of 1200 W at a leading p.f. of 0·8. What is the line current and per phase impedance ? If a balanced 600 W lighting load is added in parallel, find the line current.

Practice Questions

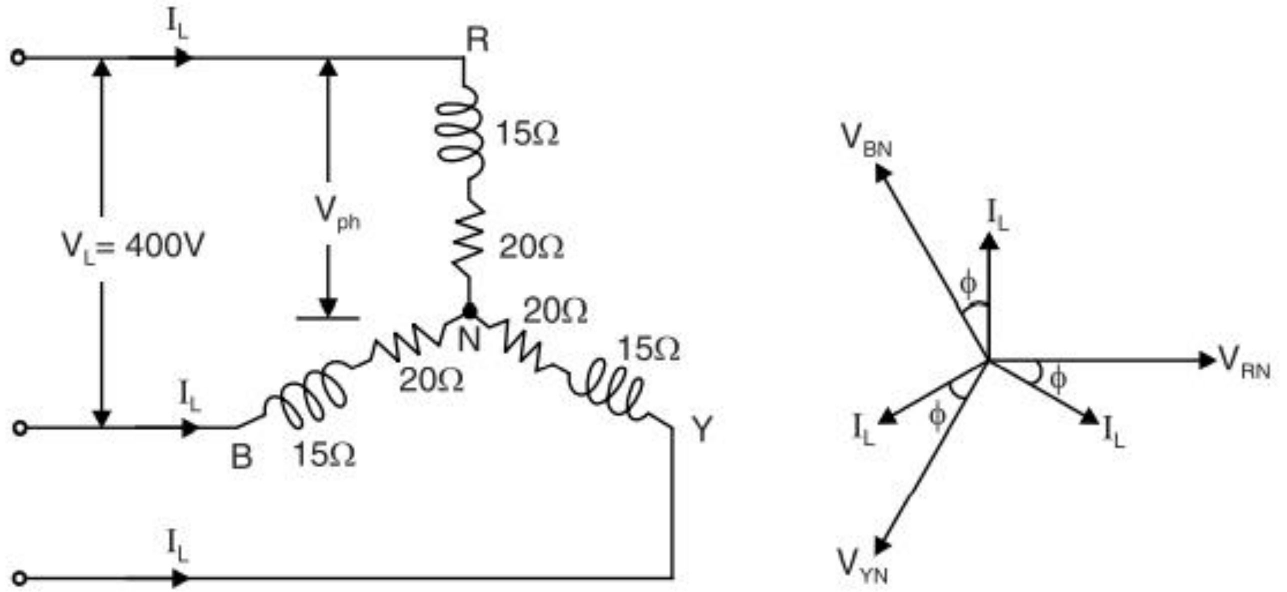
Qn. 5: Three-phase star-connected load when supplied from 400 V, 50 Hz source takes a line current of 10 A at 66.86^{o} w.r.t. its line voltage. Calculate (i) impedance parameters (ii) power factor and active power consumed.

Qn. 6: The load to a 3-phase supply consists of three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find (i) the phase and line voltages (ii) the kVAR input (iii) resistance and reactance of each coil.

Qn. 7: Each phase of a star-connected load consists of a non-reactive resistance of 100 Ω in parallel with a capacitance of 31.8 μF. Calculate the line current, the power absorbed, the total kVA and power factor when connected to a 416 V, 3-phase, 50 Hz supply.

Qn. 8: Three similar coils, arranged symmetrically in space, are fed from a 400 V, 3-phase, 50 Hz supply. The coils are connected in star and each coil has a resistance of 100 Ω and inductive reactance of 250 Ω. The mutual reactance (ωM) between each pair of coils is 95 Ω. Find the current taken by each coil and its power factor

Solution Qn. 1



(*i*) Phase voltage, $*V_{ph} = V_L/\sqrt{3} = 400/\sqrt{3} = 231$ V

Impedance / phase, $Z_{ph} = \sqrt{20^2 + 15^2} = 25\,\Omega$

Phase current, $I_{ph} = V_{ph} / Z_{ph} = 231/25 = 9.24$ A

∴ Line current, $I_L = I_{ph} =$ **9.24 A**

(*ii*) Power factor, $\cos\phi = \dfrac{\text{Resistance/phase}}{\text{Impedance/phase}} = 20 / 25 =$ **0.8 *lag***

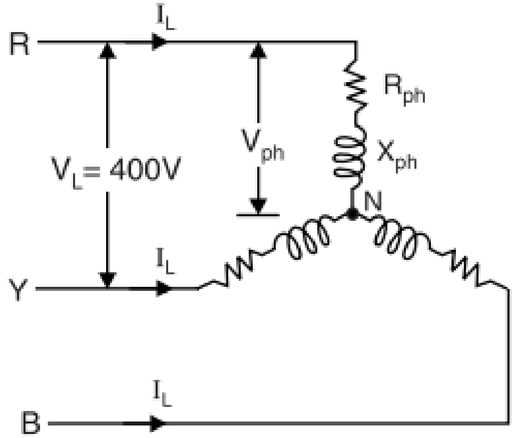
(*iii*) Power supplied, $P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 9.24 \times 0.8 =$ **5121 W**

Alternatively. $P = 3\, I^2_{ph} R_{ph} = 3 \times (9.24)^2 \times 20 =$ **5121 W**

Solution Qn. 3

Phase voltage, $V_{ph} = V_L/\sqrt{3} = 400/\sqrt{3} = 231$ V

(*i*) When the three coils are star-connected



$$\text{Total power taken, } P = \sqrt{3}\, V_L I_L \cos\phi$$

$$\text{Line current, } I_L = \frac{P}{\sqrt{3}\, V_L \cos\phi} = \frac{1500}{\sqrt{3}\times 400\times 0.2} = 10.83 \text{ A}$$

$$\text{Phase current, } I_{ph} = I_L = 10.83 \text{ A}$$

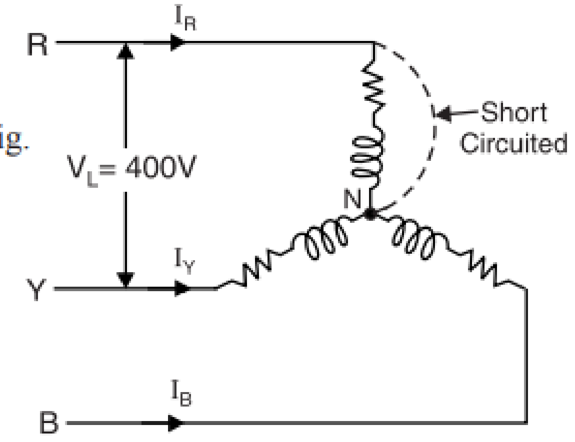
$$\text{Impedance/phase, } Z_{ph} = V_{ph}/I_{ph} = 231/10.83 = 21.33\ \Omega$$

$$\text{Resistance/phase, } R_{ph} = Z_{ph}\cos\phi = 21.33\times 0.2 = \mathbf{4.266\ \Omega}$$

$$\text{Reactance/phase, } X_{ph} = \sqrt{(21.33)^2 - (4.266)^2} = 20.9\ \Omega$$

$$\therefore \quad \text{Inductance/phase, } L_{ph} = \frac{X_{ph}}{2\pi f} = \frac{20.9}{2\pi\times 50} = \mathbf{0.0665\ H}$$

