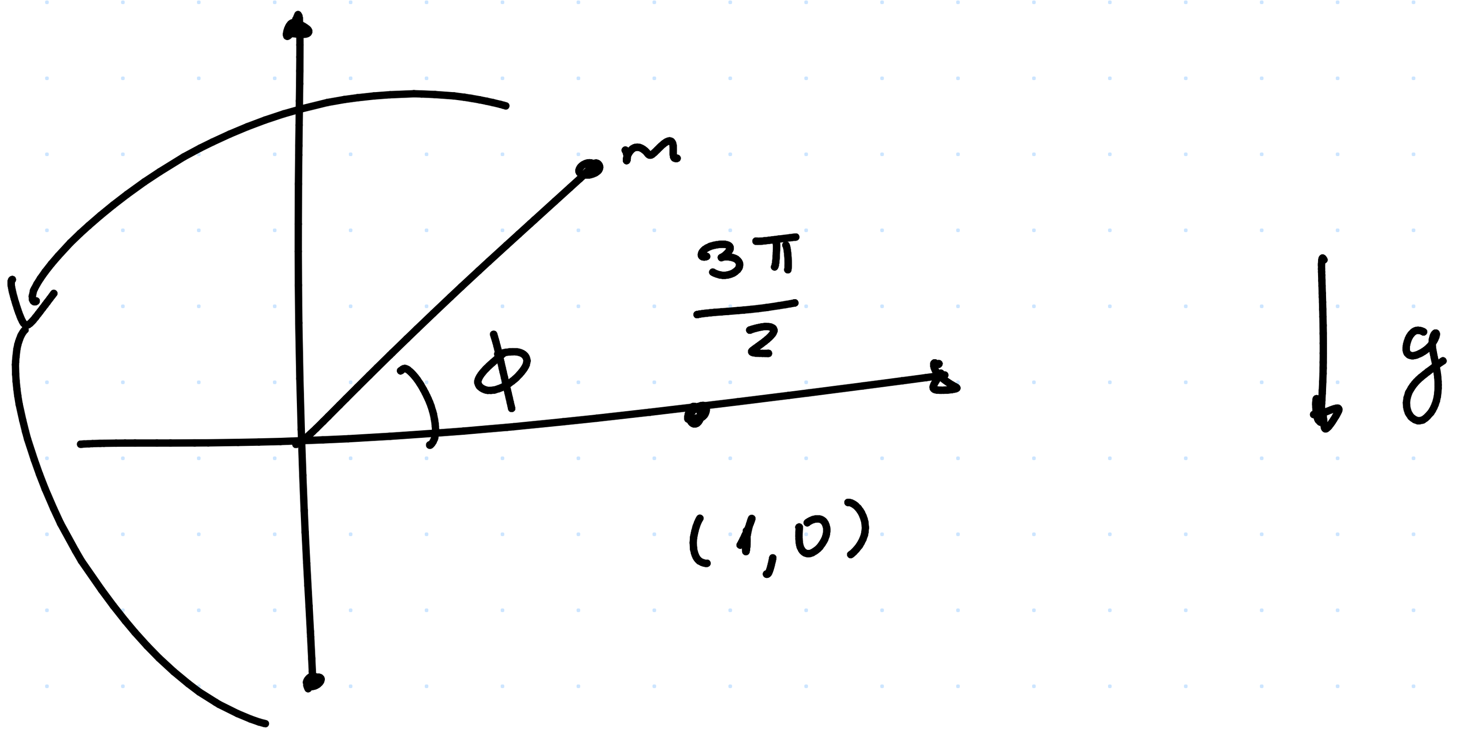
Funcions especiales — Clase # 2 — 07/08/2025

$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\phi}\,\hat{\phi}$$

$$\ddot{\vec{r}} = \left[\ddot{r} - r\dot{\phi}^2\right]\hat{r} + \left[\ddot{\phi} + \dot{r}\dot{\phi}\right]\hat{\phi}$$

