CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Final- Standard 30

Due Date .... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed John Blackburn. □

3 Standard 30- Computational Complexity: Structure

Problem 2.

(a) Suppose that $\mathcal{C}$ is a complexity class such that $\mathsf{P} \subseteq \mathcal{C}$ (so every problem in P is contained in $\mathcal{C}$). Suppose that there is a problem L that is $\mathcal{C}$-complete under polynomial-time reductions. Recall that this means:

- $L \in \mathcal{C}$; and
- For all problems $K \in \mathcal{C}$, K can be reduced to L in polynomial time.

Show that if $L \in \mathsf{P}$, then $\mathcal{C} \subseteq \mathsf{P}$. (Since we are assuming that $\mathsf{P} \subseteq \mathcal{C}$ this means that $\mathsf{P} = \mathcal{C}$.)

Proof. We know $L \in \mathsf{P}$ and that all problems $K \in \mathcal{C}$ can be reduced to L. Thus, we know that $L \in \mathcal{C}$ as well. We also know that we can reduce L to any problem in $\mathcal{C}$. Thus making any problem in C also in P. From there we can determine that $\mathcal{C} \subseteq \mathsf{P}$ $\square$

(b) We say that a language $L \in \mathsf{EXPTIME}$ if there is an algorithm A that (i) correctly decides whether a given input string x belongs to L, and (ii) A runs in time at most 2^{n^c} where c is a constant (depends only on L) and $n := |x|$ is the size of the input in bits. It is known that $\mathsf{P} \subseteq \mathsf{EXPTIME}$ but also that $\mathsf{P} \neq \mathsf{EXPTIME}$. Use these facts and part (a) to argue that no $\mathsf{EXPTIME}$-complete language belongs to P.

Proof. Any EXPTIME-complete language is a problem that any problem in EXPTIME can be reduced to. Thus, if all problems in P can be reduced to an EXPTIME-complete problem we would be able to prove that P = EXPTIME. Therefore we can see that no EXPTIME-complete problem can belong to P because that would mean P = EXPTIME. $\square$