(*ii*) When one of the coils is short-circuited. Suppose the phase load connected between terminals *R* and *N* is short-circuited as shown in Fig. Clearly, the terminal *N* will be at the same potential as the terminal *R*. Therefore, the phase voltages V_{YN} and V_{BN} become equal to the line voltages.



$$\text{Current in phase } Y, I_Y = V_L/Z_{ph} = 400/21.33 = 18.75 \text{ A}$$

$$\text{Current in phase } B, I_B = V_L/Z_{ph} = 400/21.33 = 18.75 \text{ A}$$

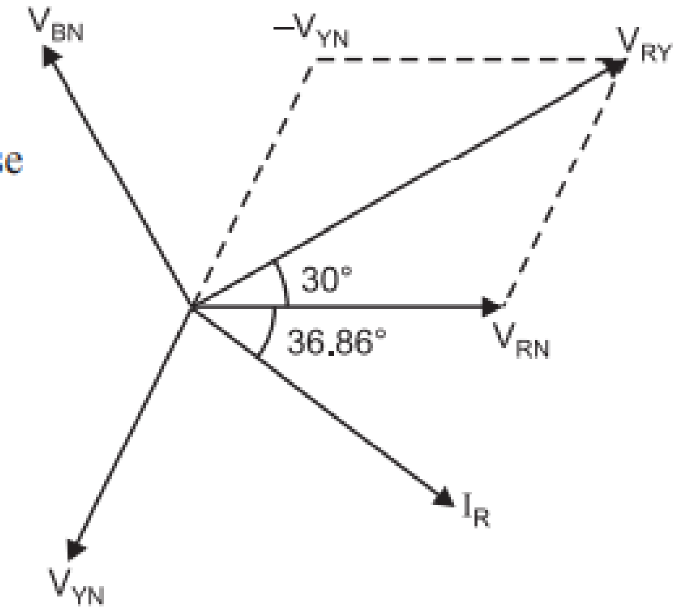
Therefore, line currents in each of the unfaulted sections (*i.e.*, lines *Y* and *B*) are **18.75 A.**

The magnitudes of the two phase currents are equal (being 18.75 A) and they are 60° apart. The current in phase *R* is equal to the phasor sum of the two.

$$\therefore \quad \text{Current in phase } R, I_R = 2\, I_{ph}\cos(60°/2) = 2\times 18.75\times \sqrt{3}/2 = \mathbf{32.47\ A}$$

Solution Qn. 5

We know that line voltages are 30° ahead of their respective phase voltages. This fact is illustrated in Fig.



It is clear that phase angle between phase voltage and phase current is $\phi = 66.86° - 30° = 36.86°$.

(*i*) $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$I_L = I_{ph} = 10 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{231}{10} = 23.1\ \Omega$$

$\therefore$ $$R_{ph} = Z_{ph} \cos\phi = 23.1 \times \cos 36.86° = \mathbf{18.48\ \Omega}$$

$$X_{Lph} = Z_{ph} \sin\phi = 23.1 \times \sin 36.86° = \mathbf{13.86\ \Omega}$$

(*ii*) Power factor $= \cos\phi = \cos 36.86° =$ **0.8 lagging**

Alternatively. $$\text{p.f.} = \frac{R_{ph}}{Z_{ph}} = \frac{18.48}{23.1} = \mathbf{0.8\ lagging}$$

Active power consumed, $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 10 \times 0.8 =$ **5544 W**

Solution Qn.7

Fig. shows the conditions of the problem. The phase voltage $V_{ph} = 416/\sqrt{3} = 240$ V.

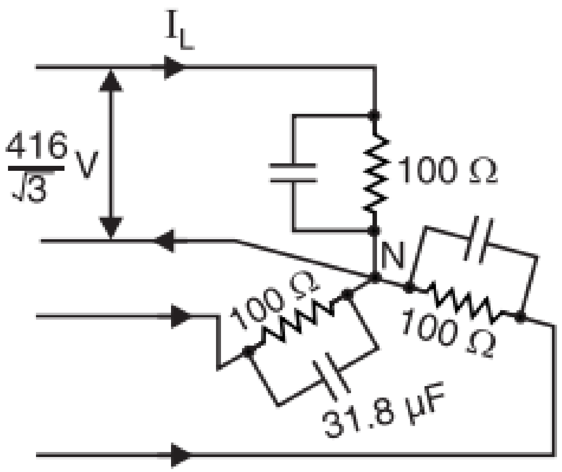
Let us take *R*-phase as the reference. Its phase voltage is given by ;

$$\boldsymbol{V_{ph}} = 240 \angle 0° \text{ V} = (240 + j\,0) \text{ V}$$

$$\text{Admittance/phase, } \boldsymbol{Y_{ph}} = \frac{1}{R} + j\omega C$$

$$= \frac{1}{100} + j \times 314 \times 31.8 \times 10^{-6}$$

$$= (0.01 + j\,0.01)\text{S}$$



$$\therefore \quad \text{Phase current, } \boldsymbol{I_{ph}} = \boldsymbol{V_{ph}}\,\boldsymbol{Y_{ph}} = 240\,(0.01 + j\,0.01)$$

$$= (2.4 + j\,2.4) \text{ A} = 3.39 \angle 45° \text{ A}$$

$$\text{For star connection, } I_{ph} = I_L = \mathbf{3.39\ A}$$

$$\text{Circuit power factor} = \cos\phi = \cos 45° = \mathbf{0.707}\ \textit{lead}$$

$$\boldsymbol{S} = \boldsymbol{V_{ph}}\boldsymbol{I_{ph}} = (240 + j\,0)\,(2.4 + j\,2.4)$$

