Machine Learning - Regression

Professor Widom's Instructional Odyssey

www.professorwidom.org

Stanford University

Google

Google Cloud Platform

Association for Computing Machinery

Very Large Data Bases Endowment Inc.

Instabase

amazon web services

Machine Learning

Using data to make inferences or predictions

Supervised machine learning

- Set of labeled examples to learn from: training data
- Develop model from training data
- Use model to make inferences about new data

Unsupervised machine learning

- Unlabeled data, look for patterns or structure (similar to data mining)

Machine Learning

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Unsupervised ma

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  (similar to dat

Also…

- Reinforcement learning
  Improve model as new data arrives
- Semi-supervised learning
  Labeled + unlabeled
- Active learning
  Semi-supervised, ask user for labels

Regression

Using data to make inferences or predictions

- Supervised
- Training data, each example:
  - Set of predictor values - “independent variables”
  - Numeric output value - “dependent variable”
- Model is function from predictors to output
  - Use model to predict output value for new predictor values
- Example
  - Predictors: mother height, father height, current age
  - Output: height

Back to Regression

- Set of predictor values - "independent variables"
- Numeric output value - "dependent variable"
- Model is function from predictors to output

Training data

$w_1, x_1, y_1, z_1 \rightarrow o_1$
$w_2, x_2, y_2, z_2 \rightarrow o_2$
$w_3, x_3, y_3, z_3 \rightarrow o_3$
……

Model

$f(w, x, y, z) = o$

Back to Regression

Goal: Function f applied to training data should produce values as close as possible in aggregate to actual outputs

Training data

$w_1, x_1, y_1, z_1 \rightarrow o_1$
$w_2, x_2, y_2, z_2 \rightarrow o_2$
$w_3, x_3, y_3, z_3 \rightarrow o_3$
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Model

$f(w, x, y, z) = o$

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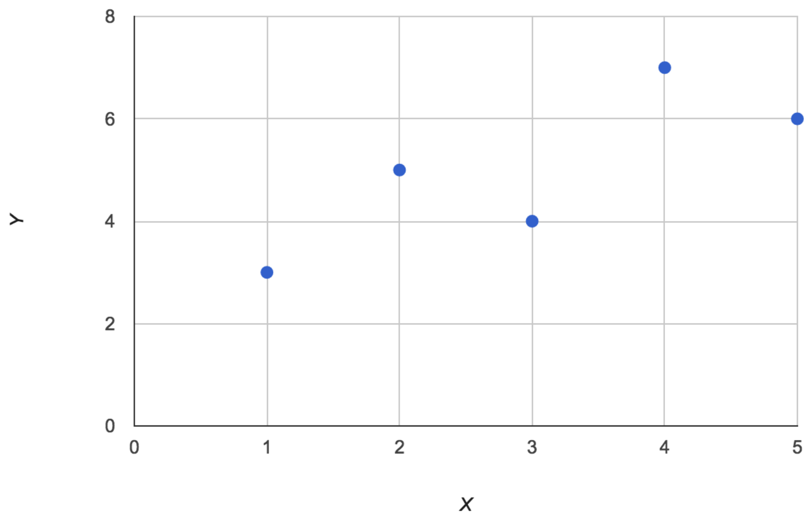
$f(w_2, x_2, y_2, z_2) = o_2'$
$f(w_3, x_3, y_3, z_3) = o_3'$

Simple Linear Regression

We will focus on:

- One numeric predictor value, call it *x*
- One numeric output value, call it *y*

