EEE/ETI3105
ELECTRICAL MACHINES I

LECTURE 1: MAGNETIC CIRCUITS

Introduction

- **Electrical machines** are energy converting devices
  1. Mechanical energy to Electrical energy (generators)
  2. Electrical energy to Mechanical energy (motors)
  3. Electrical energy to Electrical energy (transformers)

✓ Use of the principle of electromechanical energy conversion.

$$W_E = W_e + W_{eL} + W_{eS}$$

Energy supplied by an electric source	*Energy transferred to the coupling field by the electric system*	*Energy losses of the electric system. Basically, I^2R*	*Energy stored in the electric or magnetic field*

- The interaction between the electric and mechanical terminals, i.e., the electromechanical energy conversion, occurs through the medium of the electric/magnetic stored energy.

Basic Principles Concerning Electrical Machines

- Before going into understanding what an electrical machine is or even their operation it is necessary to look into some of the basic principles that govern their operation.
- The basic principles that will be briefly highlighted are:
  1. Terminology of magnetic circuits
  2. Faraday's Laws of Electromagnetic Induction
  3. Lenz's Law
  4. Fleming Left Hand rule
  5. Fleming Right Hand rule
  6. Electromagnetic Inductance / Mutual Inductance

Magnetic Circuits

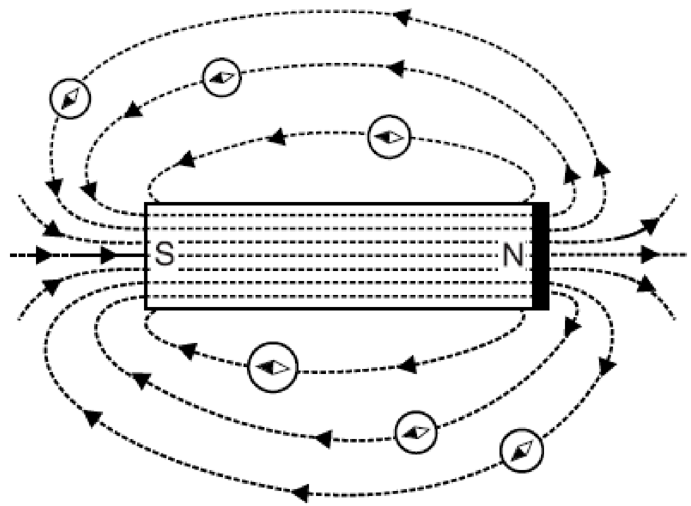
- The operation of all the electrical machines such as DC machines, transformers, synchronous machines, induction motors, etc., rely upon their *magnetic circuits*.
- The closed path followed by the magnetic lines of force is called a magnetic circuit.
- The operation of all the electrical devices (e.g., transformers, generators, motors, etc.) depends upon the magnetism produced by their magnetic circuits.
- Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

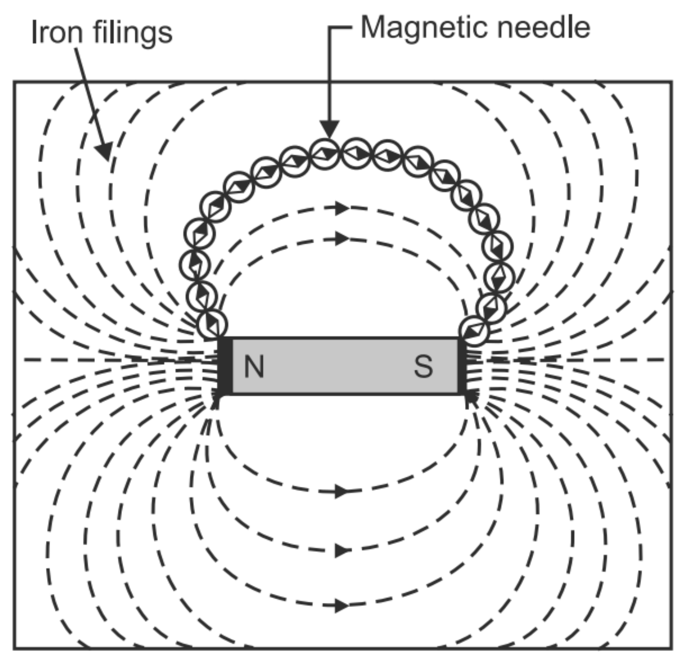
Magnetic Circuits

- *The space (or field) in which a magnetic pole experiences a force is called a* **magnetic field.**

Properties of magnetic lines of force.

- **(*i*)** *Each magnetic line of force forms a closed loop* *i.e.* outside the magnet, the direction of a magnetic line of force is from north pole to south pole and it continues through the body of the magnet to form a closed loop.
- **(*ii*)** *No two magnetic lines of force intersect each other*.
- **(*iii*)** *Where the magnetic lines of force are close together, the magnetic field is strong and where they are well spaced out, the field is weak.*
- **(*iv*)** *Magnetic lines of force contract longitudinally and widen laterally.*
- **(*v*)** *Magnetic lines of force are always ready to pass through magnetic materials like iron in preference to pass through non-magnetic materials like air.*





Magnetic field around a bar magnet

Magnetic Flux (Φ)

- The **total number of magnetic lines of force** produced by a magnetic source is called magnetic flux. It is denoted by Greek letter Φ(phi)..
- Units are in Weber (Wb)
- The more the magnetic lines of force, the greater the magnetic flux and the stronger the magnetic field.

