DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

PRIVATE BAG 10143, DEDAN KIMATHI, NYERI KENYA

ELECTRICAL & ELECTRONIC ENGINEERING DEPARTMENT

EEE 3103 ELECTROMAGNETICS II

Prerequisites
EEE 2205 Electromagnetic I

Expected Learning Outcomes
By the end of this course, the learner should be able to;
1. Carry out analysis relating to electric and magnetic fields using the classical laws of electricity and magnetism
2. Understand the principles of electromagnetic theory
3. Apply the principles of electromagnetic theory to modern science and technology

Course Content
- **Static Magnetic Fields:** BiotSavart's Law, Ampere's Circuit Law,Applications of Ampere's Law,
- Magnetic Flux Density, Maxwell's Equations for Static EM Fields,
- Magnetic Scalar and Vector Potentials, Forces due to Magnetic Fields,
- Magnetic laws in vectorial form. Magnetic Torque and Moment,
- A Magnetic Dipole, Magnetization in Materials, auxiliary field, susceptibility, ferromagnetism. Classification of Magnetic Materials,
- Boundary conditions for static magnetic fields.
- Earth's magnetic field,
- Magnetic field energy, Energy stored by inductors. B.H. curve, hysteresis. Inductors and Inductances,
- Magnetic Circuits, Force on Magnetic Materials. Force on current carrying elements. Laws on static magnetic fields.
- Numerical methods of magnetic field computations.
- **High Frequency Measurements**: R.F. instruments, measurement of R, L and C at high frequencies.

Laboratory/Practical Exercises
i) Magnetization in materials
ii) Ac inductive circuits
iii) Magnetic fields

Course Assessment

Cats	10%
Assignments	5%
Labs	15%
Exam	70%
Total	**100%**

Ref Books:

Dipak, L. S., & Valdis V. L. (2006). Applied Electromagnetics and Electromagnetic Compatibility, John Wiley & Sons.

Bhag S. G., & Hüseyin R. H. (2004). Electromagnetic field theory fundamentals, Cambridge University Press, 2nd Ed.

Markus, Z. (2003). Electromagnetic Field Theory: A Problem Solving Approach, Krieger Publishing Company, reprint Ed.

Edminister, J.A. (1994). Schaum's Outline of Theory and Problems of Electromagnetics, McGraw-Hill, 2nd Ed.

Introduction

We now focus our attention on static magnetic fields, which are characterized by **H** or **B**. There are similarities and dissimilarities between electric and magnetic fields. As **E** and **D** are related according to **D=εE** for linear, isotropic material space, **H** and **B** are related according to **B =μH.**
Table 1 further shows the analogy between electric and magnetic field quantities.
The analogy is presented here to show that most of the equations we have derived for the electric fields may be readily used to obtain corresponding equations for magnetic fields if the equivalent analogous quantities are substituted.
A magnetostatic field is produced by a constant current flow (or direct current). This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vacuum tubes, or conduction currents as in current-carrying wires.
Our study of magnetostatics is not a dispensable luxury but an indispensable necessity. Motors, transformers, microphones, compasses, telephone bell ringers, television focusing controls, advertising displays, magnetically levitated high-speed vehicles, memory stores, magnetic separators, and so on, which play an important role in our everyday life, could not have been developed without an understanding of magnetic phenomena.

TABLE 1 Analogy between Electric and Magnetic Fields

Term	Electric	Magnetic
Basic laws	$\mathbf{F} = \dfrac{Q_1Q_2}{4\pi\varepsilon R^2}\mathbf{a}_R$	$d\mathbf{B} = \dfrac{\mu_o I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2}$
	$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$	$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$
Force law	$\mathbf{F} = Q\mathbf{E}$	$\mathbf{F} = Q\mathbf{u} \times \mathbf{B}$
Source element	dQ	$dQ\mathbf{u} = \mathrm{I}d\mathbf{l}$
Field intensity	$E = \dfrac{V}{\ell}\,(\mathrm{V/m})$	$H = \dfrac{I}{\ell}\,(\mathrm{A/m})$
Flux density	$D = \dfrac{\psi}{S}\,(\mathrm{C/m^2})$	$B = \dfrac{\psi}{S}\,(\mathrm{Wb/m^2})$
Relationship between fields	$\mathbf{D} = \varepsilon\mathbf{E}$	$\mathbf{B} = \mu\mathbf{H}$
Potentials	$\mathbf{E} = -\nabla V$	$\mathbf{H} = -\nabla V_m$ (**J** = (