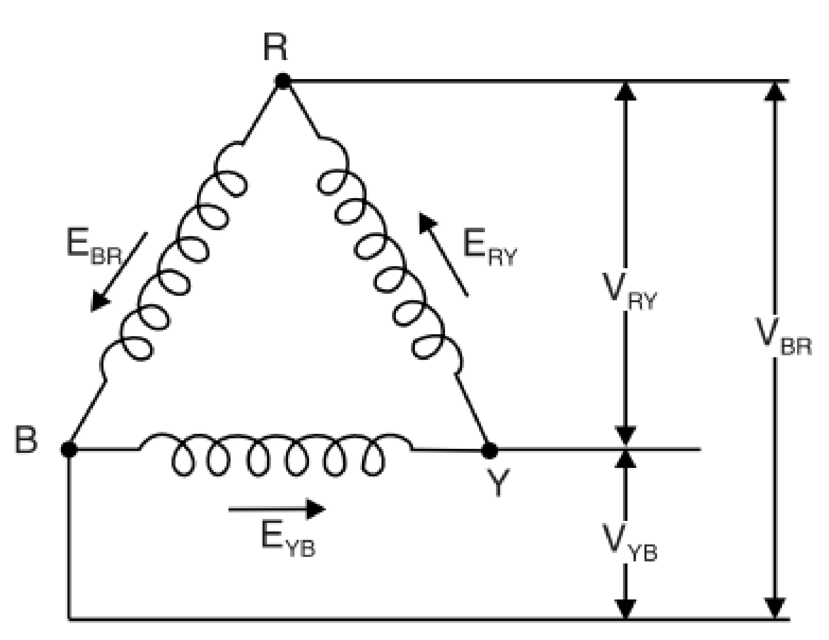
$$= (576 + j\,576) \text{ VA} = 814.4 \angle 45° \text{ VA}$$

$$\therefore \quad \text{Total active power} = 3 \times 576 = 1728 \text{ W} = \mathbf{1.728\ kW}$$

$$\text{Total apparent power} = 3 \times 814.4 = 2443 \text{ VA} = \mathbf{2.443\ kVA}$$

Delta (Δ) or Mesh Connected System

- In this method of interconnection, the dissimilar ends of the three phase windings of the alternator are joined together i.e., finishing end of one phase is connected to the starting end of the other phase and so on to obtain mesh or delta as shown in Fig. The three line conductors are taken from the three junctions of the mesh or delta and are designated as R, Y and B. This is called 3-phase, 3-wire, delta connected system.



- The arrangement is referred to as mesh connection because it forms a closed circuit. It is also known as delta connection because the diagram has the appearance of Greek letter delta (Δ).
- In delta connection, no neutral exists and, therefore, only 3-phase, 3-wire system can be formed.

Points to note

(i) As can be seen from Fig. only one phase is included between any two lines.

Hence magnitude of voltage between any two lines (*i.e.* **line voltage**) *is equal to the magnitude of* **phase voltage** *i.e.*

Line voltage magnitude, V_L = Phase voltage magnitude (E_{ph}) $\quad V_L = V_p$

The three phase voltages (= line voltages) are equal in magnitude but displaced 120° from one another.

(ii) When 3-phase load (star or delta connected) is connected to the 3-phase Δ-supply, currents flow through the phases (called **phase currents**) as well as through the lines (called **line currents**). An examination of Fig. shows that current in any line is equal to the *phasor difference* of the currents in the two phases connected to that line. Therefore, magnitude of line currents is different from the magnitude of phase currents.

(iii) For balanced load, the three phase currents (I_R, I_Y and I_B) are equal in magnitude but displaced 120° from one another.

Line current = $\sqrt{3}$ × Phase current ... *in magnitude* $\quad I_L = \sqrt{3}\, I_p$

The three line currents will be equal in magnitude but displaced 120° from one another.

Power

$$\text{Total power, } P = 3 \times \text{Power per phase} = 3 \times V_{ph}\, I_{ph} \cos\phi$$

$$\text{For a delta connection, } V_{ph} = V_L \quad ; \quad I_{ph} = I_L/\sqrt{3}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$$

$\therefore$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

where $\cos\phi$ is the power factor of each phase. Note that in either case, star or delta, the expression for the total power is the same provided that the system is balanced.

$$\text{Reactive power, } Q = \sqrt{3}\, V_L I_L \sin\phi$$

$$\text{Apparent power, } S = \sqrt{3} V_L I_L$$

The relationship between active power (P), reactive power (Q) and apparent power (S) is the same for balanced 3-phase circuits as for single-phase circuits.

$$\therefore \quad S = \sqrt{P^2 + Q^2} \quad \text{where power factor } \cos\phi = \frac{P}{S}$$

Advantages of Delta connections

i. This type of connection is most suitable for rotary convertors.

ii. Most of the 3-phase loads are Δ-connected rather than Y-connected. One reason for this, at least for the case of an unbalanced load, is the flexibility with which loads may be added or removed on a single phase.

This is difficult (or impossible) to do with a Y-connected 3-wire load.

iii. Most of 3-phase induction motors are delta-connected.

Practice Questions

- Three similar coils each having a resistance of 5Ω and an inductance of 0.02H are connected in delta to a 440V, 3-phase, 50Hz supply. Calculate the line current and total power absorbed.

Solution. Reactance of coil, $X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$

$$\text{Impedance / phase, } Z_{ph} = \sqrt{R^2 + X_L^2} = \sqrt{(5)^2 + (6.28)^2} = 8.05\ \Omega$$

$$\text{Power factor of each coil, } \cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{5}{8.05} = 0.622\ lag$$

$$\text{Phase voltage, } V_{ph} = V_L = 440\ \text{V}$$

$$\therefore \quad \text{Phase current, } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{8.05} = 54.6\ \text{A}$$

$$\therefore \quad \text{Line current, } I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 54.6 = \mathbf{94.8\ A}$$

$$\text{Power absorbed, } P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 94.8 \times 0.622 = \mathbf{45000\ W}$$

Note that if the coils had been star-connected and connected to the same supply, then,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440/\sqrt{3}}{8.05} = 31.6\text{A} = I_L \quad ; \cos\phi = 0.622 \text{ lag}$$

$$\therefore \quad P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 31.6 \times 0.622 = 15000\ \text{W}$$

Here is an important conclusion. *The kW delivered to three equal impedances which are delta connected to a 3-phase supply is always three times the kW delivered to them if they are star-connected.*

Practice Question

- A 3-phase, 400 V, 50 Hz a.c. supply is feeding a 3-phase delta-connected load with each phase having a resistance of 25 Ω, an inductance of 0.15 H and a capacitor of 120 μF in series. Find line current, volt-amp, active power and reactive volt-amp.