Magnetic Flux Density (B)

- *The* **magnetic flux density** *is defined as the magnetic flux passing normally per unit area. Units wb/m^2 or Tesla (T)*

$$\text{Magnetic flux density, } B = \frac{\phi}{A} \text{ Wb/m}^2$$

$$\text{where} \quad \phi = \text{flux in Wb}$$

$$A = \text{area in m}^2 \text{ normal to flux}$$

Absolute and Relative Permeability

- Permeability of a material means its conductivity for magnetic flux.
- The greater the permeability of a material, the greater is its conductivity for magnetic flux and *vice-versa*.

$$\mu_r = \frac{\mu}{\mu_0}$$

μ = absolute (or actual) permeability of the material

μ_0 = absolute permeability of air or vacuum

μ_r = relative permeability of the material

- The absolute (or actual) permeability *μ_0 = of air or vacuum is $4\pi \times 10^{-7}$ H/m.
- μ_r = 1 for air or non-ferrous materials

Magneto-motive force (mmf) (Fs)

- *This is the source of magnetic flux in a magnetic circuit*. E.g. Permanent magnet or a current carrying conductor. Provided there is a coil of N turns with current (I) passing through it, $\mathbf{F_s = IN}$ (AT).

Magnetising Force (H)

- *The* **magnetising force** (*H*) *produced by an electric current is defined as the m.m.f. set up per unit length of the magnetic circuit*

$$\text{Magnetising force, } H = \frac{NI}{l}\ \text{AT/m}$$

$$NI = \text{m.m.f. (AT)}$$

$$l = \text{length of magnetic circuit in m}$$

Relation Between B and H

- The flux density *B* produced in a material is directly proportional to the applied magnetising force *H*.
- The greater the magnetising force, the greater is the flux density and *vice-versa*

$$B \propto H$$

$$\frac{B}{H} = \text{Constant} = \mu$$

$$B = \mu_0 \mu_r H \qquad \text{...in a medium}$$

$$= \mu_0 H \qquad \text{...in air}$$

$$H \times l = NI$$

$$H = \frac{NI}{l} \text{ AT/m or A/m}$$

$$\text{Incidentally, } B = \mu_0 H = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \qquad \text{...in air}$$

$$= \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2 \qquad \text{...in a medium}$$

- *Hence* **relative permeability** *of a material is equal to the ratio of flux density produced in that material to the flux density produced in air by the same magnetising force.*

Reluctance

- *The opposition that the magnetic circuit offers to the establishment magnetic flux is called reluctance.*
- The reluctance of a magnetic circuit depends upon its length, area of *X*-section and permeability of the material that makes up the magnetic circuit.

$$S = \frac{l}{\mu A} \quad \text{(AT/wb)}$$

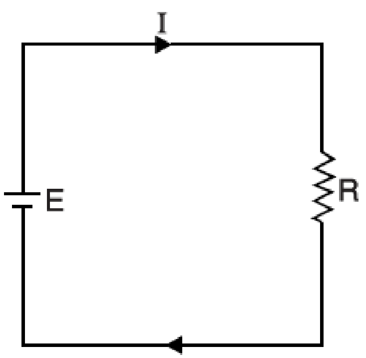
$$S = \frac{F_M}{\Phi} = \frac{NI}{\Phi} = \frac{Hl}{BA} = \frac{l}{(B/H)A} = \frac{l}{\mu_0 \mu_r A}$$

- **Permeance.** *It is the reciprocal of reluctance* and is a measure of the ease with which flux can pass through the material. Its unit is Wb/AT.

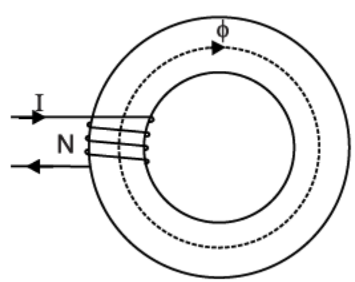
Comparison Between Magnetic and Electric Circuits

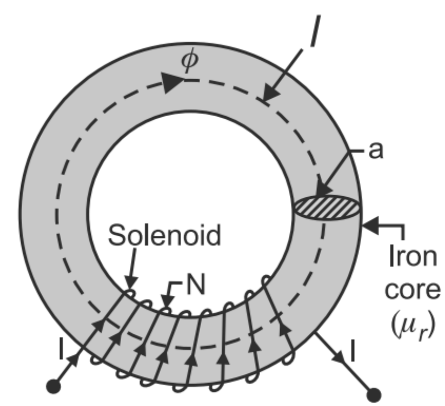
1. The closed path for magnetic flux is called a magnetic circuit.	1. The closed path for electric current is called an electric circuit.
2. Flux, $\phi = \dfrac{\text{m.m.f.}}{\text{reluctance}}$	2. Current, $I = \dfrac{\text{e.m.f.}}{\text{resistance}}$
3. m.m.f. (ampere-turns)	3. e.m.f. (volts)
4. Reluctance, $S = \dfrac{l}{a\mu_0\mu_r}$	4. Resistance, $R = \rho\,\dfrac{l}{a}$
5. Flux density, $B = \dfrac{\phi}{a}\,\text{Wb/m}^2$	5. Current density, $J = \dfrac{I}{a}\,\text{A/m}^2$
6. m.m.f. drop $= \phi\, S$	6. Voltage drop $= I\,R$
7. Magnetic intensity, $H = N\,I/l$	7. Electric intensity, $E = V/d$
8. Permeance	8. Conductance.
9. Permeability	9. Conductivity