	$V = \int_L \frac{\rho_L dl}{4\pi\varepsilon R}$	$\mathbf{A} = \int_L \frac{\mu I \, d\ell}{4\pi R}$
Flux	$\psi = \int \mathbf{D} \cdot d\mathbf{S}$	$\psi = \int_s \mathbf{B} \cdot d\mathbf{S}$
	$\psi = Q = CV$	$\psi = LI$
	$I = C\frac{dV}{dt}$	$V = L\frac{dI}{dt}$
Energy density	$w_E = \frac{1}{2}\mathbf{D} \cdot \mathbf{E}$	$w_m = \frac{1}{2}\mathbf{B} \cdot \mathbf{H}$
Poisson's equation	$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$	$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$

BIOT–SAVART'S LAW

Biot–Savart's law states that the differential magnetic field intensity dH produced at a point P, as shown in Figure 1, by the differential current element $I\,dl$ is proportional to the product $I\,d\mathbf{l}$ and the sine of the angle a between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.

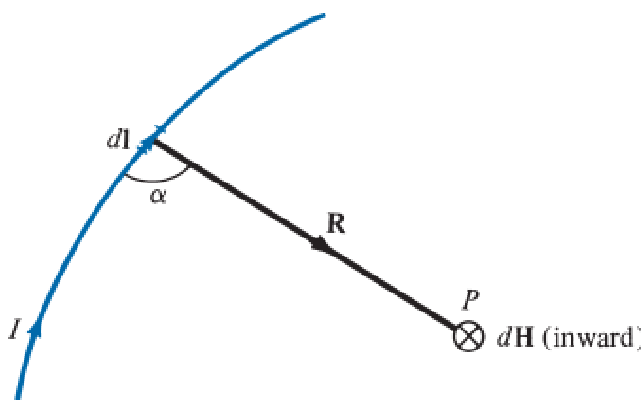

FIGURE 1 Magnetic field $d\mathbf{H}$ at P due to current element $I\,d\mathbf{l}$.

$$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

Or

$$dH = \frac{kI\,dl \sin\alpha}{R^2}$$

Where k is the constant of proportionality. In SI units, $k = 1/4\pi$.

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

In vector form

$$dH = \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3} \quad (1)$$

Where $R = |\boldsymbol{R}|$ and $a_R = \boldsymbol{R}/_R$ **: R** and d**l** are illustrated in Figure 1.
Thus the direction of d**H** can be determined by the right-hand rule with the right-hand thumb pointing in the direction of the current and the right-hand fingers encircling the wire in the direction of d**H**

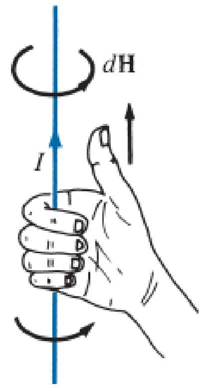

FIGURE 2 Determining the direction of d**H** using (**a**) the right-hand rule

We can have different current distributions: line current, surface current, and volume current as shown in Figure 3

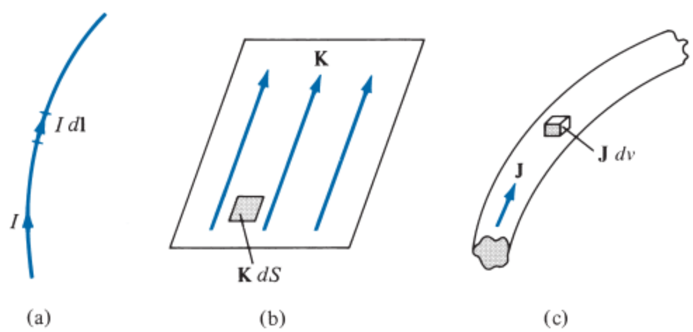

FIGURE 3 Current distributions: (**a**) line current, (**b**) surface current, (**c**) volume current.

