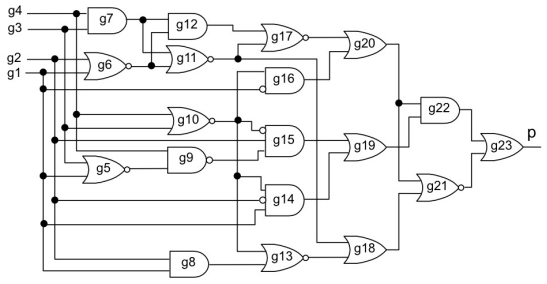
1. In the circuit below, the bubble "○" means "inversion", and the dot "●" is for the wire connection. The ID for the gate is the "number" on its name. For example, the ID of gate "g8" is 8.



Please answer the following sub-problems with sufficient details.

(a) What are the CNF formulae for the following primitive gates using Tseitin transformation?

(i) f = inv(a);

(ii) f = and(a, b'); // b' is the inversion of b

(iii) f = nand (a, b);

(iv) f = or(a, b);

(v) f = nor(a, b);

(b) Using Tseitin transformation to generate the CNF formula for "p = 1". What are the numbers of clauses and literals of the formula?

(c) Using Plaisted-Greenbaum (PG) Encoding to generate the CNF formula for "p = 1". Note that in order to enact the polarity-cared encoding, please follow the descending order of gate IDs in examining the polarities (i.e. g23, g22,…, g1). What are the numbers of clauses and literals of the formula? Compared to (b), what are the percentages of reductions in numbers of clauses and literals?

(d) [Literal counts of a gate] The literal counts are the number of times a literal appears in the clauses. Given a variable, its positive and negative literal counts are represented as a pair "(negLitCount, posLitCount)". For example, given the following clauses,

(a + f') (b + f') (a' + b' + f)

the literal counts for a, b, and f are (1, 1), (1, 1), and (2, 1), respectively.

Please list the literal counts (as pairs) for all the gates in the circuit (including PIs) using PG encoding.

(e) [Decision order] We will determine the decision order of the gates in the circuit based on the "literal counts" of their corresponding CNF formulae (as in sub-problem (d)). The detailed rules are as follows:

(i) The "decision score" of a gate is the sum of its negative (0) and positive literal (1) counts.

(ii) The "decision value" is the opposite polarity of the literal with the bigger count. If the counts for the positive and negative literals are the same, choose 0 as the decision value. For example, if the (0, 1)-literal counts of the gate 'f' and 'g' are (5, 2) and (3, 3), then their decision scores are 7 and 6, and their decision values are 1 and 0.

(iii) The "decision order" of the gates is determined by the decision scores, with the bigger scores in the front. If the scores of two gates are tied, the gate with the smaller difference between (0, 1)-literal counts wins. If tied again, compare their IDs (bigger ID wins). For example, if the literal counts of gates 'f', 'g' and 'h' are (5, 2) and (3, 4), (4, 1), the decision order will be (g = 0) → (f = 1) → (h = 1).

(iv) The orders remain unchanged throughout the decision process.

Please derive the top 7 decision gates in the circuit (including PIs).

(f) [Conflict-driven learning] We will try to witness p(g23) = 1. Please follow the decision orders and values in (e) to make the decisions, perform logic

(a)

