

# BatchLayout: A Batch-Parallel Force-Directed Graph Layout Algorithm in Shared Memory

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## ABSTRACT

Force-directed algorithms are widely used to generate aesthetically-pleasing layouts of graphs or networks arisen in many scientific disciplines. To visualize large-scale graphs, several parallel algorithms have been discussed in the literature. However, existing parallel algorithms do not utilize memory hierarchy efficiently and often offer limited parallelism. This paper addresses these limitations with BatchLayout, an algorithm that groups vertices into minibatches and processes them in parallel. BatchLayout also employs cache blocking techniques to utilize memory hierarchy efficiently. More parallelism and improved memory accesses coupled with force approximating techniques, better initialization, and optimized learning rate make BatchLayout significantly faster than other state-of-the-art algorithms such as ForceAtlas2 and OpenOrd. The visualization quality of layouts from BatchLayout is comparable or better than similar visualization tools. All of our source code, links to datasets, results and log files are available at <https://github.com/khaled-rahman/BatchLayout>.

**Index Terms:** Human-centered computing—Visualization—Visualization techniques—Graph drawings; Human-centered computing—Visualization—Visualization systems and tools—Visualization toolkits

## 1 INTRODUCTION

Networks or graphs are common representations of scientific, social and business data. In a graph, a set of vertices represents entities (e.g., persons, brain neurons, atoms) and a set of edges indicates relationships among entities (friendship, neuron synapses, chemical bonds). A key aspect of data-driven graph analytics is to visually study large-scale networks, such as biological and social networks, with millions or even billions of vertices and edges. In network visualization, the first step is to generate a layout in a 2D or 3D coordinate system which can be fed into visualization tools such as Gephi [4] and Cytoscape [29]. Therefore, the quality and computational complexity of network visualization are often dominated by the graph layout algorithms.

Force-directed layout algorithms are among the most popularly used techniques to generate the layout of a graph. Following the philosophy of the spring energy model, these algorithms calculate attractive and repulsive forces among vertices in a graph and iteratively minimize an energy function. Classical force-directed algorithms, such as the Fruchterman and Reingold (FR) algorithm [9], require  $O(n^2)$  time per iteration where  $n$  is the number of vertices in a graph. By approximating the repulsive force between non-adjacent nodes, we can get a faster  $O(n \log n)$  algorithm. In this paper, we used the Barnes-Hut approximation [3] based on the quad-tree data structure.

Even though the layout quality (measured by stress, neighborhood preservation, and other well established quality metrics as shown in Table 4) from the Barnes-Hut approximation can be worse than an exact  $O(n^2)$  algorithm, the former is often used to visualize large-scale networks. In this paper, we carefully analyzed the quality and runtime of both exact and approximate force-directed algorithms.

To visualize large graphs, several prior work discussed parallel algorithms for multicore processors [23], graphics processing unit (GPU) [6, 17], and distributed clusters [1, 2]. While we summarize state-of-the-art algorithms and software in the Related Work section, our primary focus lies in parallel algorithms for multicore servers. Some state-of-the-art force-directed algorithms such as ForceAtlas2 [17] and OpenOrd [23] have parallel implementations for multicore processors. However, these algorithms do not utilize full advantage of present memory hierarchy efficiently and offer limited parallelism because they calculate forces for one vertex at a time. We aim to improve these two aspects of parallel algorithms. We develop a parallel algorithm called *BatchLayout* that offers more parallelism by grouping vertices into minibatches and processing all vertices in a minibatch in parallel. This approach of increasing parallelism via minibatches is widely used in training deep neural networks in the form of minibatch Stochastic Gradient Descent (SGD) [11]. We adapt this approach to graph layout algorithms for better parallel performance.

Existing algorithms also access memory irregularly when processing sparse graphs with skewed degree distributions. Irregular memory accesses do not efficiently utilize multi-level caches available in current processors, impeding the performance of algorithms. In BatchLayout, we regularized memory accesses by using linear algebra operations similar to matrix-vector multiplication and employing “cache blocking” techniques to utilize memory hierarchy efficiently. More parallelism and better memory accesses made BatchLayout faster than existing parallel force-directed algorithms. On average, the exact  $O(n^2)$  version of BatchLayout is  $15\times$  faster than ForceAtlas2. The Barnes-Hut approximation in BatchLayout is about  $4\times$  and  $2\times$  faster than ForceAtlas2 (with BH approximation) and OpenOrd, respectively (the quality of an OpenOrd layout is usually worse than BatchLayout and ForceAtlas2).

BatchLayout attains high performance without sacrificing the quality of the layout. We used three aesthetic metrics to quantitatively measure the quality of layouts and also visualized the networks in Python for human verification. According to all quality measures, BatchLayout generates similar or better-quality layouts than ForceAtlas2 and OpenOrd.

Overall, BatchLayout covers all important aspects of force-directed layout algorithms. We investigated four energy models, two initialization techniques, different learning rates and convergence criteria, various minibatch sizes, and three aesthetic metrics to study the performance and quality of our algorithms. All of these options are available in an open source software that can be run on any multicore processor. Therefore, similar to ForceAtlas2, BatchLayout provides plenty of options to users to generate a readable layout for an input graph.

In this paper, we present new insights on parallel force-directed graph layout algorithms. We summarize key contributions below:

- **Improved parallelism:** We develop a class of exact and approxi-

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mate parallel force-directed graph layout algorithms called BatchLayout. Our algorithms expose more parallel work to keep hundreds of processors busy.

- **Better quality layouts:** BatchLayout generates layouts with better or similar qualities (according to various metrics) compared to other force-directed algorithms.
- **Comprehensive coverage:** BatchLayout provides multiple options for all aspects of force-directed algorithms. Especially, we cover new initialization techniques and four energy models.
- **High-performance implementation:** We provide an implementation of BatchLayout for multicore processors. BatchLayout improves data locality and decreases thread synchronizations for better performance.
- **Faster than ForceAtlas2 and OpenOrd:** On a diverse set of graphs, BatchLayout is significantly faster than ForceAtlas2 and OpenOrd.

## 2 BACKGROUND

### 2.1 Notations

We present an unweighted graph by  $G = (V, E)$  on the set of vertices  $V$  and set of edges  $E$ . The number of vertices and edges are denoted by  $n$  and  $m$ , respectively. In this paper, we only considered undirected graphs, but directed graphs can be used by ignoring the directionality of the edges. The degree of vertex  $i$  is denoted by  $\deg(i)$ . We denote the coordinate of vertex  $i$  with  $c_i$ , which represents  $(x, y)$  Cartesian coordinate in a 2D plane. The distance between two vertices  $i$  and  $j$  is represented by  $\|c_i - c_j\|$ .

In our algorithm, we used the standard Compressed Sparse Row (CSR) data structure to store the adjacency matrix of a sparse graph. For unweighted graphs, the CSR format consists of two arrays: *rowptr* and *colids*. *rowptr*, an array of length  $n + 1$ , stores the starting indices of rows. *colids* is an array of length  $m$  that stores the column indices in each row. Here, the CSR data structure is used for space and computational efficiency, but any other data structure would also work with our algorithm.

