

1a)

| i | j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|----|----|----|---|----|---|----|----|----|----|----|
| 0 | 10 | 11 | 12 | 4 | 13 | 2 | 5 | 11 | 5 | 16 | 14 |
| 1 | 7 | 11 | 12 | 4 | 13 | 2 | 5 | 11 | 5 | 16 | 14 |
| 1 | 7 | 11 | 5 | 4 | 13 | 2 | 5 | 11 | 12 | 16 | 14 |
| 3 | 6 | 11 | 5 | 4 | 13 | 2 | 5 | 11 | 12 | 16 | 14 |
| 3 | 6 | 11 | 5 | 4 | 11 | 2 | 5 | 13 | 12 | 16 | 14 |
| 6 | 5 | 11 | 5 | 4 | 11 | 2 | 5 | ! | 13 | 12 | 16 |
| Final | 5 | 5 | 5 | 4 | 11 | 2 | 11 | 13 | 12 | 16 | 14 |

1b)

| lb | pivPos | ub | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|--------|----|---|---|---|----|----|----|----|----|----|----|
| 0 | 5 | 10 | 5 | 5 | 4 | 11 | 2 | 11 | 13 | 12 | 16 | 14 |
| 0 | 2 | 5 | 4 | 2 | 5 | 11 | 5 | 11 | 13 | 12 | 16 | 14 |
| 0 | 1 | 2 | 2 | 4 | 5 | 11 | 5 | 11 | 13 | 12 | 16 | 14 |
| 3 | 4 | 5 | 2 | 4 | 5 | 5 | 11 | 11 | 13 | 12 | 16 | 14 |
| 6 | 7 | 10 | 2 | 4 | 5 | 5 | 11 | 11 | 12 | 13 | 16 | 14 |
| 8 | 9 | 10 | 2 | 4 | 5 | 5 | 11 | 11 | 12 | 13 | 14 | 16 |

1c)

The problem arises when there are long sequences of the same number, resulting in a `StackOverflowError`. With the less/more than or equal to requirement for *i* and *j*, the maximum number of function calls on the stack would be something like $\log(n)$ with *n* being the length of the same-number-sequence. While for the strictly less or more requirement, it would be *n*. The former requirement ultimately returns a `pivotPosition` somewhere in the middle of the sequence, while the latter returns `lb`. The problem then arises when `quickSort(arr, pivotPosition + 1, ub)` is called repeatedly as it goes through the same-number-sequence one by one position.

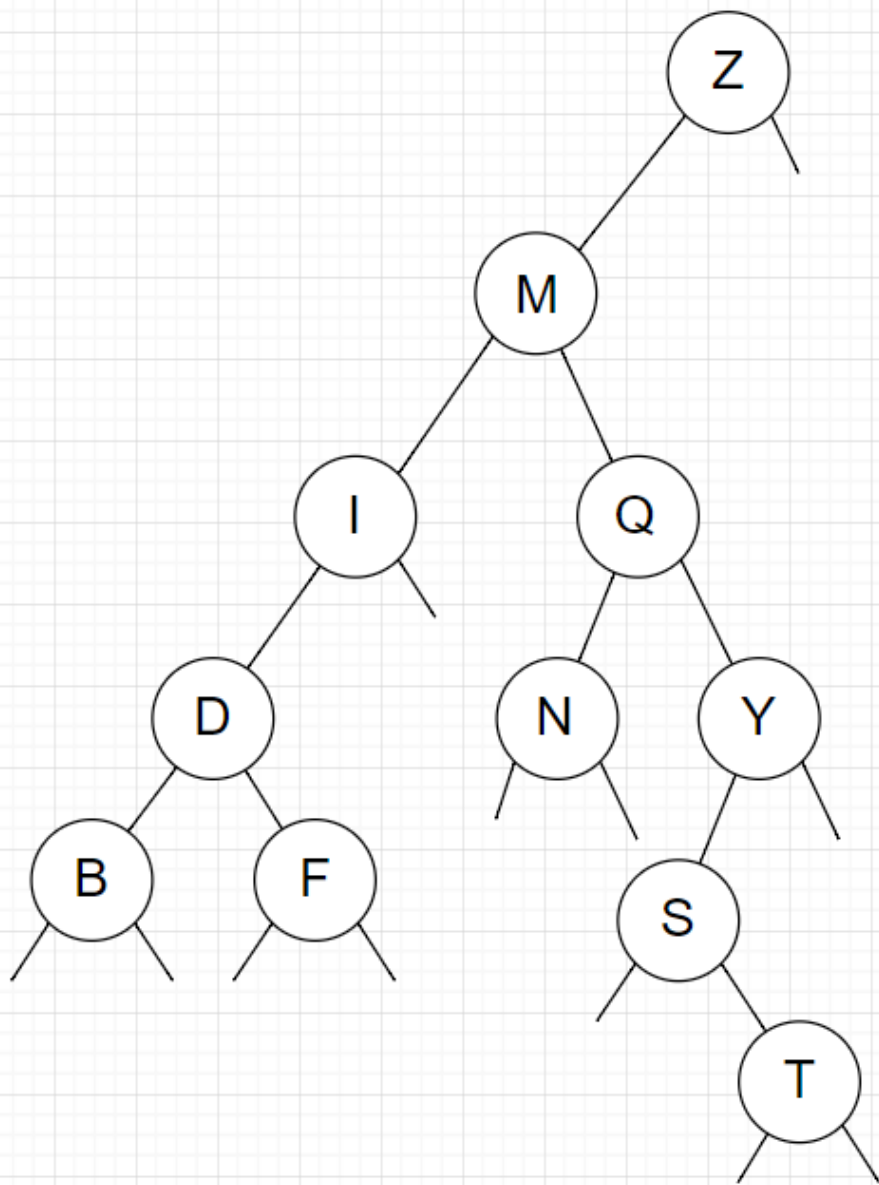
2a)

