DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

PRIVATE BAG 10143, DEDAN KIMATHI, NYERI KENYA

ELECTRICAL & ELECTRONIC ENGINEERING DEPARTMENT

EEE 3208 ELECTROMAGNETICS III

Prerequisites

EEE 3103 ELECTROMAGNETICS II

Purpose of the Course
The aim of this course is to enable the student to understand advanced analytical techniques for formulating and solving problems in applied electromagnetics.

Expected Learning Outcomes
By the end of this course, the learner should be able to;
1. Understand Time Varying Electromagnetics Fields.
2. Understand boundary value problems, with applications to electrostatics and magnetostatics, time varying fields , and radiating systems.
3. Solve Spectral domain field representations - complex multi-valued functions.
4. Understand Radio wave propagation in stratified media.

Course Content
Introduction to Time Varying Electromagnetic Fields: The electromagnetic spectrum. Maxwell's equations - differential and integral forms.

Applications to wave propagation in dielectrics and good conductors.

Uniform plane waves: magnitude and direction; in vacuum, conducting and non-conducting media.

Waves propagation, Wave Propagation in Lossy Dielectrics, Plane Waves in Lossless Dielectrics, Plane Waves in Free Space, Plane Waves in Good Conductors,

Power and the Poynting Vector, Reflection of a Plane Wave at Normal Incidence, and at Oblique Incidence.

Laboratory/Practical Exercises
i) Radio wave propagation
ii) Wave propagation in Dielectric materials
iii) Planes in free space

Course Assessment

Cats	10%
Assignments	5%
Labs	15%
Exam	70%
Total	**100%**

Reference Text Books

Bhag S. G., & Hüseyin R. H. (2004) Electromagnetic field theory fundamentals, Cambridge University Press, 2nd Ed.

Zahn, M. (2003). Electromagnetic Field Theory: A Problem Solving Approach, Krieger Publishing Company, reprint Ed.

Edminister, J.A. (1994). Schaum's Outline of Theory and Problems of Electromagnetics, McGraw-Hill, 2nd Ed.

Dipak L. S, & Valdis V. L. (2006). Applied Electromagnetics and Electromagnetic Compatibility, John Wiley & Sons

Introduction: to Time Varying Electromagnetic Fields:

In Electromagnetics I & II, focus was mainly on **static electric and magnetic fields** (electrostatics and magnetostatics).

Now, we extend to **time-varying fields**, which are the foundation for understanding **wave propagation, radiation, and radio communication systems**.

Key **concept**: When fields vary with time, **electric and magnetic fields are coupled** → a changing electric field induces a magnetic field, and vice versa.

The electromagnetic spectrum.

Electromagnetic (EM) waves are classified based on frequency f or wavelength λ.

Table 1; The Electromagnetic Spectrum

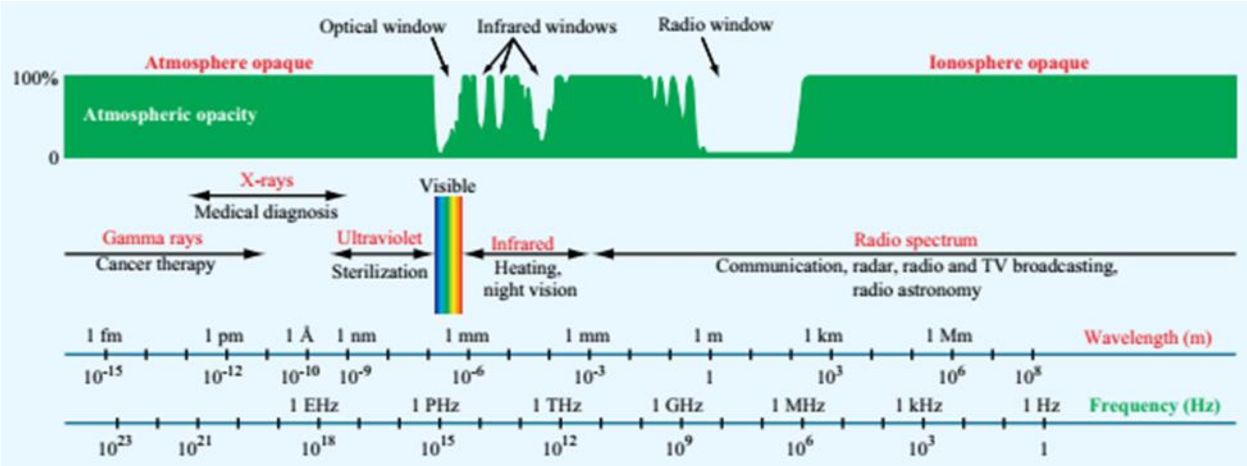
Band Designation	Frequency	Wavelength	Example Uses
ELF (Extremely Low Frequency)	3 to 30 Hz	100 to 10 Mm	
SLF (Super Low Frequency)	30 to 300 Hz	10 to 1 Mm	Power lines
ULF (Ultra Low Frequency)	300 to 3 kHz	1 Mm to 100 km	
VLF (Very Low Frequency)	3 to 30 kHz	100 to 10 km	Submarine comm.
LF (Low Frequency)	30 to 300 kHz	10 to 1 km	RFID
MF (Medium Frequency)	300 kHz to 3 MHz	1 km to 100 m	AM broadcast
HF (High Frequency)	3 to 30 MHz	100 to 10 m	Shortwave broadcast
VHF (Very High Frequency)	30 to 300 MHz	10 to 1 m	FM and TV broadcast
UHF (Ultra High Frequency)	300 MHz to 3 GHz	1 m to 10 cm	TV, WLAN, GPS, Microwave ovens
SHF (Super High Frequency)	3 to 30 GHz	10 to 1 cm	Radar, WLAN, Satellite comm.
EHF (Extremely High Frequency)	30 to 300 GHz	10 to 1 mm	Radar, Radio astronomy, Point-to-point high rate data links, Satellite comm.
Microwaves	1 to 300 GHz	30 cm to 1 mm	
Millimeter waves	30 to 300 GHz	10 to 1 mm	
Submillimeter waves	>300 GHz	<1 mm	

EM waves because they share the following fundamental properties:

- A ***monochromatic*** (single frequency) EM wave consists of electric and magnetic fields that oscillate at the same frequency *f*.
- The phase velocity of an EM wave propagating in vacuum is a universal constant given by the velocity of light.