If we define **K** as the surface current density in amperes per meter and **J** as the volume current density in amperes per meter squared, the source elements are related a;

$$I \quad d\mathbf{l} \equiv \mathbf{K}\,dS \equiv \mathbf{J}\,dv$$

Thus in terms of the distributed current sources, the Biot–Savart's law as in eq. (1) becomes;

$$\mathbf{H} = \int_L \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} \quad \text{(line current)} \quad (2)$$

$$\mathbf{H} = \int_S \frac{\mathbf{K}\, dS \times \mathbf{a}_R}{4\pi R^2} \quad \text{(surface current)} \tag{3}$$

$$\mathbf{H} = \int_v \frac{\mathbf{J}\, dv \times \mathbf{a}_R}{4\pi R^2} \quad \text{(volume current)} \tag{4}$$

Where a_R is a unit vector pointing from the differential element of current to the point of interest.

Example:

As an example, let us apply eq. (2) to determine the field due to a *straight* current carrying filamentary conductor of finite length *AB* as in Figure 4.

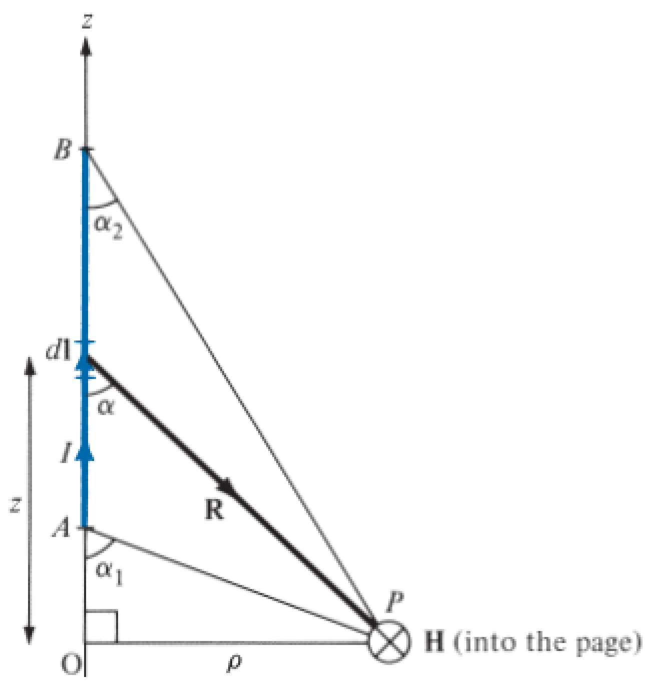

FIGURE 4 Field at point *P* due to a straight filamentary conductor.

We assume that the conductor is along the *z*-axis with its upper and lower ends, respectively, subtending angles α_1 and α_2 at *P*, the point at which **H** is to be determined.
Note that current flows from point *A*, where $\alpha = \alpha_1$, to point *B*, where $\alpha = \alpha_2$ If we consider the contribution *d***H** at *P* due to an element *d***l** at (0, 0, *z*),

$$d\mathbf{H} = \frac{I\, d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

But $d\mathbf{l} = dz\, \mathbf{a}_z$ and $\mathbf{R} = \rho \mathbf{a}_\rho - z\mathbf{a}_z$, so

$$d\mathbf{l} \times \mathbf{R} = \rho\, dz\, \mathbf{a}_\phi$$

$$\mathbf{a}_\rho \times \mathbf{a}_\phi = \mathbf{a}_z$$

$$\mathbf{a}_\phi \times \mathbf{a}_z = \mathbf{a}_\rho$$

$$\mathbf{a}_z \times \mathbf{a}_\rho = \mathbf{a}_\phi$$

Hence

$$\mathbf{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}} \mathbf{a}_\phi \tag{5}$$

