Seminar 4

1. **Binomial distribution:**

   (a) Use R functions `dbinom`, `pbinom` and `qbinom` to create a density plot, distribution function plot and quantile function plot of the random variable X with the binomial distribution $X \sim Bi(100, 0.5)$.

   (b) Generate a random vector of the length 1000 with independent elements, where each element is a realization of the random variable X_i following the same binomial distribution $X_i \sim Bi(100, 0.5)$. Produce a histogram and compare it to the density plot of the same distribution.

2. A **Poisson process** is a model describing a stream of events that occur at random times. The rate of occurrence of events λ (expected number of events per unit of time) is constant. The times of events form a sequence where the *waiting times* between events are independent and identically distributed following the exponential distribution.

   The *number of events* recorded between times s, t ($s < t$) is a random variable with the Poisson distribution $Po(\lambda\,(t - s))$ with the mean proportional to the length of the time interval: especially, the probability that there are exactly n events between times 0 and t is:

   $$p_n\,(t) = \frac{(\lambda t)^n}{n!} \mathrm{e}^{-\lambda t},$$

   so the total number of events to the time t is following $Po(\lambda t)$. Furthermore, the numbers of events occurring in non-overlapping intervals are independent.

   (a) Generate and plot a random trajectory of the Poisson process with $\lambda = 3$ on the interval $[0, 100]$. Specifically, generate the event times in $[0, 100]$ and plot a step function of time $t \in [0, 100]$ that gives the cumulative number of events that has occurred by time t.

   (b) Run the program from (a) 500 times and record the cumulative number of events at $t = 100$ for each trajectory. Plot the histogram of these values.

   (c) Plot Poisson density function for random variable X that follows Poisson distribution $X \sim Po\,(\lambda t)$, where $\lambda = 3$ and $t = 100$. Compare it with histogram from the previous task (b).

   (d) How many events do you expect to observe during the next 20 units of time? Could you provide an upper bound for this number that will be exceeded only with probability 10 %?

3. **Central limit theorem and Pascal triangle:**

   (a) Generate vector of 1000 random values following the binomial distribution $X \sim Bi\,(500, 0.5)$. Normalize the random values and plot the histogram. Compare it with a standardized normal distribution (*Moivre-Laplace* central limit theorem).

(b) Plot the probability function of the random variable X following the binomial distribution: $X \sim Bi\,(n = 20, 0.5)$ (using `dbinom()`) together with the density of the normal distribution with the same expectation value and variance like the random variable X. Try different values of n.

(c) EXTRA task: compute the fifth row of the Pascal triangle using `dbinom()` function.