- In vacuum, the wavelength λ of an EM wave is related to its oscillation frequency f by $\lambda = \frac{c}{f}$

Region	Frequency Range	Wavelength Range	Applications
Radio (RF)	3 Hz – 300 GHz	100 km – 1 mm	Broadcasting, communication
Microwave	300 MHz – 300 GHz	1 m – 1 mm	Radar, satellite, Wi-Fi
Infrared (IR)	300 GHz – 400 THz	1 mm – 0.7 μm	Remote control, thermal imaging
Visible Light	400 – 790 THz	700 – 380 nm	Human vision
Ultraviolet	790 THz – 30 PHz	380 – 10 nm	Sterilization
X-Rays	30 PHz – 30 EHz	10 nm – 0.01 nm	Medical imaging
Gamma Rays	>30 EHz	<0.01 nm	Nuclear processes



Electromagnetic spectrum

We shall examine situations in which electric and magnetic fields are dynamic, or time varying. It should be mentioned first that in static EM fields, electric and magnetic fields are independent of each other, whereas in dynamic EM fields, the two fields are interdependent. In other words, a time-varying electric field necessarily involves a corresponding time-varying magnetic field. Second, time-varying EM fields, represented by $\boldsymbol{E}(x, y, z, t)$ and $\boldsymbol{H}(x, y, z, t)$, are of more practical value than static EM fields.

In summary:

Stationary charges → electrostatic fields

Steady currents → magnetostatic fields

Time-varying currents → electromagnetic fields (or waves). Time-varying EM fields are governed by **Maxwell's Equations**.

FARADAY’S LAW

Induced emf, *V*emf equal to the time rate of change of the magnetic flux linkage by the circuit.

It can be expressed as

$$V_{\text{emf}} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt} \qquad 1$$

Where $\lambda = N\psi$ is the flux linkage, N is the number of turns in the circuit, and ψ is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This behavior is described as Lenz’s law.

Lenz’s law states the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current opposes change in the original magnetic field.

The variation of flux with time as in eq. (1) may be caused in three ways:

1. By having a stationary loop in a time-varying **B** field

2. By having a time-varying loop area in a static **B** field

3. By having a time-varying loop area in a time-varying **B** field

1. Stationary Loop in Time-Varying B Field (Transformer emf)

In Figure 1 a stationary conducting loop is in a time-varying magnetic **B** field

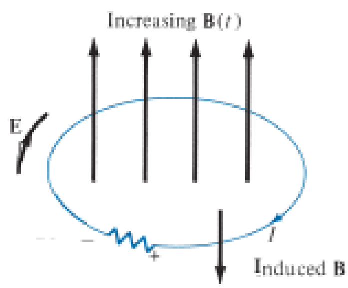


FIGURE 1 Induced emf due to a stationary loop in a time varying **B** field. From equation

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

we have;

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \qquad 2$$

This emf induced by the time-varying current (producing the time-varying **B** field) in a stationary loop is often referred to as *transformer emf* in power analysis, since it is due to transformer action. By applying Stokes's theorem to the middle term in eq. (2), we obtain

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \qquad 3$$

For the two integrals to be equal, their integrands must be equal; that is,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad 4$$

This is one of the Maxwell's equations for time-varying fields. It shows that the timevarying **E** field is not conservative$(\nabla \times E \neq 0)$. This does not imply that the principles of energy conservation are violated. The work done in taking a charge about a closed path in a time-varying electric field, for example, is due to the energy from the time-varying magnetic field. Observe that Figure 1 obeys Lenz's law: the induced current *I* flows such as to produce a magnetic field that opposes the change in **B**(*t*).

B. Moving Loop in Static B Field (Motional emf)

When a conducting loop is moving in a static **B** field, an emf is induced in the loop. We recall) that the force on a charge moving with uniform velocity **u** in a magnetic field **B** is;

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

We define the *motional electric field* **E***m* as

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

If we consider a conducting loop, moving with uniform velocity **u** as consisting of a large number of free electrons, the emf induced in the loop is;

$$V_{\text{emf}} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \qquad 5$$

This type of emf is called *motional emf* or *flux-cutting emf* because it is due to motional action. It is the kind of emf found in electrical machines such as motors, generators, and alternators.

By applying Stokes's theorem to eq. (5), we have

$$\int_S (\nabla \times \mathbf{E}_m) \cdot d\mathbf{S} = \int_S \nabla \times (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{S}$$

Or

$$\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B}) \qquad 6$$

C. Moving Loop in Time-Varying Field

In the general case, a moving conducting loop is in a time-varying magnetic field. Both transformer emf and motional emf are present. Combining eqs. (2) and (5) gives the total emf as;

$$V_{\mathrm{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \qquad 7$$

Or

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B}) \qquad 8$$

DISPLACEMENT CURRENT

We have reconsidered Maxwell's curl equation for electrostatic fields and modified it for time-varying situations to satisfy Faraday's law. We shall now reconsider Maxwell's curl equation for magnetic fields (Ampère's circuit law) for timevarying conditions.

For static EM fields, we recall that

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad 9$$

But the divergence of the curl of any vector field is identically zero. Hence,

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} \qquad 10$$

The continuity of current requires that

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \qquad 11$$

Thus eqs. (10) and (11) are obviously incompatible for time-varying conditions. We must modify eq. (9) to agree with eq. (11). To do this, we add a term to eq. (9) so that it becomes;

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d \qquad 12$$

where $\mathbf{J}_d$ is to be determined and defined. Again, the divergence of the curl of any vector is zero. Hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d \qquad 13$$

In order for eq. (13) to agree with eq. (11),

$$\nabla \cdot \mathbf{J}_d = -\nabla \cdot \mathbf{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

Or

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt} \quad 14$$

Substituting eq. (14) into eq. (12) results in

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad 15$$

This is Maxwell's equation (based on Ampère's circuit law) for a time-varying field. The insertion of $\mathbf{J}_d$ into eq.(9) was one of the major contributions of Maxwell. Without the term $\mathbf{J}_d$, the propagation of electromagnetic waves (e.g., radio or TV waves) would be impossible. At low frequencies, $\mathbf{J}_d$ is usually neglected compared with $\mathbf{J}$. However, at radio frequencies, the two terms are comparable.

MAXWELL'S EQUATIONS IN FINAL FORMS

TABLE 2 Generalized Forms of Maxwell's Equations

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$	Ampère's circuit law

*This is also referred to as Gauss's law for magnetic fields.

Where;

$\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ (***magnetic field intensity***, A/m),

$\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ (***electric field intensity***, V/m),

$\mathbf{D} = \mathbf{D}(\mathbf{r}, t)$ (***electric flux density***, C/m^2),

$\mathbf{B} = \mathbf{B}(\mathbf{r}, t)$ (***magnetic flux density***, W/m^2),

$\rho = \rho(\mathbf{r}, t)$ (***volumetric charge density***, C/m^3),

$\mathbf{J} = \mathbf{J}(\mathbf{r}, t)$ (***current density***, A/m^2),

and r is the position vector for an ordinary point in the medium. Here ordinary point refers to a point wherein within its immediate neighborhood the physical properties of the medium are continuous. In other words, the small medium around r is considered to be homogeneous.