Solution.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.1\ \Omega \quad ; \quad X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 26.54\ \Omega$$

$$\text{Net reactance/phase, } X = X_L - X_C = 47.1 - 26.54 = 20.56\ \Omega$$

$$\text{Impedance/phase, } Z_{ph} = \sqrt{R^2 + X^2} = \sqrt{(25)^2 + (20.56)^2} = 32.37\ \Omega$$

$$\text{Power factor, } \cos\phi = R/Z_{ph} = 25/32.37 = 0.772\ lag$$

$$\text{Phase current, } I_{ph} = V_{ph}/Z_{ph} = 400/32.37 = 12.36\ \text{A}$$

$$\therefore \quad \text{Line current, } I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 12.36 = \mathbf{21.4\ A}$$

$$\text{Total active power, } P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 21.4 \times 0.772 = \mathbf{11446\ W}$$

$$\text{Total apparent power, } S = P/\cos\phi = 11446/0.772 = \mathbf{14830\ VA}$$

$$\text{Total reactive power, } Q = \sqrt{S^2 - P^2} = \sqrt{(14830)^2 - (11446)^2} = \mathbf{9430\ VAR}$$

Practice Questions

- **Qn. 1:** Three identical resistances, each of 15 Ω, are connected in delta across 400 V, 3-phase supply. What value of resistance in each leg of balanced star-connected load would take the same line current ? [5 Ω]
- **Qn. 2:** Three equal impedances are connected in delta to a 440 V, 50 Hz supply. The p.f. is 0.8 lagging and $25\sqrt{3}$ kVA is drawn from the supply. Calculate the line current and total power drawn from the supply when the same impedances are star-connected.
- **Qn. 3:** A balanced D-connected load takes a line current of 18A at a p.f. of 0.85 leading from a 400 V, 3-phase, 50 Hz supply. Calculate the resistance and capacitance of each leg of the load.

Effects of Phase Sequence

- The order in which the voltages in the three phases (R, Y and B) of an alternator reach their maximum positive values is called the phase sequence.
- It is determined by the direction of rotation of the alternator. Suppose an alternator rotating in an anticlockwise direction produces voltages of phase sequence RYB.
- If this alternator rotates in the clockwise direction, the voltages produced will have a phase sequence RBY.
- Since an alternator can be rotated in either anticlockwise or clockwise direction, there can be only two possible phase sequences viz. RYB and RBY.
- By convention, phase sequence RYB is taken as positive and RBY as negative. Suppose each phase of an alternator generates 240 V (r.m.s.).
- **Phase sequence RYB**. In phasor notation, the phase voltages can be expressed as :

$$\boldsymbol{E_R} = 240 \angle 0^o \text{ V} ; \qquad \boldsymbol{E_Y} = 240 \angle -120^o \text{ V} ; \qquad \boldsymbol{E_B} = 240 \angle -240^o \text{ V}$$

- **Phase sequence RBY**. In phasor notation, the phase voltages can be expressed as :

$\boldsymbol{E_R} = 240 \angle 0^o \text{ V}$; $\boldsymbol{E_B} = 240 \angle -120^o \text{ V}$; $\boldsymbol{E_Y} = 240 \angle -240^o \text{ V}$

- Where balanced resistive loads are involved, the phase sequence is normally not important.
- The direction of rotation of a 3-phase induction motor depends upon the phase sequence of the 3-phase supply. If the phase sequence is reversed by interchanging any two lines of the 3-phase supply, the motor would rotate in the opposite direction.
- The currents in the three phases of an unbalanced Y-connected load depend on the phase sequence of the 3-phase supply. If the phase sequence is reversed, we get completely different currents in the three phases of the Y-connected load.
- Reversing the phase sequence of a 3-phase alternator which is to be paralleled with a similar alternator can cause extensive damage to both the machines.

Y/Δ or Δ/Y Conversions for Balanced Loads

- Two circuits are electrically equivalent if they draw the same line current and power when connected to the same line voltage.
- Consider a balanced Y-connected load having an impedance Z_1 in each phase as shown in Fig. (a).
- Let the equivalent Δ-connected load have an impedance of Z_2 in each phase as shown in Fig. (b).
- Since the two circuits are equivalent, the impedance between any two corresponding terminals of the two circuits is the same.
- Considering terminals 1 and 2,

For star connection, $Z_{12} = Z_1 + Z_1 = 2Z_1$

For delta connection, $Z_{12} = Z_2 \parallel (Z_2 + Z_2) = \frac{(Z_2)(2Z_2)}{Z_2 + 2Z_2} = \frac{2}{3}Z_2$

$$2Z_1 = \frac{2}{3}Z_2 \quad \text{or} \quad Z_2 = 3Z_1 \qquad \text{...}(i)$$

$$Z_\Delta = 3\,Z_Y \quad \text{or} \quad Z_Y = \frac{Z_\Delta}{3} \qquad \text{...}(ii)$$

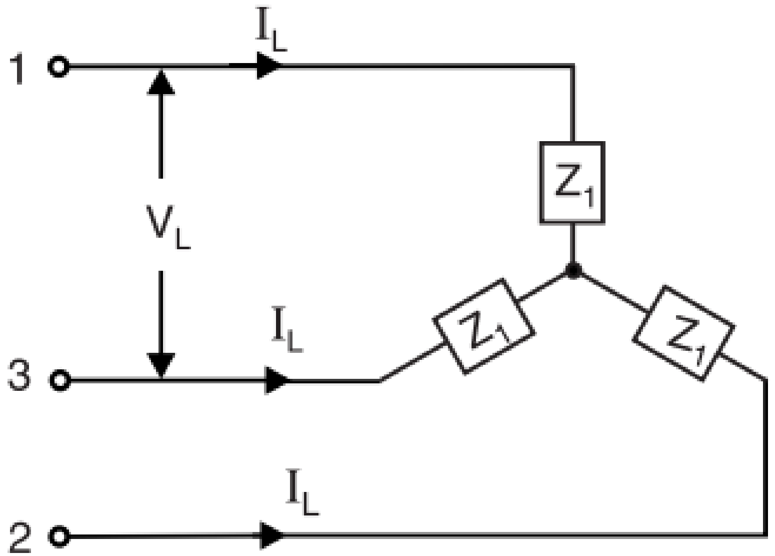


Fig. (a)

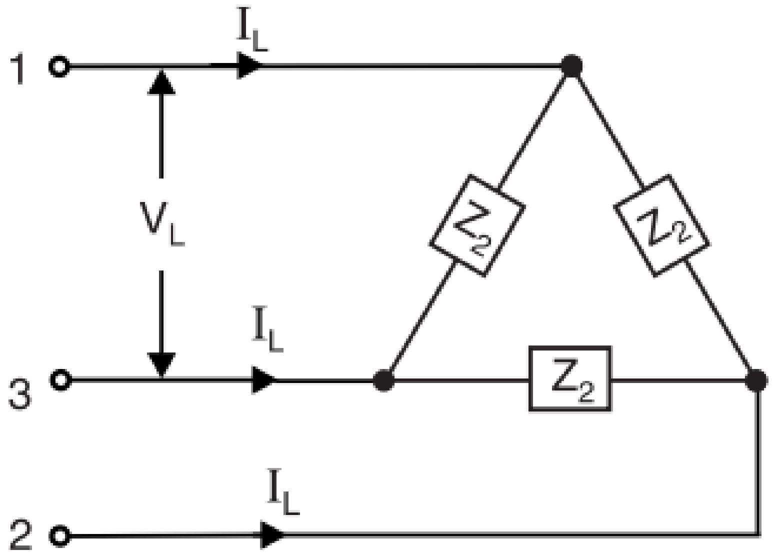


Fig. (b)

Using equations (*i*) or (*ii*), Y/Δ or Δ/Y conversions can be made for the balanced loads.