Electric Circuit



Magnetic Circuit





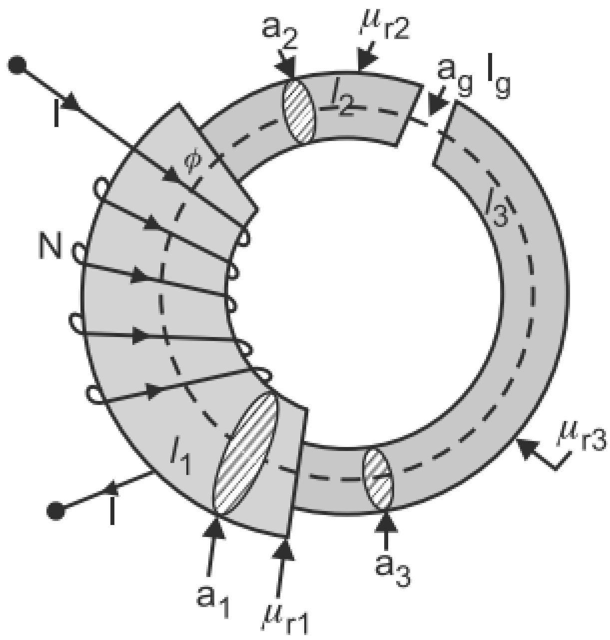
Magnetic circuit

Dissimilarities Between Magnetic and Electric Circuits

Dissimilarities	
1. In fact, the magnetic flux does not flow but it sets-up in the magnetic circuit (basically molecular poles are aligned).	1. The electric current (electrons) actually flows in an electric circuit.
2. For magnetic flux, there is no perfect insulator. It can be set-up even in the non-magnetic materials like air, rubber, glass etc. with reasonable mmf	2. For electric current, there are large number of perfect insulators like glass, air, rubber, etc., which do not allow it to follow through them under normal conditions.
3. The reluctance (***S***) of a magnetic circuit is not constant rather it varies with the value of ***B***. It is because the value of μ_r changes considerably with the change in ***B***.	3. The resistance (***R***) of an electric circuit is almost constant as its value depends upon the value of ρ which is almost constant. However, the value of ρ and ***R*** may vary slightly if temperature changes.
4. Once the magnetic flux is set-up in a magnetic circuit, no energy is expanded. However, a small amount of energy is required at the start to create flux in the circuit.	4. Energy is expanded continuously, so long as the current flows through an electric circuit. This energy is dissipated in the form of heat.

Series Magnetic Circuits

- A magnetic circuit that has a number of parts of different dimensions and materials carrying the same magnetic field is called a series magnetic circuit.
- In a series magnetic circuit, the same flux (Φ) flows through each part of the circuit.
- It can just be compared to a series electric circuit which carries the same current throughout.



Series magnetic circuit

Series Magnetic circuits

Total reluctance of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g$$

$$= \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}$$

$$\text{Total mmf} = \phi S$$

$$= \phi \left(\frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0} \right)$$

$$= \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

$(\because \quad H = B/\mu_0 \mu_r)$

Practice Questions

- Qn1: A coil of insulated wire of 500 turns and of resistance 4Ω is closely wound on iron ring. The ring has a mean diameter of 0.25m and a uniform cross-sectional area of $700mm^2$. Calculate the total flux in the ring when a DC supply of 6V is applied to the ends of the winding. Assume a relative permeability of 550.
- Qn 2: Estimate the number of ampere-turns necessary to produce a flux of 100000 lines round an iron ring of 6 cm^2 cross section and 20cm mean diameter having an air gap 2 mm wide across it. Permeability of the iron may be taken 1200. Neglect the leakage flux outside the 2 mm air gap. [3344.8 AT]
- Qn 3:A wrought iron bar 30 cm long and 2 cm in diameter is bent into a circular shape. It is then wound with 500 turns of wire. Draw the circuit and calculate the current required to produce a flux of 0.5 mWb in magnetic circuit with an air gap of 1 mm;μ_r (iron) = 4000

Practice Questions

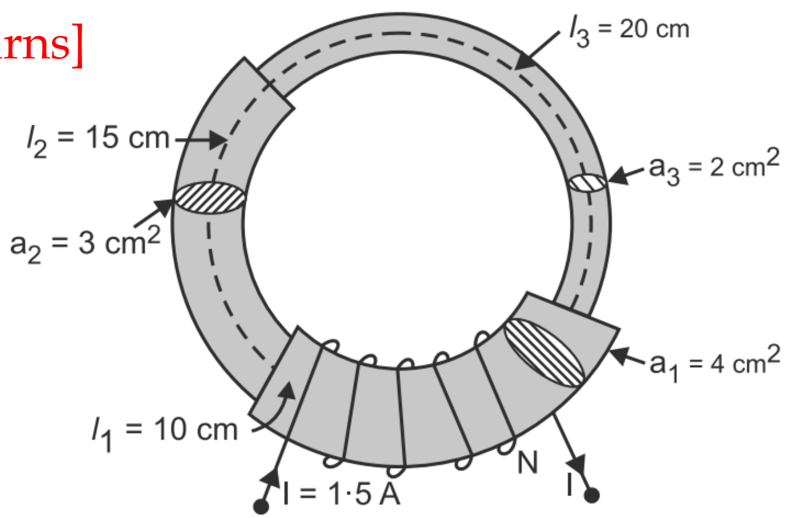
- Qn4: A circular ring 20 cm in diameter has an air gap 1 mm wide cut in it. The area of a cross-section of the ring is 3.6 cm^2. Calculate the value of direct current needed in a coil of 1000 turns uniformly wound round the ring to create a flux of 0.5 mWb in the air gap. Assume relative permeability for the iron as 650. [2.17A]

- Qn 5: An iron ring of mean length 1 m has an air gap of 1 mm and a winding of 200 turns. If the relative permeability of iron is 500 when a current of 1 A flows through the coil, find the flux density.