### 2.2 Force Calculations

Force-directed graph drawing algorithms compute *attractive* forces between every pair of adjacent vertices and *repulsive* forces between nonadjacent vertices. In the spring-electrical model, a pair of connected vertices attract each other like a spring based on Hooke's law and a pair of nonadjacent vertices repulse each other like electrically charged particles based on Coulomb's law. Following this spring-electrical model, we define the attractive force  $f_a(i, j)$  and the repulsive force  $f_r(i, j)$  between vertices  $i$  and  $j$  as follows:

$$f_a(i, j) = \frac{\|c_i - c_j\|^a}{K}, \quad f_r(i, j) = \frac{-RK^2}{\|c_i - c_j\|^r}.$$

Here,  $K$  acts as an optimal spring length and  $R$  is considered as the regulator of relative strength of forces. Different values of  $a$  and  $r$  give different energy models, known as the  $(a, r)$ -energy model [17, 27]. The combined force  $f(i, c)$  of a vertex  $i$  with respect to the current coordinates  $c$  of all vertices is computed by:

$$f(i, c) = \sum_{(i, j) \in E} f_a(i, j) \times \frac{c_j - c_i}{\|c_i - c_j\|} + \sum_{\substack{i \neq j \\ (i, j) \notin E}} f_r(i, j) \times \frac{c_j - c_i}{\|c_i - c_j\|}.$$

Here,  $c$  is an array of coordinates of all vertices i.e.,  $c = \{c_i | i \in V\}$ . We multiplied attractive and repulsive forces by a unit vector to get the direction of movement of vertex  $i$ . Hence, following the spring-electrical model, the *energy* of the whole graph will be proportional to  $\sum_{i \in V} f^2(i, c)$ . The goal of a force-directed algorithm is to minimize the energy so that the layout of the graph becomes stable, readable and visually appealing.

### Algorithm 1: A Sequential Force-Directed Algorithm

```

Input:  $G(V, E)$  and an initial layout  $c$ 
1:  $Step = 1.0$                                  $\triangleright$  initial step length
2:  $Loop = 0$ 
3: while  $Loop < MaxIteration$  do
4:    $Energy = 0$ 
5:   for  $i \leftarrow 0$  to  $n - 1$  do                 $\triangleright$  force initialization
6:      $f = 0$ 
7:     for  $j \leftarrow 0$  to  $n - 1$  do
8:       if  $(i, j) \in E$  then
9:          $f = f + f_a(i, j) \times \frac{c_j - c_i}{\|c_i - c_j\|}$ 
10:      else
11:         $f = f + f_r(i, j) \times \frac{c_j - c_i}{\|c_i - c_j\|}$ 
12:      $c_i = c_i + Step \times \frac{f}{\|f\|}$ 
13:      $Energy = Energy + \|f\|^2$ 
14:    $Step = Step \times 0.999$ 
15:    $Loop = Loop + 1$ 
16: return the final layout  $c$ 
```

### 2.3 A Standard Sequential Algorithm

Algorithm 1 describes a standard sequential force-directed graph layout algorithm. We closely followed the notation of an Adaptive Cooling Scheme by Y. Hu [15]. Algorithm 1 starts with an initial layout and iteratively improves it by minimizing energy. In each iteration, vertices are chosen in a predefined order (line 5), and attractive and repulsive forces are calculated for a selected vertex with respect to all other vertices in the graph. Next, the relative position of the selected vertex is updated (line 12) and this updated position is used in processing the subsequent vertices. *Step* used in line 12 is a hyper-parameter that influences the convergence of the algorithm. We will discuss possible values of this hyper-parameter in the experimental section. A working example of this algorithm is shown in Fig. S1 of the supplementary file. Each iteration of Algorithm 1 computes  $O(m)$  attractive forces and  $O(n^2)$  repulsive forces, giving us a sequential  $O(n^2)$  algorithm. Using the Barnes-Hut approximation [3] of repulsive forces, Algorithm 1 can easily be made into a sequential  $O(n \log n)$  algorithm.

## 3 THE BATCHLAYOUT ALGORITHM

### 3.1 Toward A Scalable Parallel Algorithm

Despite its simplicity, it is hard to yield high performance from Algorithm 1. A naïve parallelization of Algorithm 1 can simply compute forces of a vertex in parallel by making line 7 of Algorithm 1 a parallel loop. However, for vertex  $i$ , this naïve parallel algorithm offers  $O(n + \deg(i))$  parallel work for the exact algorithm and  $O(\log n + \deg(i))$  parallel work for the approximate algorithm. To increase the parallel work, we followed the trend in training deep neural networks via SGD. In traditional SGD, the gradient is calculated for each training example, which is then used to update model parameters. Since SGD offers limited parallelism, most practical approaches use a batch of training examples (batch size varies from 256 to several thousands) to compute the gradient, an approach known as minibatch SGD. We follow a similar approach by calculating forces for a batch of vertices and updating their coordinates in parallel. BatchLayout revolves around the batch processing of vertices and covers all other aspects of force-directed algorithms.

### 3.2 Overview of BatchLayout

Figure 1 shows the skeleton of a typical force-directed graph layout algorithm. After starting with an initial layout, an algorithm iteratively minimizes energy by computing attractive and repulsive forces. Upon convergence, the final layout is drawn using a visualization tool. Figure 1 shows that different steps of the algorithm have various choices upon which the quality and runtime of the

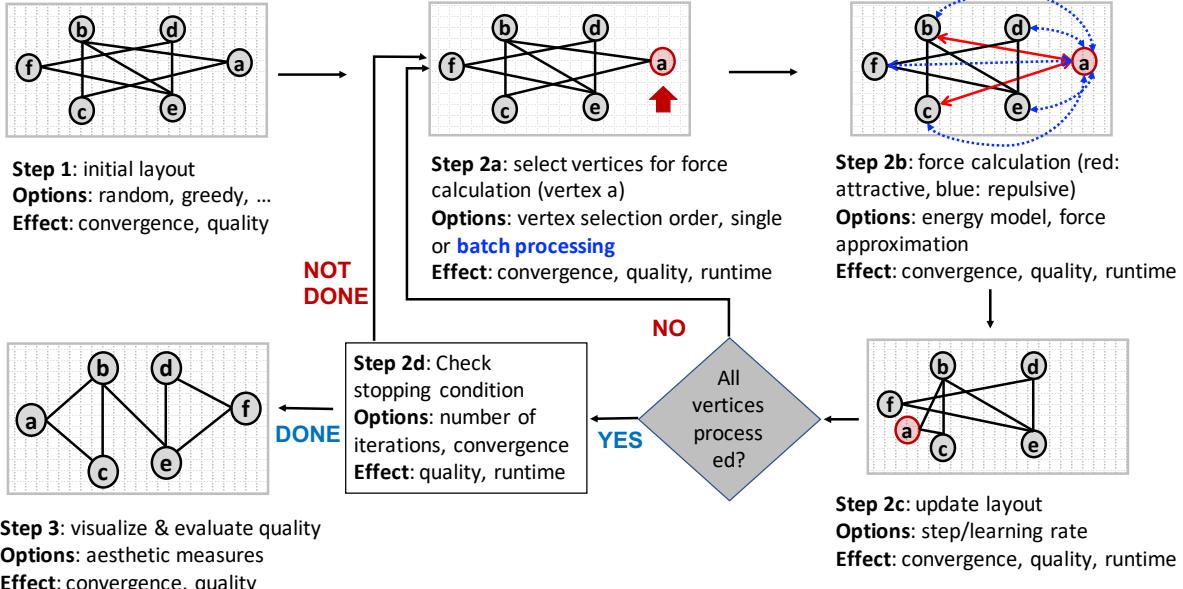


Figure 1: Steps involved in a typical force directed graph layout algorithm. An algorithm starts with an initial layout (usually a random layout) and iteratively updates the layout by calculating attractive and repulsive forces among vertices. When a stopping criterion is satisfied, the graph is visualized using the generated layout and the quality of the layout is evaluated. Each step has various algorithmic options that impact the convergence speed, the quality of the final layout, and the runtime of the algorithm. We highlighted “batch processing” in Step 2a since it is a required option for efficient parallel algorithms discussed in this paper. We explored many options within our parallel algorithm.

algorithm depend. A comprehensive software such as ForceAtlas2 provides multiple options in each step. Like ForceAtlas2, BatchLayout also provides multiple options in each of these steps. In particular, BatchLayout uses two initialization strategies, process vertices in different orders, considers four energy models, calculates exact and approximate repulsive forces, explores different learning rates and convergence conditions. We evaluated the impact of different options on the convergence and quality of the algorithm. We developed parallel algorithms for all options and evaluated their parallel performance.