The concepts of linearity, isotropy, and homogeneity of a material medium still apply for time-varying fields; in a linear, homogeneous, and isotropic medium characterized by $\sigma, \varepsilon,$ and μ, the constitutive relations

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_o\mathbf{E} + \mathbf{P} \qquad 16$$

$$\mathbf{B} = \mu\mathbf{H} = \mu_o(\mathbf{H} + \mathbf{M}) \qquad 17$$

$$\mathbf{J} = \sigma\mathbf{E} + \rho_v\mathbf{u} \qquad 18$$

Hold for time-varying fields. Consequently, the boundary conditions remain valid for timevarying fields, where $\mathbf{a}_n$ is the unit normal vector to the boundary.

$$E_{1t} - E_{2t} = 0 \quad \text{or} \quad (\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{a}_n = \mathbf{0} \qquad 19$$

$$H_{1t} - H_{2t} = K \quad \text{or} \quad (\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_n = \mathbf{K} \qquad 20$$

$$D_{1n} - D_{2n} = \rho_s \quad \text{or} \quad (\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{a}_n = \rho_s \qquad 21$$

$$B_{1n} - B_{2n} = 0 \quad \text{or} \quad (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{a}_n = 0 \qquad 22$$

However, for a perfect conductor ($\sigma = \infty$) in a time-varying field,

$$\mathbf{E} = \mathbf{0}, \quad \mathbf{H} = \mathbf{0}, \quad \mathbf{J} = \mathbf{0}$$

And hence,

$$\mathbf{B}_n = \mathbf{0}, \quad \mathbf{E}_t = \mathbf{0}$$

TIME-HARMONIC FIELDS

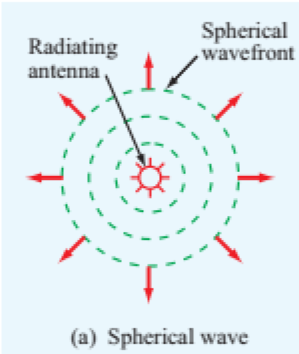
A **time-harmonic field** is one that varies periodically or sinusoidally with time.

TABLE 3 Time-Harmonic Maxwell's Equations Assuming Time Factor $e^{j\omega t}$

Point Form	Integral Form
$\nabla \cdot \mathbf{D}_s = \rho_{vs}$	$\oint \mathbf{D}_s \cdot d\mathbf{S} = \int \rho_{vs}\, dv$
$\nabla \cdot \mathbf{B}_s = 0$	$\oint \mathbf{B}_s \cdot d\mathbf{S} = 0$
$\nabla \times \mathbf{E}_s = -j\omega\mathbf{B}_s$	$\oint \mathbf{E}_s \cdot d\mathbf{l} = -j\omega \int \mathbf{B}_s \cdot d\mathbf{S}$
$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega\mathbf{D}_s$	$\oint \mathbf{H}_s \cdot d\mathbf{l} = \int (\mathbf{J}_s + j\omega\mathbf{D}_s) \cdot d\mathbf{S}$

ELECTROMAGNETIC WAVE PROPAGATION

A time-varying electric field produces a magnetic field and, conversely, a timevarying magnetic field produces an electric field. This cyclic pattern often results in electromagnetic (EM) waves propagating through free space and in material media. When a wave propagates through a homogeneous medium without interacting with obstacles or material interfaces, it is said to be unbounded. Light waves emitted by the sun and radio transmissions by antennas are good examples. Unbounded waves may propagate in both lossless and lossy media. Waves propagating in a lossless medium (e.g., air and perfect dielectrics) are similar to those on a lossless transmission line in that they do not attenuate. When propagating in a lossy medium (material with nonzero conductivity, such as water), part of the power carried by an EM wave gets converted into heat. A wave produced by a localized source, such as an antenna, expands outwardly in the form of a spherical wave, as depicted in figure 2



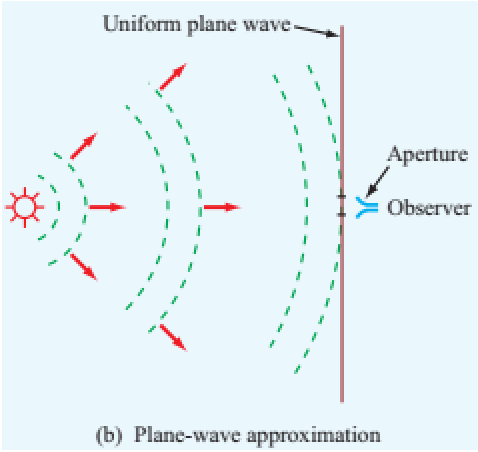


Figure 2 Waves radiated by an EM source, such as a light bulb or an antenna, have spherical wavefronts, as in (a); to a distant observer, however, the wavefront across the observer's aperture appears approximately planar, as in (b).

In general, waves are means of transporting energy or information.
Typical examples of EM waves include radio waves, TV signals, radar beams, and light rays. All forms of EM energy share three fundamental characteristics: they all travel at high velocity; in traveling, they assume the properties of waves; and they radiate outward from a source, without benefit of any discernible physical vehicles. Our major goal is to solve Maxwell's equations and describe EM wave motion in the following media:

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_0 ,\ \mu = \mu_0$)
2. Lossless dielectrics ($\sigma \approx 0,\ \varepsilon = \varepsilon_r \varepsilon_0 \ , \mu = \mu_r \mu_0 \ ,\ or\ \sigma \ll \omega\varepsilon$)
3. Lossy dielectrics ($\sigma \neq 0,\ \varepsilon = \varepsilon_r \varepsilon_0 \ , \mu = \mu_r \mu_0$)
4. Good conductors ($\sigma = \infty,\ \ \varepsilon = \varepsilon_0 \ , \mu = \mu_r \mu_0 \ , or\ \sigma \gg \omega\varepsilon$)
where ω is the angular frequency of the wave.

WAVE EQUATION

Wave equation is a differential equation describes the propagation of electric/magnetic field with distance and time

General form of wave equation for electric field;

A **wave** is a function of both space and time. Wave motion occurs when a disturbance at point *A*, at time *t*o, is related to what happens at point *B*, at time $> t_0$. In one dimension, a scalar wave equation takes the form of

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0 \qquad 23$$

Where u is the *wave velocity*.
If we assume harmonic (or sinusoidal) time dependence $e^{j\omega t}$ eq. (23) becomes;

$$\frac{d^2 E_s}{dz^2} + \beta^2 E_s = 0 \qquad 24$$

where $\beta = {}^{\omega}/_{u}$ and E_s is the phasor form of E.
the possible solutions to eq. (24) are;

$E^+ = Ae^{j(wt-\beta z)}$ 25

$E^- = Be^{j(wt+\beta z)}$ 26

Where E^+ means positive z-travel and E^- means negative travel. Combining E^+ and E^- leads to;

