5. 3

Ще докажем Church-Rosser с индукция по дефиницията на терм $M \in \Lambda^{\omega\beta}$. Нека $\Gamma \vdash M : \sigma$ и $\Gamma \vdash M : \tau$

1) $M \equiv x$, x е променлива.

$$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma} \quad \text{и} \quad \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \Rightarrow \sigma \equiv \tau$$

2) $M \equiv M_1 M_2$, $M_1, M_2 \in \Lambda^{\omega\beta}$

$$\frac{\Gamma \vdash M_1 : \rho_1 \Rightarrow \sigma \quad \Gamma \vdash M_2 : \rho_1}{\Gamma \vdash M_1 M_2 : \sigma} \quad \text{и} \quad \frac{\Gamma \vdash M_1 : \rho_2 \Rightarrow \tau \quad \Gamma \vdash M_2 : \rho_2}{\Gamma \vdash M_1 M_2 : \tau}$$

$\Gamma \vdash M_2 : \rho_1$ и $\Gamma \vdash M_2 : \rho_2$ от ИП: $\rho_1 \equiv \rho_2$

$\Gamma \vdash M_1 : \rho_1 \Rightarrow \sigma$ и $\Gamma \vdash M_1 : \rho_2 \Rightarrow \tau$ от ИП: $\rho_1 \Rightarrow \sigma \equiv \rho_2 \Rightarrow \tau$

$\rho_2 \Rightarrow \sigma \equiv \rho_1 \Rightarrow \sigma \equiv \rho_2 \Rightarrow \tau \Rightarrow \sigma \equiv \tau$

3) $M \equiv \lambda x^{\alpha} N$, $N \in \Lambda^{\omega\beta}$; $\sigma \equiv \alpha \Rightarrow \beta_1$ $\tau \equiv \alpha \Rightarrow \beta_2$

$$\frac{\Gamma, x : \alpha \vdash N : \beta_1}{\Gamma \vdash \lambda x^{\alpha} N : \alpha \Rightarrow \beta_1} \qquad \frac{\Gamma, x : \alpha \vdash N : \beta_2}{\Gamma \vdash \lambda x^{\alpha} N : \alpha \Rightarrow \beta_2}$$

$\Gamma, x : \alpha \vdash N : \beta_1$ и $\Gamma, x : \alpha \vdash N : \beta_2$ $\Rightarrow$ от ИП $\beta_1 \equiv \beta_2$

$\Rightarrow \alpha \Rightarrow \beta_1 \equiv \alpha \Rightarrow \beta_2 \Rightarrow \sigma \equiv \tau$