[null, T, S, R, N, P, O, A, E, I, G, H]

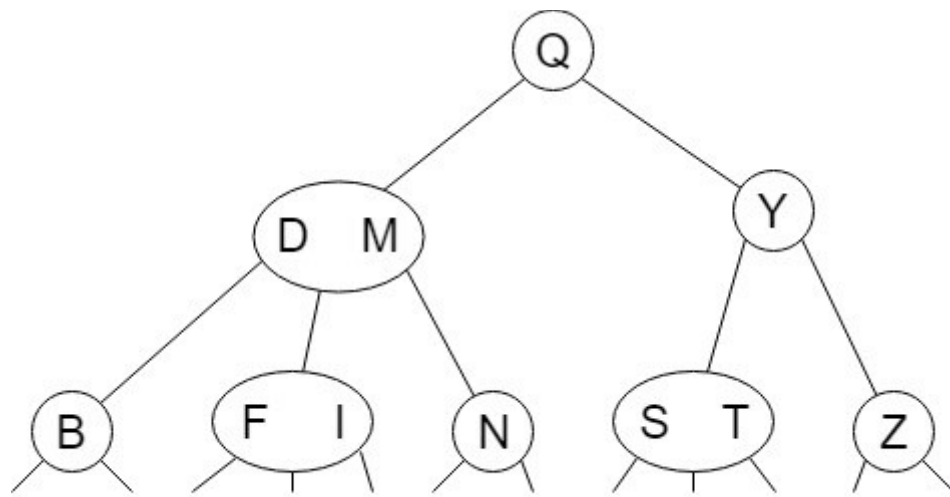
2b)

If only one maximum/minimum value is tracked, this value would of course change when the maximum/minimum is removed. Then one would have to find a new maximum/minimum, which in the worst case would take linear time for each remove. It would then be better to keep the data structure sorted in the first place.

3a)



4a)



4b)

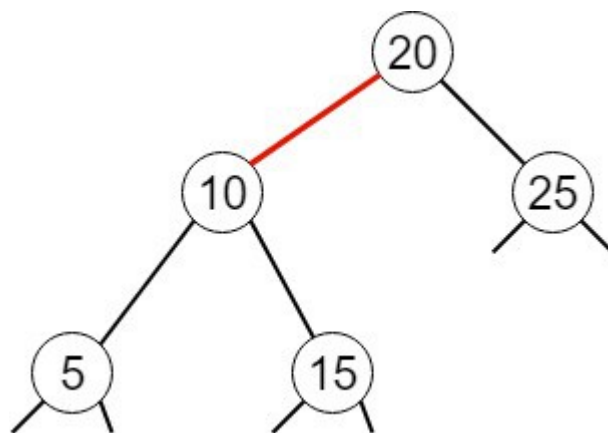
B F Y N S T I D

4c)