- Qn 6: An iron ring of cross sectional area 6 cm^2 is wound with a wire of 100 turns and has a saw cut of 2 mm. Calculate the magnetizing current required to produce a flux of 0.1 mWb if mean length of magnetic path is 30 cm and relative permeability of iron is 470. [3.15A]

Practice Questions

- Qn 7: Determine magnetomotive force, magnetic flux, reluctance and flux density in case of a steel ring 30 cm mean diameter and a circular cross-section 2 cm in diameter has an air gap 1 mm long. It is wound uniformly with 600 turns of wire carrying a current of 2.5 A. Neglect magnetic leakage. The iron path takes 40% of the total magnetomotive force.
- Qn 5: The ring shaped core shown in Fig. below is made of a material having a relative permeability of 1000. The flux density in the smallest area of cross-section is 2 T. If the current through the coil is not to exceed 1.5 A, compute the number of turns of the coil. [N = 371 turns]



Solution Qn 1

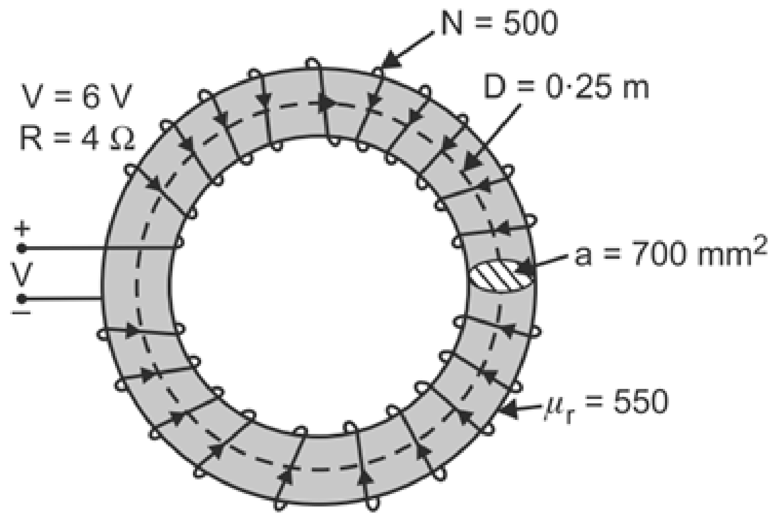
Mean length of iron ring, $l = \pi D = \pi \times 0{\cdot}25 = 0{\cdot}25\,\pi$ m

Area of cross-section, $a = 700\text{ mm}^2 = 700 \times 10^{-6}\text{ m}^2$

Current flowing through the coil,

$$I = \frac{\text{Voltage applied across coil}}{\text{Resistance of coil}}$$

$$= \frac{6}{4} = 1 \cdot 5\text{ A}$$



Total flux in the ring, $\phi = \dfrac{N\,I}{l\,/\,a\,\mu_0\mu_r} = \dfrac{N\,I \times a\,\mu_0\mu_r}{l}$

$$= \frac{500 \times 1 \cdot 5 \times 700 \times 10^{-6} \times 4\pi \times 10^{-7} \times 550}{0 \cdot 25\,\pi} = \mathbf{0{\cdot}462\ mWb}$$

Solution Qn 3

$l_i = 30\text{ cm} = 0{\cdot}3\text{ m};$

Diameter, $d = 2$ cm

$$\text{Area, } a = \frac{\pi}{4}d^2 = \frac{\pi(2)^2}{4} \times 10^{-4}\ \text{m}^2$$

