Lab Assignment 4, IC-252

Problem 1. In this problem, the random variable X is the sum of the numbers appearing on two die.

1. (a) Write a code that calculates the probabilities $P(X = i)$ where $i \in \{1, 2, 3, \ldots, 13\}$. Do all these probabilities add to 1, i.e., $\sum_i P(X = i) = 1$? (b) Your program should throw an error when this isn't true. (c) Make a plot of $P(X = i)$ as a function of i.
2. The *cumulative distribution function* $F(x)$ gives you the probability that the random variable X takes values $\leq x$. (a) Calculate $F(x)$ using a computer program. Your program should be able to calculate $F(x)$ for any given $x \in \mathbb{R}$. (b) Calculate in particular $F(\pi)$ and $F(\sqrt{30})$. (c) Make a plot of $F(x)$ as a function of $x \in [-1, 20]$.

Problem 2. Consider the *random walk problem* in one dimension in which a person takes steps (*step length* 1 m) randomly either to the left (with probability p) or right (with probability $q = 1 - p$). Let us say the person starts from the origin, i.e., $x = 0$, shown as Lamp post in Figure 1. In this case, the random variable X is *displacement* of the person from the origin after a given number of steps, say n.

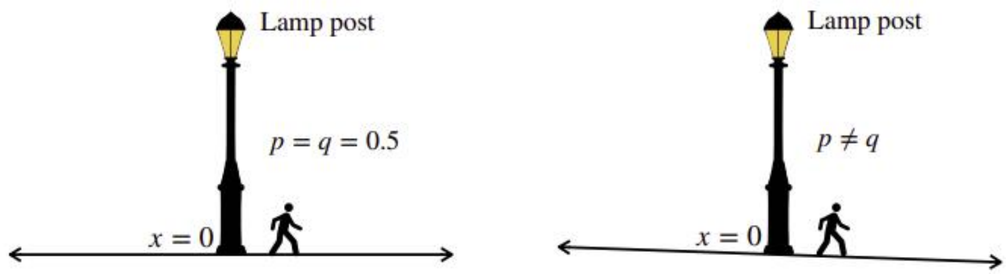

Figure 1: The random walk problem. *Left:* Equal probability of moving in both directions. *Right:* Because of the inclination, the person has more probability of moving towards right as compared to the left direction.

1. Write a computer program and calculate $P(X = -n)$, $P(X = -n + 1)$, …, $P(X = n - 1)$ and $P(X = n)$ after n steps. Put a check condition in your code that ensures all these probabilities add to 1.
2. Check your program for $n = 5$ and $n = 10$ fixing $p = 0.5$. Note that this corresponds to the left panel of Figure 1 where the walker has equal probability of going to the left or right.
3. Using `Matplotlib`, plot the *probability mass function* $P(X)$ as a function of X. In the present case, X is BINOMIAL RANDOM VARIABLE.
4. Calculate the cumulative distribution function $F(x) = P(X \leq x)$. Make a plot of $F(x)$.
5. Repeat the above analysis when $p = 0.7$. This situation might arise when there is a slope as shown in the *right* panel of Figure 1.

Problem 3.

A discrete random variable X follows a probability mass function (PMF), with parameter $\lambda > 0$, given by:

$$f[k;\lambda] = Pr[X = k] = \frac{\lambda^k e^{-\lambda}}{k!} \tag{1}$$

where k is the number of occurrences $k \in \{0, 1, 2, \ldots\}$, e is Euler's number, and ! is the factorial function.

(a) Obtain one plot of $Pr[X = k]$ vs. k with the following values of $\lambda \in \{1, 4, 10\}$.

(b) Obtain one plot of $Pr[X \leq k]$ vs. k with the following values of $\lambda \in \{1, 4, 10\}$.

(c) Calculate the mean and the variance of the random variable X (experimentally, i.e., running the experiments multiple times and not mathematically).

(d) Reason and infer the name of the PMF in (1).

Problem 4.

A discrete random variable X, probability of getting exactly k successes in n independent trials is given by the following probability mass function, with parameter $n \in \mathbb{N}$, $p \in [0, 1]$:

$$f[k;n,p] = Pr[X = k] = \frac{n!}{k!(n-k)!}\, p^k (1-p)^{n-k} \tag{2}$$

for $k \in \{0, 1, 2, \ldots n\}$, and ! is the factorial function.

(a) Obtain one plot of $Pr[X = k]$ vs. k with the following value pairs of $(p, n) \in \{(0.5, 20), (0.7, 20), (0.5, 40)\}$.

(b) Obtain one plot of $Pr[X \leq k]$ vs. k with the following value pairs of $(p, n) \in \{(0.5, 20), (0.7, 20), (0.5, 40)\}$.

(c) Calculate the mean and the variance of the random variable X (experimentally, i.e., running the experiments multiple times and not mathematically).

(d) Reason and infer the name of the PMF in (2).