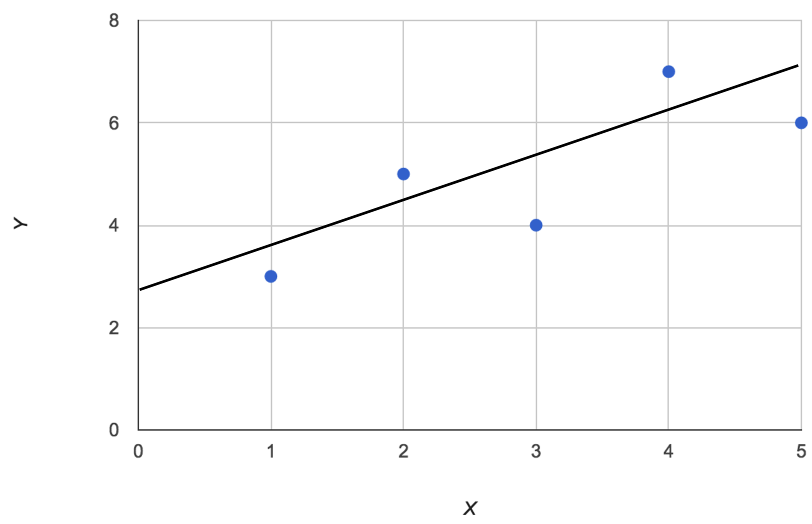
➢ Data items are points in two-dimensional space



Simple Linear Regression

We will focus on:

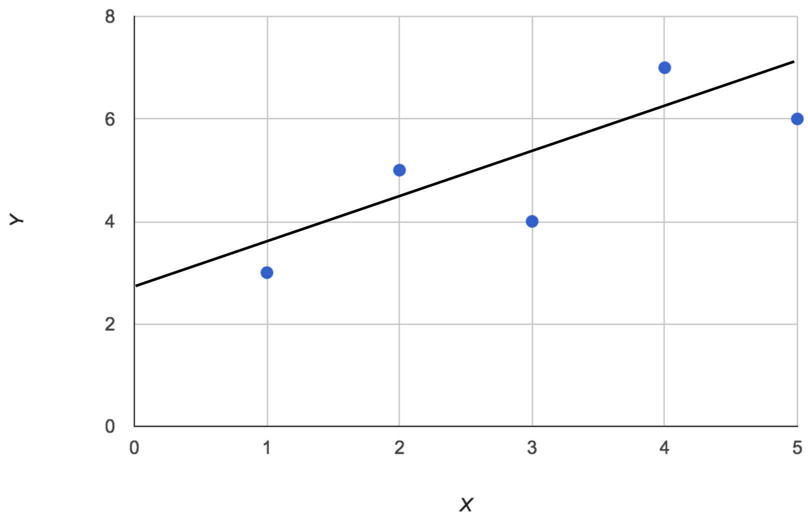
- One numeric predictor value, call it *x*
- One numeric output value, call it *y*
- Functions *f(x)=y* that are lines (for now)