Letting $z = \rho \cot \alpha$, $dz = -\rho \csc^2 \alpha\, d\alpha$, $[\rho^2 + z^2]^{3/2} = \rho^3 \csc \alpha^3$,

$$\mathbf{H} = -\frac{1}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \csc^2 \alpha\, d\alpha}{\rho^3 \csc^3 \alpha} \mathbf{a}_\phi$$

$$= -\frac{I}{4\pi\rho} \mathbf{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin \alpha\, d\alpha$$

Or

$$\mathbf{H} = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \mathbf{a}_\phi \tag{6}$$

This expression is generally applicable for any straight filamentary conductor. The conductor need not lie on the *z*-axis, but it must be straight.

Special cases:

(1) When the conductor is *semi-infinite* (with respect to *P*) so that point *A* is now at *O (*10, 0, 0) while *B* is at (0, 0, ∞), $\alpha_1 = 90^0, \alpha_2 = 0^0$ the above equation (6) becomes;

$$\mathbf{H} = \frac{I}{4\pi\rho} \mathbf{a}_\phi \tag{7}$$

(2). When the conductor is *infinite* in length. For this case, point *A* is at (0,0,-∞) while *B* is at (0, 0, ∞); $\alpha_1 = 180^0, \alpha_2 = 0^0$and eq. (6) reduces to

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi \tag{8}$$

Where;

$$\mathbf{a}_\phi = \mathbf{a}_\ell \times \mathbf{a}_\rho$$

Where a_l is a unit vector along the line current and a_p is a unit vector along the perpendicular line from the line current to the field point.

Example

The conducting triangular loop in Figure 5(a) carries a current of 10 A. Find **H** at (0, 0, 5) due to side 1 of the loop.

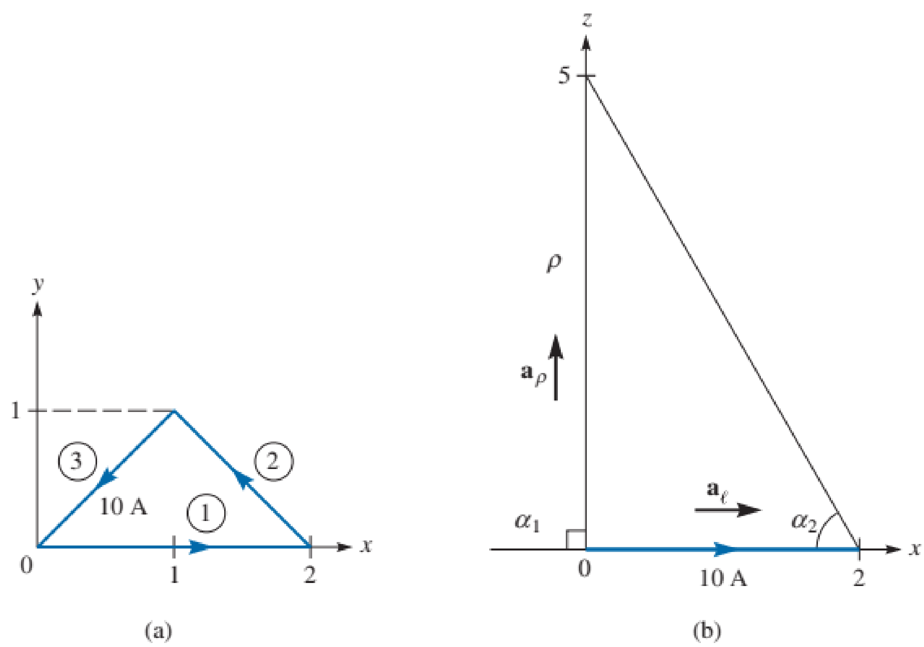

FIGURE 5: (**a**) conducting triangular loop, (**b**) side 1 of the loop.