### 3.3 Batch processing of vertices

Since force calculations consume almost all computing time of our algorithm, we discuss parallel force calculation techniques first. As mentioned before, calculating forces for a single vertex does not provide enough work to keep many processors busy. Hence, the BatchLayout algorithm selects a subset of vertices called a *minibatch* and calculates forces for each vertex in the minibatch independently. After all vertices in a minibatch finish force calculations, we update their coordinates in parallel. Therefore, unlike Algorithm 1, the minibatch approach delays updating coordinates until all vertices in a minibatch finish their force calculations. Fig. 2 illustrates this approach with a minibatch size of 2.

Batched force calculation has a benefit and a drawback. As we process more vertices simultaneously, we can keep more processors busy by providing ample parallel work. However, batch processing may slow down the convergence because the updated coordinates of a vertex cannot be utilized immediately. In our experiment, we observed that the benefit of concurrent work is significantly more than the disadvantage caused by slower convergence. Hence, batch-processing makes a parallel algorithm significantly faster.

### 3.4 Improved Memory Accesses Via Cache Blocking

While batch processing of vertices provides ample parallel work, processors can still remain idle if they have to wait for data to be fetched from memory. Memory stalling can happen if an algorithm does not utilize spatial and temporal locality in accessing data. We addressed this issue by using cache blocking, a technique frequently used in

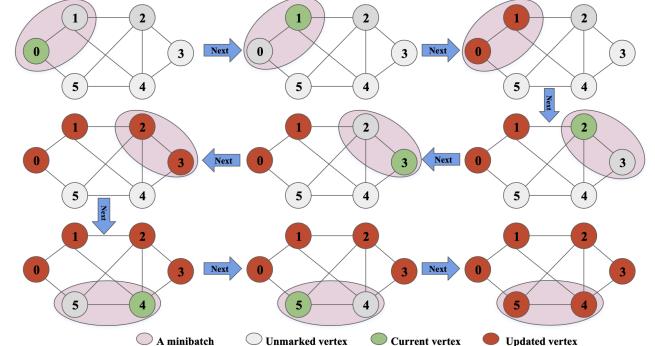


Figure 2: Force calculation with minibatch size 2. We start from vertex 0 and consider two consecutive vertices in a minibatch shown in shaded regions. This example finishes in three steps (each row in the figure), and each step processes a minibatch of two vertices. Green color denotes a vertex whose force is currently being computed, and red denotes a vertex whose coordinate is being updated. In the first step (first row), vertex 0 and vertex 1 form a minibatch whose forces are first computed and then their coordinates are updated. Similarly, in the second step (second row, from right to left), we calculate forces for vertex 2 and 3, and then update their coordinates. Notice that in step 2, updated coordinates of the previous minibatch {0,1} are used.

linear algebra operations to utilize the cache hierarchy available in modern processors.

We noticed that force calculations can be viewed as a matrix-vector multiplication. Fig. 3 shows a simple example where the adjacency matrix of the graph used in Fig. 2 is considered. The vector of current coordinates are shown twice in dense vectors  $A$  and  $B$ .  $A'$  holds the updated coordinates. In our minibatch model, we compute the forces for all vertices in a minibatch with respect to all other vertices in the graph. Suppose, in batch 0, we compute forces for vertices 0 and 1 with respect to vertices 0...2 (blue). When forces are calculated for vertex 0, coordinates of vertices 0...2

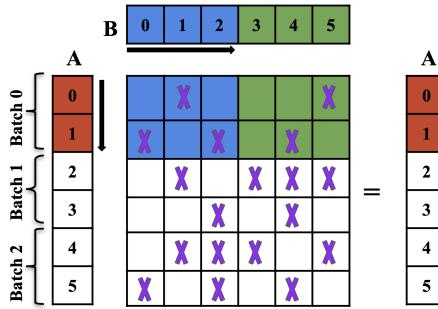


Figure 3: Computation of forces for the graph in Fig. 2 using cache blocking technique. Here, the graph is stored as an adjacency matrix with ‘X’ denoting edges between vertices. Vectors represent coordinates of vertices, and force calculations are viewed as a matrix–vector multiplication.

are brought in from the main memory and stored in a CPU cache. The cumulative sum of force calculations are stored in a temporary vector,  $A'$ . When we compute forces for the next vertex 1, all necessary coordinates are already available in cache (i.e., using data locality), which reduces the memory traffic. Batch 0 gets a similar performance advantage when forces are computed with respect to vertices 3...5 (green). BatchLayout with the cache-blocking scheme is presented in Algorithm 2. Here,  $G.\text{rowptr}$  and  $G.\text{colids}$  are two arrays which hold the row pointers and column IDs, respectively, of the graph in a CSR format.  $BS$  is the size of minibatch and  $Step$  length is initialized to 1.0. In line 6 of Algorithm 2, we iterate over a subset of vertices (minibatch) and in line 10, we iterate over all vertices to calculate  $f_a$  and  $f_r$ . In line 14, when two vertices are connected by an edge we calculate  $f_a$ ; otherwise, we calculate  $f_r$  (line 18). After calculating forces for a vertex, we store it in a temporary location (see line 19). This temporary storage helps us to calculate forces in parallel. Next, we iterate over all vertices in a minibatch to update relative position (see line 22). We provide a flexible rectangular cache blocking area which is bounded by variables  $p$  and  $q$ . This increases the data locality in the L1 cache of each core, improving the performance of our algorithm. Loop counters  $i$  and  $j$  are increased by  $p$  and  $q$ , respectively. Variable  $k_x$  holds the starting index of neighbors for  $p$  vertices (see line 8). After the loop completes, in line 24, we update step length. To keep Algorithm 2 simple, we assume that the number of vertices in a graph is multiple of  $(b + 1) * BS$ . In our implementation, we incorporated other cases. The overall running time of this procedure is same as Algorithm 1 i.e.,  $O(n^2)$ . However, the main advantage of this technique is in parallel computation of forces. Note that Algorithm 1 is a special case of Algorithm 2 when the size of the minibatch is 1.

### 3.5 Repulsive Force Approximation

Exact repulsive force computations need  $O(n^2)$  time per iteration, which is too expensive for large graphs. Therefore, we have developed a parallel Barnes-Hut algorithm [3] for approximating repulsive forces. Barnes-Hut takes  $O(n \log n)$  time per iteration. For 2D visualization, a quad-tree data structure is often used to approximate repulsive forces. An example is shown in Fig. 4. Given the coordinates of vertices in Fig. 4(a), a high-level block is divided into four equal blocks as shown in Fig. 4(b). In this example, we split up to 2 levels, creating  $4^2 = 16$  small blocks in the 2D plane. Empty blocks are merged with their adjacent nonempty blocks. We show how the algorithm approximates the repulsive force  $f_r$  for the green vertex. First, a centroid (shown in red) is calculated for each block by averaging the coordinates of all vertices in the block. Next, an approximate repulsive force is computed with respect to the centroids, as shown by the dashed gray lines.