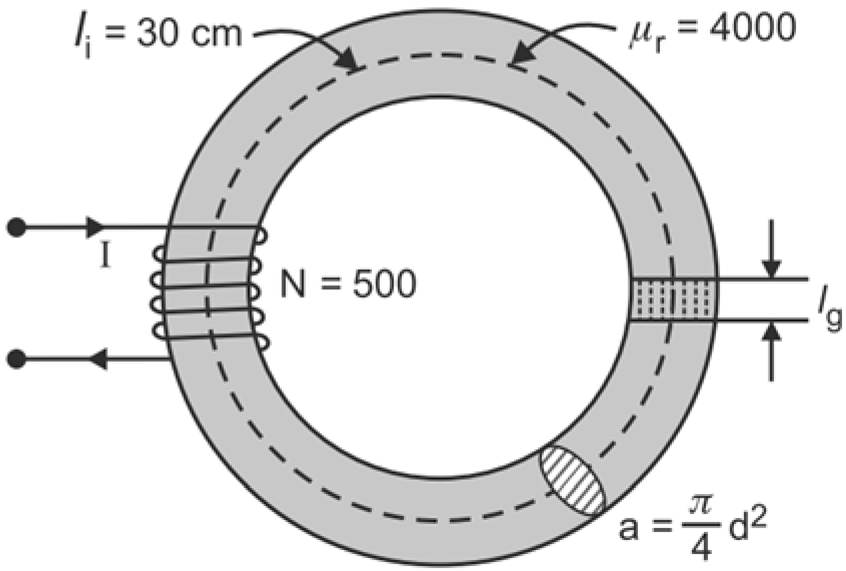
$$= \pi \times 10^{-4}\ \text{m}^2$$

$$\phi = 0{\cdot}5\text{ mWb} = 0{\cdot}5 \times 10^{-3}\text{ Wb}$$

$N = 500$ turns

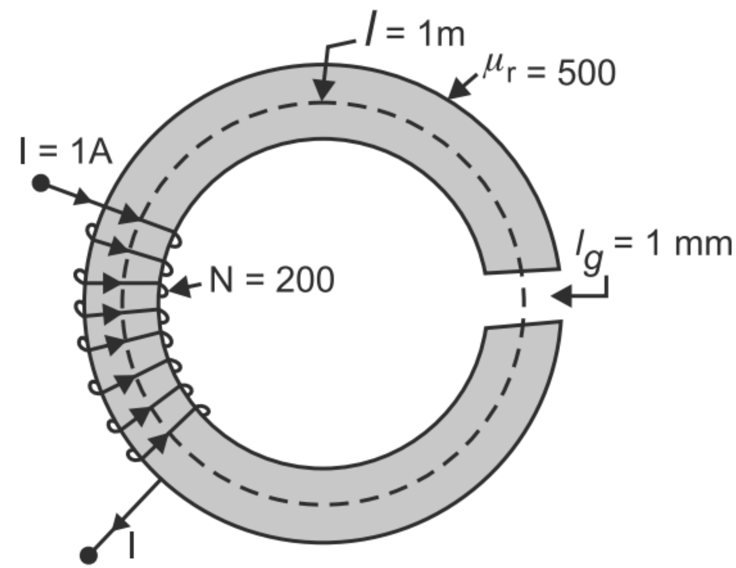
$$NI = \frac{\phi}{a}\left[\frac{l_i}{\mu_0\mu_r} + \frac{l_g}{\mu_0}\right]$$

$$I = \frac{0\cdot 5 \times 10^{-3}}{500 \times \pi \times 10^{-4}}\left[\frac{0\cdot 3}{4\pi \times 10^{-7} \times 400} + \frac{0\cdot 001}{4\pi \times 10^{-7}}\right]$$



Solution Qn 5

The magnetic circuit is shown in Fig.



Magnetic circuit

Now, mmf = flux × reluctance

$$NI = \phi\left(\frac{l_i}{a\,\mu_0\,\mu_r} + \frac{l_g}{a\,\mu_0}\right)$$

$$NI = B\left(\frac{l_i}{\mu_0\;\mu_r} + \frac{l_g}{\mu_0}\right)$$

where $N = 200$ turns; $I = 1$ A; $\mu_r = 500$

$$l_g = 1 \text{ mm} = 0{\cdot}001 \text{ m};$$

$$l_i = (1 - 0{\cdot}001) = 0{\cdot}999 \text{ m}$$

$$\therefore \qquad 200 \times 1 = B\left(\frac{0\cdot 999}{4\pi \times 10^{-7} \times 500} + \frac{0\cdot 001}{4\pi \times 10^{-7}}\right)$$

$$B = \frac{200}{2385\cdot 73} = \mathbf{0{\cdot}0838\ Wb/m^2}$$

Solution Qn 7

Mmf of the magnetic circuit = $NI = 600 \times 2{\cdot}5 =$ **1500 ATs** (*Ans.*)

As iron path takes 40% of the total mmf, the reluctance of iron in 40% and the rest of the reluctance (60%) is of air path.

$$\therefore \quad \frac{S_a}{S_i} = \frac{60}{40} = \frac{3}{2} = 1{\cdot}5$$

$$\text{Reluctance of air path, } S_a = \frac{l_a}{a\,\mu_0}$$

where $l_a = 1 \times 10^{-3}$ m; $a = \dfrac{\pi}{4}(2)^2 \times 10^{-4} = \pi \times 10^{-4}$ m^2;

$$\therefore \quad S_a = \frac{1 \times 10^{-4}}{\pi \times 10^{-4} \times 4\pi \times 10^{-7}} = 2{\cdot}533 \times 10^6 \text{ ATs/Wb}$$

$$\text{Reluctance of iron path, } S_i = \frac{S_a}{1\cdot 5} = \frac{2\cdot 533 \times 10^6}{1\cdot 5} = 1{\cdot}688 \times 10^6 \text{ ATs/Wb}$$

$$\text{Total reluctance} = S_a + S_i = (2{\cdot}533 + 1{\cdot}688) \times 10^6 \text{ ATs/Wb}$$

$$= \mathbf{4{\cdot}221 \times 10^6\ ATs/Wb}\ (\textit{Ans.})$$

$$\text{Magnetic flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{1500}{4\cdot 221 \times 10^6} = \mathbf{0{\cdot}3554\ m\ Wb}\ (\textit{Ans.})$$

$$\text{Flux density, } B = \frac{\phi}{a} = \frac{0\cdot 3554 \times 10^{-3}}{\pi \times 10^{-4}} = \mathbf{1{\cdot}131\ Wb/m^2}\ (\textit{Ans.})$$

Electromagnetic Induction

- *The phenomenon of production of e.m.f. and hence current in a conductor or coil when the magnetic flux linking the conductor or coil changes is called* **electromagnetic induction.**
- Flux Linkages
- *The product of number of turns (N) of the coil and the magnetic flux (Φ) linking the coil is called* **flux linkages** *i.e.*
    - Flux linkages = $N\,\Phi$
- Experiments show that the magnitude of e.m.f. induced in a coil is directly proportional to the rate of change of flux linkages. If N is the number of turns of the coil and the magnetic flux linking the coil changes (say increases) from Φ_1 to Φ_2 in t seconds, then,

Induced e.m.f., $e \propto$ Rate of change of flux linkages

$$e \propto \frac{N\phi_2 - N\phi_1}{t}$$

Faraday's Laws of Electromagnetic Induction

Faraday's First Law.