(i): $f(f \leftrightarrow a')$

$\equiv f(f \rightarrow a')(f' \rightarrow a)$

$\equiv f(f' + a')(f + a')$

(ii): $f(f \leftrightarrow ab')$

$\equiv f(f \rightarrow a)(f \rightarrow b')(f' \rightarrow a' + b)$

$\equiv f(f' + a)(f' + b')(f' + a' + b)$

(iii): $f(f \leftrightarrow (ab)')$

$\equiv f(f' \rightarrow a)(f' \rightarrow b)(f \rightarrow a' + b')$

$\equiv f(f + a)(f + b)(f' + a' + b')$

(iv): $f(f \leftrightarrow a + b)$

$\equiv f(a \rightarrow f)(b \rightarrow f)((a+b)' \rightarrow f')$

$\equiv f(f + a')(f + b')(f' + a + b)$

(v): $f(f \leftrightarrow (a+b)')$

$\equiv f(a \rightarrow f')(b \rightarrow f')((a+b)' \rightarrow f)$

$\equiv f(f' + a)(f' + b)(f + a + b)$

p=1		p			
p1 ↔	g23	p'+g23	p+g23'		
g23 ↔	g22+g21'	g23'+g22+g21'	g22'+g23	g21+g23	
g22 ↔	g19*g20	g22'+g19	g22'+g20	g19'+g20'+g22	
g21 ↔	g18+g20	g21'+g18+g20	g18'+g21	g20'+g21	
g20 ↔	g16+g17'	g20'+g16+g17'	g16'+g20	g17+g20	
g19 ↔	g14+g15	g19'+g14+g15	g14'+g19	g15'+g19	
g18 ↔	g11'+g13'	g18'+g11'+g13'	g11+g18	g13+g18	
g17 ↔	g11'+g12	g17'+g11'+g12	g11+g17	g12'+g17	
g16 ↔	g1'*g10'	g16'+g1'	g16'+g10'	g1+g10+g16	
g15 ↔	g9'*g10*g2	g15'+g9'	g15'+g10	g15'+g2	g9+g10'+g2'+g15
g14 ↔	g1*g2'*g10'	g14'+g1	g14'+g2'	g14'+g10'	g1'+g2+g10+g14
g13 ↔	g8+g10'	g13'+g8+g10'	g8'+g13	g10+g13	
g12 ↔	g6'*g7	g12'+g6'	g12'+g7	g6+g7'+g12	
g11 ↔	g6'+g7	g11'+g6'+g7	g6+g11	g7'+g11	
g10 ↔	g3+g4	g10'+g3+g4	g3'+g10	g4'+g10	
g9 ↔	g4*g5'	g9'+g4	g9'+g5'	g4'+g5+g9	
g8 ↔	g1*g2	g8'+g1	g8'+g2	g1'+g2'+g8	
g7 ↔	g3*g4	g7'+g3	g7'+g4	g3'+g4'+g7	
g6 ↔	g1+g2	g6'+g1+g2	g1'+g6	g2'+g6	
g5 ↔	g1+g3	g5'+g1+g3	g1'+g5	g3'+g5	

clauses contained in PG encoding is highlighted in yellow

(b) # of clauses: 62

# of literals: 144

(c) # of clauses: 45 ⇒ reduce 27.4%

# of literals: 104 ⇒ reduce 27.8%

(d) [Literal counts of a gate] The literal counts are the number of times a literal appears in the clauses. Given a variable, its positive and negative literal counts are represented as a pair "(negLitCount, posLitCount)". For example, given the following clauses,

(a + f') (b + f') (a' + b' + f)

the literal counts for a, b, and f are (1, 1), (1, 1), and (2, 1), respectively.

Please list the literal counts (as pairs) for all the gates in the circuit (including PIs) using PG encoding.

(e) [Decision order] We will determine the decision order of the gates in the circuit based on the "literal counts" of their corresponding CNF formulae (as in sub-problem (d)). The detailed rules are as follows:

(i) The "decision score" of a gate is the sum of its negative (0) and positive literal (1) counts.

(ii) The "decision value" is the opposite polarity of the literal with the bigger count. If the counts for the positive and negative literals are the same, choose 0 as the decision value. For example, if the (0, 1)-literal counts of the gate 'f' and 'g' are (5, 2) and (3, 3), then their decision scores are 7 and 6, and their decision values are 1 and 0.