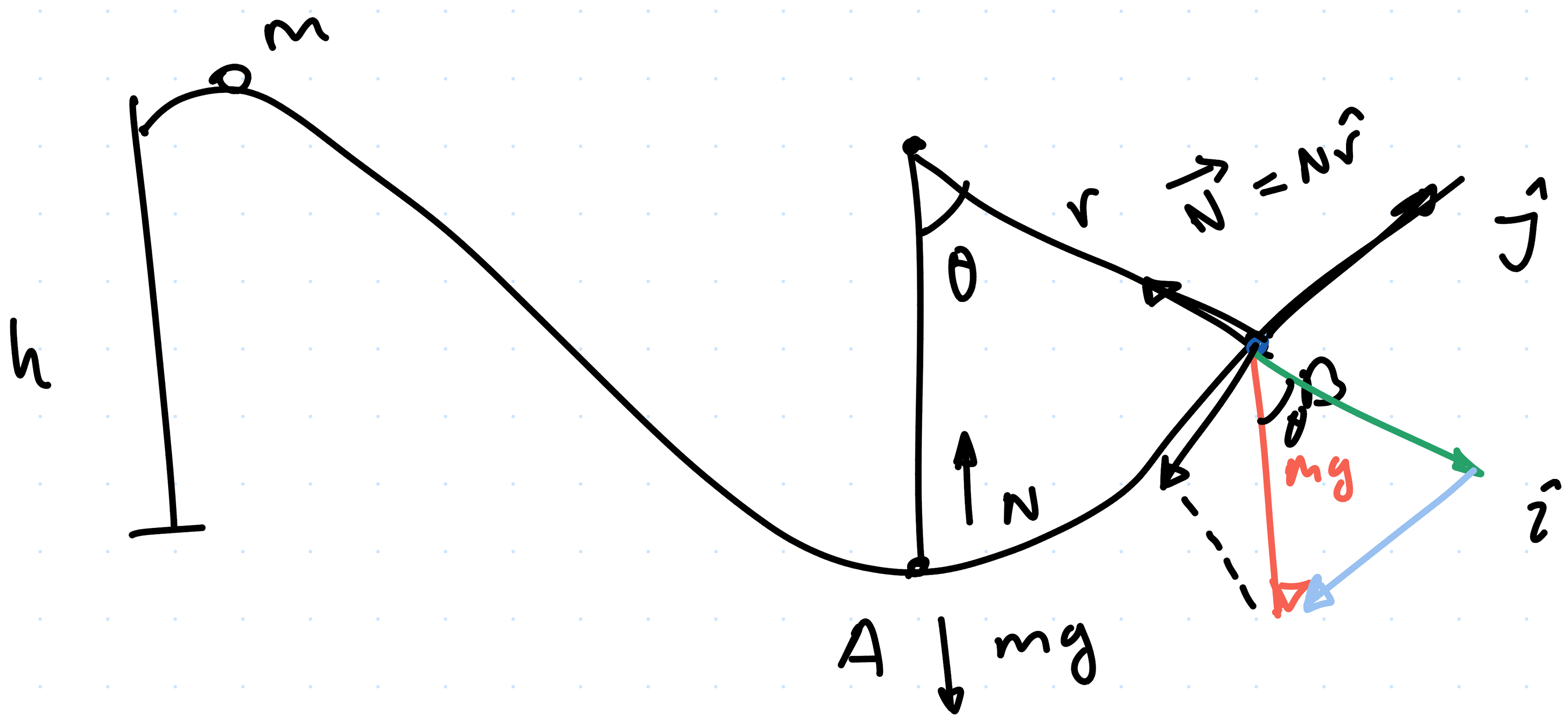
$$\sum_i \vec{F}_{i}^{\,ext} = m\vec{a}$$



$$(N - mg)\hat{j} = m\left[-r\dot{\phi}^2\right]\hat{r} + \ddot{\phi}\,\hat{\phi}$$

$$\hat{r} = \cos\phi\,\hat{\imath} + \operatorname{sen}\phi$$

$$\hat{\phi} = -\operatorname{sen}\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$$



$$\begin{pmatrix} \hat{r} \\ \hat{\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi & \operatorname{sen}\phi \\ -\operatorname{sen}\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \hat{\imath} \\ \hat{\jmath} \end{pmatrix}$$