### Algorithm 2: Cache Blocking BatchLayout

```

Input:  $G(V, E)$ 
1:  $Step = 1.0$                                       $\triangleright$  initial step length
2:  $Loop = 0$ 
3: while  $Loop < MaxIteration$  do
4:    $Energy = 0$ 
5:   for  $b \leftarrow 0$  to  $\frac{n}{BS} - 1$  do in parallel
6:     for  $i \leftarrow b * BS$  to  $(b + 1) * BS - 1$  by  $p$  do
7:       for  $x \leftarrow 0$  to  $p - 1$  do
8:          $k_x = G.\text{rowptr}[i + x]$ 
9:          $ct_{i+x} = 0$ 
10:        for  $j \leftarrow 0$  to  $n - 1$  by  $q$  do
11:          for  $x \leftarrow 0$  to  $p - 1$  do
12:             $f = 0$ 
13:            for  $y \leftarrow 0$  to  $q - 1$  do
14:              if  $j + y == G.\text{colids}[k_x]$  then
15:                 $f += f_a(i, j) \times \frac{c_j - c_i}{\|c_i - c_j\|}$ 
16:                 $k_x += 1$ 
17:              else
18:                 $f += f_r(i, j) \times \frac{c_j - c_i}{\|c_i - c_j\|}$ 
19:                 $ct_{i+x} += f$ 
20:      end parallel for
21:      for  $i \leftarrow b * BS$  to  $(b + 1) * BS - 1$  do
22:         $c_i += Step \times \frac{ct_i}{\|ct_i\|}$ 
23:         $Energy += \|ct_i\|^2$ 
24:       $Step = Step \times 0.999$ 
25:       $Loop += 1$ 
26: return the final layout  $c$ 

```

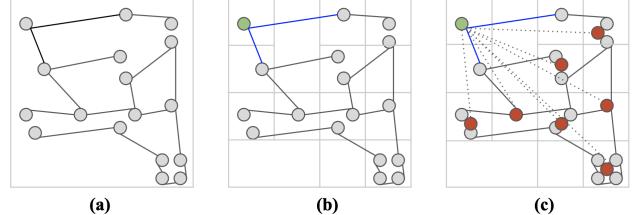


Figure 4: (a) An example graph. (b) Calculating forces of the green vertex using a quad-tree. (c) Blue solid lines show attractive forces, and dashed gray lines denote approximate repulsive forces with respect to centroids (red).

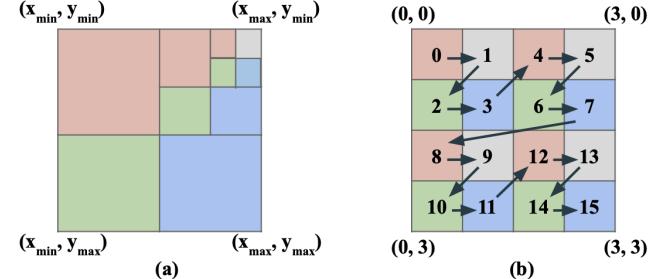


Figure 5: (a) A quad-tree structure. (b) Morton order of traversal for 16 vertices in 16 different blocks.

Among several choices of quad-tree implementation [31], we adopted an approach based on Warren and Salmon [30]. Fig. 5(a) shows that we define a drawing plane by bounding coordinates  $(x_{max}, y_{max})$  and  $(x_{min}, y_{min})$ . This bounding plane is the root of the quad-tree. Next, we split the bounding plane (root) into four equal blocks, each of which represents a child node (shown by a unique color) of the root. We sort the positions of all vertices based on Morton code (also called Z-order curve) [25] and obtain an ordered

### Algorithm 3: Greedy Initialization

```

Input: G(V, E)
1:  $S = \emptyset$                                  $\triangleright$  initial empty stack
2:  $visited = [False, \dots, False]$ 
3:  $V_0 = (0, 0)$                              $\triangleright V_{0x} = 0, V_{0y} = 0$ 
4:  $S.push(V_0)$ 
5:  $visited_0 = True$ 
6: while  $S$  is not empty do
7:    $U = S.pop()$ 
8:    $d = \frac{360}{deg(U)}$ 
9:    $D = 0$ 
10:  for  $u$  neighbors of  $U$  do
11:    if  $visited_u$  is False then
12:       $V_u = (U_x + \cos(\frac{PI*D}{180.0}), U_y + \sin(\frac{PI*D}{180.0}))$ 
13:       $visited_u = True$ 
14:       $S.push(V_u)$ 
15:       $D = D + d$ 
16:  return  $V$ 

```

list of vertices. This ordering basically represents the Depth First Search (DFS) traversal of a quad-tree. Fig. 5(b) shows a possible Morton ordering of a graph with 16 vertices. From the Morton ordering, we recursively build the tree, where leaf nodes represent vertices in the graph, and an internal node stores its centroid which is computed from its children.

Basically, nearer non-adjacent vertices contribute more than far vertices to  $f_r$ . For this reason, a threshold ( $\theta$ ) is set to determine relative distance between two vertices while calculating  $f_r$ . A Multipole Acceptance Criteria (MAC),  $\theta > \frac{D}{\|c_i - c_j\|}$  is evaluated while traversing the tree for  $f_r$  calculations. If MAC is *true* then two vertices are located at far enough distance such that the current centroid can approximate that repulsive force. In this way,  $f_r$  calculation can be approximated in  $O(\log n)$  time which has been discussed elaborately in [12, 15], thus, the overall running time becomes  $O(n \log n)$  per iteration. We use the term BatchLayoutBH to describe this version and provide full pseudo-code in supplementary file, Algorithm S3.

### 3.6 Initialization

The initial layout plays an important role in the convergence and visualization quality of the final layout. Many existing algorithms start with random layouts where vertex positions are assigned randomly [15]. In addition to random initial layouts, we developed a novel greedy initialization technique as shown in Algorithm 3. We maintain a *stack*  $S$  to keep track of visited vertices. The algorithm starts with a randomly selected vertex  $V_0$ , and  $(0, 0)$  is used as its coordinate. In every iteration, the algorithm extracts a vertex  $U$  from the stack and places  $U$ 's neighbors on a unit circle centered at  $U$  (line 10 – 14 in Algorithm 3). Additionally, the distance between every pair of  $U$ 's neighbors is also equal. In line 12,  $PI$  is a constant whose value is 3.1416. In section 3.7, we will discuss that uniform edge length is a good aesthetic metric for assessing the quality of a layout. Hence, we consider this unit-radius approach to be a better initializer. Our experimental results does show that the greedy initialization indeed converge faster than the random initialization. The greedy approach is similar to a DFS with  $O(n+m)$  complexity.

### 3.7 Performance Metrics

There are several aesthetic metrics in the literature to assess the quality of a layout generation algorithm [8, 21, 28]. Though there does not exist a single metric that can uniquely determine the effectiveness of a layout, it gives a quantitative value that tells us how layout algorithms perform. For the sake of completeness, we have used the following aesthetic metrics along with running time analysis.

**Stress (ST):** Stress computes the difference between geometric distance and graph theoretic distance for any pair of vertices in a

graph [5,8]. Since different algorithms may require different drawing space based on which stress may change, the drawing space is scaled before computing the stress for all algorithms. Thus, the reported value is the minimum value achievable after scaling the layout. A lower ST value indicates a better layout, and it is computed using  $\sum_{i,j \in V} w_{ij} (\|c_i - c_j\| - d_{ij})^2$  where,  $c_i$  and  $c_j$  are coordinates of vertices  $i$  and  $j$ , respectively. The value of  $d_{ij}$  represents a graph theoretic distance between vertices  $i$  and  $j$ , and  $w_{ij} = \frac{1}{d_{ij}^2}$ .

**Edge Uniformity (EU):** Sometimes the uniformity of edge lengths represents better readability of a layout [16]. We define EU in a similar way as defined in [8, 13]. It finds the normalized standard deviation of edge length which can be computed using  $\sqrt{\frac{\sum_{e \in E} (l_e - l_\mu)^2}{|E| \cdot l_\mu^2}}$  where,  $l_\mu$  is the average length of all edges and  $|E|$  represents the number of edges in the graph. Lower value of EU represents better quality of the layout.

**Neighborhood Preservation (NP):** This measure computes the number of neighbors that are close to a vertex in a graph as well as in the driven layout. It has been used to compare graph layouts in many tools like tsNET [20] and Maxent [10]. This is a normalized similarity measure where 0 means that the adjacency of vertices are not preserved in the layout whereas 1 means that all vertices preserve their respected neighbors.

