

1.39

Лема 1

Use induction on  $P[x \mapsto M][x \mapsto N] \equiv P[x \mapsto M[x \mapsto N]]$

Use induction on  $y$  by cases on term  $P$ .

1]  $P \equiv y, y \in V$

1.1]  $y \neq x$

$$P[x \mapsto M][x \mapsto N] \equiv y[x \mapsto N] \equiv y$$

$$P[x \mapsto M[x \mapsto N]] \equiv y$$

1.2]  $y \equiv x$

$$P[x \mapsto M][x \mapsto N] \equiv M[x \mapsto N]$$

$$P[x \mapsto M][x \mapsto N] \equiv M[x \mapsto N]$$

2]  $P \equiv P_1 P_2$

$$P[x \mapsto M][x \mapsto N] \equiv (P_1[x \mapsto M][x \mapsto N])(P_2[x \mapsto M][x \mapsto N])$$

or by

$$\equiv (P_1[x \mapsto M[x \mapsto N]])(P_2[x \mapsto M[x \mapsto N]]) \equiv$$

$$\equiv P[x \mapsto M[x \mapsto N]]$$

3]  $P \equiv \lambda y. Q$

3.1]  $y \equiv x$

$$P[x \mapsto M][x \mapsto N] \equiv P[x \mapsto N] \equiv P \equiv P[x \mapsto M[x \mapsto N]]$$

3.2]  $y \neq x$

or  $y$  is not a variable in the Barendregt name, so  $y \notin \text{FV}(Q)$

$y \notin \text{FV}(M)$  and  $y \notin \text{FV}(N)$ , so  $y \in \text{BV}(P)$

$$\begin{aligned}
 P \llbracket X \mapsto M \rrbracket \llbracket X \mapsto N \rrbracket &\equiv \lambda y (Q \llbracket X \mapsto M \rrbracket) \llbracket X \mapsto N \rrbracket \equiv \\
 &\equiv \lambda y (Q \llbracket X \mapsto M \rrbracket \llbracket X \mapsto N \rrbracket) \stackrel{\text{or } \lambda n}{=} \lambda y (Q \llbracket X \mapsto M \llbracket X \mapsto N \rrbracket \rrbracket) \\
 &\equiv P \llbracket X \mapsto M \llbracket X \mapsto N \rrbracket \rrbracket
 \end{aligned}$$

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Уже написано ~~не~~  $\beta$ -выражения  $\lambda y y$  и  $\lambda x x$  не  
 являются  $\beta$ -нормальными  $\beta \rightarrow \beta^2$

$$\beta / M \equiv (\lambda y A) \beta, M \equiv A \llbracket y \mapsto \beta \rrbracket$$

~~$\lambda y A \llbracket y \mapsto \beta \rrbracket$~~

$$\begin{aligned}
 \beta.1) \quad y \neq x \\
 M \llbracket X \mapsto N \rrbracket &\equiv (\lambda y A) \llbracket \beta \llbracket X \mapsto N \rrbracket \rrbracket \stackrel{\beta}{\rightarrow} A \llbracket y \mapsto \beta \llbracket X \mapsto N \rrbracket \rrbracket \\
 &\stackrel{\beta}{\rightarrow} A \llbracket X \mapsto N \rrbracket \llbracket y \mapsto \beta \llbracket X \mapsto N \rrbracket \rrbracket
 \end{aligned}$$

lemma 2 (for 1.35)

$$\begin{aligned}
 M' &\equiv A \llbracket y \mapsto \beta \rrbracket \llbracket X \mapsto N \rrbracket \equiv A \llbracket X \mapsto N \rrbracket \llbracket y \mapsto \beta \llbracket X \mapsto N \rrbracket \rrbracket \\
 M \llbracket X \mapsto N \rrbracket &\equiv (\lambda y A \llbracket X \mapsto N \rrbracket) (\beta \llbracket X \mapsto N \rrbracket) \stackrel{\beta}{\rightarrow} \\
 &\stackrel{\beta}{\rightarrow} A \llbracket X \mapsto N \rrbracket \llbracket y \mapsto \beta \llbracket X \mapsto N \rrbracket \rrbracket
 \end{aligned}$$

$$\beta.2) \quad y \equiv x$$

$$M \llbracket X \mapsto N \rrbracket \equiv (\lambda x A) (\beta \llbracket X \mapsto N \rrbracket) \stackrel{\beta}{\rightarrow} A \llbracket x \mapsto \beta \llbracket X \mapsto N \rrbracket \rrbracket \equiv$$

lemma 1

$$\equiv A \llbracket x \mapsto \beta \rrbracket \llbracket X \mapsto N \rrbracket \equiv M \llbracket X \mapsto N \rrbracket$$



2.1)

$$M \equiv AP, M' \equiv BP, A \xrightarrow{P} B$$

$$M[x \mapsto N] \equiv (A[x \mapsto N])(P[x \mapsto N]) \xrightarrow{P} (B[x \mapsto N])(P[x \mapsto N]) \equiv M'[x \mapsto N]$$

$\left\{ \begin{array}{l} A[x \mapsto N] \xrightarrow{P} B[x \mapsto N] \\ \text{or} \quad \text{40} \end{array} \right.$

$\xrightarrow{P} (B[x \mapsto N])(P[x \mapsto N]) \equiv M'[x \mapsto N]$   
 $\hookrightarrow \text{or 2.1)}$

2.2)

$$M \equiv PA, M' \equiv PB, A \xrightarrow{P} B$$

$$M[x \mapsto N] \equiv (P[x \mapsto N])(A[x \mapsto N]) \xrightarrow{P} (P[x \mapsto N])(B[x \mapsto N]) \equiv M'[x \mapsto N]$$

$\left\{ \begin{array}{l} A[x \mapsto N] \xrightarrow{P} B[x \mapsto N] \\ \text{or} \quad \text{40} \end{array} \right.$

$\xrightarrow{P} (P[x \mapsto N])(B[x \mapsto N]) \equiv M'[x \mapsto N]$   
 $\hookrightarrow \text{or 2.2)}$

2.3)

$$M \equiv \lambda y. A, M' \equiv \lambda y. B, A \xrightarrow{P} B$$

2.3.1)  $y \equiv x$

$$M[x \mapsto N] \equiv \lambda y. A \xrightarrow{P} \lambda y. B \equiv M'[x \mapsto N]$$

2.3.2)  $y \neq x$

$$M[x \mapsto N] \equiv \lambda y. (A[x \mapsto N]) \xrightarrow{P} \lambda y. (B[x \mapsto N]) \equiv M'[x \mapsto N]$$

$\left\{ \begin{array}{l} A[x \mapsto N] \xrightarrow{P} B[x \mapsto N] \\ \text{or} \quad \text{40} \end{array} \right.$

$\xrightarrow{P} \lambda y. (B[x \mapsto N]) \equiv M'[x \mapsto N]$   
 $\hookrightarrow \text{or 2.3.1)}$