- *Any change in the magnetic field of a conducting coil causes an emf to be induced in the coil.*
- If the conductor circuit is closed, the induced emf will cause current to circulate through the circuit and this current is called induced current.
- The induction of emf requires a conductor, a magnetic field and linking or cutting of flux by the conductor. The linking of magnetic field by the conductor can occur in three ways:
  1. By moving a conductor in a stationary permanent magnet or dc electromagnet. This configuration is used in all dynamos, generators and motors.
  2. By moving an electromagnet with respect to a stationary conductor. This configuration is used in large ac generators (especially synchronous generators)
  3. Having a stationary conductor and a stationary electromagnet and variation of flux by feeding an alternating current to the magnet. This is used in transformers.

Faraday's Second Law of EMI

- *It states that the magnitude of the induced emf is equal to the rate of change of flux-linkages.*
- Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time, t seconds.

∴ induced emf:

$$e = \frac{N\Phi_2 - N\Phi_1}{t} = N\frac{\Phi_2 - \Phi_1}{t} \quad \text{Wb/s or Volt}$$

- Putting the above expression in its differential form, we get

$$e = \frac{d}{dt}(N\Phi) = N\frac{d\Phi}{dt} \quad \text{Volt}$$

How to increase emf induced in a coil

1. By increasing the number of turns in the coil i.e N-from the formulae derived above it is easily seen that if number of turns of coil is increased, the induced emf also increases.

2. By increasing magnetic field strength surrounding the coil. Mathematically if magnetic field increases, flux increases and if flux increases emf induced will also get increased.
   - Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for coil to cut and hence there will be more emf induced.

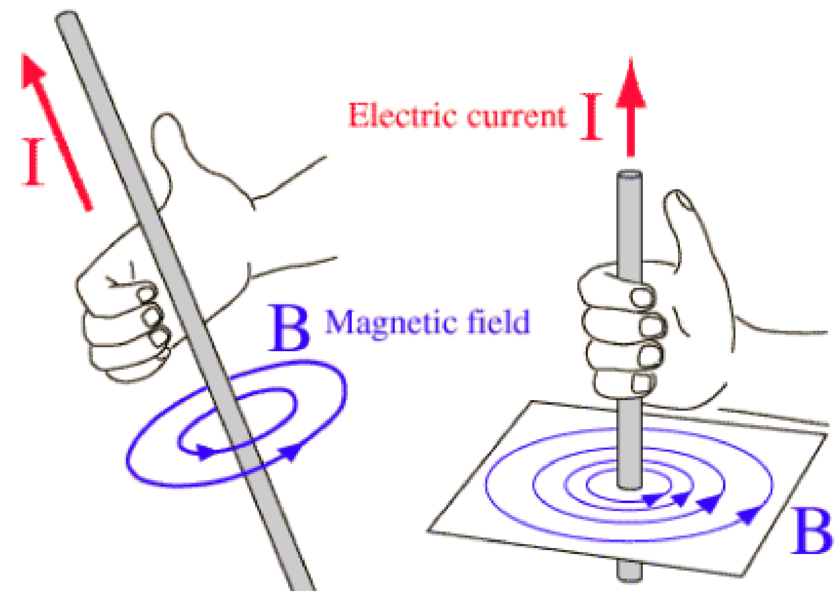
3. By increasing the speed of the relative motion between the coil and the magnet - If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

Lenz's Law

- This law states that the electromagnetically induced current (due to Faraday's law) always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.
- Usually, a negative sign is used in Faraday's law of electromagnetic induction, to indicate that the induced emf (e) and the change in magnetic flux (dΦ) have opposite signs.

$$e = -N\frac{d\Phi}{dt} \quad \text{Volt}$$

NOTE : *For finding the directions of magnetic field or current, use right hand thumb rule i.e if the fingers of the right hand are placed around the wire so that the thumb points in the direction of current flow, then the curling of fingers will show the direction of the magnetic field produced by the wire.*

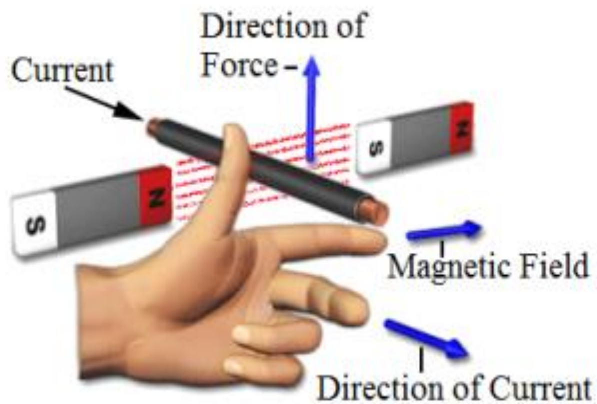


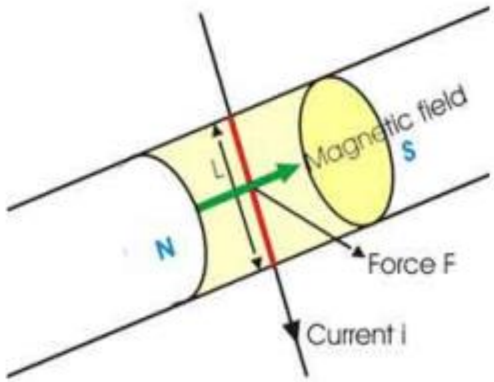
Fleming Left Hand and Right Hand rules

- Whenever, a current carrying conductor comes under a magnetic field, there will be force acting on the conductor and on the other hand, if a conductor is forcefully brought under a magnetic field, there will be an induced current in that conductor.
- In both of the phenomenon, there is a relation between magnetic field, current and force.
- This relation is directionally determined by Fleming Left Hand rule and Fleming Right Hand rule, respectively.
- ‘Directionally’ means these rules do not show the magnitude but show the direction of any of the three parameters (magnetic field, current, force) if the direction of other two are known.
- Fleming Left Hand rule is mainly applicable for electric motors and Fleming Right Hand rule is mainly applicable for electric generators.

