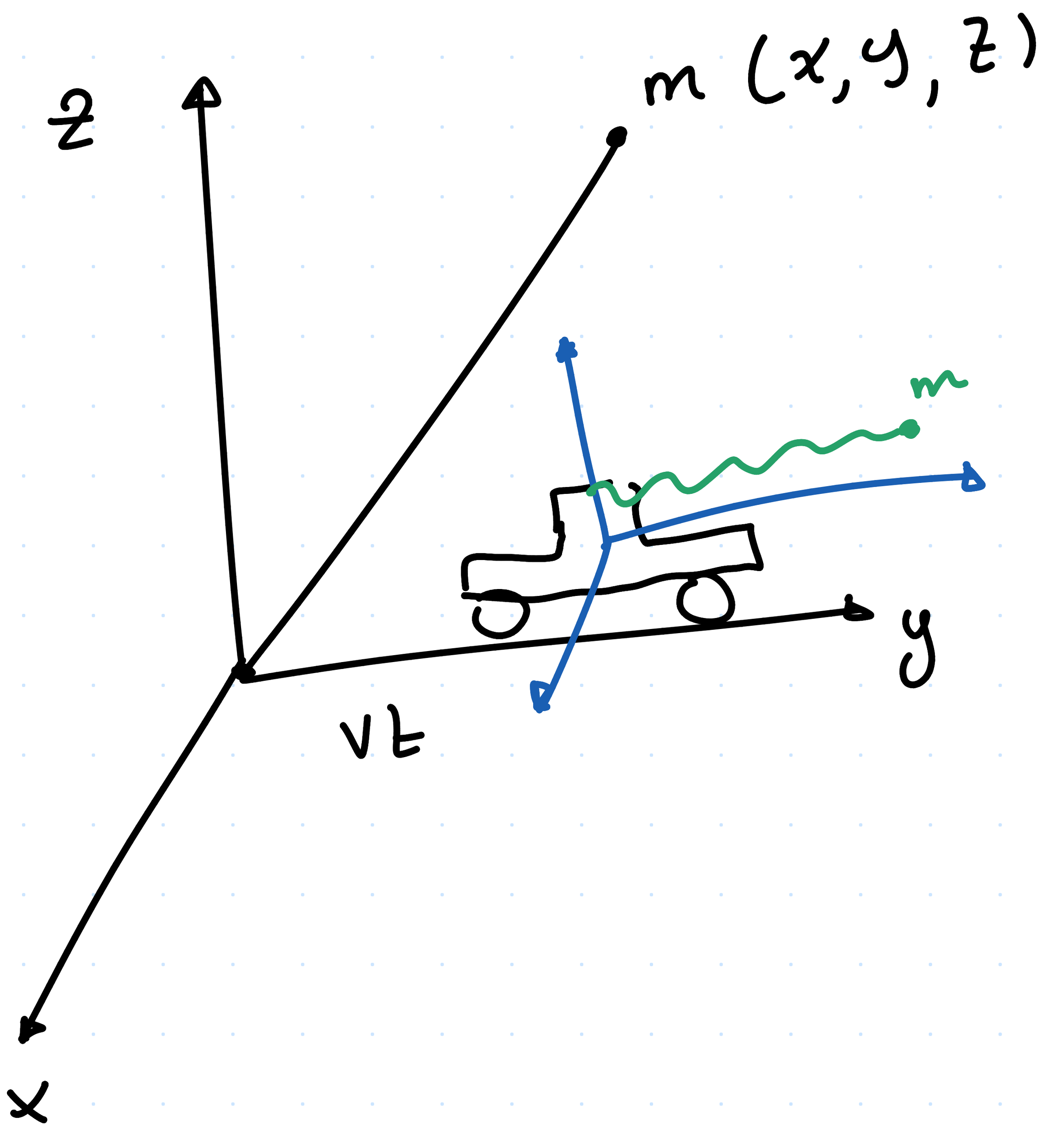
Funciones especiales clase # 1 05/08/2025



- Transformaciones de Galileo

$v = cte$

- Marco de referencia situado en puntos de reposo de preferencia.

↑ sistema de referencia que no se mueve.

