

# Notes

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## Articles

### Displacement Interpolation Using Lagrangian Mass Transport

[Link pdf](#)

**Mass Transport Problem :**

Transportation Simplex  $\rightarrow$  Earth Moving Distance

Network simplex algorithm (*with block search pivoting ??*) from LEMON graph library  $\rightarrow$  general min-cost flow problems Transportation simplex have worst case complexity in  $O(n^3)$  but generally behaves in  $O(n^2)$  in some context.

**Limitation :** Problem size  $\rightarrow$  cost matrix storage too heavy for GPU memory. Presented method working for interpolation between two distributions, future work idea : interpolation between N distribution (texture mixing)

Radial basis function ?

### Minimum-cost flow algorithms: An experimental evaluation

[Link pdf](#)

**Minimum-cost flow algorithms in the LEMON library :**

**Spanning tree data structures :**

ATI (Augmented Threaded Index ) vs XTI (eXtended Threaded Index ) XTI apparently have better performance for network simplex.

Additional improvement : a reverse thread index is also stored for each node to represent the depth-first traversal as a doubly-linked list.

For initialization of the initial spanning tree solution, adding an artificial root with additionnals arcs between the nodes and the new roots provides better performances.

**Pivot rules :**

simplest pivot rules : *best eligible* and *first eligible*.

*block search pivot rule* : cyclically examines certain subsets (blocks) of the arcs and select best candidate at each iteration. Block size seems quite important parameter, set the size proportionally to  $|A(G)|$  between 1% and 10%. Article experiments suggest  $block\_size = \lfloor \sqrt{m} \rfloor$  with ( $m = |A(G)|$ , the number of arc in graph  $G$ )

*candidate list pivot rule* : method that examines arcs and build a list of most eligible arcs. The list is then used for at most  $K$  iterations. If an arc becomes non-eligible, it is removed from the list. Article suggest :  $L = \lfloor \sqrt{m}/4 \rfloor$  and  $K = \lfloor L/10 \rfloor$ .

*altering candidate list pivot rule* : improved version of *candidate list pivot rule* presented in the article. It maintain a list of size  $K$  of eligible arcs at each iteration, extending and removing arcs only searching in one arc block with size  $B$ . Article suggest  $B = \lfloor \sqrt{m} \rfloor$  and  $K = \lfloor B/100 \rfloor$ .

## Ressources

repository of Optimized version of Simplex Network from LEMON : [https://github.com/nbonneel/network\\_simplex](https://github.com/nbonneel/network_simplex)

Simplex algo for mincost flow problem : *Network Optimization: Continuous and Discrete Models*, p. 201