Fleming Left Hand Rule (FLHR)

- It is found that whenever an current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field.
- If you hold out your left hand with forefinger, second finger and thumb at right angle to one another.
- Then the fore finger represents the direction of the field and the second finger that of the current, and thumb gives the direction of the force.

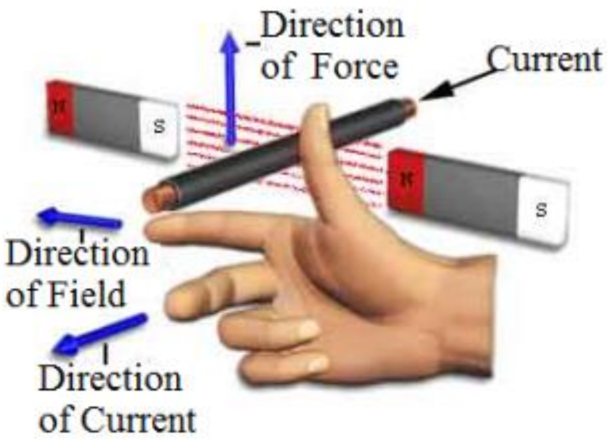




- In the figure it is shown that, a portion of a conductor of length L placed vertically in a uniform horizontal magnetic field strength H, produced by two magnetic poles N and S.

Fleming Right Hand Rule (FRHR)

- The FLHR is used to get the direction of force experienced by the conductor carrying current, placed in a magnetic field while FRHR is used to get direction of induced e.m.f. when conductor is moving in a magnetic field
- Hold out the right hand with the first finger, second finger and thumb at right angle to each other.
- If forefinger represents the direction of the field, the thumb points in the direction of motion or applied force, then second finger points in the direction of the induced current.



- If $\boldsymbol{i}$ is the current flowing through this conductor, the magnitude of the force acts on the conductor is, $F = BiL$

Self & Mutual Inductance

- *Inductance is the name given to the property of a circuit whereby there is an e.m.f. induced into the circuit by the change of flux linkages produced by a current change.*
- Self inductance **(L)** is when the e.m.f. is induced in the same circuit as that in which the current is changing.

$$E = -L\frac{dI}{dt} \text{ volts} \qquad L = \frac{N\Phi}{I} \qquad L = \frac{N^2}{S}$$

- Mutual inductance **(M)** is when an e.m.f. is induced in a circuit by a change of flux due to current changing in an adjacent circuit.

$$E_2 = -M\frac{dI_1}{dt} \text{ volts} \qquad M = \frac{N_2\Phi_1}{I_1} \qquad M = \frac{N_1 N_2}{S}$$

Practice Question

Qn. The mean diameter of a steel ring is 40cm and flux density of 0.9T is produced by 3500AT/m. If the cross-section of the ring is 15cm^2 and number of turns is 440, calculate

- i) The exciting current
- ii) The self inductance
- iii) Exciting current and inductance when air gap of 1cm is cut in the ring.

Solution

Solution : $d_{mean} = 40$ cm, $B = 0.9$ Wb/m^2, $H = 3500$ AT/m, $a = 15$ cm^2, $N = 440$

$$l_T = \pi \times d_{mean} = \pi \times 40 = 125.6637 \text{ cm}$$

$$B = \mu_0 \mu_r H = \mu H$$

$$\therefore \quad \mu = \frac{B}{H} = \frac{0.9}{3500} = 2.5714 \times 10^{-4}$$

$$S_T = \frac{l_T}{\mu a}$$

$$\therefore \quad S_T = \frac{125.6637 \times 10^{-2}}{2.5714 \times 10^{-4} \times 15 \times 10^{-4}} = 3.2579 \times 10^6 \text{ AT/Wb}$$

$$\text{m.m.f.} = NI = H \times l_T = 3500 \times 125.6637 \times 10^{-2} = 4398.2295 \text{ AT}$$

i) $$I = \frac{4398.2295}{N} = \frac{4398.2295}{440} = 9.99 \approx 10 \text{ A}$$

ii) $$L = \frac{N^2}{S_T} = \frac{(440)^2}{3.2579 \times 10^6} = 0.0594 \text{ H}$$

Solution

iii) Now air gap $l_g = 1$ cm is cut.

$$\therefore \quad l_i = l_T - l_g = 124.6637 \text{ cm}$$

$$\therefore \quad S_T = S_i + S_g = \frac{l_i}{\mu a} + \frac{l_g}{\mu_0 a}$$

$$= \frac{124.6637 \times 10^{-2}}{2.5714 \times 10^{-4} \times 15 \times 10^{-4}} + \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 15 \times 10^{-4}}$$

$$= 3.232 \times 10^6 + 5.3051 \times 10^6 = 8.5371 \times 10^6 \text{ AT/Wb}$$

$$\phi = B \times a = 0.9 \times 15 \times 10^{-4} = 1.35 \times 10^{-3} \text{ Wb}$$

Now $$\phi = \frac{\text{m.m.f}}{\text{Reluctance}}$$

$$\therefore \quad 1.35 \times 10^{-3} = \frac{\text{m.m.f}}{8.5371 \times 10^6}$$

$$\therefore \quad \text{m.m.f} = NI = 11525.085$$

$$\therefore \quad I = \frac{11525.1494}{N} = 26.1935 \text{ A}$$

and $$L = \frac{N^2}{S_T} = \frac{(440)^2}{8.5371 \times 10^6} = 0.0226 \text{ H}$$

Practice Question

Qn. The mean diameter of a steel ring is 40cm and flux density of 0.9T is produced by 3500AT/m. If the cross-section of the ring is $15cm^2$ and number of turns is 440, calculate

- i) The exciting current
- ii) The self inductance
- iii) Exciting current and inductance when air gap of 1cm is cut in the ring.