Solve:

$$\mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$$

The key point to keep in mind in applying the above equation is figuring out $\alpha_1, \alpha_2,\ \rho\ and\ a_\varphi$. To find **H** at (0, 0, 5) due to side 1 of the loop in Figure 5(a), consider Figure 5(b), where side 1 is treated as a straight conductor. Notice that we join the point of interest (0, 0, 5) to the beginning and end of the line current. $\alpha_1, \alpha_2,\ and\ \rho$ are assigned as follows;

$$\cos\alpha_1 = \cos 90° = 0, \quad \cos\alpha_2 = \frac{2}{\sqrt{29}}, \quad \rho = 5$$

To determine a_φ,

$\mathbf{a}_\ell = \mathbf{a}_x$ and $\mathbf{a}_\rho = \mathbf{a}_z$, and so;

$\mathbf{a}_\phi = \mathbf{a}_x \times \mathbf{a}_z = -\mathbf{a}_y$

Hence;

$$\mathbf{H}_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi = \frac{10}{4\pi(5)}\left(\frac{2}{\sqrt{29}} - 0\right)(-\mathbf{a}_y)$$
$$= -59.1\mathbf{a}_y \text{ mA/m}$$

AMPERE'S CIRCUIT LAW

Ampère's circuit law states that the line integral of H around a *closed* path is the same as the net current *I*enc enclosed by the path.

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{enc} \qquad (9)$$

The eq. (9) always holds regardless of whether the current distribution is symmetrical or not, but we can use the equation to determine **H** only when a symmetrical current distribution exists. Ampère's law is a special case of Biot–Savart's law.

By applying Stokes's theorem to the left-hand side of eq. (9), we obtain;

$$I_{enc} = \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S} \qquad (10)$$

But

$$I_{enc} = \int_S \mathbf{J} \cdot d\mathbf{S} \qquad (11)$$

Comparing the surface integrals in the two10 and 11 equations reveal that;

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad (12)$$

This is the third Maxwell's equation to be derived; it is essentially Ampère's law in differential (or point) form.

<u>Applications of ampere's law</u>

We now apply Ampère's circuit law to determine **H** for some symmetrical current distributions. For symmetrical current distribution, **H** is either parallel or perpendicular to *d***l**. When **H** is parallel to *d***l**, $|H|$= constant.

A. Infinite Line Current

Consider an infinitely long filamentary current *I* along the *z*-axis as in Figure 6. To determine **H** at an observation point *P*, we allow a closed path to pass through *P*. This path, on which Ampère's law is to be applied, is known as an *Amperian path*. We choose a concentric circle as the Amperian path in which shows that **H** is constant provided r is constant. Since this path encloses the whole current *I*, according to Ampère's law,

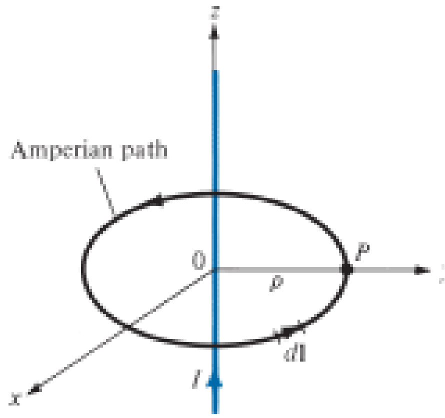

FIGURE 6 Ampère's law applied to an infinite filamentary line current.

$$I = \int_L H_\phi \mathbf{a}_\phi \cdot \rho \, d\phi \, \mathbf{a}_\phi = H_\phi \int_L \rho \, d\phi = H_\phi \cdot 2\pi\rho$$

Or

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi \tag{13}$$

B. Infinite Sheet of Current

Consider an infinite current sheet in the $z = 0$ plane. If the sheet has a uniform current density $K = K_y a_y$ A/m as shown in Figure 7