3-phase Balanced Loads in Parallel

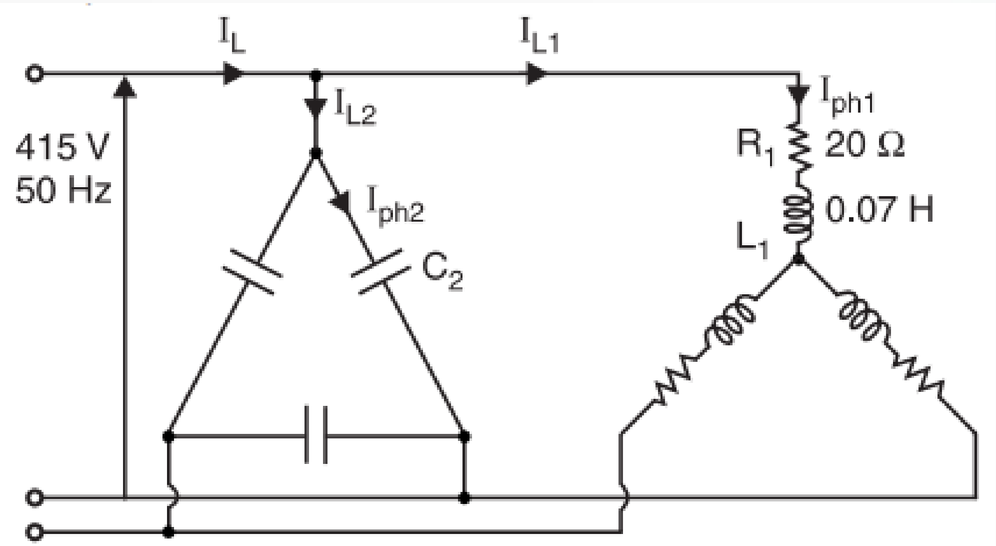
- In a 3-phase supply system, it is quite common to have balanced Δ and Y loads connected in parallel. Rarely the loads are all Δ -connected or all Y-connected. The problems on such parallel loads can be solved by one of the following three ways :
- (i) All the Y-loads in the problem may be converted into equivalent Δ loads. Then these loads in parallel (all now being Δ loads) can be treated on a single-phase basis (line-to-neutral) to find the various circuit values e.g., currents, power factor etc.
- (ii) All the Δ loads in the problem may be converted into equivalent Y loads and treated as in (i) above.
- Out of the above two methods, usually the latter is preferred because it is more convenient to handle Y-connection.
- (iii) A still shorter and more commonly used method is to treat each load on a complete 3-phase basis. The active, reactive and apparent powers in each 3-phase load are determined.

$$S = \sqrt{P^2 + Q^2}$$

- The various characteristic quantities (e.g., power factor, line current etc.) can be easily determined for the combined loads.

Practice Question

- **Qn. 1:** Three star-connected impedances $Z_1 = (20 + j\ 37.7)\ \Omega$ per phase are connected in parallel with three delta connected impedances $Z_2 = (30 - j\ 159.3)\ \Omega$ per phase. The line voltage is 398 V. Find (i) the total line current (ii) circuit power factor (iii) active power and reactive power taken by the combination.
- **Qn. 2**: Each phase of a star-connected load consists of a coil of resistance 20Ω and inductance 0.07 H. The load is connected to a 415V, 3-phase, 50Hz supply. Calculate :(i) The line current, the power and the power factor (ii) The capacitance per phase of a delta-connected capacitor bank which would improve the overall power factor to unity.



Solution Qn. 1

(*i*) The phase voltage $= 398/\sqrt{3} = 230$ V

$$\boldsymbol{Z_1} = (20 + j\,37.7)\ \Omega = 42.7 \angle 62.05°\ \Omega$$

$$\therefore \quad \text{Line current, } \boldsymbol{I_1} = \frac{230\angle 0°}{\boldsymbol{Z_1}} = \frac{230\angle 0°}{42.7\angle 62.05°} = 5.39 \angle -62.05°\text{ A} = (2.52 - j\,4.76)\text{A}$$

Equivalent star impedance per phase of Δ-load

$$= \frac{\boldsymbol{Z_2}}{3} = \frac{30 - j159.3}{3} = (10 - j\,53.1)\ \Omega = 54 \angle -79.3°\ \Omega$$

$$\therefore \quad \text{Line current, } \boldsymbol{I_2} = \frac{230 \angle 0°}{54 \angle -79.3°} = 4.26 \angle 79.3°\text{ A} = (0.79 + j\,4.19)\text{A}$$

$$\text{Total line current, } \boldsymbol{I} = \boldsymbol{I_1} + \boldsymbol{I_2} = (2.52 - j\,4.76) + (0.79 + j\,4.19)$$

$$= (3.31 - j\,0.57)\text{A} = \mathbf{3.36 \angle -9.8°A}$$

(*ii*) Power factor $= \cos 9.8° =$ **0.985** ***lag***

(*iii*) Total active power, $P = 3 \times$ power/phase $= 3 \times 230 \times 3.36 \times 0.985 =$ **2284 W**

Total reactive *VA*, $Q = 3\ VI \sin\phi = 3 \times 230 \times 3.36 \times 0.17 =$ **401 VAR**

Solution Qn. 2

(*i*) Without capacitor bank

$$X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times 0.07 = 22\ \Omega$$

$$Z_{ph1} = \sqrt{R_1^2 + X_{L1}^2} = \sqrt{20^2 + 22^2} = 29.8\ \Omega$$

$$I_{L1} = I_{ph1} = \frac{V_{ph1}}{Z_{ph1}} = \frac{415/\sqrt{3}}{29.8} = \mathbf{8.05\ A}$$

$$P_1 = 3I^2_{ph1}R_1 = 3 \times (8.05)^2 \times 20 = \mathbf{3890\ W}$$

$$\cos\phi_1 = R_1/Z_{ph1} = 20/29.8 = \mathbf{0.672}\ lag$$

or $P_1 = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 415 \times 8.05 \times 0.672 = 3890\ \text{W}$

(*ii*) With capacitor bank. In order that the overall power factor becomes unity, the reactive component of line current should be zero *i.e.*, $I_L \sin\phi = 0$. This will be so if $I_{L2} = I_{L1} \sin\phi_1$ *i.e.*,

$$I_{L2} = I_{L1}\sin\phi_1 = I_{L1}\frac{X_{L1}}{Z_{ph1}} = 8.05 \times \frac{22}{29.8} = 5.95\ \text{A}$$

