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Author(s): Timothy J. Feddersen and Wolfgang Pesendorfer

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Abstention in Elections with Asymmetric Information and Diverse Preferences

TIMOTHY J. FEDDERSEN *Northwestern University*

WOLFGANG PESENDORFER *Princeton University*

We analyze a model of a two-candidate election with costless voting in which voters have asymmetric information and diverse preferences. We demonstrate that a strictly positive fraction of the electorate will abstain and that, nevertheless, elections effectively aggregate voters' private information. Using examples, we show that more informed voters are more likely to vote than their less informed counterparts. Increasing the fraction of the electorate that is informed, however, may lead to higher levels of abstention. We conclude by showing that a biased distribution of information can lead to a biased voting population but does not lead to biased outcomes.

The following stylized facts occupy a central place in the empirical literature on voting and elections in the United States. (1) Better educated and wealthy individuals participate in elections more frequently than others (Wolfinger and Rosenstone 1980). (2) Over the last forty years, participation in elections has declined significantly while education levels have increased (Brody 1978). In addition, recent research suggests that there is a correlation between participation and policy outcomes.¹ These facts, together with the likely correlation between participation and outcomes, have led some to conclude that election outcomes are increasingly biased toward the wealthy and better educated (Lijphart 1997).

We analyze a game-theoretic model of participation in large elections. We derive necessary and sufficient conditions for abstention and show how the level of abstention depends upon the information environment. We then show that for a variety of model specifications our model gives results that are consistent with the stylized facts mentioned above without producing biased election outcomes.

The model presented here is an extension of a model developed in Feddersen and Pesendorfer (1996). In that article, we examined a setting with two alternatives and two states of the world. Voters have identical state-dependent preferences and prefer alternative 1 in state 1 and alternative 2 in state 2.² A subset of voters knows the true state, while others have no information. We showed that equilibrium behavior in large elections has three features. First, uninformed voters may have a strict incentive to abstain even though they are not indifferent between the alternatives.³ Second, abstention levels may be high even in large electorates.

Third, even with strategic abstention, elections satisfy "full information equivalence," that is, the winning alternative is the alternative that would win an election in which all voters were fully informed and voted for their preferred alternative.

The current model generalizes the earlier framework in three ways. First, we introduce a continuum of voter preference types; hence, in a typical electorate no pair of voters has exactly identical preferences. Preference diversity allows us to analyze participation when information and preferences are correlated. Second, we consider environments with more than two states. Third, we consider the effects of noisy information.

We assume that voter preferences depend on a preference type and on a state that represents the relative "quality" of the two alternatives. Quality refers to a characteristic of the alternatives that all voters enjoy. For example, suppose that an election is held to decide whether a bridge should be built. Each voter's preference type is determined by how frequently s/he would use the bridge and the cost of building it. The "state" in this example corresponds to the cost of the bridge. As the cost increases, all voters like the bridge less. In the case in which two candidates are running, we may view the candidates' "character" as a quality dimension. Candidates with "good character" stick closely to their announced position, while those with "poor character" may not. Therefore, all risk-averse voters prefer candidates with better character.

We assume that voters are asymmetrically informed about the quality of the alternatives. A variety of features of large elections makes asymmetric information a realistic assumption. For example, not all voters follow elections in the media or pay attention to campaign advertisements. Moreover, campaigns target voter groups to receive specific messages. Finally,

exactly one of them knows the true state. It is an equilibrium for those voters who do not know the state to abstain and to let the informed voter decide the outcome. To see why this is the case suppose that in the event the election is tied a coin is tossed to determine the winner. In the proposed equilibrium, uninformed voters strictly prefer to abstain because their vote only changes the outcome when they vote for the wrong candidate—the candidate not supported by the perfectly informed voter—and the wrong candidate wins the coin toss.

Timothy J. Feddersen is Professor of Managerial Economics and Decision Sciences, J. L. Kellogg Graduate School of Management, Northwestern University, Evanston, IL 60208-2009 (tfed@nwu.edu). Wolfgang Pesendorfer is Professor of Economics, Princeton University, Princeton, NJ 08544 (pesendor@phoenix.princeton.edu).

¹ For example, Hill, Leighley, and Hinton-Andersson (1995) conclude that welfare spending is correlated with participation of lower income groups. See also Bartels (1996) and Lipjhart (1997).

² We also introduce voters who are "partisans" in the sense that their preferences do not depend upon the state.

³ Suppose there are three voters who must choose between candidate 1 and 2 using majority rule. There are two states of the world: state 1 and state 2. State 1 is more likely than state 2. All the voters prefer candidate 1 in state 1 and candidate 2 in state 2. All voters know that

voters' ability to understand the messages they observe depends on their ability and expertise.