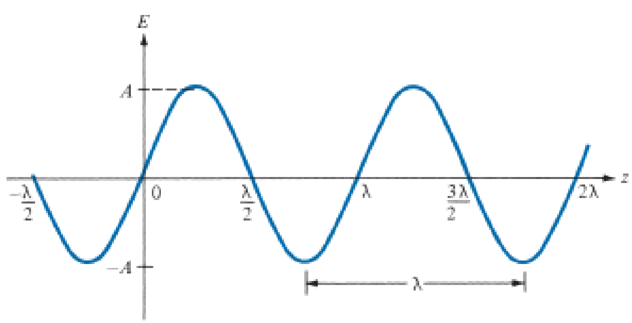
$$E = Ae^{j(\omega t-\beta z)} + Be^{j(\omega t+\beta z)} \qquad 27$$

Where A and B are real constants.
For the moment, let us consider the solution in eq. (25). Taking the imaginary part of this equation, we have

$$E = A\sin(\omega t - \beta z) \qquad 28$$

Note the following characteristics of the wave in eq. (28):
1. It is time harmonic because we assumed time dependence of the form$e^{j\omega t}$ to arrive at eq. (28).
2. The *amplitude* of the wave A has the same units as E.
3. The *phase* (in radians) of the wave depends on time t and space variable z, it is the term$(\omega t - \beta z)$.
4. The *angular frequency* ω is given in radians per second; β the *phase constant* or *wave number*, is given in radians per meter.

Because E varies with both time t and the space variable z, we may plot E as a function of t by keeping z constant and vice versa. The plots of $E(z, t = constant)$ and $E(t, z = constant)$ are shown in Figure 3(a) and (b), respectively



(a)

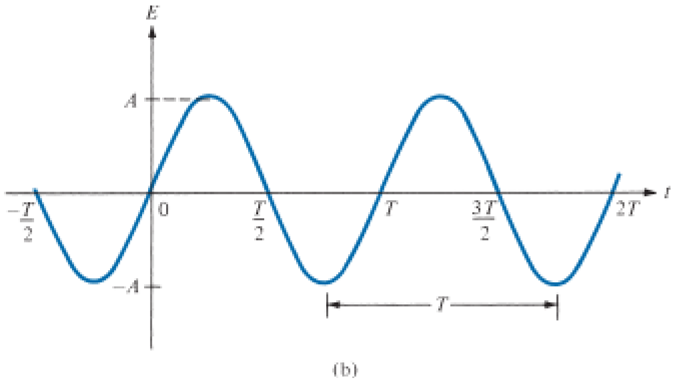


Figure 3 Plot of $E(z,t) = Asin(\omega t - \beta z)$ (a) with constant *t*, (b) with constant *z*.

From Figure 3(a), we observe that the wave takes distance λ to repeat itself and hence λ is called the *wavelength* (in meters). From Figure 3(b), the wave takes time T to repeat itself; consequently T is known as the *period*, in seconds. Since it takes time T for the wave to travel distance λ at the speed u, we expect

$$\lambda = uT \qquad 29$$

But $= 1/f$, where *f* is the *frequency* (the number of cycles per second) of the wave in hertz (Hz). Hence,

$$u = f\lambda \qquad 30$$

Because of this fixed relationship between wavelength and frequency, one can identify the position of a radio station within its band by either the frequency or the wavelength. Usually the frequency is preferred. Also, because

$$\omega = 2\pi f \qquad 31$$

$$\beta = \frac{\omega}{u} \qquad 32$$

And

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \qquad 33$$

we expect from eqs. (31) and (32) that;

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{u} \qquad 34$$

Equation (34) shows that for every wavelength of distance traveled, a wave undergoes a phase change of 2π radians.

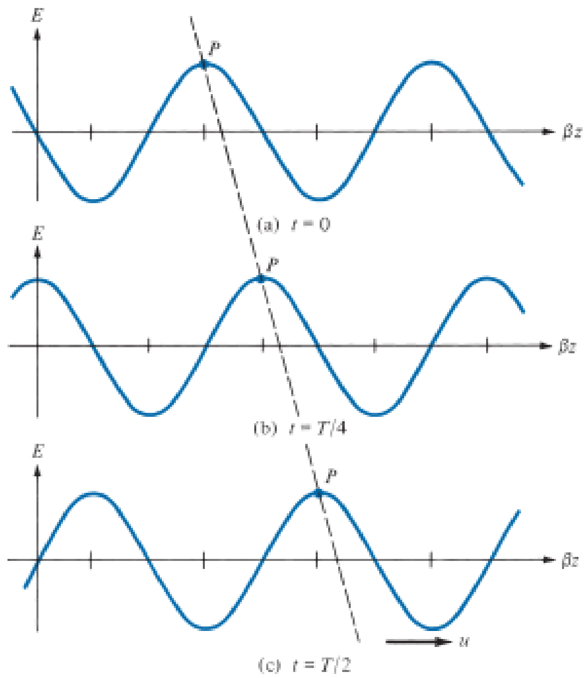


Figure 4 Plot of $E(z,t) = Asin(\omega t - \beta z)$ at time (a) t =0, (b) t = T/4, (c) t = T/2; P moves in the +z-direction with velocity u.

Point P is a point of constant phase, therefore;

$$\omega t - \beta z = \text{constant}$$

Or

$$\frac{dz}{dt} = \frac{\omega}{\beta} = u \qquad 35$$

In summary, we note the following:

1. A wave is a function of both time and space.
2. Though time t = 0 is arbitrarily selected as a reference for the wave, a wave is without beginning or end.
3. A negative sign in $(\omega t \pm \beta z)$ is associated with a wave propagating in the +z- direction (forward-traveling or positive-going wave), whereas a positive sign indicates that a wave is traveling in the - z-direction (backward-traveling or negative-going wave).
4. Since $sin(\varphi) = -sin(\varphi) = sin(\varphi \pm \pi)$, , whereas $cos(-\varphi) = \cos\varphi$.

$$\sin(\psi \pm \pi/2) = \pm\cos\psi$$

$$\sin(\psi \pm \pi) = -\sin\psi$$

$$\cos(\psi \pm \pi/2) = \mp\sin\psi$$

$$\cos(\psi \pm \pi) = -\cos\psi \qquad 36$$

where $\varphi = \omega t \pm \beta z$. One of the relations in eqs. (36) can be used to represent any time-harmonic wave in the form of sine or cosine.