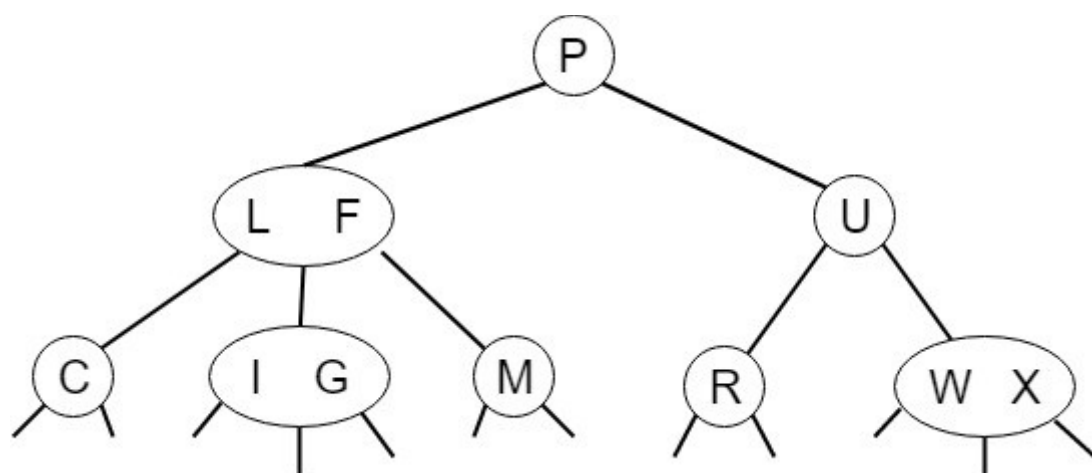
(iii) and (iv)

4d)

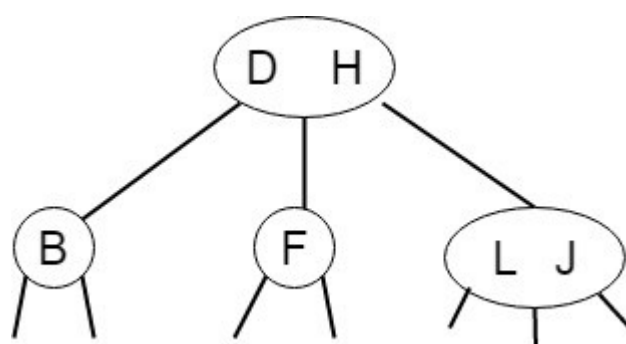
(ii)



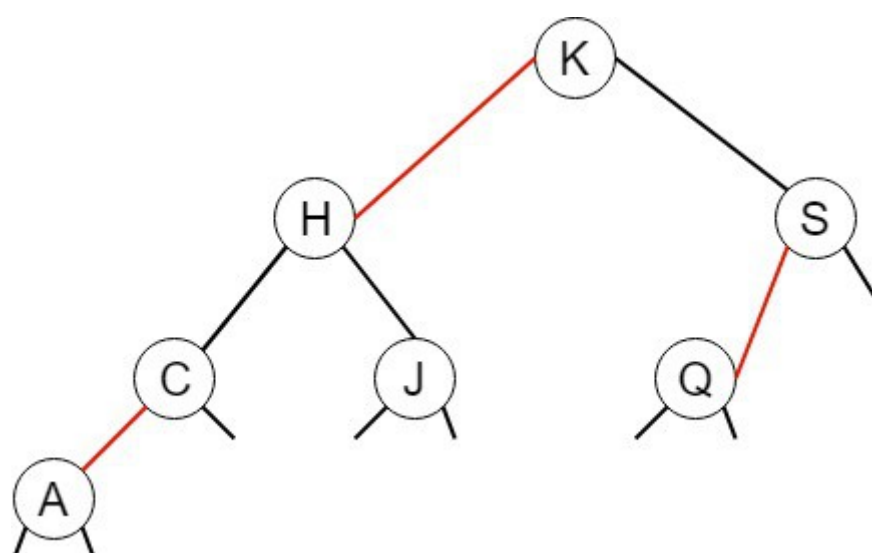
(iii)



(iv)



(v)



4e)

L R F F R F R F R F L

4f)

The maximum depth increments by 2 for every increment in height, starting at maximum depth 2 for height 0. Red-black trees are perfectly balanced. This means that the minimum number of nodes n for an increment in height k is given by $2^{k+1} - 1$. Therefore, height is a function of n by the expression $\lg_2(n + 1) - 1$. The maximum number of nodes from root to null in a red-black tree of size n is then:

$$2 + 2 * \text{height} = 2 + 2 (\lg_2(n + 1) - 1) = 2 \lg_2(n + 1).$$