- Sistema de referencia inercial

Espacio de $\mathbb{R}^3$

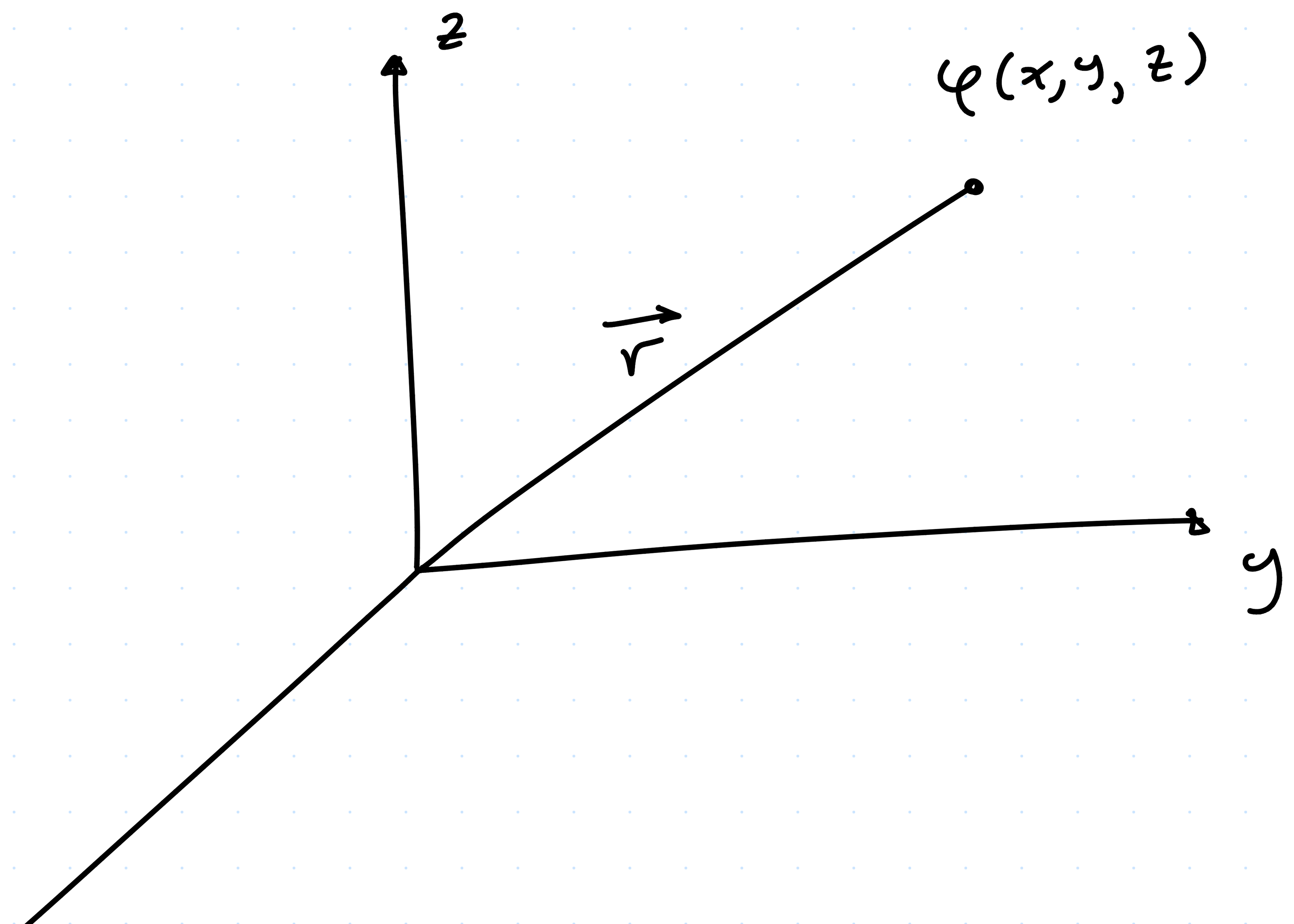
$\mathbb{R}^n \mapsto \mathbb{R}$

- Existen
  - campo escalar $\mathbb{R}^n \to \mathbb{R}$ $\{ \varphi(x, y, z)$
  - campo vectorial $\mathbb{R}^n \to \mathbb{R}^n$

- Trabajo con funciones suaves.