(iii)The "decision order" of the gates is determined by the decision scores, with the bigger scores in the front. If the scores of two gates are tied, the gate with the smaller difference between (0, 1)-literal counts wins. If tied again, compare their IDs (bigger ID wins). For example, if the literal counts of gates 'f', 'g' and 'h' are (5, 2) and (3, 4), (4, 1), the decision order will be (g = 0) → (f = 1) → (h = 1).

(iv)The orders remain unchanged throughout the decision process.

Please derive the top 7 decision gates in the circuit (including PIs).

p=1		p			
p1	g23	p'+g23	p+g23'		
g23	g22+g21'	g23'+g22+g21'	g22'+g23	g21+g23	
g22	g19*g20	g22'+g19	g22'+g20	g19'+g20'+g22	
g21	g18+g20	g21'+g18+g20	g18'+g21	g20'+g21	
g20	g16+g17'	g20'+g16+g17'	g16'+g20	g17+g20	
g19	g14+g15	g19'+g14+g15	g14'+g19	g15'+g19	
g18	g11'+g13'	g18'+g11'+g13'	g11+g18	g13+g18	
g17	g11'+g12	g17'+g11'+g12	g11+g17	g12'+g17	
g16	g1'*g10'	g16'+g1'	g16'+g10'	g1+g10+g16	
g15	g9'*g10*g2	g15'+g9'	g15'+g10	g15'+g2	g9+g10'+g2'+g15
g14	g1*g2'*g10'	g14'+g1	g14'+g2'	g14'+g10'	g1'+g2+g10+g14
g13	g8+g10'	g13'+g8+g10'	g8'+g13	g10+g13	
g12	g6'*g7	g12'+g6'	g12'+g7	g6+g7'+g12	
g11	g6'+g7	g11'+g6'+g7	g6+g11	g7'+g11	
g10	g3+g4	g10'+g3+g4	g3'+g10	g4'+g10	
g9	g4*g5'	g9'+g4	g9'+g5'	g4'+g5+g9	
g8	g1*g2	g8'+g1	g8'+g2	g1'+g2'+g8	
g7	g3*g4	g7'+g3	g7'+g4	g3'+g4'+g7	
g6	g1+g2	g6'+g1+g2	g1'+g6	g2'+g6	
g5	g1+g3	g5'+g1+g3	g1'+g5	g3'+g5	

(d)

	pos	neg	d score	difference
p	1	1	2	0
g1	5	2	7	3
g2	3	2	5	1
g3	3	2	5	1
g4	2	3	5	1
g5	1	1	2	0
g6	4	3	7	1
g7	3	4	7	1
g8	1	2	3	1
g9	1	1	2	0
g10	4	4	8	0
g11	4	2	6	2
g12	2	3	5	1
g13	1	1	2	0
g14	1	3	4	2
g15	1	3	4	2
g16	2	3	5	1
g17	3	2	5	1
g18	2	1	3	1
g19	1	1	2	0
g20	3	1	4	2
g21	2	1	3	1
g22	1	2	3	1
g23	1	1	2	0

(e)