## 4 RESULTS

### 4.1 Experimental Setup

We implemented BatchLayout in C++ with multithreading support from OpenMP. We ran all experiments in a server consisting of Intel Xeon Platinum 8160 processors (2.10GHz) arranged in two NUMA sockets. The system has 256GB memory, 48 cores (24 cores/socket), and 32MB L3 cache/socket. For comparison, we used ForceAtlas2 implemented in Gephi and OpenOrd from its GitHub<sup>1</sup> repository. We also experimented with Gephi's OpenOrd implementation (denoted by OpenOrdG), but found that the GitHub implementation produced better visualizations. We use Gephi's toolkit v0.9.2 [4] to run multi-threaded ForceAtlas2 and OpenOrdG. The Barnes-Hut variant of ForceAtlas2 is termed as ForceAtlas2BH. For BatchLayoutBH, we use the same threshold value (1.2) for MAC as in ForceAtlas2BH. Unless otherwise specified, we use the default settings for all of these tools. For OpenOrd, we set the edge-cutting parameter to 0, and we set the default time distribution in different stages of simulated annealing. While the BatchLayout software includes four variants of  $(a, r)$ -energy model considered in the ForceAtlas2 paper, we only show results from BatchLayout using  $(2, -1)$ -energy model (equivalent to the Fruchterman and Reingold algorithm). We selected this model because it usually generates better visualizations [17].

### 4.2 Datasets

Table 1 shows a diverse collection of test graphs including small world networks, scale free networks, mesh, structural problem, optimization problem, linear programming, economic problem, social networks, and road networks. These graphs were collected from the SuiteSparse matrix collection (<https://sparse.tamu.edu>). Specifically, we use Luxembourg OSM (lxOSM) and Flan\_1565 datasets to test the ability of algorithms to visualize large-scale graphs. For scalability experiments, we also generated benchmark networks by Lancichinetti et al. [22]. For this tool, we set mixing parameter, avg. degree, community minsize, community maxsize to 0.034, 80, 128 and 300, respectively, and vary maximum degree from 100 to 300. We generated a total of 5 random graphs, each of which has  $2^v$  vertices, where  $v \in \{16, 17, 18, 19, 20\}$ .

<sup>1</sup><https://github.com/SciTechStrategies/OpenOrd>

Table 1: Graphs used in our experiments.  $|V|$  and  $|E|$  represent number of vertices and edges, respectively. d means average degree.

Graph	$ V $	$ E $	d	Type
Powergrid	4,941	13,188	2.66	Small World
add32	4,960	19,848	3.00	Scale Free
ba_network	6,000	5,999	1.99	Scale Free
3elt_dual	9,000	26,556	2.95	Mesh
PGP	10,680	48,632	4.55	Scale Free
pkustk02	10,800	399,600	76	Feiyue twin tower
fe_4elt2	11,143	65,636	5.89	Mesh
bodyy6	19,366	134,208	5.93	NASA Mat.
pkustk01	22,044	979,380	45.42	Beijing bot. exhib. h.
OPF_6000	29,902	274,697	8.66	Ins. of Pow. Sys. G. U.
finance256	37,376	298,496	6.98	Lin. Prog.
finan512	74,752	261,120	6.98	Eco. Prob.
IxOSM	114,599	239,332	2.08	Op. St. Map
comYoutube	1,134,890	5,975,248	5.26	Social Net.
Flan_1565	1,564,794	114,165,372	71.95	Struc. Prob.

### 4.3 Algorithmic Options and Their Impacts

#### 4.3.1 Initialization

At first, we compare the differences between greedy initialization using Algorithm 3 and random initialization using the C++ function `rand`. Both initialization techniques generate random coordinates within a given range  $[-\text{MAXMIN}, \text{MAXMIN}]$ . We feed initial coordinates to BatchLayout, and Fig. 6(a) shows the layout energy for the *3elt\_dual* graph after various iterations. We observe that in the  $t^{\text{th}}$  iteration, the greedy-initialized layout has lower energy than the randomly-initialized layout. This result is consistent across other graphs as well. Fig. 6(a) reveals that greedy initialization accelerates the convergence of BatchLayout. Hence, we use the greedy initialization in subsequent experiments in the paper (both initializations are available in our software).

#### 4.3.2 Minibatch size

We now select a minibatch size that provides enough parallelism without affecting the convergence rate significantly. Fig. 6(b) shows layout energies for different minibatches for the *pkustk02* graph. Here, BL256 denotes BatchLayout with minibatch size of 256. For this graph, we observe that small minibatches converge faster (in terms of iterations) than large minibatches. For example, BL1 and BL256 achieve the same energy at 1000 and 1100 iterations, respectively (marked by brown squares in Fig. 6(b)). Consequently, BL256 needs to run 100 more iterations to reach the same energy achieved by BL1. However, the cost of extra iterations is offset by faster parallel runtime of BL256. Fig. 6(c) shows the runtime for different minibatches when we run BatchLayout with 48 threads for the same graph *pkustk02*. To achieve the energy level marked by brown squares, BL1 takes around 433.6 seconds, whereas BL256 takes only 9.7 seconds. That is, BL256 is  $\sim 40 \times$  faster than BL1 despite the former taking 100 more iterations than the latter to reach the same layout energy. For this reason, minibatches play a central role in attaining high performance by our parallel algorithm. However, using larger minibatches beyond 256 does not improve the performance further. For example, BL256 and BL2048 behave almost similarly as can be seen in Figs. 6(b) and 6(c). We also observe similar minibatch profiles for other graphs. Hence, we set 256 as the default minibatch size for the rest of our experiments (a user can change the minibatch size in our software).

#### 4.3.3 Batch Randomization

We conduct experiments for the randomization of batches which is a common practice in machine learning. We reported standard deviation as well as energy curves for different runs using *3elt\_dual* graph (Figure S3 in supplementary file). From our experiments, we observed that randomized batch selection achieves better energy curve than fixed sequential batch selection but results are not deterministic i.e., for each run, output is different (similar but not same) even though all hyper-parameters are same. We can see a greater difference for a smaller number of iteration. Randomization also introduces overhead in total running time for reshuffling and it can not take advantage of optimal cache usage. Moreover, randomized batch selection does not make any significant improvement in layout. So, we keep sequential batch selection as our default option in BatchLayout.

#### 4.3.4 Number of threads

Using more threads not necessarily translates to faster runtime, especially for small graphs. This is because of the overhead of thread *fork-join* model used in OpenMP. For larger graphs with more than 100,000 vertices, we recommend employing all available threads. For smaller graphs, users can consider using fewer threads. Hence, the number of threads is provided as an option in BatchLayout.

#### 4.3.5 Step length or learning rate

Line 14 of Algorithm 1 uses a multiplicative factor for *Step*, which dictates both convergence and the quality of the final layout. Intuitively, we want to adapt the step length so that the algorithm takes bigger steps in the beginning but increasingly smaller steps as we move closer to the minimum energy. Fig. 7(a) shows layout energies at convergence with various multiplicative factors for the *3elt\_dual* graph. We observe that the factor of 0.999 (that is 0.1% reduction of step length in every iteration) provides the best balance as the converged energy is minimum for this factor. Hence, we use this multiplicative factor in all experiments with BatchLayout. We note that adaptive steps are also used in other force-directed layouts such as in the algorithm developed by Yifan Hu [15]. In this paper, we empirically identify an effective adaption policy so that BatchLayout converges faster.

### 4.4 Running Time and Scalability

We run all algorithms for 500 iterations using 47 threads (one thread per core) and report their runtime in Table 2. Since OpenOrdG leaves out a core for kernel-specific tasks, we also leave a core unutilized for fairness in comparison. In Table 2, we include time for all steps except file input/output operations. If we consider exact force calculations, BatchLayout is on average  $15 \times$  faster than ForceAtlas2 (min:  $7.6 \times$ , max:  $21.8 \times$ ). For approximate force calculation, BatchLayoutBH is on average  $2 \times$  faster than OpenOrd (min:  $1.4 \times$ , max:  $9.5 \times$ ). Both BatchLayoutBH and OpenOrd are faster than ForceAtlas2BH. Notice that OpenOrdG (that is Gephi’s OpenOrd) also runs fast, but its layout quality is very low as seen in the next section. Overall, BatchLayoutBH is almost always the fastest algorithm as denoted by bold numbers in Table 2. We also tested tsNET [20] to conduct some experiments (with perplexity and learning rate set to 800 and 6000, respectively). We observe that tsNET can generate good layouts for small graphs, but failed to generate any layout for medium or bigger graphs in our experiments. For small graphs like *3elt\_dual*, tsNET took around 1 hour whereas other multi-threaded tools generated layout within few seconds. Hence, we did not include tsNET runtimes in Table 2.