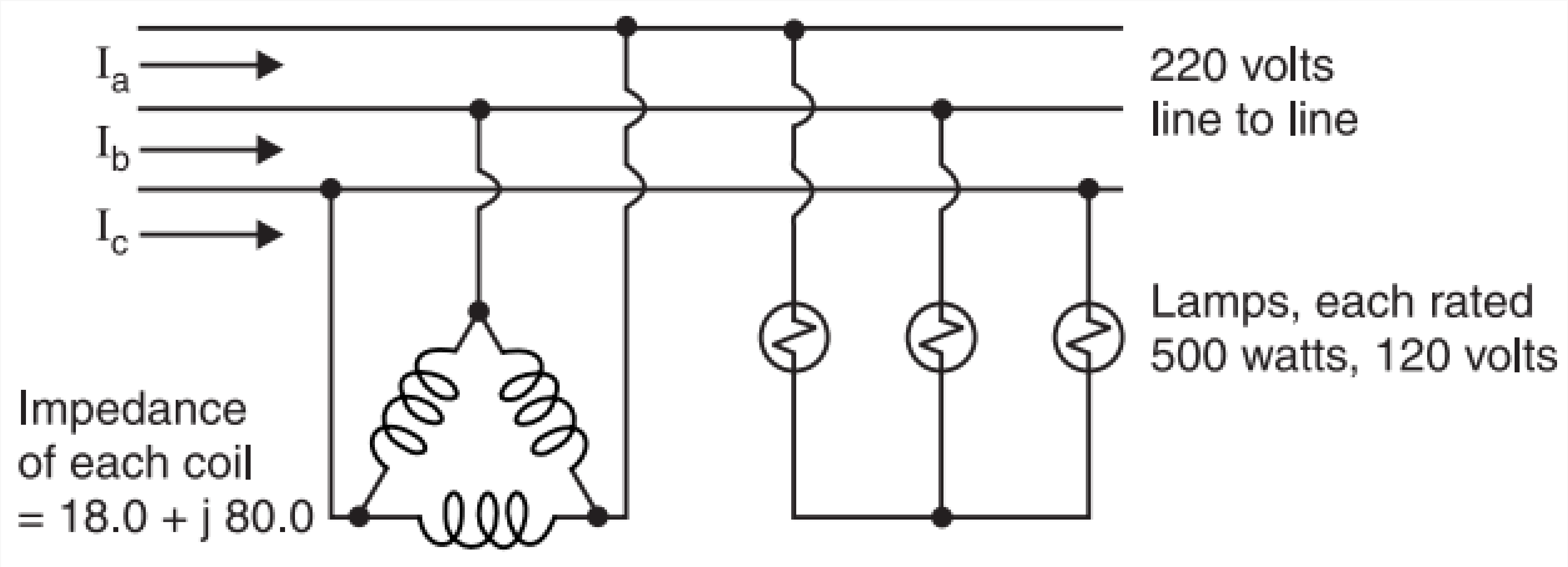
$$I_{ph2} = I_{L2}/\sqrt{3} = 5.95/\sqrt{3} = 3.44\ \text{A}$$

$$X_{C2} = V_{ph}/I_{ph2} = 415/3.44 = 121\ \Omega$$

$$C_2 = \frac{1}{2\pi f X_{C_2}} = \frac{1}{2\pi \times 50 \times 121} = 26.3 \times 10^{-6}\ \text{F} = \mathbf{26.3\ \mu F}$$

Practice Question

- In the circuit shown in Fig. below, find the line current and total power.



Solution

The phase voltage $V_{an} = 220\angle 0° / \sqrt{3} = 127\angle 0°$ V

$$Z_1 = R + j\,0 = \frac{(\text{Rated } V)^2}{P} = \frac{120^2}{500} = 28.8\ \Omega$$

Coil impedance, $Z_2 = (18 + j\,80)\ \Omega$

$$\text{Equivalent star impedance/phase} = \frac{Z_2}{3} = \frac{18 + j\,80}{3} = \frac{82\angle 77.6°}{3} = 27.3\angle 77.6°\ \Omega$$

Since lamp and coil impedances are in parallel, it is convenient to work considering admittances.

$$\text{Total admittance/phase, } Y_{ph} = \frac{1}{R} + \frac{1}{27.3\angle 77.6°} = \frac{1}{28.8} + 0.0366\angle -77.6°$$

$$= (0.0347) + (0.008 - j\,0.0357) = (0.0427 - j\,0.0357)\text{ S}$$

$$\text{Line current, } I_a = Y_{ph}V_{an} = (0.0427 - j\,0.0357)\,(127)$$

$$= (5.42 - j\,4.54)\text{ A} = \mathbf{7.06\angle -39.9°\ A}$$

$$\text{Power per phase} = 127 \times 7.06 \times \cos 39.9° = 688\text{ W}$$

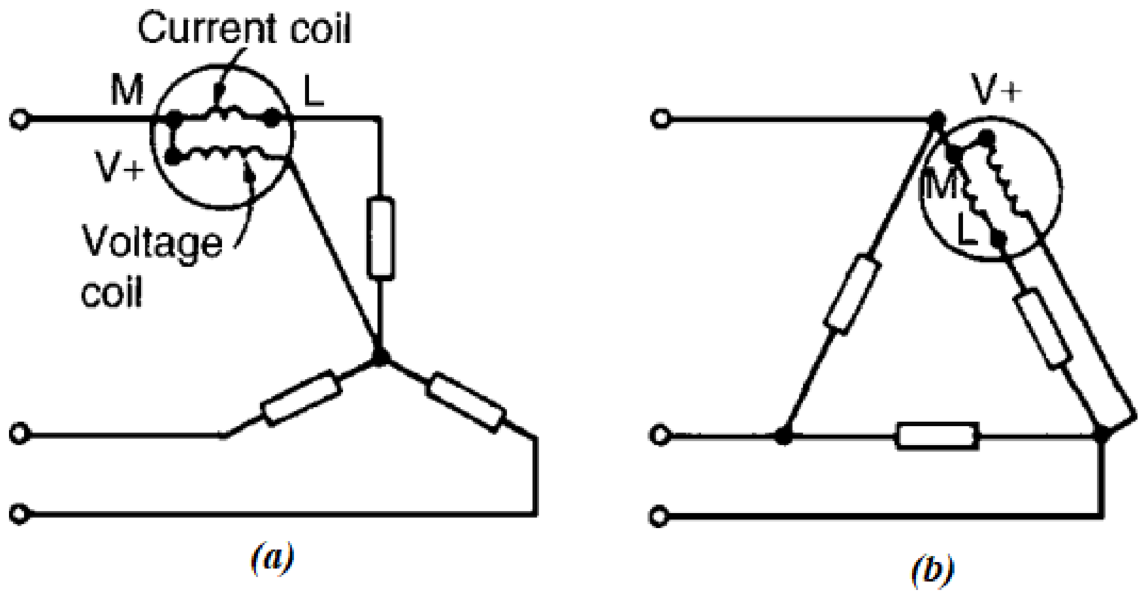
$$\text{Total power, } P = 3 \times 688 = \mathbf{2064\ W}$$

Power Measurement in 3-phase Circuits

- Power in three-phase loads may be measured by the following methods:

<u>One-wattmeter method for a balanced load</u>

- Wattmeter connections for both star and delta are shown in Fig (a) & (b), respectively.