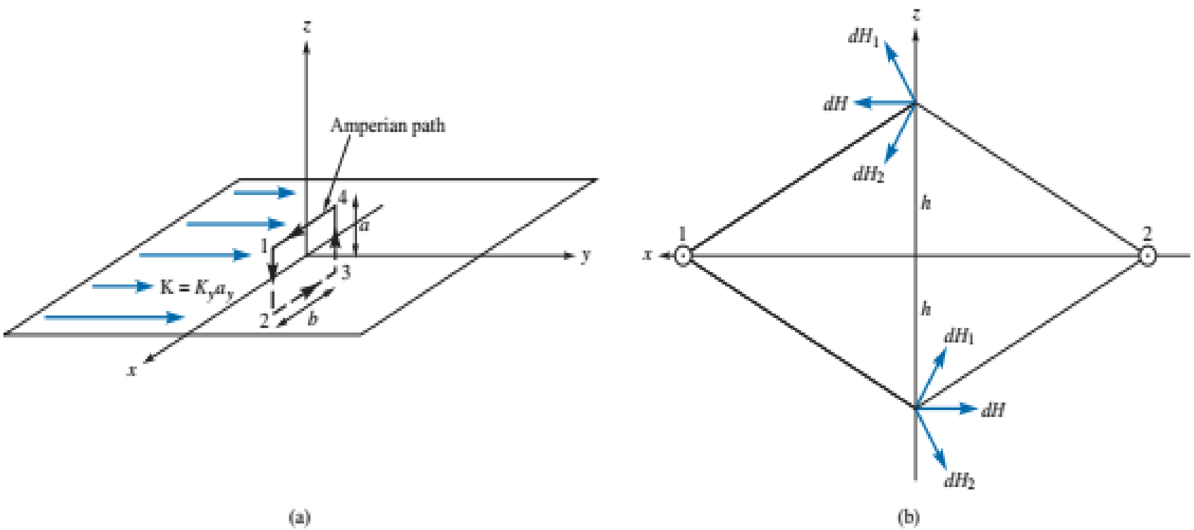

FIGURE 7 Application of Ampère's law to an infinite sheet: (**a**) closed path 1-2-3-4-1, (**b**) symmetrical pair of current filaments with current along **a***y*.

Applying Ampère's law to the rectangular closed path 1-2-3-4-1 (Amperian path) gives

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = K_y b \tag{14}$$

We regard the infinite sheet as comprising filaments; *d***H** above or below the sheet due to a pair of filamentary currents. As evident in Figure 7(b), the resultant *d***H** has only an *x*-component. Also, **H** on one side of the sheet is the negative of that on the other side. Owing to the infinite extent of the sheet, the sheet can be regarded as consisting of such filamentary pairs so that the characteristics of **H** for a pair are the same for the infinite current sheet, that is,

$$\mathbf{H} = \begin{cases} H_o \mathbf{a}_x & z > 0 \\ -H_o \mathbf{a}_x & z < 0 \end{cases} \tag{15}$$

Where *H*o is yet to be determined. Evaluating the line integral of **H** in eq. (13) along the closed path in Figure 7(a) gives;

$$\begin{aligned} \oint \mathbf{H} \cdot d\mathbf{l} &= \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \mathbf{H} \cdot d\mathbf{l} \\ &= 0(-a) + (-H_o)(-b) + 0(a) + H_o(b) \\ &= 2H_o b \end{aligned} \tag{16}$$

From eqs. (14) and (16), we obtain

$$H_o = \frac{1}{2} K_y$$

Substituting *H*o in eq. (15) gives

$$\mathbf{H} = \begin{cases} \dfrac{1}{2} K_y \mathbf{a}_x, & z > 0 \\ -\dfrac{1}{2} K_y \mathbf{a}_x, & z < 0 \end{cases} \tag{17}$$

In general, for an infinite sheet of current density **K** A/m,

$$\mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{a}_n \tag{18}$$

Where $\mathbf{a}_n$ is a unit normal vector directed from the current sheet to the point of interest.