5. E and H are called uniform waves if they lie in a plane and are constant over such planes.

Wave Equation Derivation (From Maxwell's Equations)

Faraday's law	**Ampere's law**
$\Downarrow$	$\Downarrow$
$\nabla \times \mathbf{E} = -\mu \dfrac{\partial \mathbf{H}}{\partial t}$	$\nabla \times \mathbf{H} = \epsilon \dfrac{\partial \mathbf{E}}{\partial t}$
$\Downarrow$	$\Downarrow$
for plane waves $E_y = E_z = 0$, $H_x = H_z = 0$	for plane waves $E_y = E_z = 0$, $H_x = H_z = 0$
$\Downarrow$	$\Downarrow$
$\dfrac{\partial E_x}{\partial z} = -\mu \dfrac{\partial H_y}{\partial t}$	$\dfrac{\partial H_y}{\partial z} = -\epsilon \dfrac{\partial E_x}{\partial t}$
$\Downarrow$	$\Downarrow$
differentiation with respect to z	differentiation with respect to t
$\Downarrow$	$\Downarrow$
$\dfrac{\partial^2 E_x}{\partial z^2} = -\mu \dfrac{\partial^2 H_y}{\partial t \partial z}$	$\dfrac{\partial^2 H_y}{\partial z \partial t} = -\epsilon \dfrac{\partial^2 E_x}{\partial t^2}$
$\searrow$	$\swarrow$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}}$$

Worked Example:

An electric field in free space is given by;

$$\mathbf{E} = 50 \cos(10^8 t + \beta x)\mathbf{a}_y \text{ V/m}$$

(a) Find the direction of wave propagation.

(b) Calculate β and the time it takes to travel a distance of $\lambda/2$

(c) Sketch the wave at $t = 0$, $T/4$, and $T/2$.

Solution:

(a) From the positive sign in $(\omega t + \beta x)$, we infer that the wave is propagating along $-a_x$.

(b) In free space, $u = c$:

$$\beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

$$\beta = 0.3333 \text{ rad/m}$$

If T is the period of the wave, it takes T seconds to travel a distance λ at speed c. Hence to travel a distance of λ/2 will take

$$t_1 = \frac{T}{2} = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{10^8} = 31.42 \text{ ns}$$

Alternatively, because the wave is traveling at the speed of light c,

$$\frac{\lambda}{2} = ct_1 \quad \text{or} \quad t_1 = \frac{\lambda}{2c}$$

But

$$\lambda = \frac{2\pi}{\beta} = 6\pi$$

Hence

$$t_1 = \frac{6\pi}{2(3 \times 10^8)} = 31.42 \text{ ns}$$

(c.)

At $\quad t = 0, \quad E_y = 50 \cos \beta x$

At $\quad t = T/4, E_y = 50 \cos\left(\omega \cdot \frac{2\pi}{4\omega} + \beta x\right) = 50 \cos(\beta x + \pi/2)$

$$= -50 \sin \beta x$$

At $\quad t = T/2, E_y = 50 \cos\left(\omega \cdot \frac{2\pi}{2\omega} + \beta x\right) = 50 \cos(\beta x + \pi)$

$$= -50 \cos \beta x$$

$\boldsymbol{E_y}$ at t =0, T/4, T/2 is plotted against x as shown in Figure 5. Notice that a point P (arbitrarily selected) on the wave moves along $-a_x$ as t increases with time. This shows that the wave travels along $-a_x$.

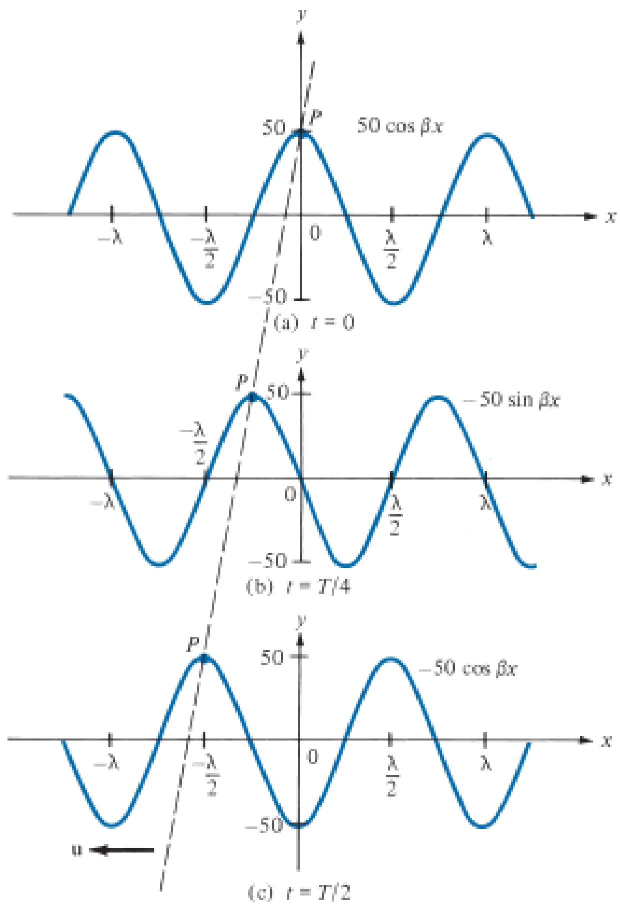


FIGURE 5 For Example 1; wave travels along $-a_x$.

WAVE PROPAGATION IN LOSSY DIELECTRICS

Wave propagation in lossy dielectrics is a general case from which wave propagation in media of other types can be derived as special cases. A **lossy dielectric** is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric. ie, a lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with $\sigma \neq 0,$ as distinct from a lossless dielectric (perfect or good dielectric) in which $\sigma = 0$.

Consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free macroscopic $\rho_v = 0$. Assuming and suppressing the time factor $e^{i\omega t}$, Maxwell's equations

(see Table 3) becomes

$$\nabla \cdot E_s = 0 \qquad 1$$

$$\nabla \cdot H_s = 0 \qquad 2$$

$$\nabla \times E_s = -j\omega\mu H_s \qquad 3$$

$$\nabla \times H_s = (\sigma + j\omega\varepsilon) E_s \qquad 4$$

Taking the curl of both sides of $\nabla \times E_s$ gives

$$\nabla \times \nabla \times \mathbf{E}_s = -j\omega\mu(\nabla \times \mathbf{H}_s) \qquad 5$$

Applying the vector identity

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} \qquad 6$$

to the left-hand side of eq(5) and invoking eqs. (1) and (4), we obtain

$$\nabla(\underbrace{\nabla \cdot \mathbf{E}_s}_{0}) - \nabla^2\mathbf{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}_s$$

Or

$$\nabla^2\mathbf{E}_s - \gamma^2\mathbf{E}_s = 0 \qquad 7$$

Where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \qquad 8$$

and Υ, in reciprocal meters, is called the *propagation constant* of the medium. By a similar procedure, it can be shown that for the **H** field,

$$\nabla^2\mathbf{H}_s - \gamma^2\mathbf{H}_s = 0 \qquad 9$$

Equations (7) and (9) are known as homogeneous vector *Helmholtz's equations* or simply vector *wave equations.* In Cartesian coordinates, eq. (7), for example, is equivalent to three scalar wave equations, one for each component of **E** along $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.
Since Υ in eqs. (7) to (9) is a complex quantity, we may let

$$\gamma = \alpha + j\beta \qquad 10$$

We obtain α and β from eqs. (8) and (10) by noting that

$$-\text{Re}\,\gamma^2 = \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \qquad 11$$