**Thread and vertex scalability.** We now discuss the thread and vertex scalability of the three fastest algorithms: BatchLayoutBH (FLBH), ForceAtlas2BH (FA2BH), and OpenOrd (OO). As discussed in section 4.2, we use a benchmark tool [22] to generate graphs for the scalability experiment. Fig. 7(b) shows the thread

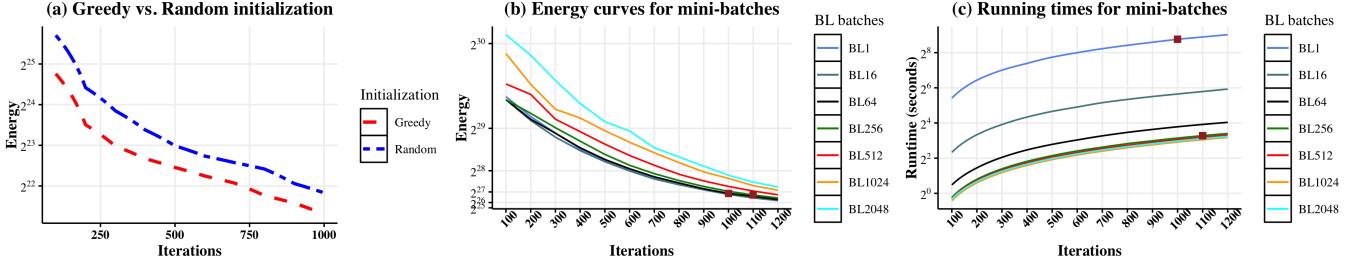


Figure 6: (a) Energy curves for greedy and random initialization (*3elt\_dual* graph). (b) Energy curves (log scale) for different batches (*pkustk02* graph). (c) Running time (log scale) for different batches of BatchLayout (*pkustk02* graph).

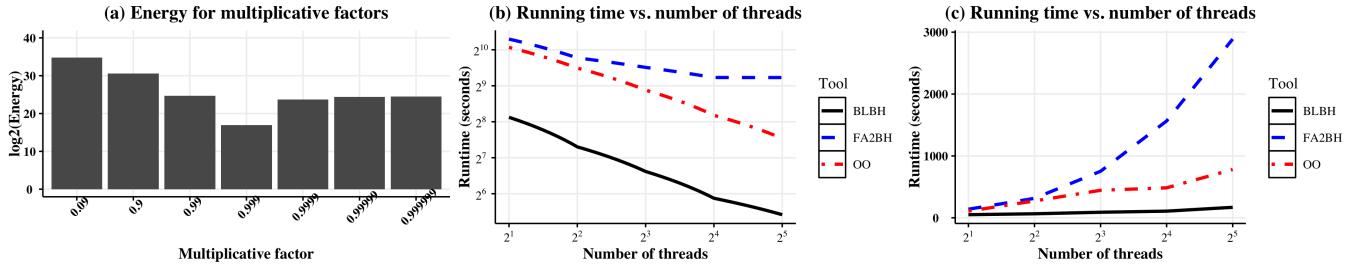


Figure 7: (a) Energy values (log scale) for different multiplicative factor of *Step* using *3elt\_dual* graph. (b) Strong scaling: Running time (log scale) for different number of threads using a synthetic graph of  $2^{18}$  vertices generated by [22]. (c) Weak scaling: Running time (seconds) for different number of threads and graph sizes.

scalability for a graph with  $2^{18}$  vertices. We observe that both BatchLayoutBH and OpenOrd scale linearly with threads (cores), but BatchLayoutBH is  $\sim 4 \times$  faster than OpenOrd on all thread counts. ForceAtlas2BH does not scale linearly, and it is generally not faster than OpenOrd. Fig. 7(c) shows the weak scalability where we use increasingly larger graphs varying number of threads. Fig. 7(c) starts with a graph of  $2^{16}$  vertices running on 2 threads. Then, we increasingly double the problem size as well as computing resource, run experiments and report the runtime. Among all three algorithms, we observe that BatchLayoutBH’s work distribution remains balanced (i.e., curve remains horizontal). Hence, BatchLayoutBH shows superior weak scaling performance compared to other two algorithms.

**Memory usage.** We measured the memory consumption of all algorithms using the `memory-profiler` python package. For the *IxOSM* graph, BatchLayout, BatchLayoutBH, ForceAtlas2, ForceAtlas2BH, OpenOrdG, and OpenOrd consumed maximum memory of 16.36MB, 30.34MB, 676.38MB, 1348.38MB, 1265.32MB and 14.12MB, respectively. Gephi’s ForceAtlas2, ForceAtlas2BH and OpenOrdG consumed higher memory than others possibly because of the high memory requirements of Gephi itself. In fact, ForceAtlas2BH failed to generate layouts for two large graphs due to its high memory consumption. In our algorithm, Barnes-Hut force approximation requires additional space to store the quad-tree data structure, and hence it consumed more memory than the exact force computation. Overall, BatchLayoutBH can compute layouts faster without consuming significant memory.

#### 4.5 Visualization

We now visualize the layouts generated by all algorithms considered in the paper. At first, we run BatchLayout until convergence (when the energy difference between successive iterations becomes less than  $10^{-6}$ ). Then, all other algorithms including BatchLayoutBH were run for the same number of iterations that BatchLayout took to converge. The visualization from these layouts are shown in Table 3. The runtime and converged energy are provided in supplementary Table S2, which shows that BatchLayoutBH runs much faster than other tools. Table 3 demonstrates that BatchLayout and BatchLayoutBH produce readable layouts that are comparable or better than ForceAtlas2 and ForceAtlas2BH, respectively. Generally,

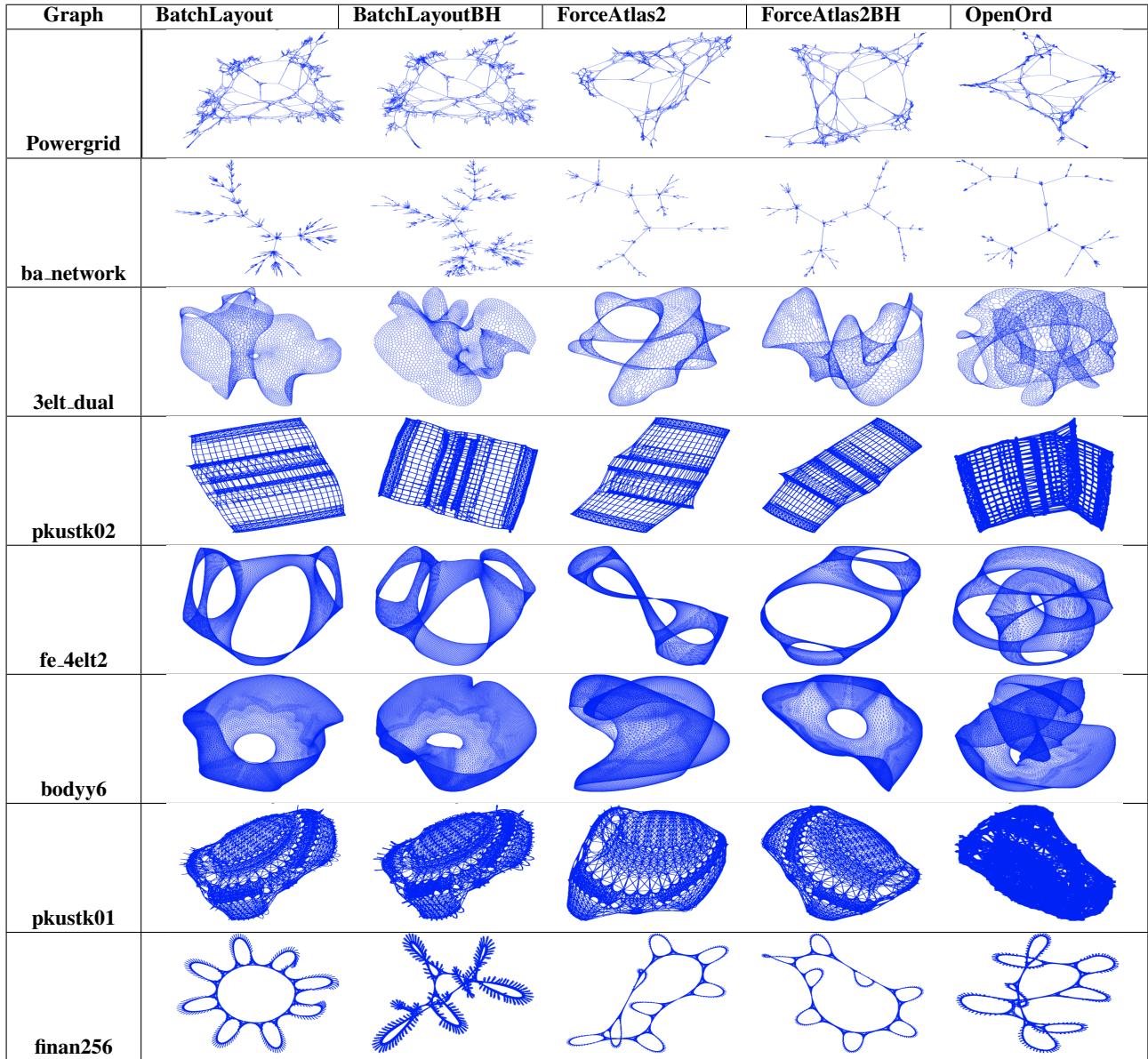
Table 2: Runtime for different methods. Lower value represents better result. BL, BLBH, FA2, FA2BH, OOG and OO represent BatchLayout, BatchLayoutBH, ForceAtlas2, ForceAtlas2BH, OpenOrdG and OpenOrd, respectively. Better results are shown in bold font.