Normal en dirección radial

$\det = 1$

matrix rot

• Polares

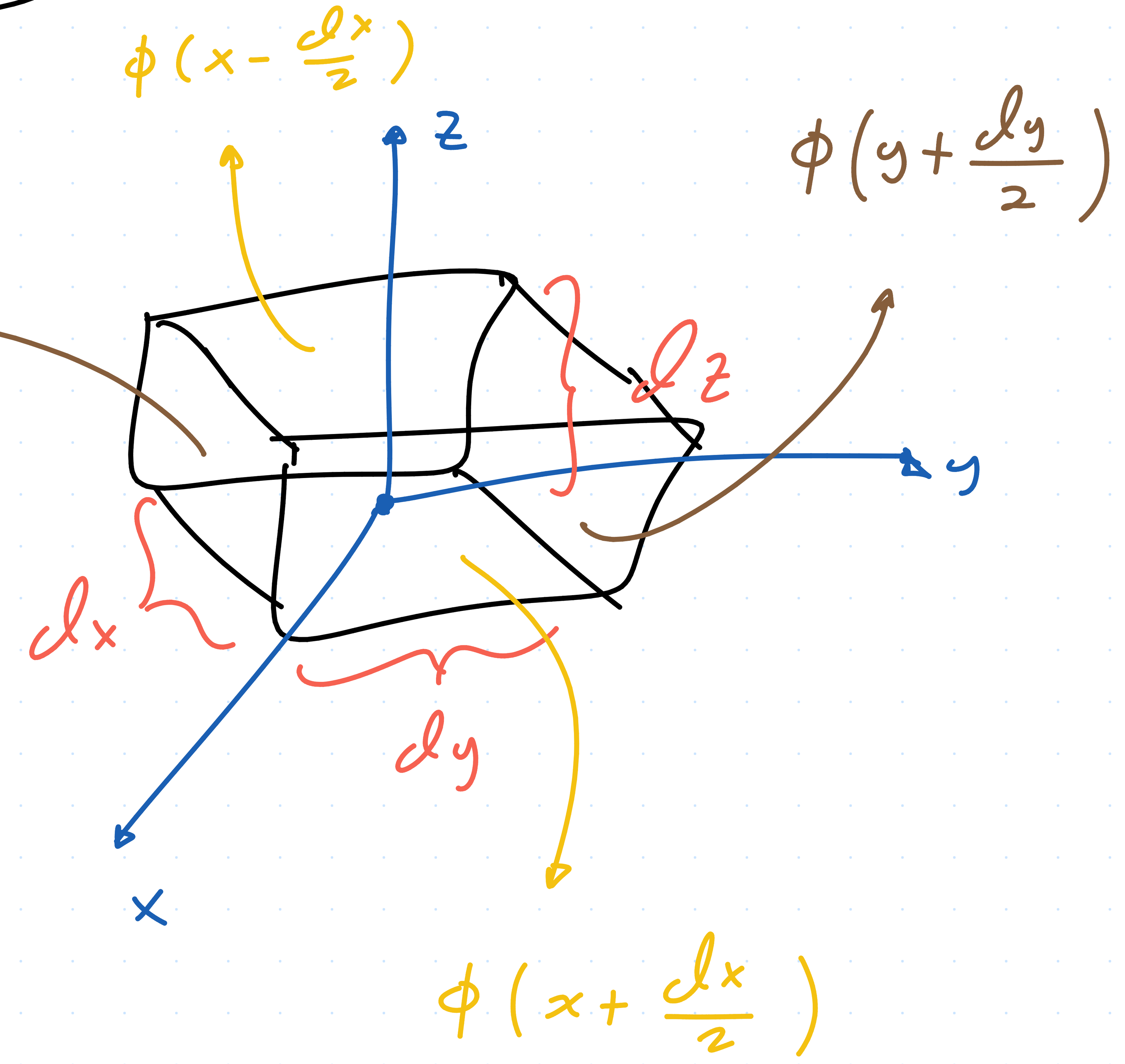
Radio vector

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\nabla\phi(x,y,z) = \frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k}$$

Def

$$\lim_{V\to 0} \oint_S \frac{\phi(x,y,z)}{V}\, d\vec{S}$$



- superficie infinitesimal

$$\Rightarrow \left(\int \phi(x + \tfrac{dx}{2})\, dy\, dz - \int \phi(x - \tfrac{dx}{2})\, dy\, dz \right) \hat{i}$$

$$+ \left(\int \phi(y + \tfrac{dy}{2})\, dx\, dz - \int \phi(y - \tfrac{dy}{2})\, dx\, dz \right) \hat{j}$$

$$+ \left(\int \phi(z + \tfrac{dz}{2})\, dx\, dy - \int \phi(z - \tfrac{dz}{2})\, dx\, dy \right) \hat{k}$$

Aproximamos mediante serie de Taylor.

