

(1)

$$\text{RW} \frac{\frac{}{A \Rightarrow A} Ax}{A \Rightarrow A, B}$$

$$\text{R} \rightarrow \frac{}{\Rightarrow A, A \rightarrow B}$$

$$\text{R} \exists \frac{}{\Rightarrow A, \exists x (A \rightarrow B)}$$

$$\text{L} \rightarrow \frac{}{A \rightarrow \exists x B \Rightarrow \exists x (A \rightarrow B)}$$

$$A \rightarrow \exists x B \Rightarrow \exists x (A \rightarrow B)$$

$$\text{R} \rightarrow \frac{}{\Rightarrow (A \rightarrow \exists x B) \Rightarrow \exists x (A \rightarrow B)}$$

$$\Rightarrow (A \rightarrow \exists x B) \Rightarrow \exists x (A \rightarrow B) \quad x \notin FV(A)$$

$$\text{LW} \frac{\frac{}{B \Rightarrow B} Ax}{A, B \Rightarrow B}$$

$$\text{R} \rightarrow \frac{}{B \Rightarrow A \rightarrow B}$$

$$\text{R} \exists \frac{}{B \Rightarrow \exists x (A \rightarrow B)}$$

$$\text{L} \exists \frac{}{\exists x B \Rightarrow \exists x (A \rightarrow B)}$$

$$(2) \quad \frac{\frac{}{A \Rightarrow A} Ax}{A \Rightarrow C, A} Rw \quad \frac{\frac{}{C \Rightarrow C} Ax}{A, C \Rightarrow C} Lw$$

$$L \rightarrow \frac{}{A \rightarrow C, A \Rightarrow C}$$

$$R \rightarrow \frac{}{A \Rightarrow (A \rightarrow C) \rightarrow C}$$

$$Lw \frac{}{A \Rightarrow (A \rightarrow C) \rightarrow C, B}$$

$$R \rightarrow \frac{}{\Rightarrow (A \rightarrow C) \rightarrow C, A \rightarrow B}$$

$$L \rightarrow \frac{}{(A \rightarrow B) \rightarrow C \Rightarrow (A \rightarrow C) \rightarrow C}$$

$$R \rightarrow \frac{(A \rightarrow B) \rightarrow C \Rightarrow (A \rightarrow C) \rightarrow C}{\Rightarrow ((A \rightarrow B) \rightarrow C) \rightarrow (A \rightarrow C) \rightarrow C}$$

$$Lw \frac{\frac{}{C \Rightarrow C} Ax}{A \rightarrow C, C \Rightarrow C}$$

$$R \rightarrow \frac{}{C \Rightarrow (A \rightarrow C) \rightarrow C}$$

(3)

$$\frac{Ax}{A \Rightarrow A}$$

$$\frac{Rw}{A \Rightarrow B, A}$$

$$\frac{Rw}{A, B \Rightarrow A, B}$$

$$\frac{R \rightarrow}{A \Rightarrow B \rightarrow A, B}$$

$$\frac{R \rightarrow}{\Rightarrow A \rightarrow B, B \rightarrow A}$$

$$\frac{Rv}{\Rightarrow A \rightarrow B, (A \rightarrow B) \vee (B \rightarrow A)}$$

$$\frac{Rv}{\Rightarrow (A \rightarrow B) \vee (B \rightarrow A), (A \rightarrow B) \vee (B \rightarrow A)}$$

$$\frac{Rc}{\Rightarrow (A \rightarrow B) \vee (B \rightarrow A)}$$