And

$$|\gamma^2| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} \qquad 12$$

From eqs. (11) and (12), we obtain

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]} \qquad 13$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]} \qquad 14$$

Without loss of generality, if we assume that a wave propagates along $+\mathbf{a}_z$ and that $\mathbf{E}_s$ has only an *x*-component, then;

$$\mathbf{E}_s = E_{xs}(z)\mathbf{a}_x$$

15

We then substitute into eq. (7), which yields

$$(\nabla^2 - \gamma^2)E_{xs}(z) = 0$$

16

Without loss of generality, if we assume that a wave propagates in an unbounded medium along $\mathbf{a}_z$ and that **E** has only an *x*-component that does not vary with *x* and *y*, then;

$$\underbrace{\cancel{\frac{\partial^2 E_{xs}(z)}{\partial x^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 E_{xs}(z)}{\partial y^2}}}_{0} + \frac{\partial^2 E_{xs}(z)}{\partial z^2} - \gamma^2 E_{xs}(z) = 0$$

Or

$$\left[\frac{d^2}{dz^2} - \gamma^2\right]E_{xs}(z) = 0$$

17

This is a scalar wave equation, a linear homogeneous differential equation, with solution

$$E_{xs}(z) = E_o e^{-\gamma z} + E_o' e^{\gamma z}$$

18

Where E_o and E_o' are constants. The fact that the field must be finite at infinity requires that $E_o' = 0$.

Using eq. (10), we obtain;

$$\mathbf{E} + z, t) = \mathrm{Re}[\mathrm{E}_{xs} + z)e^{j\omega t}\mathbf{a}_x] = \mathrm{Re} + \mathrm{E}_o e^{-\alpha z} e^{j + \omega t - \beta z)}\mathbf{a}_x)$$

Or

$$\mathbf{E}(z, t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$

19

A sketch of $|E|$ 0 at times $t = 0$ and $t = \Delta t$ is portrayed in Figure 6, where it is evident that **E** has only an *x*-component and it is traveling in the +*z*-direction.

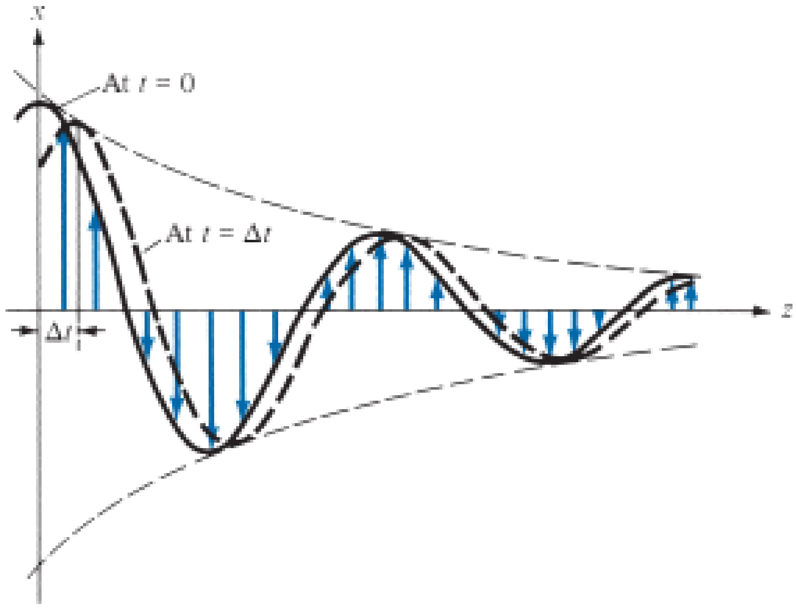


FIGURE 6 An *E*-field with an *x*-component traveling in the +*z*-direction at times $t = 0$ and $t = \Delta t$; arrows indicate instantaneous values of *E*.

We obtain **H** (*z*, *t*) either by taking similar steps to solve eq. (9) or by using eq. (19) in conjunction with Maxwell's equations,

$$\mathbf{H}(z, t) = \text{Re}(H_o e^{-\alpha z} e^{j(\omega t - \beta z)} \mathbf{a}_y) \qquad 20$$

Where

$$H_o = \frac{E_o}{\eta} \qquad 21$$

and η is a complex quantity known as the *intrinsic impedance*, in ohms, of the medium.
It can be shown that;

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \theta_\eta = |\eta| e^{j\theta_\eta} \qquad 22$$

With

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}, \quad \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} \qquad 23$$

where $0 \leq 45^o$. Substituting eqs. (21) and (22) into eq. (20) gives;

$$\mathbf{H} = \text{Re}\left[\frac{E_o}{|\eta| e^{j\theta_\eta}} e^{-\alpha z} e^{j(\omega t - \beta z)} \mathbf{a}_y\right]$$

Or

$$\mathbf{H} = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \mathbf{a}_y \quad (24)$$

As the wave propagates along $\mathbf{a}_z$, it decreases or attenuates in amplitude by a factor $e^{\alpha z}$, and hence α is known as the *attenuation constant*, or attenuation coefficient, of the medium. It is a measure of the spatial rate of decay of the wave in the medium, measured in nepers per meter (Np/m), and can be expressed in decibels per meter (dB/m). An attenuation of 1 neper denotes a reduction to $e^{-1}$of the original value, whereas an increase of 1 neper indicates an increase by a factor of e. Hence, for voltages

$$1 \text{ Np} = 20 \log_{10} e = 8.686 \text{ dB} \quad (25)$$

The quantity β is a measure of the phase shift per unit length in radians per meter and is called the *phase constant* or wave number.
In terms of β, the wave velocity u and wavelength λ are, respectively, given by

$$u = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta} \quad (26)$$

We also notice from eqs. (19) and (24) that **E** and **H** are out of phase by θ_η at any instant of time due to the complex intrinsic impedance of the medium. Thus at any time, **E** leads **H**(or **H** lags **E**) by θ_η. Finally, we notice that the ratio of the magnitude of the conduction current density $\mathbf{J}_c$ to that of the displacement current density $\mathbf{J}_d$ *in a lossy medium is;*

$$\frac{|\mathbf{J}_{cs}|}{|\mathbf{J}_{ds}|} = \frac{|\sigma \mathbf{E}_s|}{|j\omega\varepsilon \mathbf{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

Or

$$\tan\theta = \frac{\sigma}{\omega\varepsilon} \quad (27)$$

where $\tan\theta$ is known as the *loss tangent* and θ is the *loss angle* of the medium.