(1) $g_{10}=0$

(2) $g_7=0$

(3) $g_6=1$

(4) $g_1=1$

(5) $g_{11}=1$

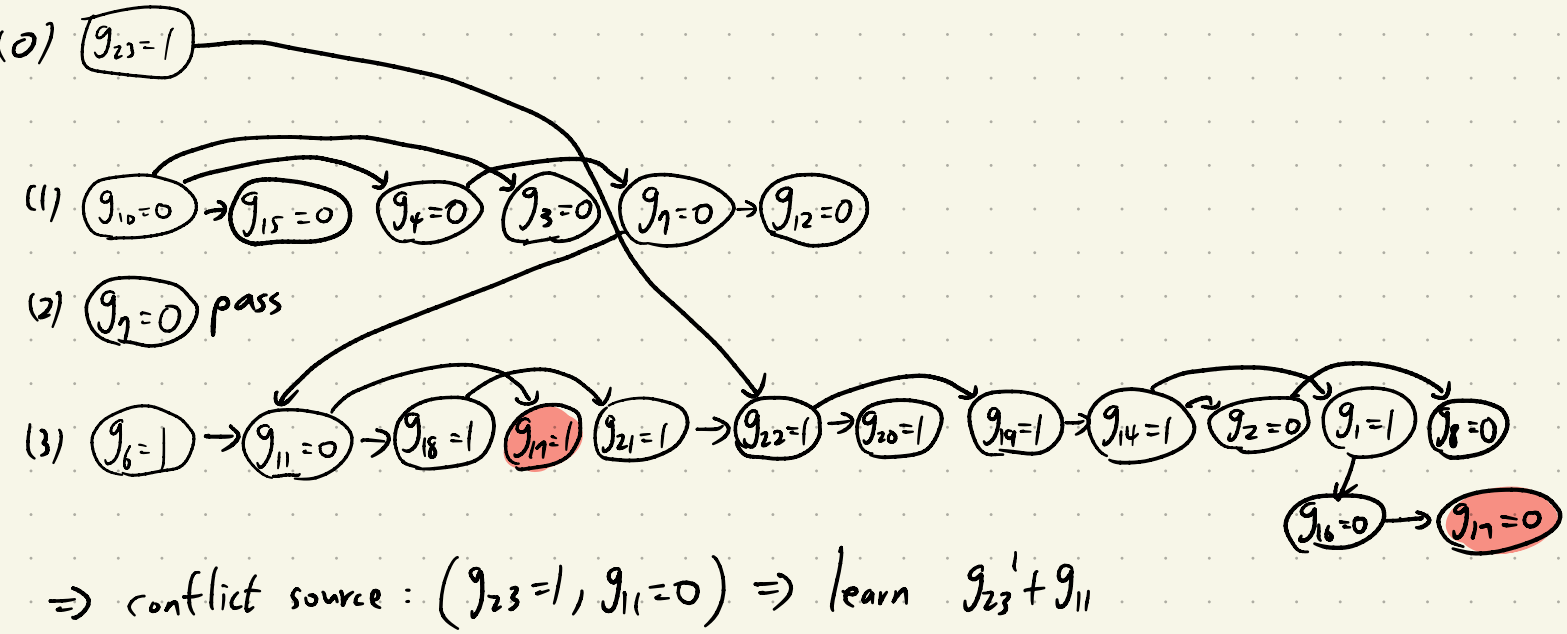
(6) $g_{17}=1$

(7) $g_{16}=0$

(f) [Conflict-driven learning] We will try to witness p(g23) = 1. Please follow the decision orders and values in (e) to make the decisions, perform logic implications, and construct the implication graph. Do not make decision on a gate if it has been implied in an earlier decision level. List the implications by decision levels or draw the implication graph in your report.

You may encounter a conflict after a few decisions. Please perform the conflict analysis to derive the conflict sources on the first UIP cut and construct a learned gate (i.e. AND gate with constrained value '0' on its output) for it.