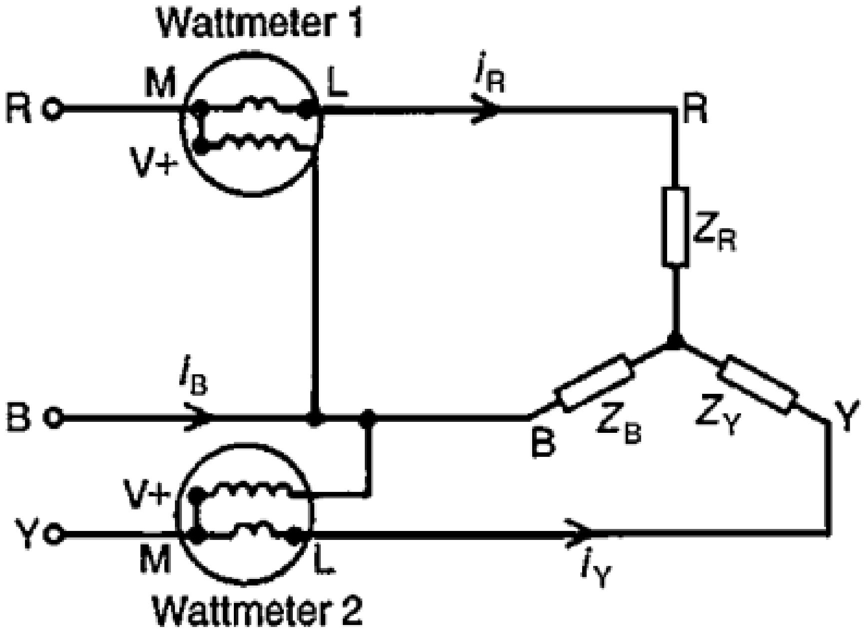
$$\left.\begin{array}{c}\text{Total}\\\text{power}\end{array}\right\} = 3 \times \text{wattmeter reading}$$

Two-wattmeter method for balanced or unbalanced loads

- A connection diagram for this method is shown in Fig. below for a star-connected load. Similar connections are made for a delta-connected load.

Total power = sum of wattmeter readings

$$= P_1 + P_2$$



The power factor may be determined from:

$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

$$\tan\phi = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} \quad \text{...lagging p.f.}$$

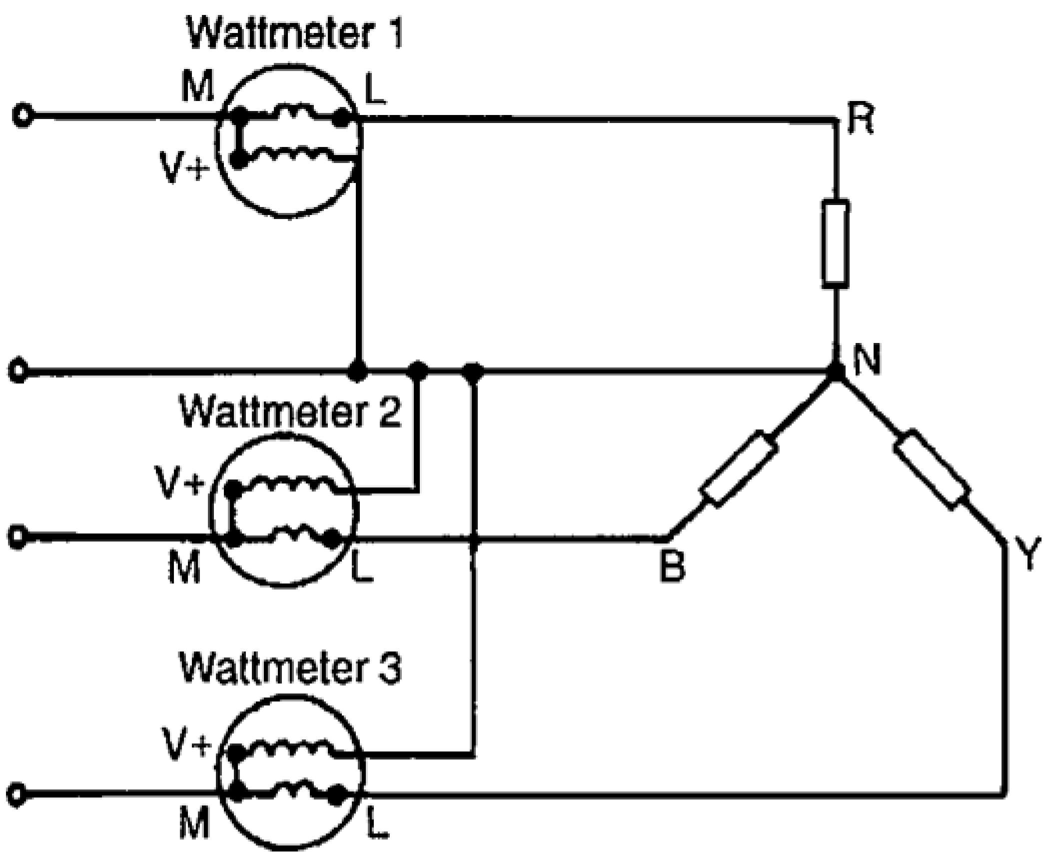
$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} \quad \text{...leading p.f.}$$

$$\tan\phi = \sqrt{3}\,\frac{(\text{Higher reading}) - (\text{Lower reading})}{(\text{Higher reading}) + (\text{Lower reading})}$$

Three-wattmeter method for a three-phase

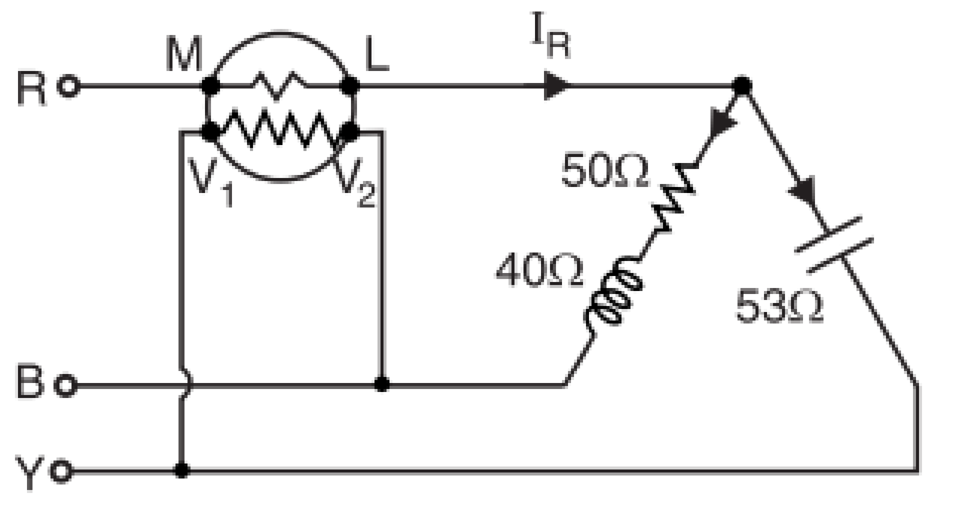
- Three-wattmeter method for a three-phase, 4- wire system for balanced and unbalanced loads

$$\text{Total power} = P_1 + P_2 + P_3$$



Practice Question

- Find the reading on the wattmeter when the network shown in Fig. below is connected to a symmetrical 440 V, 3-phase supply. The phase sequence is RYB. Neglect electrostatic effects and instrument losses.