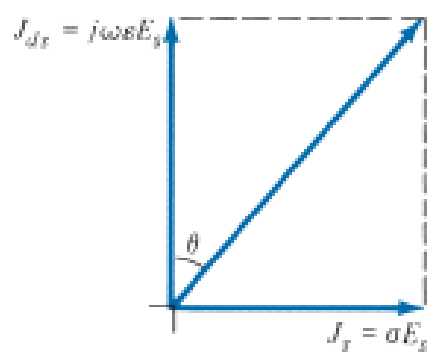


FIGURE 7 Loss angle of a lossy medium.

A medium is said to be a good (lossless or perfect) dielectric if $\tan\theta$ is very small ($\sigma \ll \omega\varepsilon$) or a good conductor if $\tan\theta$ is very large ($\sigma \gg \omega\varepsilon$). From the viewpoint of wave propagation, the characteristic behavior of a medium depends not only on its constitutive parameters σ, ε *and* μ,

but also on the frequency of operation. A medium that is regarded as a good conductor at low frequencies may be a good dielectric at high frequencies.
Note from eqs. (23) and (27) that

$$\theta = 2\theta_\eta \qquad 28$$

From eq. (4)

$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s = j\omega\varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]\mathbf{E}_s \qquad 29$$

$$= j\omega\varepsilon_c\mathbf{E}_s$$

Where

$$\varepsilon_c = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right] = \varepsilon[1 - j\tan\theta] \qquad 30$$

Or

$$\varepsilon_c = \varepsilon' - j\varepsilon'' \qquad 31$$

With $\varepsilon' = \varepsilon$, $\varepsilon'' = \sigma/\omega$, $\varepsilon = \varepsilon_0\varepsilon_r, \varepsilon_c$ is called the *complex permittivity* of the medium. We observe that the ratio of ε'' *to* ε' is the loss tangent of the medium; that is,

$$\tan\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} \qquad 32$$

In subsequent sections, we will consider wave propagation in media of other types that may be regarded as special cases of what we have considered here. Thus we will simply deduce the governing formulas from those obtained for the general case treated in this section.

Example:
A lossy dielectric has an intrinsic impedance of $200\angle 30^0\ \Omega$ at a particular radian frequency ω. If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component.

$$\mathbf{H} = 10\, e^{-\alpha x}\cos\left(\omega t - \frac{1}{2}x\right)\mathbf{a}_y \text{ A/m}$$

Find **E** and α.
Solution:
The given wave travels along $\mathbf{a}_x$ so that $\mathbf{a}_k = \mathbf{a}_x$; $\mathbf{a}_H = \mathbf{a}_y$, so

$$-\mathbf{a}_E = \mathbf{a}_k \times \mathbf{a}_H = \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

Or
$\mathbf{a}_E = -\mathbf{a}_z$ also
$H_o = 10$, so

$$\frac{E_o}{H_o} = \eta = 200\,\angle 30^\circ = 200\,e^{j\pi/6} \rightarrow E_o = 2000e^{j\pi/6}$$

Except for the amplitude and phase difference, **E** and **H** always have the same form. Hence

$$\mathbf{E} = \mathrm{Re}\left(2000e^{j\pi/6}e^{-\gamma x}e^{j\omega t}\mathbf{a}_E\right)$$

Or

$$\mathbf{E} = -2e^{-\alpha x}\cos\left(\omega t - \frac{x}{2} + \frac{\pi}{6}\right)\mathbf{a}_z \text{ kV/m}$$

Knowing that $\beta = 1/2$, we need to determine α. Since

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

And

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1}{\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1}\right]^{1/2}$$

But $\dfrac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = \tan 60^\circ = \sqrt{3}$. Hence,

$$\frac{\alpha}{\beta} = \left[\frac{2 - 1}{2 + 1}\right]^{1/2} = \frac{1}{\sqrt{3}}$$

Or

$$\alpha = \frac{\beta}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0.2887 \text{ Np/m}$$

PLANE WAVES IN LOSSLESS DIELECTRICS

In a lossless dielectric, $\sigma \ll \omega\varepsilon$.

$$\sigma \simeq 0, \quad \varepsilon = \varepsilon_o\varepsilon_r, \quad \mu = \mu_o\mu_r$$

33

Substituting the above in the following equations,

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

We have:

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu\varepsilon} \qquad 34$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \quad \lambda = \frac{2\pi}{\beta} \qquad 35$$

Also:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^\circ \qquad 36$$

and thus **E** and **H** are in time phase with each other.

PLANE WAVES IN FREE SPACE

Plane waves in free space comprise a special case. In this case

$$\sigma = 0, \quad \varepsilon = \varepsilon_o, \quad \mu = \mu_o \qquad 37$$

we obtain,;

$$\alpha = 0, \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} \qquad 38$$

$$u = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c, \lambda = \frac{2\pi}{\beta} \qquad 39$$

Where $c \cong 3 \times 10^8$ m/s, the speed of light in a vacuum. The fact that EM waves travel in free space at the speed of light is significant. It provides some evidence that light is the manifestation of an EM wave. In other words, light is characteristically electromagnetic.

By substituting the constitutive parameters in eq. (37) into eq. (23), $\theta_\eta = 0$ and$\eta = \eta_0$, where η_0 is called the *intrinsic impedance of free space* and is given by;

$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \simeq 377\ \Omega \qquad 40$$

$$\mathbf{E} = E_o\cos(\omega t - \beta z)\,\mathbf{a}_x$$

Then

$$\mathbf{H} = H_o \cos(\omega t - \beta z)\mathbf{a}_y = \frac{E_o}{\eta_o}\cos(\omega t - \beta z)\mathbf{a}_y \tag{41}$$

The plots of **E** and **H** are shown in Figure 8(a). In general, if $\mathbf{a}_E$, $\mathbf{a}_H$, and $\mathbf{a}_k$ are unit vectors along the **E** field, the **H** field, and the direction of wave propagation