Graph	Running time (seconds)					
	BL	BLBH	FA2	FA2BH	OOG	OO
Powergrid	<b>0.95</b>	0.98	20.73	3.04	1.67	1.43
add32	1.03	<b>1.02</b>	20.75	3.15	1.71	1.45
ba_network	1.42	<b>1.22</b>	25.47	3.61	2.18	1.78
3elt_dual	2.92	<b>1.54</b>	54.6	4.87	3.17	2.67
PGP	4	<b>1.83</b>	68.67	6.26	4.18	3.26
pkustk02	4.23	<b>1.98</b>	76.69	10.72	10.32	3.83
fe_4elt2	4.33	<b>1.86</b>	75.4	6.34	4.21	3.35
bodyy6	12.3	<b>2.95</b>	143.08	9.69	6.75	5.85
pkustk01	16.23	<b>3.43</b>	242.72	18.49	15.33	7.25
OPF_6000	29.75	<b>4.69</b>	266.96	18.31	12.45	8.95
finance256	45.49	<b>5.4</b>	412.6	20.37	14.69	11.18
finan512	185.8	<b>10.53</b>	1408	46.98	31.74	22.57
IxOSM	-	<b>21.59</b>	-	72.30	-	33.62
comYout.	-	<b>174.5</b>	-	1504.7	-	1670.52
Flan_1565	-	<b>224.3</b>	-	-	-	609.56

BatchLayoutBH and ForceAtlas2BH layouts are much better than OpenOrd. Gephi’s OpenOrd generates inferior layouts; hence, we did not show them in Table 3. Note that BatchLayout uses Fruchterman and Reingold as its default energy model, which is different from the default model in ForceAtlas2. Hence, BatchLayout’s superior layouts for some graphs (e.g., finan256) are not surprising because the FR model usually generates better layouts as was also reported in the ForceAtlas2 paper [17]. ForceAtlas2 does not use FR as its default model possibly because of its higher computational cost (also reported in [17]). In this paper, we show that the FR algorithm runs much faster in our BatchLayout framework and can generate better layouts for some graphs.

In the scalability and runtime experiments, we showed results with a fixed 500 iterations. Supplementary Table S1 shows the visualization from all algorithms after 500 iterations. In many cases, layouts after 500 iterations are close to the layouts at convergence.

Table 3: Layouts of different graphs generated by different algorithms. BatchLayout convergence criteria was set to 1E-6. Number of iterations and running time are shown in supplementary file, Table S2.



Hence, many algorithms including BatchLayout provide “number of iterations” as an option to users.

Both BatchLayout and ForceAtlas2 provide many options beside default options used in our experiments. It is possible that tuning these parameters will generate better-quality layouts for some graphs. However, we stick to default parameters in this paper for simplicity and fairness. OpenOrd may generate different layouts for the same graph when different numbers of threads are used. By contrast, BatchLayout and BatchLayoutBH generate the same layout for a given graph irrespective of number of threads.

Finally, Fig. 8 shows the layout of *Flan\_1565*, the largest graph in our dataset. We show layouts from BatchLayoutBH and OpenOrd after running them for 5000 iterations (ForceAtlas2BH went out of memory for this graph). For this graph, OpenOrd’s layout is not good enough even though we allow both algorithms to run for 5000 thousands of iterations. Furthermore, OpenOrd took 1 hour and 39 minutes whereas BatchLayoutBH took 49 minutes to

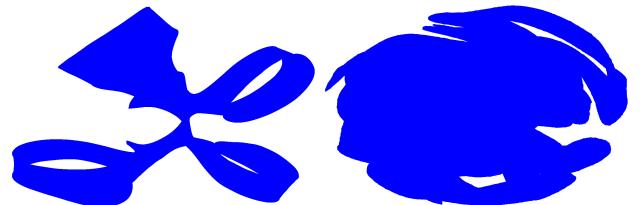


Figure 8: Layouts of *Flan\_1565* graph (1.56M vertices and 114.2M edges) generated by BatchLayoutBH (left) and OpenOrd (right).

finish 5000 iterations. This clearly demonstrates the effectiveness of BatchLayoutBH to visualize graph with hundreds of millions of edges. However, generating layouts in half an hour may not be good enough for certain applications. We plan to address this by implementing BatchLayoutBH for distributed systems.

Table 4: Comparison of Stress (ST) and Neighborhood preservation (NP) measures among all tools. For ST, a lower value means a better result and for NP, a higher value represents a better result. Better results are shown in bold font.

Graph	ST						NP					
	BL	BLBH	FA2	FA2BH	OO	OOGH	BL	BLBH	FA2	FA2BH	OOG	OO
Powergrid	<b>1.3E+6</b>	1.6E+6	2.4E+6	2.2E+6	2.5E+6	2.4E+6	0.39	0.266	0.403	<b>0.408</b>	0.324	0.336
add32	2.2E+6	<b>1.9E+6</b>	5.3E+6	5.4E+6	6.2E+6	2.3E+6	0.453	0.347	<b>0.538</b>	0.523	0.362	0.409
ba_network	2.8E+6	<b>2.5E+6</b>	3.4E+6	3.4E+6	3.9E+6	2.9E+6	0.357	0.295	0.457	0.452	0.384	<b>0.465</b>
3elt_dual	<b>5.6E+6</b>	6.6E+6	9.2E+6	8.0E+6	1.4E+7	1.1E+7	<b>0.38</b>	0.271	0.253	0.334	0.171	0.174
PGP	<b>8.2E+6</b>	1.2E+7	9.9E+6	9.8E+6	1.1E+7	8.8E+6	0.18	0.119	0.24	0.243	<b>0.266</b>	0.231
pkustk02	<b>3.7E+6</b>	4.4E+6	1.0E+7	1.1E+7	1.9E+7	8.1E+6	<b>0.649</b>	0.583	0.549	0.544	0.38	0.513
fe_4elt2	<b>1.0E+7</b>	1.7E+7	1.2E+7	1.5E+7	2.1E+7	1.8E+7	<b>0.36</b>	0.25	0.319	0.289	0.222	0.207
bodyy6	<b>3.1E+7</b>	4.1E+7	3.4E+7	4.3E+7	6.1E+7	5.2E+7	0.213	0.205	<b>0.278</b>	0.251	0.217	0.124
pkustk01	<b>1.7E+7</b>	1.9E+7	1.8E+7	2.0E+7	4.1E+7	3.1E+7	0.513	0.398	<b>0.528</b>	0.523	0.373	0.396

Table 5: Comparison of Edge uniformity (EU) measures among all tools. For this measure, lower value means better result. Better result of each graph is shown in bold font.