- Derivada de campo escalar → Gradiente

Tenemos



x

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$; $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$

$$d\varphi = \nabla\phi \cdot d\vec{r}$$

$$d\varphi = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy + \frac{\partial\varphi}{\partial z}dz$$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$$

Gradiente de función escalar de $\underline{\mathbb{R}^3 \longrightarrow \mathbb{R}}$

Definición de divergencia

si tenemos un campo escalar

$\varphi(x, y, z)$

<u>Gradiente</u>

$$\lim_{V\to 0} \frac{\int \varphi \, d\vec{s}}{V} = \nabla\phi$$

volumen

- Teorema de Gauss
  - ↳ superficies cerradas
- Stokes → Abiertas

Divergencia

$$\lim_{V \to 0} \frac{\oint \vec{F} \cdot ds}{V} = \nabla \cdot \vec{F}$$

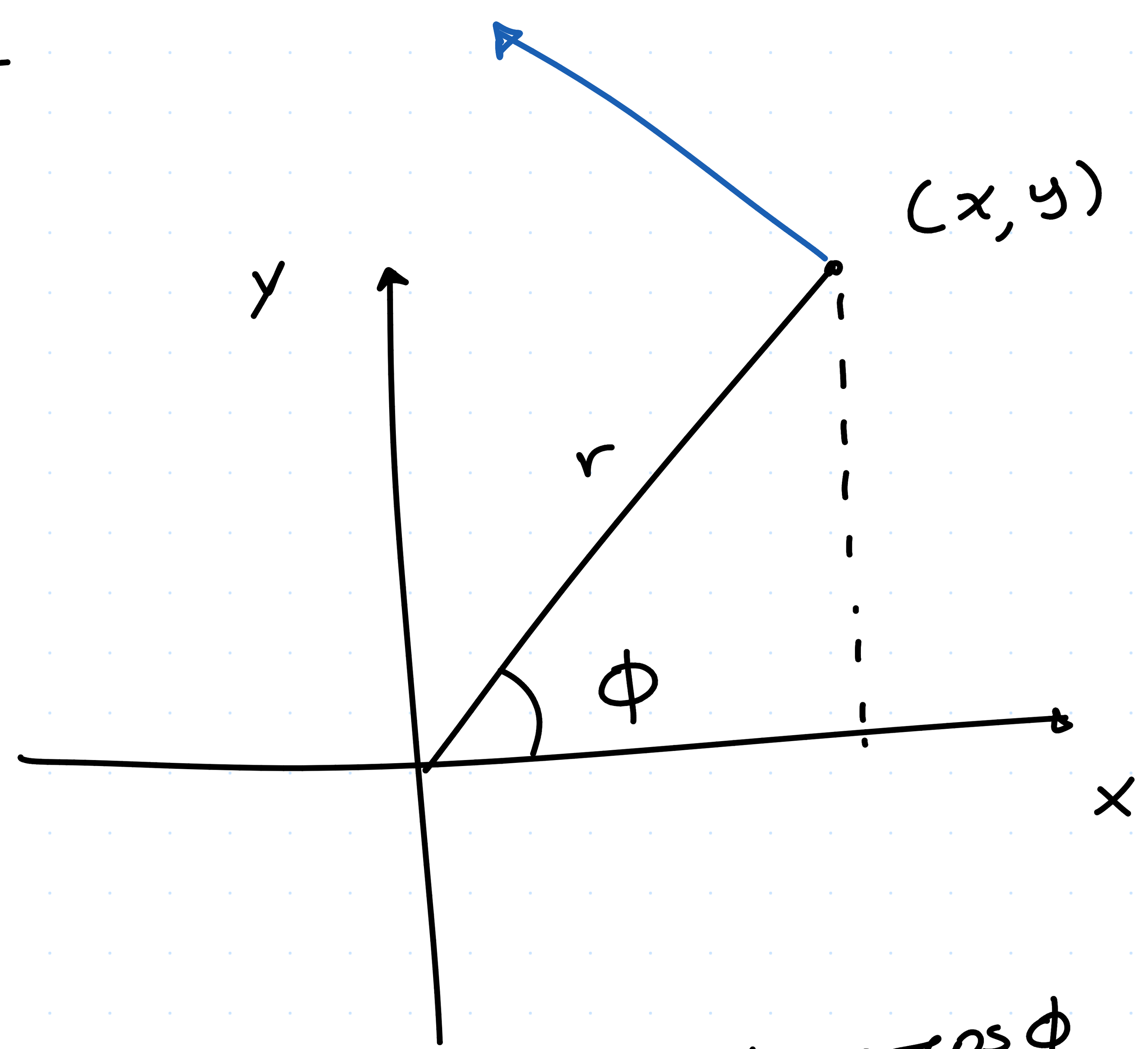
$$\lim_{V \to 0} \frac{\int (\nabla \cdot \vec{F})\, dv}{V} =$$

