CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Quiz- Standard 23

Due Date .... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (john blackburn). □

3 Standard 23- DP: Use Recurrence to Solve

Problem 2. Recall the Rod Cutting Problem from class.

- Instance: Let $n \geq 0$ be an integer where n is the length of the rod, and let $p_1, p_2, \ldots, p_n$ be non-negative real numbers. Here, p_i is the price of selling a rod of length i.

- Solution: The maximum revenue, which we denote r_n, obtained by cutting the rod into pieces of integer lengths and selling the pieces.

Now suppose that our cutting tool is **malfunctioning** and **we are limited in the cuts we can make**. We can always cut off a piece of length 1, and additionally we can cut rods into halves (even n) or nearly halves (odd n): $\lfloor \frac{n}{2} \rfloor$ and $\lceil \frac{n}{2} \rceil$. A recurrence relation describing r_n is:

$$r_n = \begin{cases} 0 & \text{for } n = 0, \\ \max(p_n, r_1 + r_{n-1}, 2r_{n/2}) & \text{for even } n > 0, \\ \max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2}) & \text{for odd } n > 0. \end{cases}$$

Suppose we have a rod of length 6, with prices given as follows:

$$p_1 = 1, \quad p_2 = 6, \quad p_3 = 8, \quad p_4 = 10, \quad p_5 = 12, \quad p_6 = 15.$$

Using a bottom-up dynamic programming approach, determine the maximum revenue r_6 obtained by cutting up this rod **under the limited cut assumption described above**. For your convenience, we have provided the lookup table for you to use. You may also hand-draw your lookup tables.
Clearly show all work for how you filled in each cell of the lookup table.

Answer.

r_0	r_1	r_2	r_3	r_4	r_5	r_6
0	1	6	8	12	14	16

To fill in the lookup table I started from the base case r_0 which is equivalent to 0 which is defined above in the recurrence relation. The next case was r_1 which is equivalent to $\max(p_1, r_1 + r_0, r_0 + r_1)$ which comes out to $\max(1, 1 + 0, 0 + 1) = 1 = r_1$. we can now move to r_2, $r_2 = \max(p_n, r_1 + r_{n-1}, 2r_{n/2}) = \max(p_2, r_1 + r_1, 2r_1) = \max(6, 2, 2) = 6$.

Now for r_3: $r_3 = \max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2}) = \max(p_3, r_1 + r_2, r_1 + r_2) = max(8, 7, 7) = 8$.
$r_4 = \max(p_n, r_1 + r_{n-1}, 2r_{n/2}) = \max(p_4, r_1 + r_3, 2r_2) = \max(10, 9, 12) = 12$
$r_5 = \max(p_n, r_1 + r_{n-1}, r_{(n-1)/2} + r_{(n+1)/2}) = \max(p_5, r_1 + r_4, r_2 + r_3) = max(12, 13, 14) = 14$
$r_6 = \max(p_n, r_1 + r_{n-1}, 2r_{n/2}) = \max(p_6, r_1 + r_5, 2r_3) = \max(15, 15, 16) = 16$

Thus completes all entries for the lookup table solved in a bottom up fashion. So, the maximum revenue r_6 can aquire is 16 units.

□