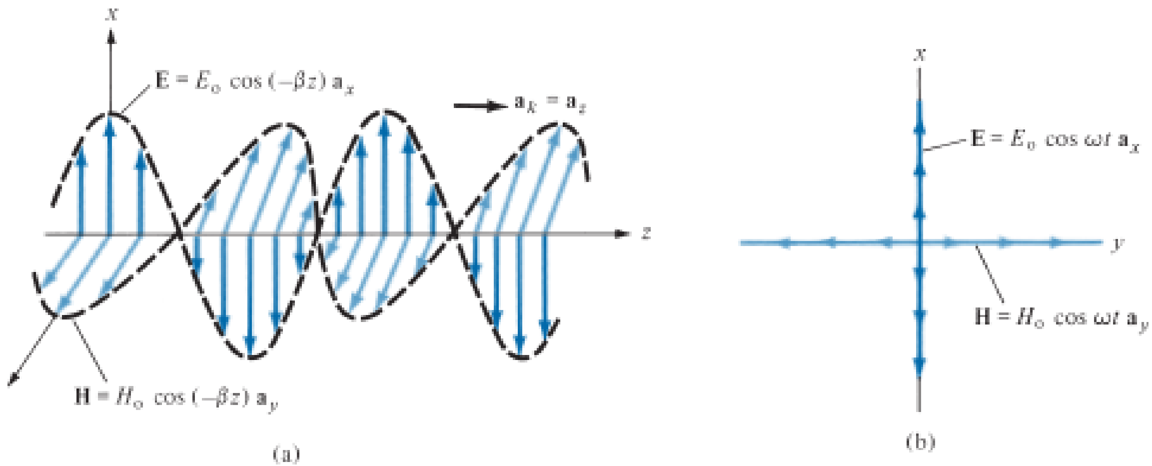


FIGURE 8 Plots of **E** and **H** (**a**) as functions of z at $t = 0$; and (**b**) at $z = 0$. The arrows indicate instantaneous values

$$\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H$$

Or

$$\mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E$$

Or

$$\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k \tag{42}$$

Both **E** and **H** fields (or EM waves) are everywhere normal to the direction of wave propagation, $\mathbf{a}_k$. That means that the fields lie in a plane that is transverse or orthogonal to the direction of wave propagation. They form an EM wave that has no electric or magnetic field components along the direction of propagation; such a wave is called a *transverse electromagnetic* (TEM) wave. A combination of **E** and **H** is called a *uniform plane wave* because **E** (or **H**) has the same magnitude throughout any transverse plane, defined by z = constant. The direction in which the electric field points is the *polarization* of a TEM wave.

Worked Example:

In a lossless dielectric for which $\eta = 60\pi, \mu_r = 1$, and $H = -0.1cos(\omega t - z)a_x + 0.5sin(\omega t - z)a_y \ \frac{A}{m}$,, calculate $\varepsilon_{r,}\omega$, and E.

Solution

In this case, $\sigma = 0, \alpha = 0,$ and $\beta = 1,$ so

$$\eta = \sqrt{\mu/\varepsilon} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

Or

$$\sqrt{\varepsilon_r} = \frac{120\pi}{\eta} = \frac{120\pi}{60\pi} = 2 \quad \rightarrow \quad \varepsilon_r = 4$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{4} = \frac{2\omega}{c}$$

Or

$$\omega = \frac{\beta c}{2} = \frac{1\,(3 \times 10^8)}{2} = 1.5 \times 10^8 \text{ rad/s}$$

Applying Maxwells Equations;

$$\nabla \times \mathbf{H} = \underbrace{\cancel{\sigma}}_{0}\mathbf{E} + \varepsilon\frac{\partial \mathbf{E}}{\partial t} \quad \rightarrow \quad \mathbf{E} = \frac{1}{\varepsilon}\int \nabla \times \mathbf{H}\, dt$$

because $\sigma = 0$. But

$$\nabla \times \boldsymbol{H} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x(z) & H_y(z) & 0 \end{vmatrix} = -\frac{\partial H_y}{\partial z}\mathbf{a}_x + \frac{\partial H_x}{\partial z}\mathbf{a}_y$$

$$= H_{2o}\cos(\omega t - z)\mathbf{a}_x + H_{1o}\sin(\omega t - z)\mathbf{a}_y$$

where $H_{1o} = -0.1$ and $H_{2o} = 0.5$. Hence

$$\mathbf{E} = \frac{1}{\varepsilon}\int \nabla \times \mathbf{H}\, dt = \frac{H_{2o}}{\varepsilon\omega}\sin(\omega t - z)\mathbf{a}_x - \frac{H_{1o}}{\varepsilon\omega}\cos(\omega t - z)\mathbf{a}_y$$

$$= 94.25\sin(\omega t - z)\mathbf{a}_x + 18.85\cos(\omega t - z)\mathbf{a}_y \text{ V/m}$$

Example 2

A uniform plane wave propagating in a medium has

$$\mathbf{E} = 2e^{-\alpha z}\sin(10^8 t - \beta z)\mathbf{a}_y \text{ V/m}$$

If the medium is characterized by $\varepsilon_r = 1, \mu_r = 20$, and $\sigma = 3\ S/m,$ find $\alpha, \beta,$ and **H**.

Solution:
We need to determine the loss tangent to be able to tell whether the medium is a lossy dielectric or a good conductor.

$$\frac{\sigma}{\omega\varepsilon} = \frac{3}{10^8 \times 1 \times \dfrac{10^{-9}}{36\pi}} = 3393 \gg 1$$

Showing that the medium may be regarded as a good conductor at the frequency of operation. Hence,

$$\alpha = \beta = \sqrt{\frac{\mu\omega\sigma}{2}} = \left[\frac{4\pi \times 10^{-7} \times 20(10^8)(3)}{2}\right]^{1/2}$$
$$= 61.4$$

$$\alpha = 61.4 \text{ Np/m}, \quad \beta = 61.4 \text{ rad/m}$$

Also

$$|\eta| = \sqrt{\frac{\mu\omega}{\sigma}} = \left[\frac{4\pi \times 10^{-7} \times 20(10^8)}{3}\right]^{1/2}$$
$$= \sqrt{\frac{800\pi}{3}}$$
$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 3393 \quad \rightarrow \quad \theta_\eta = 45^\circ = \frac{\pi}{4}$$

Hence

$$\mathbf{H} = H_o e^{-\alpha z} \sin\left(\omega t - \beta z - \frac{\pi}{4}\right)\mathbf{a}_H$$

Where;

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

And

$$H_o = \frac{E_o}{|\eta|} = 2\sqrt{\frac{3}{800\pi}} = 69.1 \times 10^{-3}$$

Thus

$$\mathbf{H} = -69.1\, e^{-61.4z} \sin\left(10^8 t - 61.42z - \frac{\pi}{4}\right)\mathbf{a}_x \text{ mA/m}$$

Example 3.

Given $E = E_m sin(\omega t - \beta z) a_y$ in free space, Find:

(i) Electric flux density
(ii) Magnetic flux density
(iii) (Magnetic field intensity

Solve.

(i) $D = \varepsilon_o E = \varepsilon_o E_m sin(\omega t - \beta z) a_y$

(ii) The Maxwell equation $\nabla \times E = -\frac{\partial B}{\partial t}$ gives

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_m sin(\omega t - \beta z) & 0 \end{vmatrix} = -\frac{\partial B}{\partial t}$$

Or $-\frac{\partial B}{\partial t} = \beta E_m cos(\omega t - \beta z) a_x$

Intergrating $B = -\frac{\beta E_m}{\omega} sin(\omega t - \beta z) a_x$

Where the consent of integration has been neglected.

(iii) From $B = \mu H$,
Then
$H = -\frac{\beta E_m}{\omega \mu_o} sinsin(\omega t - \beta z) a_x$