Tenemos

$$\int \left\{ \phi(x) + \frac{\partial \phi}{\partial x} \frac{dx}{2} \right\} dy\, dz - \int \left\{ \phi(x) + \frac{\partial \phi}{\partial x} \frac{dx}{2} \right\} dy\, dz$$

$$\frac{\partial \phi}{\partial x} dx\, dy\, dz\, \hat{\imath} + \frac{\partial \phi}{\partial y} dy\, dx\, dz\, \hat{\jmath} + \frac{\partial \phi}{\partial z} dx\, dy\, dz\, \hat{k}$$

Gradiente

$$\nabla \cdot \vec{F} = \lim_{V \to 0} \oint \frac{\vec{F} \cdot d\vec{s}}{V} \quad ; \quad \nabla \times \vec{F} = \lim_{V \to 0} \oint \frac{\vec{F} \times d\vec{s}}{V}$$

Cartesianas (operadores)

$$\nabla \phi = \frac{\partial \phi}{\partial A} \hat{\imath} + \frac{\partial \phi}{\partial y} \hat{\jmath} + \frac{\partial \phi}{\partial z} \hat{k}$$

$$\nabla \cdot \vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$(\nabla \times \vec{F})_i = \varepsilon_{ijk}\, \partial_j\, f_k$$

$$\vdots$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Polares

$$\hat{r} = \rho \hat{\rho}$$

$$\nabla \phi = \frac{\partial \phi}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial \phi}{\partial \varphi} \hat{\varphi}$$

$$\nabla \cdot \vec{F} = ?$$

$$\nabla \times \vec{F} = 0$$

$$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$$

$$\vec{r} = \vec{r}(x, y, z)$$

$$\vec{r} = \vec{r}(q_1, q_2, q_3)$$

$$d\vec{r} = \frac{\partial \vec{r}}{\partial q_1} dq_1 + \frac{\partial \vec{r}}{\partial q_2} dq_2 + \frac{\partial \vec{r}}{\partial q_3} dq_3$$

$$d\vec{r} = \vec{e}_1\, dq_1 + \vec{e}_2\, dq_2 + \vec{e}_3\, dq_3$$

$$d\vec{r} = \frac{\vec{e}_1}{\|\vec{e}_1\|} \|\vec{e}_1\|\, dq_1 + \frac{\vec{e}_2}{\|\vec{e}_2\|} \|\vec{e}_2\|\, dq_2 + \frac{\vec{e}_3}{\|\vec{e}_3\|} \|\vec{e}_3\|\, dq_3$$

Tenemos

$$d\vec{r} = \hat{e}_1 h_1\, dq_1 + \hat{e}_2 h_2\, dq_2 + \hat{e}_3 h_3\, dq_3$$

$h_1, h_2, h_3 =$ factores de escala

$$d\phi(q_1, q_2, q_3) = \frac{\partial \phi}{\partial q_1} dq_1 + \frac{\partial \phi}{\partial q_2} dq_2 + \frac{\partial \phi}{\partial q_3} dq_3$$

$$\nabla\phi \cdot d\vec{r} = \nabla\phi \cdot \left\{ \hat{e}_1 h_1 dq_1 + \hat{e}_2 h_2 dq_2 + \hat{e}_3 h_3 dq_3 \right\}$$

$$\nabla\phi = \frac{1}{h_1}\frac{\partial\phi}{\partial q_1}\hat{e}_1 + \frac{1}{h_2}\frac{\partial\phi}{\partial q_2}\hat{e}_2 + \frac{1}{h_3}\frac{\partial\phi}{\partial q_3}\hat{e}_3$$