Final- Standard 29

Due Date ....TODO
Name ....**John Blackburn**
Student ID **Jobl2177**

Contents

1   Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.

- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.

- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).

- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.

- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.
- My submission is in my own words and reflects my understanding of the material.

- Any collaborations and external sources have been clearly cited in this document.

- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.

- I have neither copied nor provided others solutions they can copy.

Agreed John Blackburn. □

3 Standard 29- Computational Complexity: Problems in **NP**

Problem 2. A *rank-3 hypergraph* $\mathcal{H}(V, E)$ consists of a set of vertices V, together with a set of hyperedges E where each hyperedge is a set $\{a, b, c\}$ of distinct vertices. [**Note:** In this language, a simple graph would be a rank-2 hypergraph.]

We say that two rank-3 hypergraphs on n vertices $\mathcal{H}_1(V_1, E_1)$ and $\mathcal{H}_2(V_2, E_2)$ are *isomorphic* if there exists a one-to-one map $\varphi : V_1 \to V_2$ such that $\{a, b, c\} \in E_1$ if and only if $\{\varphi(a), \varphi(b), \varphi(c)\} \in E_2$.

Show that the problem of deciding whether two rank-3 hypergraphs are isomorphic

Answer. We take as certificate two isomorphic rank-3 hypergraphs $\mathcal{H}_1(V_1, E_1)$ and $\mathcal{H}_2(V_2, E_2)$ with n vertices each. We now find a polynomial time algorithm to verify that the hypergraphs are isomorphic. We know that V_1 and V_2 both have n elements. In order to verify our solution we must show that if $\{\varphi(a), \varphi(b), \varphi(c)\} \in E_2$ then $\{a, b, c\} \in E_1$. To do this we compare each set of three vertices in E_2 to find the corresponding mappings. There are $n(n-1)/3$ sets to compare thus taking $O(n^2)$ time to compute all the mappings in E_2. We then must confirm that those sets of vertices than are in E_2 are also in E_1. There can be at most $n(n-1)/3$ sets to check meaning this step will also $O(n^2)$ time to compute. We must also check that the map is one-to-one. To do this we check that for each distinct element pair $x, y \in V_1$ that $\varphi(x) = \varphi(y)$. There are $n(n-1)/2$ pairs to compare thus taking $O(n^2)$ time to compute. Thus with the certificate and a verifier algorithm that runs in polynomial time we can determine that this problem is indeed in NP. $\square$