Rotacional

$$\lim_{V \to 0} \frac{\int \vec{F} \times d\vec{s}}{V}$$

vector normal

Simetria polar



$x = r\cos\phi$

$y = r\,\text{sen}\,\phi$

- Reescribimos la base.

Ayudar a reescribir la parte polar.

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{r} = r\cos\phi\,\hat{\imath} + r\,\text{sen}\,\phi\,\hat{\jmath}$$

$$\vec{r} = r\left[\cos\phi\,\hat{\imath} + \operatorname{sen}\phi\,\hat{\jmath}\right]$$

$$\hat{r} = \frac{\vec{r}}{r} = \underbrace{\cos\phi\,\hat{\imath} + \operatorname{sen}\phi\,\hat{\jmath}}_{\hat{r}}$$

$$\boxed{\vec{r} = r\,\hat{r}}$$

Base $\longrightarrow$ L.I y genera a todo el espacio

$\hat{\imath}$ y $\hat{\jmath}$ son ortonormales.

Necesitamos un vector $\hat{\phi}$ ortonormal a $\hat{r}$

vectores tangentes a una superficie:

$$\frac{\partial \vec{r}}{\partial r} = \vec{r}$$

$$\frac{\partial \vec{r}}{\partial \phi} = \vec{\phi}$$

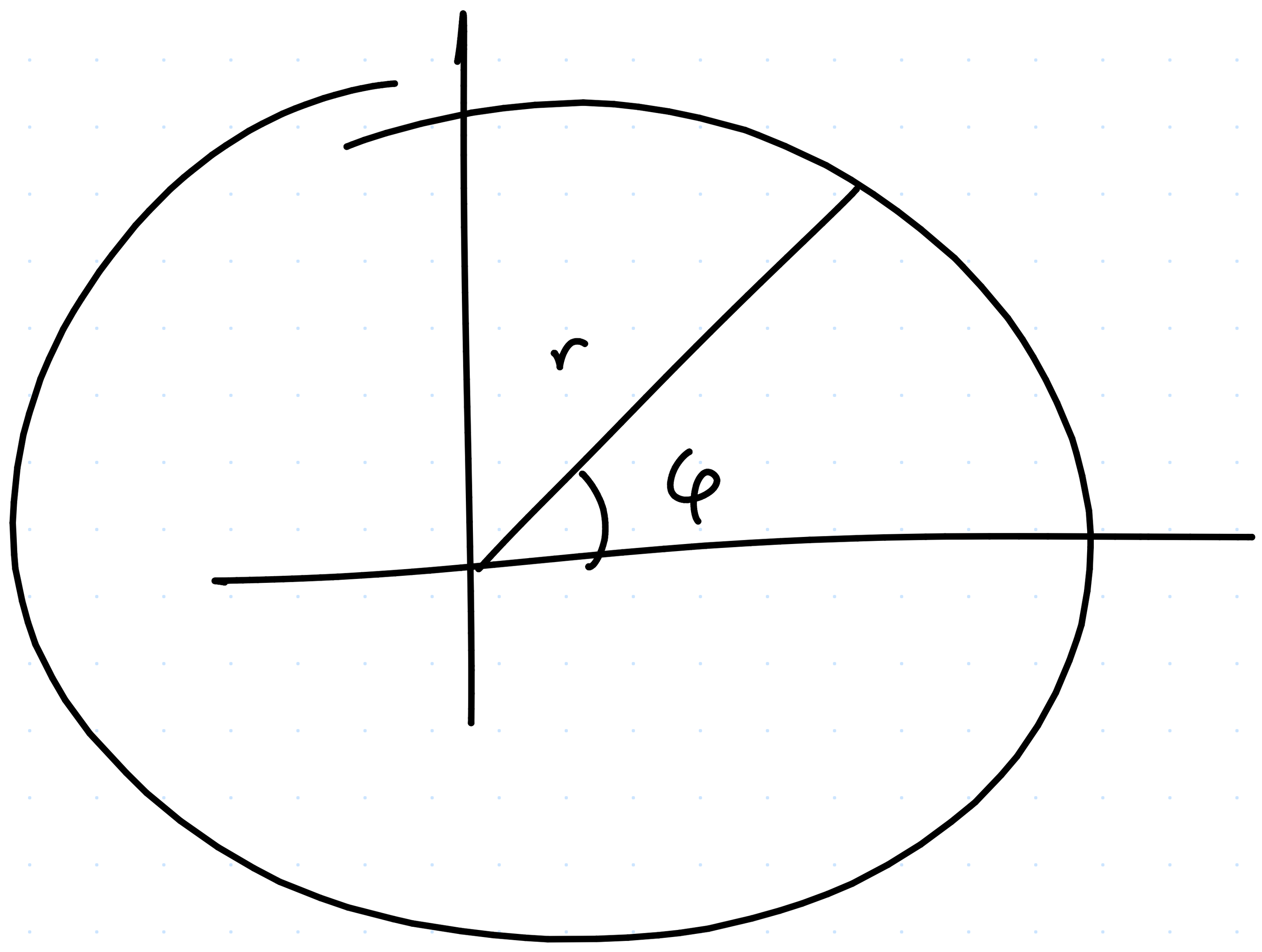
$$\vec{\phi} = -r\operatorname{sen}\phi\,\hat{\imath} + r\cos\phi\,\hat{\jmath}$$

