SECM Model Equations Summary

Equation 1

$$\text{sigmoid}(x) = \frac{1}{1+e^{-x}}; \quad \Phi_{\text{edu}}(t) = \text{sigmoid}(10 \cdot (\text{PISA}_{\text{math}} - 400)) \cdot \left[1 - e^{-N_{\text{STEM}}/N_{\text{crit}}}\right] \quad (1)$$

Equation 2

$$\begin{aligned} f(\text{TER}_t, \text{STEM}_t, \text{PISA}_{\text{math}}) = 1 - \text{sigmoid}(10 \cdot (\text{TER}_t - 0.2)) \\ \cdot \left(1 - \exp\left(-\frac{\text{STEM}_t}{N_{\text{crit}}}\right)\right) \\ \cdot \text{sigmoid}(10 \cdot (\text{PISA}_{\text{math}} - 400)) \end{aligned} \quad (2)$$

Equation 3

$$\texttt{Y\_t+1} = \texttt{Y\_t} \cdot \exp\left(\texttt{rho\_t} \cdot [1 + \texttt{kappa\_d} \cdot \texttt{D\_t}] \cdot \left[1 - 0.4 \cdot \sqrt{\texttt{TER\_t}/0.3}\right]\right) + \texttt{sigma} \cdot [\ldots] + \texttt{alpha} \cdot \texttt{Z\_t} \cdot [\ldots] \quad (3)$$

Equation 4

$$X_{t+1} = X_t \cdot [1 + \Phi_{\text{growth}} - \eta \cdot Y_t] + \Delta X_{\text{net}} \quad (4)$$

Equation 5

$$\Phi_{\text{growth}} = a_1 \cdot \texttt{PopDens\_t} + a_2 \cdot \texttt{INFRA\_t} + a_3 \cdot \texttt{RES\_t} \quad (5)$$

Equation 6

$$\Delta X_{\text{net}} = \Delta X_{\text{diff}} + \Delta X_{\text{trade}} + \Delta X_{\text{bonus\_eff}} \quad (6)$$

Equation 7

$$Y\_Limit_t = \theta \cdot X_t \quad (7)$$

Equation 8

$$X_{t+1} = X_t \cdot (1 + \lambda \cdot \Delta X_{\text{bonus},t}) - \eta \cdot Y_t + \delta \cdot Z_t \quad (8)$$

Equation 9

$$Y_t = Y_{\text{base},t} + Y_{\text{adj},t} + \beta_Z \cdot Z_t \quad (9)$$

Equation 10

$$Z_t = \zeta_1 \cdot |X_t - X_{t-1}| + \zeta_2 \cdot \max(0, X_{t-1} - X_t) + \zeta_3 \cdot \text{Noise}_t \quad (10)$$

Equation 11

$$\Delta X_{\text{bonus},t} = \text{user-defined (or 0 if unknown)} \tag{11}$$

Equation 12

$$Y_{\text{limit},t} = \theta \cdot X_t \tag{12}$$

Equation 13

$$S_t = \sum_{\tau=1}^{t} \max(0, Y_\tau - Y_{\text{limit},\tau}) \tag{13}$$

Equation 14

$$\text{If } S_t > \phi \cdot \sqrt{X_t}, \text{ then collapse/reset is triggered} \tag{14}$$

Equation 15

$$Y_{\text{base},t} = a_0 + \frac{a_1}{X_t} \tag{15}$$

Equation 16

$$Y_{\text{adj},t} = b_1 \cdot \frac{N_t}{X_t} \tag{16}$$

Equation 17

$$Z_t = \zeta_1 \cdot |X_t - X_{t-1}| + \zeta_2 \cdot \max(0, X_{t-1} - X_t) \tag{17}$$

Equation 18

$$S_t = S_{t-1} + \max(0, Y_t - Y_{\text{limit},t}) \tag{18}$$

Equation 19

$$\text{If } S_t > \phi \cdot \sqrt{X_t}, \text{ then collapse/reset} \tag{19}$$

Equation 20

$$X_{t+1} = X_t \times \left[1 + \lambda_d \cdot \tanh\left(\frac{\text{PopDens}_t}{D_{\text{opt}}}\right) - \lambda_r(t) \cdot \max\left(0, \frac{\text{PopDens}_t - D_{\text{opt}}}{D_{\text{opt}}}\right)\right] \tag{20}$$

$$+ \Delta X^{\text{eff}}_{\text{bonus}}(t) + \Delta X_{\text{diff}}(t) + \Delta X_{\text{trade}}(t) - \eta_t \cdot Y_t + \Delta X_Z(t) \tag{21}$$

Equation 21

$$
\begin{aligned}
Y_{t+1} = Y_t \cdot \exp\left(\rho_t^{\text{new}} + \varepsilon_0^{\text{buff}}\right) &\quad (22)\\
+ \sigma^{\text{new}} \cdot X_t &\quad (23)\\
+ \alpha \cdot Z_t \cdot \mathbb{I}\left\{\Delta X_{\text{bonus}}^{\text{eff}}(t) \geq 0\right\} &\quad (24)
\end{aligned}
$$

Equation 22

$$
\begin{aligned}
Z_t = \Gamma(\Psi_t) \cdot [\beta_t \cdot Z_{\text{ext}}(t) + \kappa_t \cdot |\Delta X_{\text{bonus}}(t-1)| + \varepsilon \cdot (\Delta X_{\text{bonus}}(t) - \Delta X_{\text{bonus}}(t-1))] &\quad (25)\\
+ \nu_t \cdot (1 - \Psi_t) + \varphi_{\text{dens}} \cdot D_t + \varphi_{\text{edu}} \cdot f(\text{TER}_t, \text{STEM}_t, \text{PISA}_{\text{math}}) &\quad (26)
\end{aligned}
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Equation 23

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\begin{aligned}
Xpc_t &= \frac{X_t}{N_t} && (27)\\
Y_{\text{base},t} &= a_0 + \frac{a_1}{X_t} && (28)\\
Y_{\text{adj},t} &= b_1 \cdot \frac{N_t}{X_t} && (29)\\
Y_t &= Y_{\text{base},t} + Y_{\text{adj},t} && (30)\\
Z_t &= \zeta_1 \cdot |X_t - X_{t-1}| + \zeta_2 \cdot \max(0, X_{t-1} - X_t) && (31)\\
Y_{\text{limit},t} &= \theta \cdot X_t && (32)\\
S_t &= S_{t-1} + \max(0, Y_t - Y_{\text{limit},t}) && (33)
\end{aligned}
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