CSCI 3104 Fall 2021
Instructor: Profs. Grochow and Waggoner

Problem Set 3

Due Date .... **September 21, 2021**
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Collaborators .... **None**

Contents

1 Instructions

- The solutions **must be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

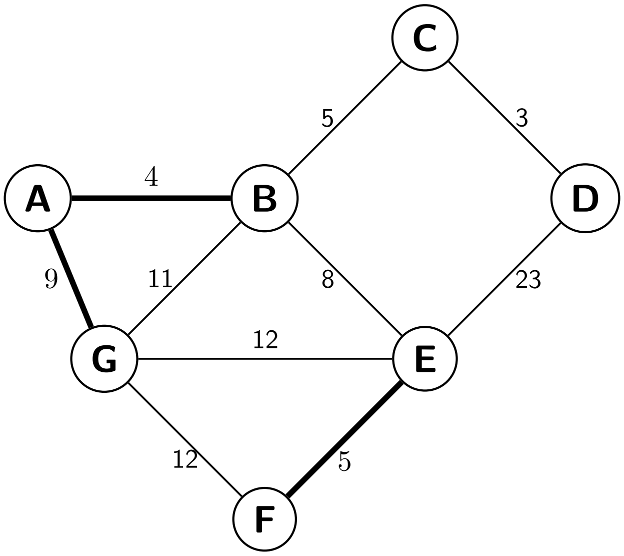
Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (John Blackburn). □

3 Standard 6 - MST: safe and useless edges

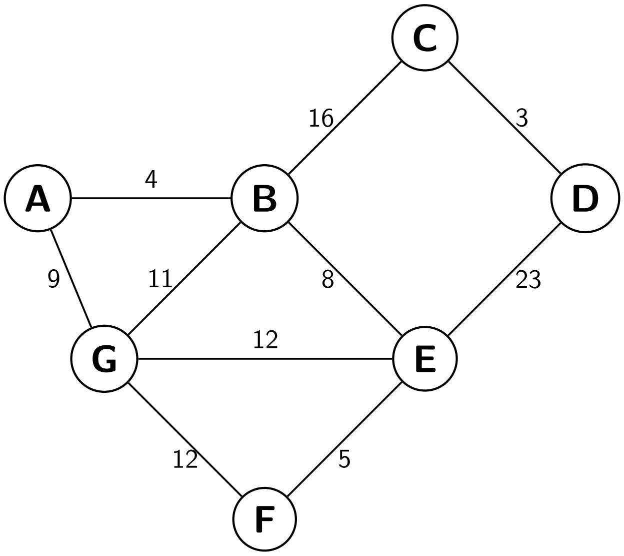
Problem 2. Consider the weighted graph $G(V, E, w)$ below. Let $\mathcal{F} = \{\{A, B\}, \{A, G\}, \{E, F\}\}$ be an intermediate spanning forest (indicated by the thick edges below). Label each edge that is **not** in $\mathcal{F}$ as safe, useless, or undecided. Provide a 1-2 sentence explanation for each such edge.



Answer. (B, G) is useless, because it creates a cycle within F. (B, C) is safe because it is the lightest edge coming from a component of F. (B, E) is safe because it is the lightest edge coming from a component of F. Edges $(G, E), (G, F), (E, D), (C, D)$ are all undecided because they aren't the lightest edges coming from components of F and they don't create any cycles. They aren't useless nor light therefore making them undecided. □

4 Standard 7- Kruskal's Algorithm

Problem 3. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Kruskal's algorithm adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Kruskal's algorithm takes as it selects the first **three** edges.

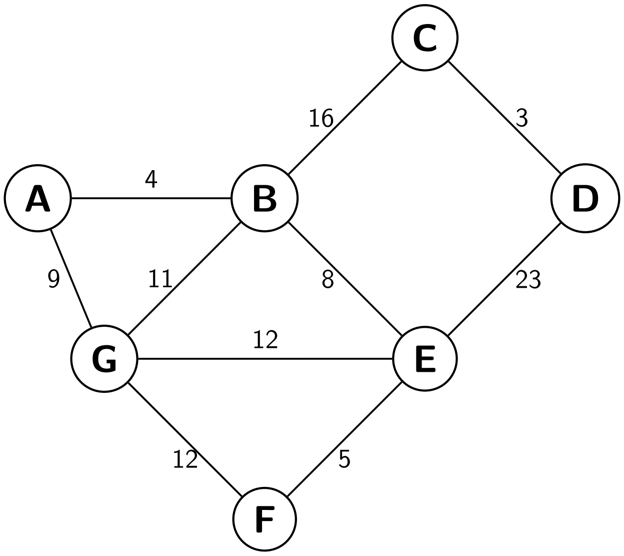


Answer. Kruskal's first sorts all the edges in G by least weight and places them all in a priority queue. Then the algorithm adds the lightest edge in all of G. Then adds the next lightest edge available until a MWST has been reached. It only skips edges if they are useless. Otherwise they next lightest edge available is always added no matter if it is a part of the MWST. For this graph the path Kruskal's takes is: First Kruskal's would add the lowest weight edge in the graph (C, D), Then it would add the next lightest edge in the entire graph G, that being edge (A, B). Next it adds (E, F). Then (B, E). Then (A, G). It skips (B, G), then (G, E), and (G, F) because each of those edges are useless ie they would create cycles within the MWST. And Finally edge (B, C) is added completing the MWST.

□

5 Standard 8- Prim's Algorithm

Problem 4. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Prim's algorithm, **using the source vertex** A, adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Prim's algorithm takes as it selects the first **three** edges.



Answer. Prim's Algorithm is very similar to kruskall's, the only difference being that prims creates the MWST growing out from a source vertex, always adding the lightest edge connected to the MWST. Kruskalls adds the lowest weight edge in the entire graph no matter if it is connected to the MWST. So for this graph the path Prim's takes is: First it adds the lightest edge coming from vertex A, that being edge (A, B). Next the lightest edge coming from the MWST is (B, E) so that edge is added. The third edge added is (E, F). The next edge added is (A, G). Edges $(G, B), (G, E), (G, F)$ are skipped over because they would create cycles within the MWST. Therefore the next edge added is (B, C) because it is the lightest edge connected to the MWST. Finally, completing the MWST is edge (C, D).

□