CSCI 3104 Fall 2021
Instructor: Profs. Grochow and Waggoner

Requizzing Period 1- Standard 5

Due Date .... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (John Blackburn). □

3 Standard 5- Exchange Arguments

3.1 Problem 2

Problem 2. Consider the interval scheduling problem from class. You are given a set of intervals $\mathcal{I}$, where each interval has a start and finish time $[s_i, f_i]$. Your goal is to select a subset S of the given intervals such that (i) no two intervals in S overlap, and (ii) S contains as many intervals as possible subject to condition (i).

Suppose we have two intervals with the same start time but different finish times. That is, let $I_1 = [s, f_1]$ and $I_2 = [s, f_2]$ with $f_2 > f_1$.

(a) Let overlap($[s, f]$) denote the number of intervals of $\mathcal{I}$ (excluding $[s, f]$) with which $[s, f]$ overlaps. Explain carefully why overlap(I_1) $\leq$ overlap(I_2).

Answer. We have that overlap(I_1) denotes the number of intervals of $\mathcal{I}$ with which $[s, f_1]$ overlaps. So, any interval that is within $[s, f_1]$ is a part of overlap(I_1). We know that I_2 starts at the same point as I_1 and ends at a farther point $f_2 > f_1$. Therefore, if we exchange overlap(I_1) for overlap(I_2) we can determine that we now have overlap(I_1) plus any intervals that start in the difference between f_2 and f_1. If there are no more intervals overlapping in this new added space we get that overlap(I_1) = overlap(I_2). And if there are new intervals overlapping in this space we get that overlap(I_1) < overlap(I_2). So, overall we can see that overlap(I_1) $\leq$ overlap(I_2). □

(b) Suppose that overlap(I_1) < overlap(I_2). Explain carefully why I_2 can safely be exchanged for I_1 (that is, in any non-overlapping set of intervals containing I_2, replacing I_2 by I_1 always results in another non-overlapping set of intervals, no smaller than the one we started with).

Answer. We have that I_2 is in a non-overlapping set. We know that I_1 starts at the same point as I_2 and that the finishing point of I_1 is less than that of I_2. Therefore, if we exchange I_2 for I_1, we know that the starting point of the interval will be the same and not overlap with any other interval already in the set. start of I_1 is equivalent to the start of I_2. The end of I_1 is less than the end of I_2. So, any set that isn't overlapping with the end of I_2 will also not overlap with the end of I_1. We also know that overlap(I_1) < overlap(I_2). So we can determine that I_1, will not overlap with any interval that I_2 does not overlap with. We even know that I_1 will overlap with even less intervals than I_2. Therefore, any exchange from I_2 to I_1 will safely hold any non-overlapping set and possibly even make the set larger. □