THEORETICAL RESULTS

In the first part of the article we present our model and analytical results. We show that the full-information equivalence result mentioned above is robust. Candidate 1 wins with high probability in states in which a majority of the electorate strictly prefers alternative 1 to alternative 2 and loses with high probability otherwise.

We also give necessary and sufficient conditions for abstention in a large electorate. Because we assume the set of states is finite, there is a critical state in which the fraction of the electorate that prefers alternative 1 to alternative 2 is closest to one-half. The critical margin denotes the absolute value of the difference in the expected fraction of voters who prefer alternative 1 to alternative 2 in the critical state. Our main theoretical result is that in a large electorate the fraction of voters who abstain is proportional to the critical margin.

The fact that abstention can occur under some model specifications but not others may raise concerns about our model as an explanation for abstention. The concern is as follows: Surely, there is always some state of the world in which exactly half of any population would prefer one alternative to the other. Our results would then be taken as evidence that asymmetric information cannot provide an explanation for abstention. Such an argument misunderstands what a state represents in our model. A state represents what a very large number of people may learn about the relative quality of the two alternatives. Thus, in the bridge example, it may be the case that the true cost of the bridge ranges continuously from \$50 to \$50 million, but the most any very large number of people could ever learn is that the true cost is either above or below \$25 million. In this case, even though the true cost of the bridge is a continuous random variable, it is appropriate to assume there are two states, and hence the margin of victory in the critical state may be large.

EMPIRICAL PREDICTIONS

In the second part of the article we provide comparative statics results for a series of examples that are consistent with the stylized facts mentioned above.

First, we analyze an example of a model in which some voters are perfectly informed and others have no information. Informed voters always participate, and uninformed voters, depending on their preference type, abstain. In this setting we show that increasing the probability a voter is perfectly informed may result in higher levels of abstention. Thus, although informed voters always participate, an increase in the fraction of informed voters changes the behavior of the uninformed, so that the overall level of abstention rises.

Next, we provide examples with noisy signals. Our findings are as follows. (1) Individuals with better information are more likely to participate than individuals with worse information. At the same time, increas-

ing the fraction of the electorate that receives a noisy signal can increase levels of abstention among both the informed and uninformed. (2) The voting electorate is biased toward the winning alternative, that is, whenever the right alternative wins, voters with preference types to the left of the median are more likely to abstain, and vice versa. (3) A biased distribution of information may result in an electorate that is systematically unrepresentative without resulting in unrepresentative outcomes.

RELATED LITERATURE

Various articles focus on the consequences of costs to vote on turnout (Feddersen 1992; Lohmann 1993; Morton 1991; Palfrey and Rosenthal 1983, 1985; Riker and Ordeshook 1968). For reviews of both the theoretical and empirical literature on turnout, see Aldrich (1993), Feddersen and Pesendorfer (1996), Grofman (1993), or Matsusaka (1992). Ghirardato and Katz (1997) develop a decision-theoretic explanation of abstention that does not rely on costs to vote. Their explanation of abstention relies on voters having non-standard utility functions.

Traditionally, models of participation in elections have been decision theoretic and have focused on costs to participate as the key variable predicting participation (Riker and Ordeshook 1968). A decision-theoretic model of participation runs into the following difficulty, however: Empirically, education level is positively correlated with turnout.⁴ To explain this cross-sectional variation, a decision-theoretic model must assume it is less costly for those with relatively more education to participate.⁵ Yet, in the past forty years education levels in the population have increased, but turnout has declined (Brody 1978). A decision-theoretic model will have trouble simultaneously explaining the longitudinal phenomena of declining overall turnout and increasing membership in high-turnout categories.⁶

A substantial empirical literature examines the relationship between participation and outcomes (Bartels 1996; Hill, Leighley, and Hinton-Andersson 1995; for a review of this literature, see Lijphart 1997). Some scholars conclude that full participation would not change election results substantially, while others argue that support for parties on the Left would substantially increase (Lijphart 1997).

The empirical literature assumes that voter behavior will stay the same as the level of participation of other voters changes. This may be appropriate in a decision-theoretic model, but in a game-theoretic voting model like ours it is not. In a game-theoretic model, optimal behavior is predicated on the behavior of other agents. In our model the behavior of all agents changes if, for example, voting is mandatory or the information levels of some voters change.⁷ The election outcomes (as a

⁴ See Wolfinger and Rosenstone (1980).

⁵ See Matsusaka (1992) for an example of such a theory.

