Theory Assignment-3 due for submission on Moodle by 12th March

Q1. Let

H = event that person has disease,

S =event that test result is positive

R1 = event that positive test result came from Machine 1

R2 = event that positive test result came from Machine 2

$$P(H)=0.001$$
, $P(R1)=0.2$, $P(R2)=0.8$, $P(R1|H)=0.89$, $P(R2|H)=0.99$, $P(R1|\overline{H})=0.025$, $P(R2|\overline{H})=0.005$

Given that person has a disease and test result is positive. What is the probability that the result came from Machine 2?

Q2. We believe there are three types of managers:

underperformers, in-line performers, and outperformers. The underperformers (MU) beat the market only 25% of the time, the in-line performers (MI) beat the market 50% of the time, and the outperformers (MO) beat the market 75% of the time.

Initially we believe a given manager is most likely to be an in-line performer, and is less likely to be an underperformer or an outperformer. Specifically, our prior belief is that a manager has a 60% chance of being an in-line performer, a 20% chance of being an underperformer, and a 20% chance of being an outperformer. We can summarize this as:

$$P[MU] = P[p=0.25] = 20\%, \quad P[MI] = P[p=0.50] = 60\%, \quad P[MO] = P[p=0.75] = 20\% \dots (1)$$

- a.) By taking the values given in (1), suppose the manager beats the market two years in a row. What should our updated beliefs be?
- b.) After updating the beliefs from part (a), what if the manager again beats the market next year, what should be the updates now?
- c.) By taking the values given in (1), suppose the manager beats the market three years in a row. What should our updated beliefs be? How is it different from part (b)?