C. Infinitely Long Coaxial Transmission Line

Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the *z*-axis. The cross section of the line is shown in Figure 8, where the *z*-axis is out of the page. The inner conductor has radius a and carries current I, while the outer conductor has inner radius b and thickness t and carries return current $-I$. We want to determine **H** everywhere, assuming that current is uniformly distributed in both conductors. Since the current distribution is symmetrical, we apply Ampère's law along the Amperian path for each of the four possible regions:

$0 \leq \rho \leq a,\ \ a \leq \rho \leq b,\ \ b \leq \rho \leq b+t$ and $\rho \geq b+t$

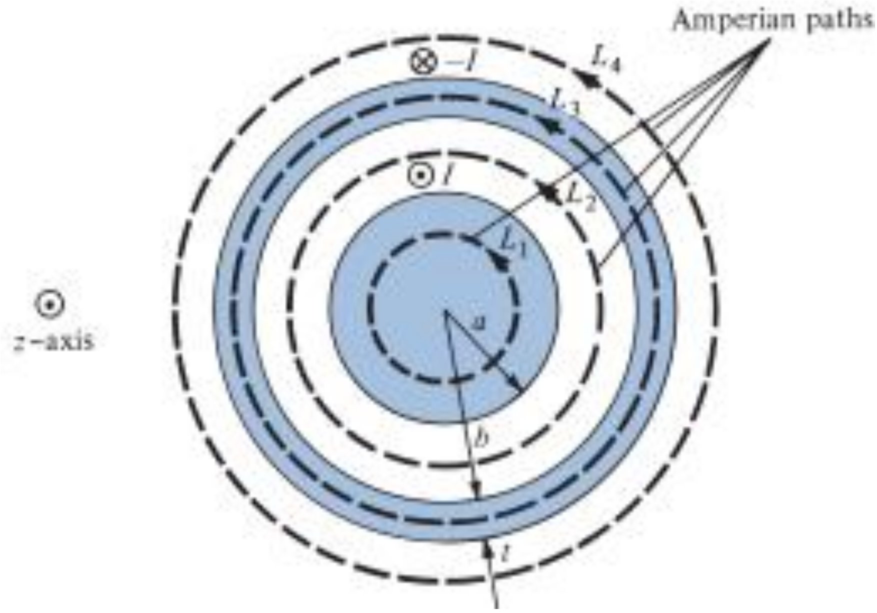

FIGURE 8 Cross section of the transmission line; the positive z-direction is out of the page.

For region, $0 \leq \rho \leq a$ we apply Ampère's law to path L_1, giving;

$$\oint_{L_1} \mathbf{H} \cdot d\mathbf{l} = I_{enc} = \int_S \mathbf{J} \cdot d\mathbf{S} \tag{19}$$

Since the current is uniformly distributed over the cross section,

$$\mathbf{J} = \frac{I}{\pi a^2}\mathbf{a}_z, \quad d\mathbf{S} = \rho\, d\phi\, d\rho\, \mathbf{a}_z$$

$$I_{enc} = \int_S \mathbf{J} \cdot d\mathbf{S} = \frac{I}{\pi a^2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \rho\, d\phi\, d\rho = \frac{I}{\pi a^2}\pi\rho^2 = \frac{I\rho^2}{a^2}$$

Hence eq. (19) becomes

$$H_\phi \int_{L_1} dl = H_\phi\, 2\pi\rho = \frac{I\rho^2}{a^2}$$

Or

$$H_\phi = \frac{I\rho}{2\pi a^2} \tag{20}$$

For region $0 \leq \rho \leq b$, we use path L_2 as the Amperian path,

$$\oint_{L_2} \mathbf{H} \cdot d\mathbf{l} = I_{enc} = I$$

$$H_\phi 2\pi\rho = I$$

Or

$$H_\phi = \frac{I}{2\pi\rho} \tag{21}$$

Since the whole current I is enclosed by L_2. Notice that eq. (21) is the same as eq. (8), and it is independent of a. For region $b \leq \rho \leq b + t$, we use path L_3, getting

$$\oint_{L_3} \mathbf{H} \cdot d\mathbf{l} = H_\phi \cdot 2\pi\rho = I_{enc} \tag{22}$$

Where

$$I_{enc} = I + \int \mathbf{J} \cdot d\mathbf{S}$$

and **J** in this case is the current density (current per unit area) of the outer conductor and is along $-\mathbf{a}_z$, that is,

$$\mathbf{J} = -\frac{I}{\pi[(b+t)^2 - b^2]}\,\mathbf{a}_z$$

Thus

$$\begin{aligned} I_{enc} &= I - \frac{I}{\pi[(b+t)^2 - b^2]} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{\rho} \rho\, d\rho\, d\phi \\ &= I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right] \end{aligned}$$