⁶ Of course, a decision-theoretic model is consistent with the data if costs to vote may vary over time.

⁷ See Bartels (1996) for an analysis in which it is assumed that the

function of the state) would remain unchanged, however.

Our model assumes that the number of voters is uncertain and distributed according to a Poisson distribution. The idea of using Poisson distributions to analyze large anonymous games is borrowed from Myerson (1997, 1998). The advantage of the Poisson distribution is that it permits easy calculation of equilibrium profiles and examples even in very large electorates.

THE MODEL

We analyze a two-alternative election. Alternatives are denoted by $j \in [1, 2]$. A voter's utility depends on a preference parameter $x \in [-1, 1] = X$, the chosen alternative j , and the state $s \in S \subset [0, 1]$. We assume the set of states S is finite and that $\{0, 1\} \subset S$, that is, the smallest state is 0, and the largest state is 1.

We denote by $u(j, s, x)$ the utility function of voters. Let

$$v(s, x) \equiv u(2, s, x) - u(1, s, x) \quad (1)$$

denote the utility difference of a voter type x between alternative 2 and alternative 1 in state s .

At the beginning of the game, nature selects a state s and an electorate. The electorate is chosen as follows. First, nature selects a number of players according to the Poisson distribution with parameter v . Thus, the probability that there are n players is given by

$$\Pr(n) = \frac{e^{-v} v^n}{n!}.$$

Second, every player is independently assigned a preference type according to the distribution function $F(x)$. Third, every player receives a signal $m \in \{\emptyset, 1, \dots, M\} = \mathcal{M}$, where \emptyset describes an uninformative signal. We assume that conditional on the state s the signal is independently distributed across agents. The probability that an agent receives the signal m in state s is $p(m|s)$.

We make the following assumptions.

ASSUMPTION 1. $v(x, s)$ is defined for all $(x, s) \in [-1, 1] \times [0, 1]$; it is continuous in x and strictly increasing in (x, s) . Furthermore, $v(-1, s) < 0$ and $v(1, s) > 0$ for all $s \in [0, 1]$.

ASSUMPTION 2. The state s is chosen according to the probability distribution $g(s)$, where $g(s) > 0$ for all $s \in S$.

ASSUMPTION 3. The distribution function $F(x)$ has a continuous density $f(x)$, and $f(x)$ is bounded away from zero on $[-1, 1]$.

ASSUMPTION 4. $p(\emptyset|s) = 1 - q$ for some $0 < q \leq 1$ and all $s \in S$; $\sum_{s \in S} p(m|s) > 0$ for all $m \neq \emptyset$.

ASSUMPTION 5. (Strict monotone likelihood ratio property) Let (m, s) and (m', s') be signal state pairs, such

that $m' > m$ and $s' > s$, where $m, m' \neq \emptyset$. Then, $p(m|s)p(m'|s') \geq p(m|s')p(m'|s)$, and the inequality is strict if $p(m|s)p(m'|s') > 0$.

Assumption 1 says that the payoff difference between the two alternatives is increasing both in the preference type and in the state. Thus, higher states make alternative 2 more attractive. Assumption 2 implies that every state occurs with positive probability, and assumption 3 implies that every open interval of preference types occurs with positive probability. Assumption 4 implies that signal \emptyset does not provide any information about the state s and that the probability any voter type receives the uninformative signal is strictly less than 1, that is, in a large electorate there is a high probability that some voters will have private information. Assumption 5 says all signals other than \emptyset satisfy the strict monotone likelihood ratio property (SMLRP). Specifically, when the ratio $p(m|s')/p(m|s)$ is defined, it is increasing in m if $s' > s$ and $m \in \{1, 2, \dots, M\}$. Informally, SMLRP says that higher signals are more likely in higher states. For a careful discussion of the monotone likelihood ratio property see Milgrom (1981).

EQUILIBRIUM

Each player chooses an action $a \in \{A, 1, 2\}$, where $a = A$ denotes abstention, $a = 1$ denotes a vote for alternative 1, and $a = 2$ denotes a vote for alternative 2. By n_a we denote the number of agents who take action a . We assume that the election outcome is determined by majority rule. Alternative 1 is the winner if and only if $n_2 \leq n_1$. This implies that in case of a tie, 1 will be the winner of the election.⁸

A symmetric strategy profile is a measurable function $\sigma: [-1, 1] \times \mathcal{M} \rightarrow \Delta^3$. The probability that a player with preference parameter x and signal m takes action a is $\sigma_a(x, m)$.