g23		~~g23~~			
g23	g22+g21'	~~g22+g21'~~	g22'+g23	g21+g23	
g22	g19*g20	~~g19+g20~~	~~g22'+g23~~	g19'+g20'+g22	
g21	g18+g20	g21'+g18+g20	~~g21'+g21~~	~~g20'+g21~~	
g20	g16+g17'	~~g20'+g16+g17'~~	~~g16'+g20~~	~~g17+g20~~	
g19	g14+g15	~~g19'+g14+g15~~	g14'+g19	g15'+g19	
g18	g11'+g13'	g18'+g11'+g13'	~~g11'+g18~~	~~g13'+g18~~	
g17	g11'+g12	~~g17'+g11'+g12~~	g11'+g17	~~g12'+g17~~	
g16	g1'*g10'	~~g16'+g1'~~	~~g16'+g10'~~	~~g1'+g16'+g16~~	
g15	g9'*g10*g2	~~g15'+g9'~~	~~g15'+g10~~	~~g15'+g2~~	g9+g10'+g2'+g15
g14	g1*g2'*g10'	~~g14'+g1'~~	~~g14'+g2~~	~~g14'+g10'~~	g1'+g2+g10+g14
g13	g8+g10'	~~g13'+g8+g10'~~	g8'+g13	g10+g13	
g12	g6'*g7	~~g12'+g6'~~	~~g12'+g7~~	~~g6+g7'+g12~~	
g11	g6'+g7	~~g11'+g6'+g7~~	~~g6'+g11~~	~~g7'+g11~~	
g10	g3+g4	~~g10'+g3+g4~~	~~g3'+g10~~	~~g4'+g10~~	
g9	g4*g5'	g9'+g4	g9'+g5'	~~g4'+g5'+g9~~	
g8	g1*g2	~~g8'+g1~~	~~g8'+g2~~	g1'+g2'+g8	
g7	g3*g4	~~g7'+g3~~	~~g3'+g4'+g7~~	~~g3'+g4'+g7~~	
g6	g1+g2	~~g6'+g1+g2~~	~~g1'+g6~~	~~g2'+g6~~	
g5	g1+g3	~~g5'+g1+g3~~	g1'+g5	g3'+g5	

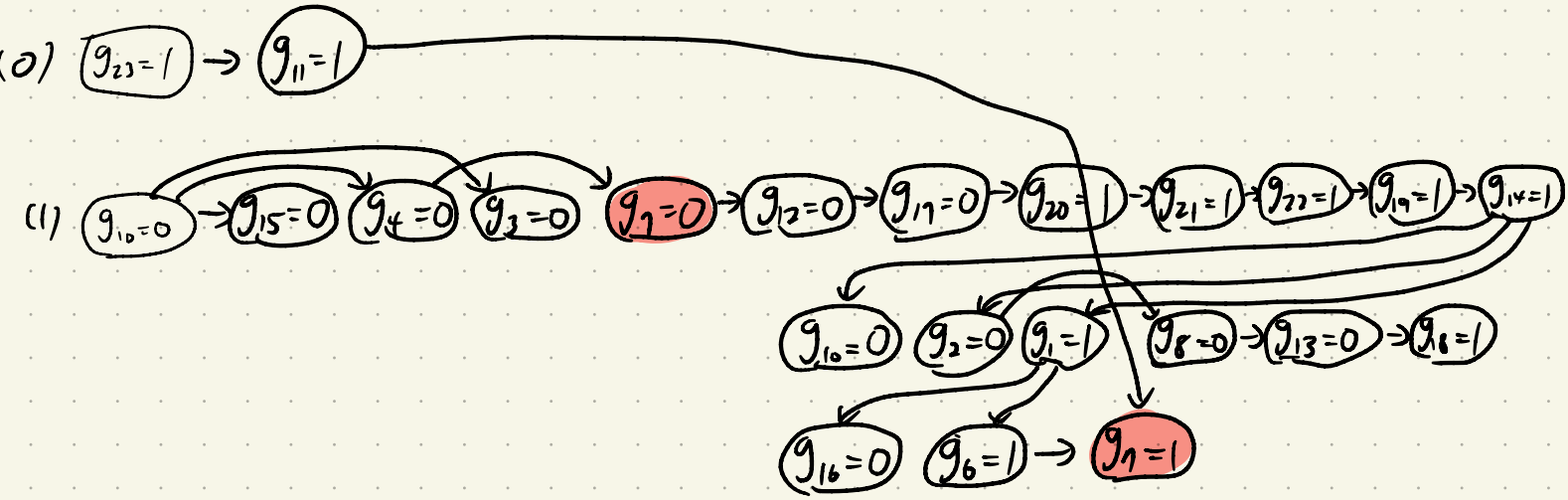


⇒ conflict source : $(g_{23}=1, g_{11}=0)$ ⇒ learn $g_{23}' + g_{11}$

g) [Witness generation] If you encountered a conflict in (f), backtrack from the learned constraint to an earlier decision level. What is the derived learned implication? At which decision level? Perform BCP on this learned implication. Will there be another conflict? If yes, perform conflict-driven learning again. If not, pick the next unassigned gate in the decision ordered list to make the next decision. Continue this process until a witness is found, or conclude that this target assignment is unsatisfiable. List the implications by decision levels or draw the implication graph in your report.

