CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Midterm 1- Standard 10

Due Date ..... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

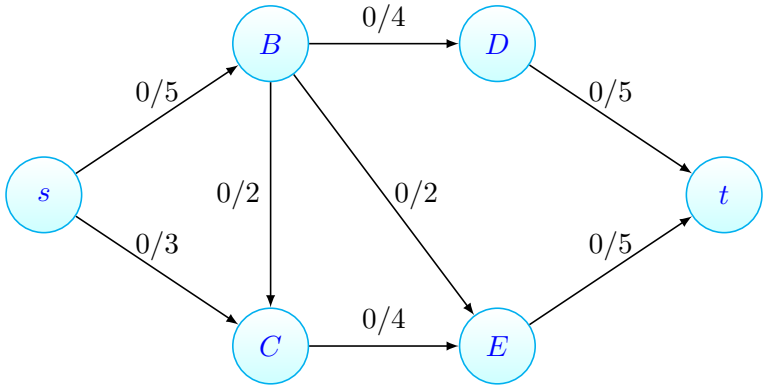
Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (John Blackburn). □

3 Standard 10- Network Flows: Ford-Fulkerson

Problem 2. Consider the following flow network, with no flow being pushed across the edges.

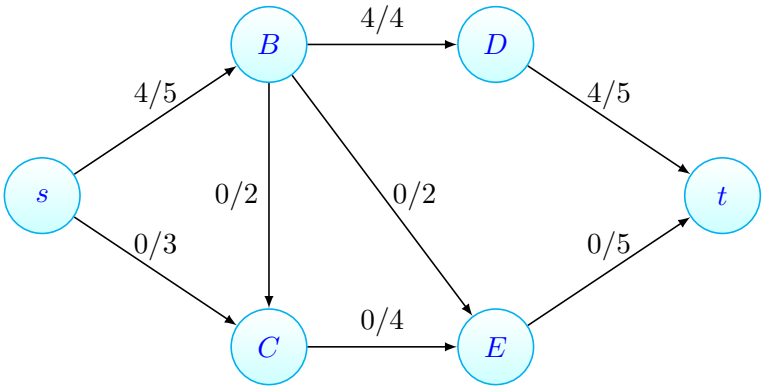


Do the following.

3.1 Problem 2(a)

(a) Consider the flow-augmenting path $s \to B \to D \to t$. Push as much flow as possible through the flow-augmenting path and draw the updated flow network below.

Answer. A max of 4 units of flow can be pushed through $s \to B \to D \to t$. The resulting flow network is below.

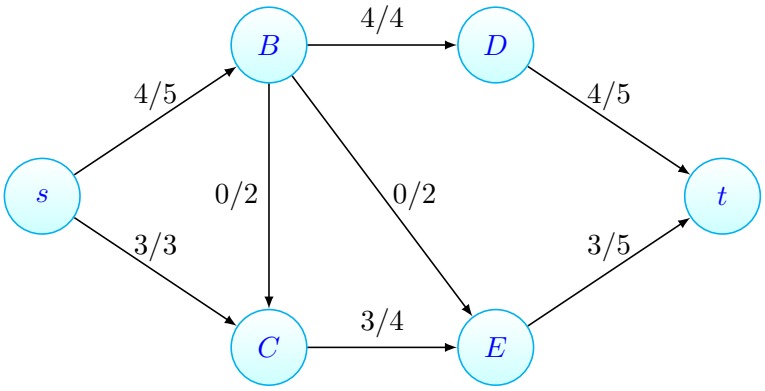


□

3.2 Problem 2(b)

(b) Find a flow-augmenting path using the updated flow configuration from part (a). Then do the following: (i) clearly identify both the flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer. A new flow augmenting path given the updated configuration is $s \to C \to E \to t$, and the max flow through this path is 3. The resulting flow network is below.

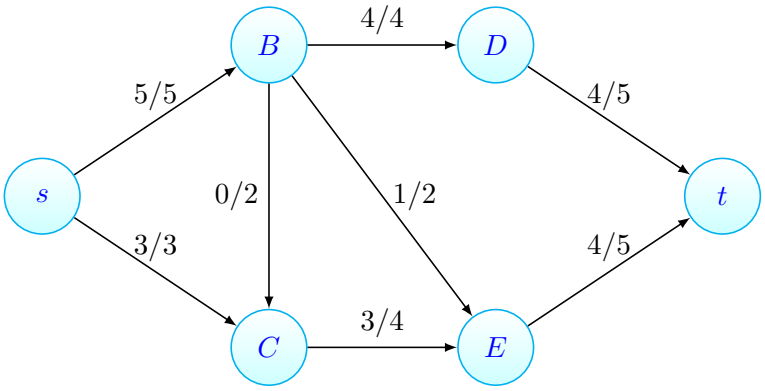


□

3.3 Problem 2(c)

(c) Find a flow-augmenting path using the updated flow configuration from part (b). Then do the following: (i) clearly identify both the flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer. A new path that can be added to our current configuration is $s \to B \to E \to t$ and the max flow that can be pushed through is 1. Resulting flow configuration is below



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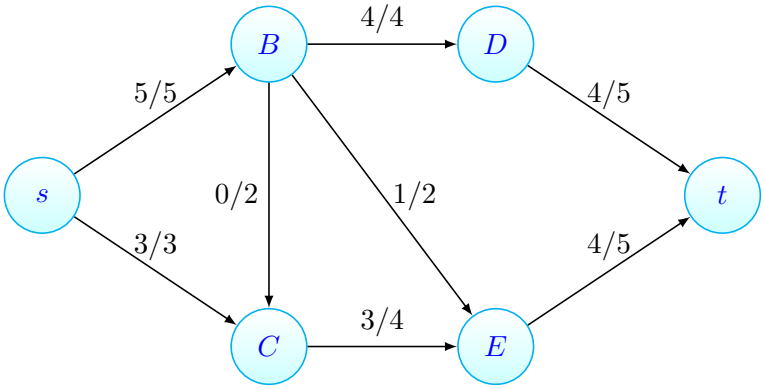
3.4 Problem 2(d)

(d) Using the flow configuration from part ((c)), finish executing the Ford-Fulkerson algorithm. Include the following here: (i) your flow network, reflecting the maximum-valued flow configuration you found, and (ii) the corresponding minimum capacity cut. There may be multiple minimum capacity cuts, but you should identify the one corresponding to your maximum-valued flow configuration. Then (iii) finally, compare the value of your flow to the capacity of the cut.

Note: You do **not** need to include the remaining steps of the Ford-Fulkerson algorithm. We will not check these steps when grading.

Answer. There are no more paths that can be added to our flow network. I have found the max flow to be 8 units of flow through the entire network. My final flow network is below. For the minimum cut I found I have two sets marking the partition between the vertices where the minimum cut can be found. The first being set $X = \{s\}$ containing only the source vertex, because starting from the source no flow can be pushed from the source to any vertex's connected to it. Leaving all other vertices in the other set $Y = \{B, C, D, E, t\}$. The cut is across edges s, B and s, C summing up to 8. Meaning my min cut found is 8. My max flow is also 8, so that means I have found the true max flow and min cut for the network.



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