Graph	EU					
	BL	BLBH	FA2	FA2BH	OOG	OO
Powergrid	0.83	<b>0.5</b>	1.46	1.33	1.93	1.18
add32	1.38	<b>1.04</b>	1.30	1.26	1.86	1.76
ba_network	1.07	<b>0.58</b>	3.23	3.30	3.01	2.93
3elt_dual	0.39	<b>0.39</b>	0.59	0.63	1.37	0.53
PGP	0.81	<b>0.65</b>	1.43	1.42	2.44	1.43
pkustk02	0.73	<b>0.67</b>	1.09	1.12	1.93	0.88
fe_4elt2	<b>0.39</b>	0.53	0.53	0.51	1.55	0.48
bodyy6	0.76	0.80	<b>0.49</b>	0.51	1.64	0.79
pkustk01	0.83	<b>0.70</b>	1.01	1.02	1.62	0.93

## 4.6 Comparison of Aesthetic Metrics

We quantitatively measure the quality of graph layouts using standard aesthetic metrics discussed in Section 3.7. Similar to the runtime analysis, we measure aesthetic quality of layouts after running each algorithm for 500 iterations and report the numbers in Tables 4 and 5. There is no clear winner according to all metrics, which is expected since different algorithms optimize different energy functions. Table 4 shows ST (lower is better) and NP measures (higher is better), where BatchLayout and BatchLayoutBH are better than other tools for ST measure and competitive with their peers for NP measure. Table 5 shows EU (lower is better), where we observe that BatchLayoutBH is winner for most of the graph instances. Overall, BatchLayout and BatchLayoutBH perform better according to ST and EU measures and competitive according to NP measure.

## 5 RELATED WORK

Graph drawing is a well studied problem in the literature. In addition to the spring-electrical approach, there are other models such as the spectral method [19], and high-dimensional embedding [14]. Recently, Kruiger et al. [20] introduced tsNET based on stochastic neighbor embedding technique. Due to space limitation, we restrict our discussion to force-directed approaches.

Kamada and Kawai [18] proposed a spring model where optimal drawing is obtained by minimizing stress. In their model, stress is represented by the difference between geometric distance and graph-theoretic shortest path distance. The computational complexity of this model is high, and quad-tree based force approximation can not be applied to it [15]. Other graph drawing algorithms that optimize a stress function also produce readable layouts [10, 24], but they are generally prohibitively expensive for large graphs.

Fruchterman and Reingold (FR) proposed a spring-electrical model where an energy function is defined in terms of *attractive* and *repulsive* forces [9]. This model is computationally faster than the spring model, and quad-tree based force approximation can also be applied. Yifan Hu introduced a multi-level version of force-

directed algorithm [15]. In his approach, repulsive force can be approximated by a quad-tree data structure, resulting in one of the fastest sequential algorithms. Other efficient multi-level algorithm includes *FM*<sup>3</sup> [12] which was later integrated in OGDF library [7]. Interestingly, Andreas Noack introduced the LinLog energy model which is very effective in capturing clusters in the graph layout [26].

OpenOrd [23] by Martin et al. is a parallel multi-level algorithm following the force-directed model. OpenOrd is reasonably fast and generate layouts of large-scale graphs. Finally, ForceAtlas2 by Jacomy et al. introduced a more general framework for graph layout combining various features (repulsive force approximation by Barnes-Hut approach, LinLog mode, etc.) [17]. ForceAtlas2 is known to generate continuous layouts as a user can stop the program anytime with valid (possibly unoptimized) layouts. Our work is influenced by ForceAtlas2 and covers many features available in ForceAtlas2. The ForceAtlas2 paper [17] demonstrated that FR model can generate readable layouts but it can be slower than other energy models. In this paper, we addressed this challenge with a multi-threaded cache-efficient algorithm that is both faster and generates readable layouts.

Recently, Zheng et al. used a sequential SGD approach for graph drawing, which optimizes stress to generate the layout of a graph [32]. However, their software is very slow as it considers graph theoretic distances in the optimization function. Both ForceAtlas2 and OpenOrd can run in parallel and are available in Gephi’s toolkit [4]. Force-directed algorithm has also been implemented in distributed platforms [1, 2] as well as for GPUs [6]. These tools cannot be run on any server because they rely on special hardware. We did not compare with these implementations because our objective in this paper is to develop a general-purpose and fast algorithm for multicore servers.

## 6 DISCUSSIONS AND CONCLUSIONS

In this paper, we present a parallel force-directed algorithm BatchLayout and its Barnes-Hut approximation BatchLayoutBH for generating 2D layouts of graphs. The presented algorithms are highly scalable, robust with respect to diverse classes of graphs, can be run on any multicore computer, and runs faster than other state-of-the-art algorithms. Aside from carefully chosen default hyper-parameters (e.g., initialization, minibatch size, energy model, learning rate, convergence conditions), BatchLayout provides flexibility to let users choose hyper-parameters that are suitable for the graph. In terms of flexibility, BatchLayout is comparable to the flexibility of ForceAtlas2 and can be integrated with any visualization tool.

The high performance of BatchLayout comes from two important optimizations that we made. First, BatchLayout exposes more parallel work to keep many processors busy. More parallel work comes from our minibatch scheme, which is also a widely-used technique in training Deep Neural Networks. Second, we incorporate cache-blocking technique commonly used to optimize linear algebra operations. Hence, this paper unifies two proven techniques from

machine learning and linear algebra and delivers a high-performance graph layout algorithm. As a result, BatchLayout is highly scalable and runs significantly faster than other state-of-the-art tools.

BatchLayout generates graph layouts without sacrificing their aesthetic qualities. We visually and analytically verified the quality of layouts for different classes of graphs covering grid networks, small world networks, scale-free networks, etc. In all cases, BatchLayout generates good layouts that are similar or better than its peers.

BatchLayout is implemented as an open-source software with detailed documentation. This software depends on standard C++ and OpenMP libraries and is portable to most computers. We use simple Python scripts to visualize layouts generated by our software. Hence, we believe that BatchLayout will benefit many users in visually analyzing complex networks from diverse scientific domains.

While this paper only focuses on parallel force-directed algorithms, other classes of algorithm might generate better layouts than the algorithms considered in this paper. For example, tsNET can generate better quality layouts when it converges. However, such tools are often prohibitively expensive for large-scale graphs. For example, tsNET took 16 hours to generate a layout for *OPF\_6000*, whereas BatchLayout generates a comparable layout in just few seconds. In fact, users can set a very low threshold in BatchLayout, decrease the batch size and increase number of iterations to get superior layouts from our software (e.g., see supplementary Table S4). We also compared our results with *FM<sup>3</sup>* available in OGDF library and found that BatchLayoutBH is always faster than *FM<sup>3</sup>* for non-engineered settings in OGDF library (see supplementary Table S9 for runtime details). For two large graphs of Table 1, BatchLayoutBH is approximately 5× faster than *FM<sup>3</sup>*.

This paper only considered shared-memory parallel algorithms since multicore computers and servers are prevalent in scientific community. However, BatchLayout can be easily implemented for GPUs and distributed-memory systems. A distributed algorithm will require larger minibatches if only “data parallelism” is used. However, if we also use graph partitioning (which is equivalent to model parallelism in deep learning), we can have enough parallelism for large-scale distributed systems. Expanding BatchLayout for other high-performance architectures and improving the aesthetic quality for massive graphs remain our future work.

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