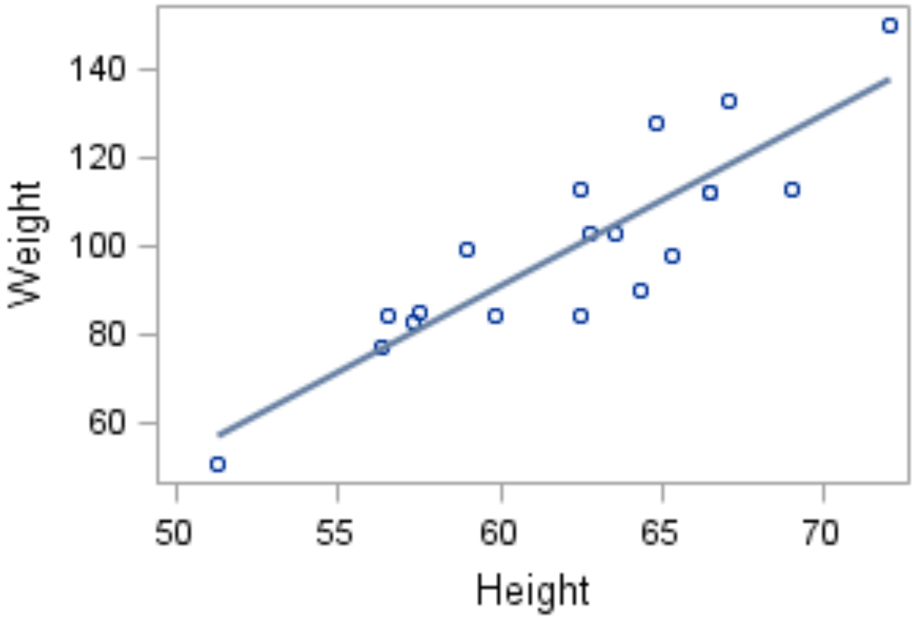
Simple Linear Regression

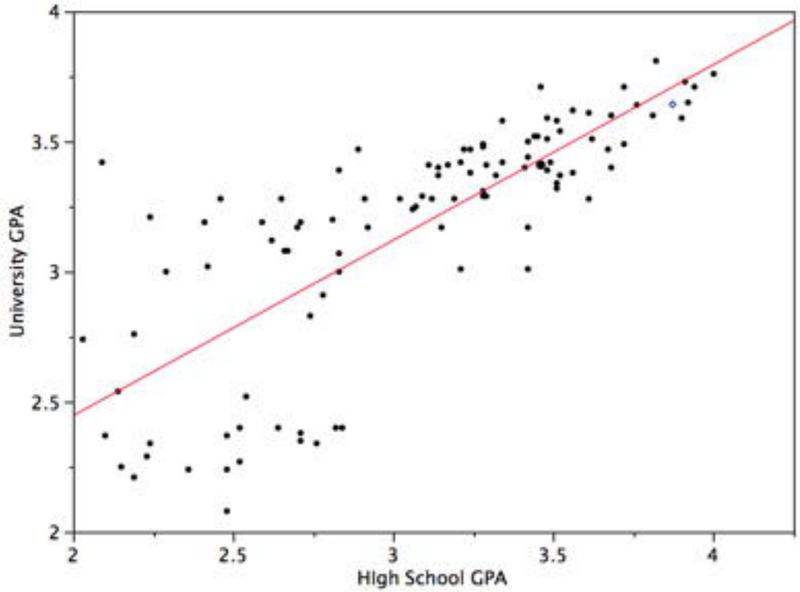
Functions $f(x)=y$ that are lines: $y = ax + b$

$y = 0.8x + 2.6$



"Real" Examples (from Overview)

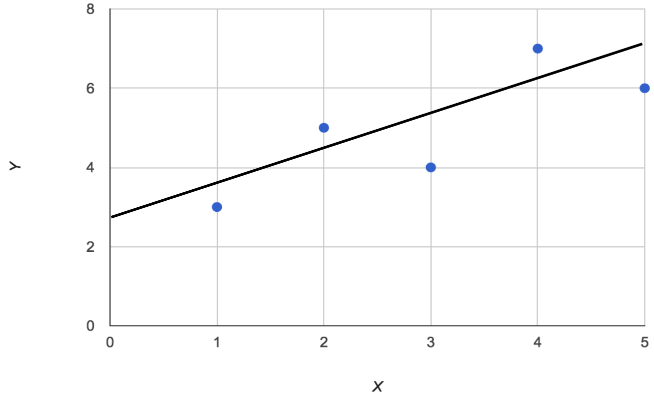




Summary So Far

- Given: Set of known (*x*,*y*) points
- Find: function *f(x)=ax+b* that "best fits" the known points, i.e., *f(x)* is close to *y*
- Use function to predict *y* values for new *x*'s

➢ Also can be used to test correlation



Correlation and Causation (from Overview)

Correlation – Values track each other

- Height and Shoe Size
- Grades and Entrance Exam Scores

Causation – One value directly influences another

- Education Level → Starting Salary
- Temperature → Cold Drink Sales

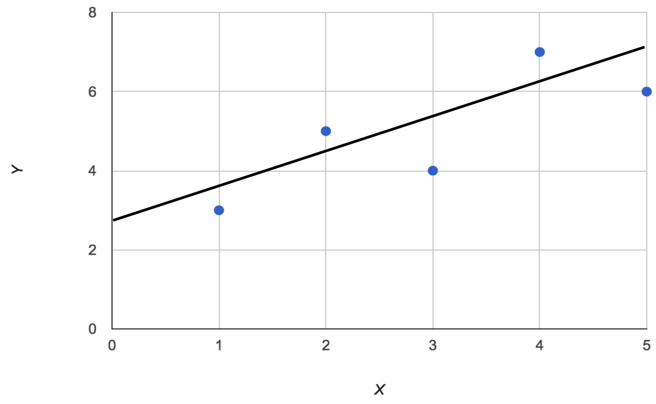
Correlation and Causation (from Overview)

