CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Quiz- Standard 11

Due Date .... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

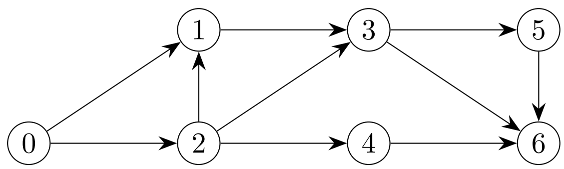
- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed John Blackburn. □

3 Standard 11- Network Flows: Reductions

Problem 2. We say that two $i \rightsquigarrow j$ paths are *edge-disjoint* if they do not share any common edges. Note however, these paths can (and in fact, often do) share common vertices (aside from i and j). As an example, consider the following graph.

- Observe that $0 \to 1 \to 3 \to 5 \to 6$ and $0 \to 2 \to 3 \to 6$ are edge-disjoint paths, as they do not share any directed edges. It is fine that they share the common vertex 3.
- Note however that $0 \to 2 \to 4 \to 6$ and $0 \to 2 \to 3 \to 6$ are **not** edge-disjoint paths, as they both share the $(0, 2)$ edge.



Consider the following problem.

- Input: A directed graph $G(V, E)$, as well as a start node i and an end node j.
- Solution: We seek to find a set $\mathcal{P}$ of $i \to j$ paths such that any two distinct paths $P_1, P_2 \in \mathcal{P}$ are edge-disjoint, and $|\mathcal{P}|$ is maximum. That is, we seek to find a maximum set of edge-disjoint $i \to j$ paths.

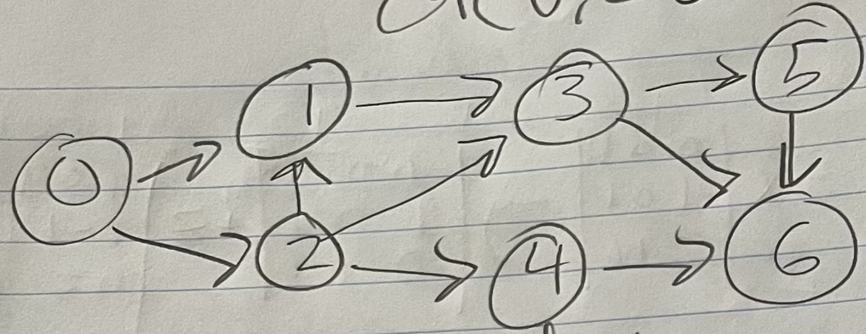
(a) Describe how to reduce the above problem to the (one-source, one-sink) max-flow problem from class. Your description should be **general**, and not tied to a specific example.

Answer. To find the maximum set of edge-disjoint $i \to j$ paths using a reduction to the max flow problem, the first step will be to reduce $G(V, E)$ to a flow network $\mathcal{N}$ that consists of the exact same edges and nodes as $G(V, E)$. Next, we add a flow capacity of 1 to each edge in $\mathcal{N}$. Now, we have a flow network that we can find the max flow in. node i is our source, and node j is our sink. Now, we would find the max flow within $\mathcal{N}$ and save the pathways that contain flow. These pathways are the key to finding set $\mathcal{P}$. These pathways are all edge disjoint because each edge can only contain 1 unit of flow and therefore only be used in a single pathway. We know that $|\mathcal{P}|$ is maximum as well because the max flow takes the maximum amount of paths available to it and only stops when there are no more paths left for it to take. So, we take our max flow pathways from $\mathcal{N}$ and claim the set of these pathways, is indeed equivalent to the maximum set of edge-disjoint $i \to j$ paths in $G(V, E)$. We can also see that the max amount of flow pushed through $\mathcal{N}$ is equivalent to $|\mathcal{P}|$. □

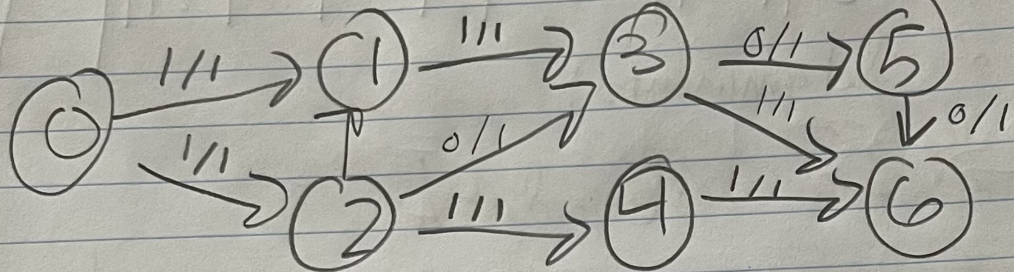
(b) Using your reduction, find a maximum set of edge-disjoint paths from $0 \rightsquigarrow 6$ in the graph above. Show your work, as well as your final answer. Note that there may be multiple maximum-size sets $\mathcal{P}$ in the graph above; you need only find one such set $\mathcal{P}$ of edge-disjoint paths, as long as it has the largest number of paths possible.

Answer. So from the above graph, I construct a flow network with the same edges and nodes, while adding flow capacities of 1 to each edge in the new network. Let this new network be called $\mathcal{N}$. The maximum flow through this flow network is 2, and let S denote the set of pathways containing flow. $S = \{\{0 \to 1 \to 3 \to 6\}, \{0 \to 2 \to 4 \to 6\}\}$. From my reduction, proved above, I know that set S, is equivalent to the max set $\mathcal{P}$ in $G(V, E)$ and that $|\mathcal{P}| = 2$ from my found max flow. Reduction work below:

$G(V,E)$



reduced to
flow network N



Max flow = 2

$S = \{\{0 \to 1, \to 3 \to 6\}, \{0 \to 2 \to 4 \to 6\}\}$

Back To $G(V,E)$

$S = P \quad |P| = $ Max flow $= 2$

PS(hope an image is ok I ran out of time) □