Practice Question

- **Qn**. A solenoid with 900 turns has a total flux of 1.33×10^{-7}Wb through its air core when the coil current is 100mA. If the flux takes 75ms to grow from zero to its maximum level, calculate the inductance of the coil. Also, calculate the induced e.m.f. in the coil during the flux growth.
- Solution

$$e = L\frac{dI}{dt} \; ; e = N\frac{d\phi}{dt}$$

$$L\frac{dI}{dt} = N\frac{d\phi}{dt} \quad \text{or} \quad L = N\frac{d\phi}{dI}$$

$$N = 900 \; ; d\phi = 1.33 \times 10^{-7} \text{ Wb} \; ; dt = 75 \text{ ms} = 75 \times 10^{-3} \text{ s} \; ;$$

$$dI = 100 \text{ mA} = 100 \times 10^{-3} \text{A}$$

$$L = 900 \times \frac{1.33 \times 10^{-7}}{100 \times 10^{-3}} = 1.2 \times 10^{-3} \text{ H} = \mathbf{1.2 \text{ mH}}$$

$$\text{Induced e.m.f., } e = N\frac{d\phi}{dt} = 900 \times \frac{1.33 \times 10^{-7}}{75 \times 10^{-3}} = 1.6 \times 10^{-3} \text{ V} = \mathbf{1.6 \text{ mV}}$$

Energy Stored in a Magnetic Field

- A component called an inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire. An inductor possesses an ability to store energy.
- When some electrical energy is supplied to a coil, it is spent in two ways:
- (i) A part of it is spent to meet I^2R loss which is dissipated in the form of heat and cannot be recovered.
- (ii) The remaining part is used to create magnetic field around the coil and is stored in the magnetic field. When this field collapses, the stored energy is released by the coil and is returned to the circuit.
- The energy stored, W, in the magnetic field of an inductor is given by:

$$W = \frac{1}{2}LI^2 \text{ Joules}$$

Practice Question

- Qn. 1: (a) A coil of 100 turns is wound on a toroidal magnetic core having a reluctance of 10^4 AT/Wb. When the coil current is 5A and is increasing at the rate of 200 A/s, determine (i) energy stored in the magnetic circuit and (ii) voltage applied across the coil. Assume coil resistance as zero.
- (b) How are your answers affected if the coil resistance is 2Ω ?
- Solution

$N = 100$ turns ; Reluctance of core, $S = 10^4$ AT/Wb

(*a*) Inductance of coil, $L = \dfrac{N^2}{S} = \dfrac{(100)^2}{10^4} = 1\text{ H}$

(*i*) Energy stored $= \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 1 \times (5)^2 = \mathbf{12.5\ J}$

(*ii*) Voltage applied across coil = Self-induced e.m.f. in the coil

$$= L\frac{dI}{dt} = 1 \times 200 = \mathbf{200\ V}$$

(*b*) If the coil resistance is 2Ω, the energy stored will remain the same *i.e.*, **12.5 J.**

Voltage across coil $= IR + L\dfrac{dI}{dt} = 5 \times 2 + 1 \times 200 = \mathbf{210\ V}$

However, there will be a loss of energy $= I^2R = (5)^2 \times 2 = 50\text{W}$

Practice Questions

- Qn. 1: An air cored coil has 400 turns, a mean length of 20cm and cross-sectional area of 6cm^2. Calculate, the inductance of the coil the average induced emf, if a current of 4A is reversed in 50ms.

- Qn. 2: A solenoid of 1 m in length and 10 cm in diameter has 5000 turns. Calculate the energy in the magnetic field when a current of 2 A flows in the solenoid.

Classification of Electrical Machines

- Electrical machines can be broadly classified as DC machines and AC machines

1. **DC Machines**
   - DC Generators
   - DC Motors
2. **AC Machines**
   - Transformers
     - - Single Phase Transformers
     - - Three Phase Transformers
   - Induction Machines
     - - Induction Generators
     - - Induction Motors
   - Synchronous Machines
     - - Synchronous Generators
     - - Synchronous Motors
   - AC Commutator Machines
   - Special Machines

Self Assessment

- What do you understand by magnetic field?
- Mention at least four properties of magnetic lines of force.
- Outline and briefly describe any five terminologies used in magnetic circuits.
- What do you understand by electromagnetic induction?
- Define Faraday's laws of electromagnetic induction
- State Fleming's Right hand rule as well as Fleming's Left hand rule.
- Distinguish between self induced and mutually induced emf.
- Estimate the inductance of a solenoid of 2500 turns wound uniformly over a length of 0.5m on a cylindrical paper tube 4 cm in diameter. The medium is air. [19.74 mH]
- Calculate the inductance of a toroid 25 cm mean diameter and 6·25 cm2 circular cross-section wound uniformly with 1000 turns of wire. Also determine the emf induced when a current increasing at the rate of 200 A/s flows in the winding. [1 mH; 0.2 V]