Correlation – Values track each other

- Height and Shoe Size
- Grades and Entrance Exam Scores

Find: function $f(x)=ax+b$ that "best fits" the known points, i.e., $f(x)$ is close to y

> The better the function fits the points, the more correlated x and y are



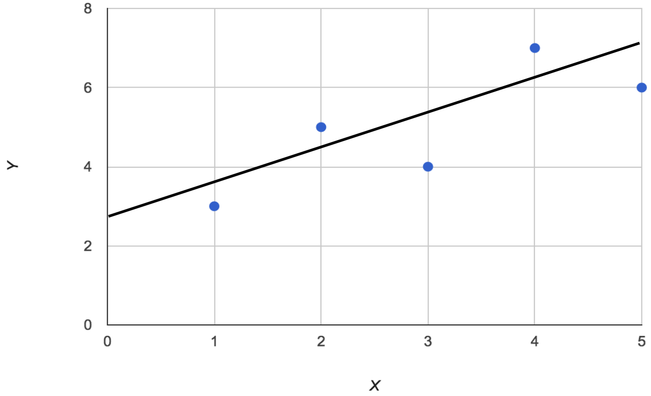
Regression and Correlation

The better the function fits the points, the more correlated *x* and *y* are

- Linear functions only
- Correlation – Values track each other

  Positively – when one goes up the other goes up
- Also negative correlation

  When one goes up the other goes down
  - Latitude versus temperature
  - Car weight versus gas mileage
  - Class absences versus final grade



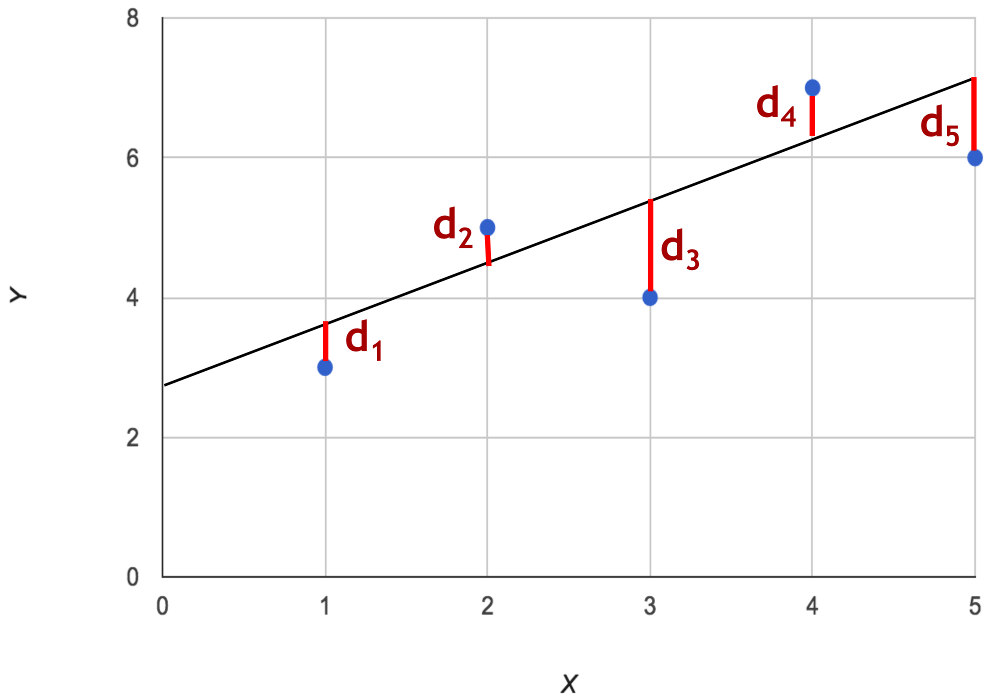
Calculating Simple Linear Regression

Method of least squares

- Given a point and a line, the error for the point is its vertical distance d from the line, and the squared error is d^2
- Given a set of points and a line, the sum of squared error (SSE) is the sum of the squared errors for all the points
- Goal: Given a set of points, find the line that minimizes the SSE