Solution

The phase sequence is *RYB*.

$V_{RY} = 440 \angle 0° \text{ V}$; $\mathbf{V_{YB}} = 440 \angle -120° \text{ V}$; $V_{BR} = 440 \angle 120° \text{ V}$

Current in current coil,

$$I_{ML} = I_R = \frac{V_{RB}}{50 + j\,40} + \frac{V_{RY}}{53\angle -90^o}$$

$$= -\frac{440\angle 120^o}{64\angle 38.7^o} + \frac{440\angle 0^o}{53\angle -90^o} \quad = j\,8.3 - 6.875 \angle 81.3°$$

$$= j\,8.3 - (1.03 + j\,6.796)$$

$$= (-1.03 + j\,1.5) \text{ A} = 1.825 \angle 124.7° \text{ A}$$

Voltage across potential coil, $V_1V_2 = V_{YB} = 440 \angle -120° \text{ V}$

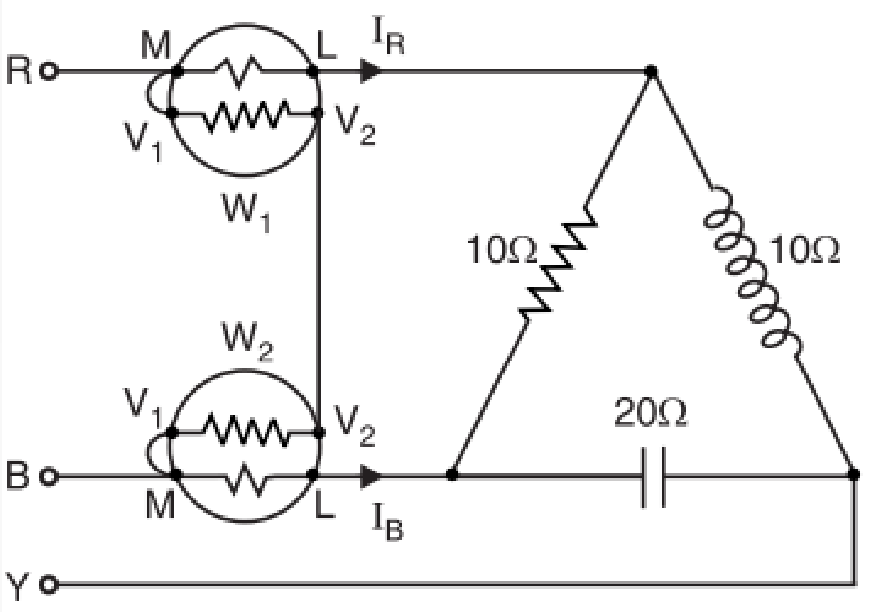
Wattmeter reading, $W = I_{ML} \times V_1V_2 = I_{ML} \cdot V_{YB}$

$$= 1.825 \times 440 \times \cos 244.7°$$

$$= -0.343 \times 10^3 \text{ W} = \mathbf{-0.343\ kW}$$

Practice Question

- Find the readings on each of the two similar watt-meters when the network shown in Fig. below is connected to 400 V, 3-phase supply. The phase sequence is RYB.



Solution

The phase sequence is RYB.

$\therefore \quad V_{RY} = 400 \angle 0° \text{ V}; \quad V_{YB} = 400 \angle -120° \text{ V}; \quad V_{BR} = 400 \angle 120° \text{ V}$

To find the reading on the wattmeter, we find current through its current coil and potential across its potential coil.

Wattmeter 1. Current in current coil, $I_{ML} = I_R$

$$= \frac{V_{RB}}{10} + \frac{V_{RY}}{10 \angle 90^o} = \frac{-400 \angle 120^o}{10} + \frac{400 \angle 0^o}{10 \angle 90^o} = -40 \angle 120° + 40 \angle -90°$$

$$= (20 - j\,74.64)\text{A} \quad = 77.3 \angle -75°\text{A}$$

$$V_1V_2 = \frac{1}{2} V_{RB} = -200 \angle 120° \text{ V}$$

$\therefore$ Wattmeter reading, $W_1 = I_{ML} \cdot V_1V_2 \cos 195° = 77.3 \times (-200) \times \cos 195°$ **= 14.93 kW**

Wattmeter 2. Current in current coil, $I_{ML} = I_B = I_{BR} + I_{BY}$

$$= \frac{V_{BR}}{10} + \frac{V_{BY}}{20 \angle 90^o} = \frac{400 \angle 120^o}{10} + \frac{-400 \angle -120^o}{20 \angle -90^o}$$

$$= 40 \angle 120° - 20 \angle -30° = 58.19 \angle 130° \text{ A}$$

Potential across potential coil is $V_1V_2 = \frac{1}{2} V_{BR} = \frac{1}{2} \times 400 \angle 120^o = 200 \angle 120° \text{ V}$

It is clear that phase angle ϕ between I_{ML} and $V_1V_2 = 130° - 120° = 10°$

$\therefore$ Wattmeter reading, $W_2 = (V_1V_2) \times I_{ML} \times \cos\phi$

$$= 200 \times 58.19 \times \cos 10° = 11.46 \times 10^3 \text{ W} = \mathbf{11.46\ kW}$$

Practice

- **Qn. 1:** Three equal impedances, each consisting of R and L in series are connected in star and are supplied from a 400 V, 50 Hz, 3-phase, 3-wire balanced supply mains. The input power to the load is measured by two watt-meters method and the two watt-meters read 3 kW and 1 kW. Determine the values of R and L connected in each phase.
- **Qn. 2**: A Y-connected balanced load is supplied from a 3-phase balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of the load consists of a resistance and a capacitor joined in series and the readings on two watt-meters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate (i) p.f. of the circuit (ii)the line current and (iii) the capacitance of each capacitor.
- **Qn. 3:** A 440 V, 3-phase, Δ-connected induction motor has an output of 14.92 kW at a p.f. of 0.82 and efficiency 85%. Calculate the readings on each of the two watt-meters connected to measure the input. If another Y-connected load of 10 kW at 0.85 p.f. lagging is added in parallel to the motor, what will be the current drawn from the line and the power taken from the line?