g23		g23			
g23	g22+g21'	g23+g22+g21'	g22'+g23	g21+g23	
g22	g19*g20	g22+g19'	g22'+g20	g19'+g20'+g22	
g21	g18+g20	g21'+g18+g20	g18'+g21	g20'+g21	
g20	g16+g17'	g20'+g16+g17'	g16'+g20	g17+g20	
g19	g14+g15	g19'+g14+g15	g14'+g19	g15'+g19	
g18	g11'+g13'	g18'+g11'+g13'	g11+g18	g13+g18	
g17	g11'+g12	g17'+g11+g12	g11+g17	g12'+g17	
g16	g1'*g10'	g16'+g1'	g10'+g16	g1'+g10'+g16	
g15	g9'*g10*g2	g15'+g9	g15'+g10	g15'+g2	g9+g10'+g2'+g15
g14	g1*g2'*g10'	g14'+g1	g14'+g2'	g14+g10'	g1'+g2+g10+g14
g13	g8+g10'	g13'+g8+g10'	g8'+g13	g10+g13	
g12	g6'*g7	g12'+g6	g12+g7	g6+g7'+g12	
g11	g6'+g7	g11'+g6'+g7	g6+g11	g7'+g11	
g10	g3+g4	g10'+g3+g4	g3'+g10	g4'+g10	
g9	g4*g5'	g9'+g4	g9'+g5'	g4+g5'+g9	
g8	g1*g2	g8'+g1	g8'+g2	g1'+g2'+g8	
g7	g3*g4	g7'+g3	g7'+g4	g3'+g4'+g7	
g6	g1+g2	g6'+g1+g2	g1'+g6	g2'+g6	
g5	g1+g3	g5'+g1+g3	g1'+g5	g3'+g5	