Calculating Simple Linear Regression

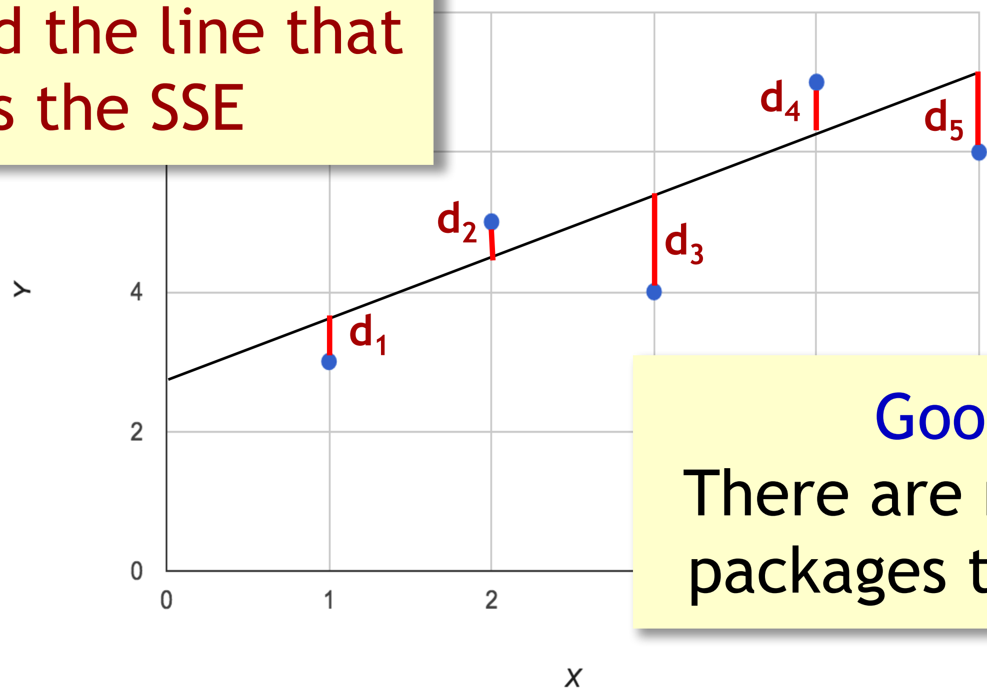
Method of least squares



$$SSE = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Method of least squares

Goal: Find the line that minimizes the SSE



Good News!
There are many software packages to do it for you

$$SSE = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Measuring Correlation

More help from software packages…

Pearson's Product Moment Correlation (PPMC)

- "Pearson coefficient", "correlation coefficient"
- Value **r** between 1 and -1

  1 maximum positive correlation

  0 no correlation

  -1 maximum negative correlation

Coefficient of determination

"The better the function fits the points, the more correlated *x* and *y* are"

- r^2, R^2, "R squared"
- Measures fit of any line/curve to set of points
- Usually between 0 and 1
- For simple linear regression $R^2 = \text{Pearson}^2$

Measuring Correlation

More h… …kages…

Swapping x and y axes yields same values

Pearson's Proc… …ion (PPMC)

- "Pearson coefficient", "correlation coefficient"
- Value **r** between 1 and -1

  1 maximum positive correlation

  0 no correlation

  -1 maximum negative correlation

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Regression Through Spreadsheets

City temperatures (using Cities.csv)

1. temperature (y) versus latitude (x)
2. temperature (y) versus longitude (x)
3. latitude (y) versus longitude (x)
4. longitude (y) versus latitude (x)

Your Turn

Correlations in the World Cup data

- Use Teams.csv and/or Players.csv *(unmodified)*
- Linear trendlines only
- What is the strongest positive correlation you can find? (highest R^2 value)
- What is the strongest negative correlation you can find? (highest R^2 value)