CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

# Midterm 2- Standard 23

Due Date ..... TODO
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## Contents



## 1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

## 2 Honor Code (Make Sure to Virtually Sign)

**Problem 1.**

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

*Agreed (john blackburn).* □

## 3 Standard 23- DP: Use Recurrence to Solve

**Problem 2.** The Subset-Sum problem is defined as follows.

- Instance: We are given $n$ items with positive weights $w_1, \ldots, w_n > 0$, as well as a target threshold $W > 0$.
- Solution: Is there a subsequence $w_{i_1}, \ldots, w_{i_k}$ such that:

$$\sum_{j=1}^{k} w_{i_j} = W.$$

That is, can we select a subsequence of items whose combined weights add to $W$?

For example, consider the input array $A = [4, 15, 8, 16, 23, 42]$. If $w = 31$ then the answer is "TRUE" since there is a subsequence $[15, 16]$ where the sum is equal to $15 + 16 = 31$. However, if $w = 13$ then the answer is "FALSE" since no subsequence of $A$ has sum equal 13.

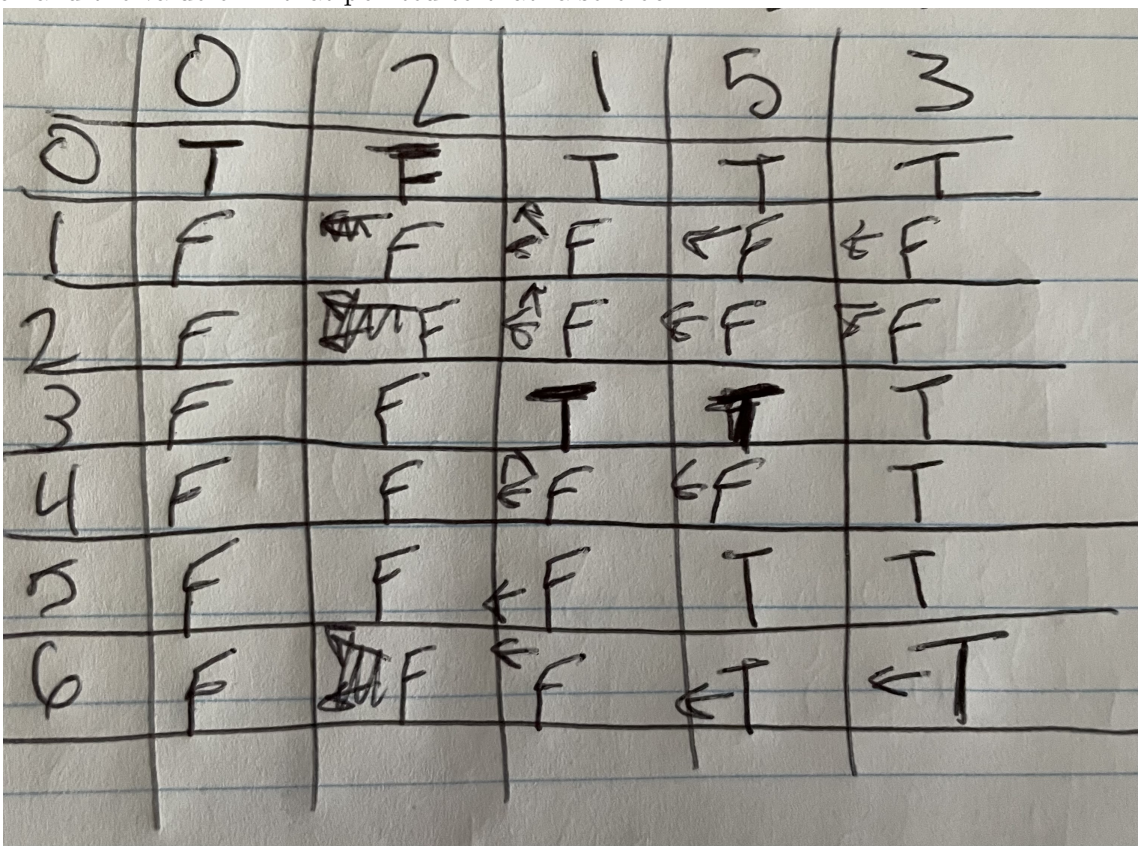
The Subset-Sum problem satisfies the following recurrence. For $0 \leq \ell \leq n$, and $0 \leq q \leq W$, define $T[\ell, q]$ be to TRUE if and only if there exists a subsequence of the first $\ell$ elements $[w_1, \ldots, w_\ell]$ that sum to $q$. Then we have:

$$T[\ell, q] = \begin{cases} TRUE & q = 0 \\ FALSE & \ell = 0 \text{ and } q > 0 \\ T[\ell - 1, q] & \ell > 0 \text{ and } w_\ell > q \\ T[\ell - 1, q] \text{ OR } T[\ell - 1, q - w_\ell] & \ell > 0 \text{ and } w_\ell \leq q \end{cases}$$

(Note that in the final case, "OR" is the Boolean OR operation.)

Consider $A = [2, 1, 5, 3]$ and $W = 6$. Design and fill in a lookup table for this problem. Include all pointers. Clearly indicate whether a solution exists; and if so, how to recover it from the lookup table.

*Answer.* There is a solution $(1, 5)$ and it can be recovered from the lookup table by starting in the bottom right corner and following the arrows until you hit the first false, the answer is then the value of A at the first false block and the value of A that pointed to that false block.

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