Substituting this in eq. (22), we have

$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right] \quad (23)$$

For region $\rho \geq b + t$, we use path L_4, getting;

$$\oint_{L_4} \mathbf{H} \cdot d\mathbf{l} = I - I = 0$$

Or

$$H_\phi = 0 \quad (24)$$

Putting eqs. (20) to (24) together gives

$$\mathbf{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\mathbf{a}_\phi, & 0 \leq \rho \leq a \\ \dfrac{I}{2\pi\rho}\mathbf{a}_\phi, & a \leq \rho \leq b \\ \dfrac{I}{2\pi\rho}\left[1 - \dfrac{\rho^2 - b^2}{t^2 + 2bt}\right]\mathbf{a}_\phi, & b \leq \rho \leq b + t \\ 0, & \rho \geq b + t \end{cases}$$

The magnitude of **H** is sketched in Figure 9.

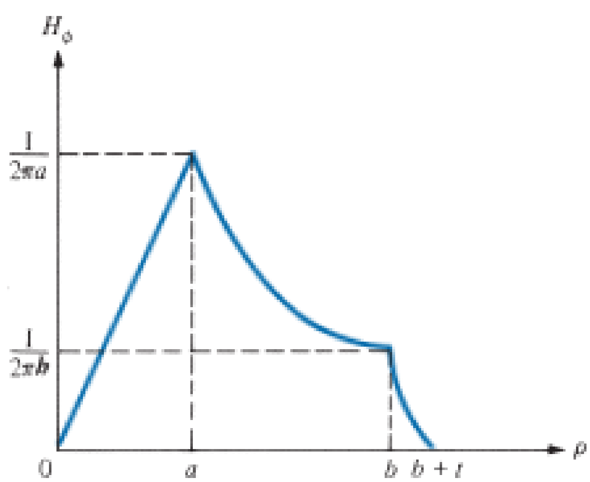

FIGURE 9 Plot of H_ϕ against ρ.

Ampère's law can be used to find **H** only due to symmetric current distributions for which it is possible to find a closed path over which **H** is constant in magnitude.

Example;

A toroid whose dimensions are shown in Figure 10 has *N* turns and carries current *I*. Determine *H* inside and outside the toroid.

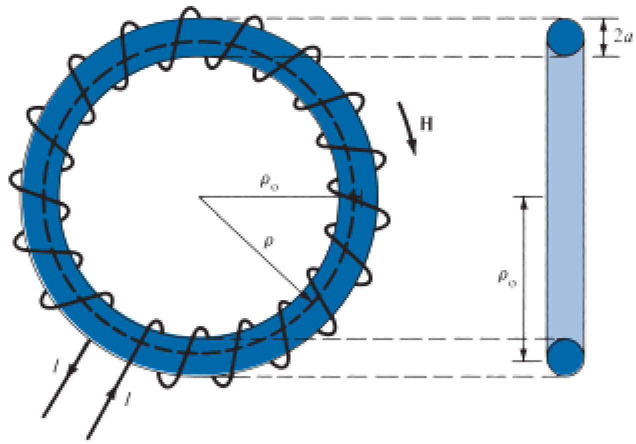

FIGURE 10: A toroid with a circular cross section.

Solve

We apply Ampère's circuit law to the Amperian path, which is a circle of radius r shown dashed in Figure 10. Since *N* wires cut through this path each carrying current *I*, the net current enclosed by the Amperian path is *NI*. Hence,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} \rightarrow H \cdot 2\pi\rho = NI$$

Or

$$H = \frac{NI}{2\pi\rho}, \quad \text{for } \rho_o - a < \rho < \rho_o + a$$

Where ρ_0 is the mean radius of the toroid as shown in Figure 10. An approximate value of *H* is

$$H_{approx} = \frac{NI}{2\pi\rho_o} = \frac{NI}{\ell}$$

Outside the toroid, the current enclosed by an Amperian path is *NI* = *NI* = 0 and hence *H* = 0.