Задача 2.41

Използваме, че $\twoheadrightarrow_{\eta} = \eta^{R,R,T}$ и правим индукция по изводите.

R) $M \equiv M' \Rightarrow M[x\mapsto N] \equiv M'[x\mapsto N] \Rightarrow M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M'[x\mapsto N]$ (рефлексив

T) $M \overset{\eta}{\twoheadrightarrow} M'$ идва от $M \overset{\eta}{\twoheadrightarrow} M''$ и $M'' \overset{\eta}{\twoheadrightarrow} M'$. От ИП
$M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M''[x\mapsto N]$ и $M'' \overset{\eta}{\twoheadrightarrow} M'[x\mapsto N]$. От транзитивност следва, че $M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M' \overset{\eta}{\twoheadrightarrow} [x\mapsto N]$

λ.1) Следва от η) случая

λ.2.1) $M \equiv QP$, $M' = Q'P$ и $Q \overset{\eta}{\twoheadrightarrow} Q'$. От ИП
$Q[x\mapsto N] \overset{\eta}{\twoheadrightarrow} Q'[x\mapsto N]$. По деф. на
$M[x\mapsto N] \equiv (Q[x\mapsto N])(P[x\mapsto N])$ и $M'[x\mapsto N] \equiv (Q[x\mapsto N])(P[x\mapsto N])$.
$M[x\mapsto N] \equiv (Q[x\mapsto N])(P[x\mapsto N]) \underset{\lambda}{\overset{\eta}{\twoheadrightarrow}} (Q'[x\mapsto N])(P[x\mapsto N]) \equiv M'[x\mapsto N]$

λ.2.2) Аналогично на λ.2.1)

λ.2.3) $M \equiv \lambda y Q$ и $M' \equiv \lambda y Q'$, $M \twoheadrightarrow M'$

λ.2.3.1) $y \equiv x \Rightarrow M[x\mapsto N] \equiv M$ и $M'[x\mapsto N] \equiv M' \Rightarrow M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M'[x\mapsto N]$

λ.2.3.2) $M[x\mapsto N] \equiv \lambda y(Q[x\mapsto N])$ и $M'[x\mapsto N] \equiv \lambda y(Q'[x\mapsto N])$
ИП $Q \overset{\eta}{\twoheadrightarrow} Q' \Rightarrow Q[x\mapsto N] \overset{\eta}{\twoheadrightarrow} Q'[x\mapsto N] \Rightarrow$ от λ затвореност
от $M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M'[x\mapsto N]$

η) $M = \lambda y Q y$; $M' = Q$; $M \overset{\eta}{\twoheadrightarrow} M'$

η.1) $y \equiv x$ $M[x\mapsto N] \equiv M$ и $M'[x\mapsto N] \equiv Q[x\mapsto N]$. От конвенцията на Barendregt знаем, че $FV(Q) \cap BV(M) = \emptyset \Rightarrow x \notin FV(Q) \Rightarrow Q[x\mapsto N] \equiv Q$
$\Rightarrow M'[x\mapsto N] \equiv M' \Rightarrow M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M'[x\mapsto N]$ по горе

η.2) $y \not\equiv x$ $M[x\mapsto N] \equiv \lambda y\, Q[x\mapsto N]y$; $M'[x\mapsto N] = Q[x\mapsto N]$
$\Rightarrow M[x\mapsto N] \overset{\eta}{\to} M'[x\mapsto N] \Rightarrow M[x\mapsto N] \overset{\eta}{\twoheadrightarrow} M'[x\mapsto N]$