CSCI 3104 Fall 2021
Instructor: Profs. Grochow and Waggoner

Requizzing Period 1- Standard 3

Due Date .... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (John Blackburn). □

3 Standard Dijkstra

3.1 Problem 2

Problem 2. Suppose we are given a finite, connected, and weighted graph $G(V, E, w)$, where the edge weights are non-negative. We define the *weight* of a path P to be the *product* (note: product, *not* sum!) of the edge weights along P. Fix vertices s, t. Our goal is to find a minimum-weight path from s to t.

(a) Suppose we construct a new graph $H(V, E, w')$ that is identical to G, with the exception that $w'((x, y)) = \log(w((x, y)))$ for all edges (x, y). That is, $V(H) = V(G)$ and $E(H) = E(G)$. So we have the same underlying graph, with the only difference being the edge weights. You may take as fact that P is a minimum-weight s to t path in G if and only if P is a shortest s to t path in H.

Suppose now that we run Dijkstra's algorithm on H, in order to find a shortest path from s to t in G. Is this approach valid? Justify your reasoning.

Answer. No, this approach is not valid. There are many cases where there can be negative edge weights. And Dijkstras algorithm cannot handle negative edge weights which causes the algorithm to produce incorrect results. For example let $w((x, y)) = .5$. Taking the logarithm of .5 gives you a negative edge weight within H and leads Dijkstras to an incorrect shortest path. Therefore, if any edge weight is less than 1 this approach will not work. □

(b) Suppose now that the edge weights of G are all positive. That is, $w((x, y)) > 0$ for all edges $(x, y) \in E(G)$. Let $H(V, E, w')$ be the graph corresponding to G, as defined in part (a). Is it now a valid approach to run Dijkstra's algorithm on H, in order to find a shortest path from s to t in G? Justify your reasoning.

Answer. No, this approach is still not valid. There are many cases where there can still be negative edge weights. And Dijkstras algorithm cannot handle negative edge weights which causes the algorithm to produce incorrect results. For example let $w((x, y)) = .5$. Taking the logarithm of .5 gives you a negative edge weight within H and leads Dijkstras to an incorrect shortest path. Therefore, if any edge weight is less than 1 this approach will not work. Just because $w((x, y)) > 0$ doesn't mean that $w'((x, y)) > 0$, so we still run into the same issue of negative edge weights. □

(c) Give conditions on the edge weights of G, so that it suffices to run Dijkstra's algorithm on H, in order to find a minimum-weight path from s to t in G. Clearly explain why your conditions are correct.

Answer. Dijkstras should be able to run a correct shortest path algorithm on H as long as the edge weights are greater than 0. So, $w((x, y)) > 1$. If the edge weights of G are all greater than 1, taking the logarithm of each edge weight will be greater than 0 and not lead to any issues. We will no longer have any negative edge weights, or weights equal to 0, therefore doing dijkstras on H will cleanly produce the shortest path in G from s to t. □