$g_{23}' + g_{11}$

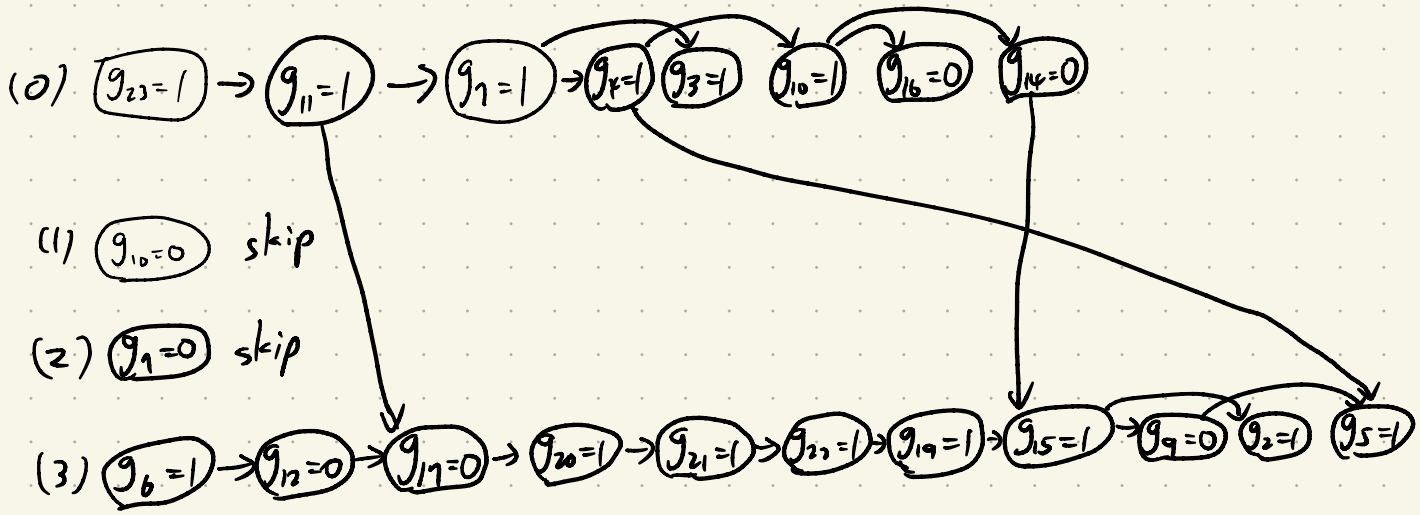


⇒ conflict source : $(g_{11}=1, g_7=0)$ ⇒ learn $g_{11}' + g_7$

g) [Witness generation] If you encountered a conflict in (f), backtrack from the learned constraint to an earlier decision level. What is the derived learned implication? At which decision level? Perform BCP on this learned implication. Will there be another conflict? If yes, perform conflict-driven learning again. If not, pick the next unassigned gate in the decision ordered list to make the next decision. Continue this process until a witness is found, or conclude that this target assignment is unsatisfiable. List the implications by decision levels or draw the implication graph in your report.

g23		~~g23~~			
g23	g22+g21'	~~g23'+g22+g21~~	g22'+g23	g21+g23	
g22	g19*g20	~~g22'+g19~~	~~g22'+g20~~	g19'+g20'+g22	
g21	g18+g20	g21'+g18+g20	~~g18'+g21~~	~~g20'+g21~~	
g20	g16+g17'	~~g20'+g16+g17'~~	~~g16'+g20~~	~~g17'+g20~~	
g19	g14+g15	~~g19'+g14+g15~~	g14'+g19	g15'+g19	
g18	g11'+g13'	g18'+g11'+g13'	~~g11+g18~~	~~g13+g18~~	
g17	g11'+g12	~~g17'+g11'+g12~~	~~g11+g17~~	~~g12'+g17~~	
g16	g1'*g10'	~~g16'+g1'~~	~~g16'+g10~~	~~g1+g10+g16~~	
g15	g9'*g10*g2	~~g15'+g9~~	~~g15'+g10~~	~~g1'+g2~~	g9+g10'+g2'+g15
g14	g1*g2'*g10'	~~g14'+g1~~	~~g14'+g2'~~	~~g14'+g10~~	g1'+g2+g10+g14
g13	g8+g10'	g13'+g8+g10	g8'+g13	g10+g13	
g12	g6'*g7	~~g12'+g6'~~	~~g12'+g7~~	~~g6+g7'+g12~~	
g11	g6'+g7	~~g11'+g6'+g7~~	~~g6'+g11~~	~~g7'+g11~~	
g10	g3+g4	~~g10'+g3+g4~~	~~g3'+g10~~	~~g4'+g10~~	
g9	g4*g5'	g9'+g4	g9'+g5'	~~g4'+g5+g9~~	
g8	g1*g2	~~g8'+g1~~	~~g8'+g2~~	g1'+g2'+g8	
g7	g3*g4	~~g7'+g3~~	~~g7'+g4~~	~~g3'+g4'+g7~~	
g6	g1+g2	~~g6'+g1+g2~~	~~g1'+g6~~	~~g2'+g6~~	
g5	g1+g3	~~g5'+g1+g3~~	g1'+g5	g3'+g5	

$g_{23}' + g_{11}$ $g_{11}' + g_7$



(4) $g_1 = 1$

(5) $g_{11} = 1$ skip

(6) $g_{17} = 1$ skip

(7) $g_{16} = 0$ skip

⋮

$g_{18} = 1$

$g_8 = 0 \rightarrow g_{13} = 0$

⇒ SAT with $g_1 = 1, g_2 = 1, g_3 = 1, g_4 = 1$