$\vdots$

$$= r\left(-\operatorname{sen}\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}\right)$$

$$\hat{\phi} = -\operatorname{sen}\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} \longrightarrow \text{vector ortonormal a } \hat{r}.$$

$$\boxed{\hat{r}\cdot\hat{\phi} = 0}$$



$$\vec{r} = r\hat{r}$$

$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\frac{d\vec{r}}{dt} \quad ?$$

$$\frac{d\hat{r}}{dt} = -\mathrm{sen}\,\phi\,\dot{\phi}\,\hat{\imath} + \cos\phi\,\dot{\phi}\,\hat{\jmath}$$

$$= \dot{\phi}\left[-\mathrm{sen}\,\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}\right]$$

$$= \dot{\phi}\,\hat{\phi}$$

$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$$

$$\frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\phi}\hat{\phi} + r\ddot{\phi}\hat{\phi} + r\dot{\phi}\frac{d\hat{\phi}}{dt}$$

$$= \ddot{r}\hat{r} + \dot{r}\dot{\phi}\hat{\phi} + \dot{r}\dot{\phi}\hat{\phi} + r\ddot{\phi}\hat{\phi} + r\dot{\phi}\frac{d\hat{\phi}}{dt}$$

$$\frac{d\hat{\phi}}{dt} = -\dot{\phi}\cos\phi\,\hat{\imath} - \dot{\phi}\,\mathrm{sen}\,\phi\,\hat{\jmath}$$

$$= -\dot{\phi}\left[\cos\phi\,\hat{\imath} + \mathrm{sen}\,\phi\,\hat{\jmath}\right]$$

$$= -\dot{\phi}\,\hat{r}$$

$$\frac{d^2\vec{r}}{dt^2} = \ddot{r}\hat{r} + 2\dot{r}\dot{\phi}\hat{\phi} + r\ddot{\phi}\hat{\phi} - r\dot{\phi}^2\hat{r}$$

$$= \underbrace{\left[\ddot{r} - r\dot{\phi}^2\right]}_{a_r}\hat{r} + \underbrace{\left[r\ddot{\phi} + 2\dot{r}\dot{\phi}\right]}_{a_T}\hat{\phi}$$

a_r

↑

Aceleración radial.

tiene en principio estas dos componentes.

V_T = vel. tangencial

$$a_r = r\dot{\phi}^2 = \frac{V_T^2}{r}$$

- sistema cartesiano
- simetría polar

↳ Gradiente y divergencia son diferentes por la distinta base.

$$\frac{d\vec{r}}{dt} = \underbrace{\dot{r}\hat{r}}_{V_y} + \underbrace{r\dot{\phi}\hat{\phi}}_{V_T}$$

$$V_T = r\dot{\phi}$$

$$\dot{\phi} = \frac{V_T}{r}$$