Given a symmetric strategy profile σ , the ex-ante probability that a voter takes action a in state s is denoted by $t_a(s)$, where

$$t_a(s) = \sum_{m \in \mathcal{M}} p(m|s) \int_{-1}^1 \sigma_a(x, m) dF(x). \quad (2)$$

For a given strategy profile σ we define the random variables $n_a(s)$ to be the number of agents who take action a in state s . Recall that the number of agents n is a Poisson random variable with parameter v . Furthermore, signals are conditionally independent, and preference types are chosen independently. It follows that $n_a(s)$, $a \in \{A, 1, 2\}$ are independent Poisson random variables (see Myerson 1997 for an extensive discussion of the properties of the Poisson distribution in large voting games). The expected number of agents who take action a in state s is denoted $v_a(s)$, where

$$v_a(s) = v \cdot t_a(s).$$

⁸ This choice of tie-breaking rule is made for technical convenience. All our results would go through if, in the case of a tie, alternative 1 wins with probability $\pi \in (0, 1)$.

behavior of informed voters stays the same when the information levels of other voters change.

It follows from the properties of the Poisson distribution and the symmetry of the strategy profile that, from the perspective of any given voter, the number of *other* voters who take action a in state s is described by the same random variable $n_a(s)$.

The results in this article depend on the fact that a voter only influences the outcome of an election when her vote is pivotal. In our model there are two ways in which a vote can be pivotal: A voter can influence the election outcome if the election is tied ($n_2 = n_1$) or if alternative 2 is ahead by one vote ($n_2 = n_1 + 1$). In the event that $n_2 = n_1 + 1$, voting for 2 and abstaining both lead to the outcome 2, and voting for 1 leads to the outcome 1. We label this event piv_1 and say that a vote is pivotal for alternative 1. In the event that $n_2 = n_1$, voting for 1 and abstaining both lead to the outcome 1, and voting for 2 leads to the outcome 2. We label this event piv_2 . We label the event that a vote is pivotal (either piv_1 or piv_2) as piv . Note that the notation $n_1 = n_2$ or $n_1 + 1 = n_2$ refers to the outcome of the election from the perspective of a particular voter *without* his vote.

Since $n_1(s)$ and $n_2(s)$ are independent Poisson random variables, we can write the probabilities of the pivotal events for a given state s and a symmetric strategy profile σ as follows:

$$\begin{aligned} \Pr\{piv_1|s\} &= \sum_{x=0}^{\infty} \Pr(n_1(s) = x) \Pr(n_2(s) = x + 1) \quad (3) \\ &= \sum_{x=0}^{\infty} \frac{e^{-v(t_1(s)+t_2(s))} (vt_2(s))^{x+1} (vt_1(s))^x}{x!(x+1)!}. \end{aligned}$$

The probability a vote is pivotal for alternative 2 is written as:

$$\begin{aligned} \Pr\{piv_2|s\} &= \sum_{x=0}^{\infty} \Pr(n_1(s) = x) \Pr(n_2(s) = x) \quad (4) \\ &= \sum_{x=0}^{\infty} \frac{e^{-v(t_1(s)+t_2(s))} (vt_2(s))^x (vt_1(s))^x}{x!x!}; \end{aligned}$$

and the probability a vote is pivotal for either alternative is written as

$$\Pr\{piv|s\} = \Pr\{piv_1|s\} + \Pr\{piv_2|s\}. \quad (5)$$

Voters form beliefs about the distribution over states conditional on their private information and the event a vote is pivotal. If the probability an agent votes for alternative 2, $t_2(s)$, is greater than zero for all s , then it follows from equations 3 and 4 that $\Pr(piv_j|s) > 0$ for all s , and $j \in \{1, 2\}$. Therefore, the probability distribution over states conditional on being pivotal for alternative $j \in \{1, 2\}$ is well defined and given by

$$\beta(s|piv_j) = \frac{\Pr(piv_j|s)g(s)}{\sum_{w \in S} \Pr(piv_j|w)g(w)}. \quad (6)$$

The probability distribution over states conditional on being pivotal and observing signal m is then given by:

$$\begin{aligned} \beta(s|piv_j, m) &= \frac{\Pr(piv_j|s)g(s)p(m|s)}{\sum_{w \in S} \Pr(piv_j|w)g(w)p(m|w)} \quad (7) \\ &= \frac{\beta(s|piv_j)p(m|s)}{\sum_{w \in S} \beta(w|piv_j)p(m|w)}. \end{aligned}$$

Our first objective is to find a simple characterization of equilibrium behavior. Recall that we assumed the payoff of alternative 2 is increasing in the preference type. Hence, a natural conjecture is for strategies to be monotone in the following sense: If type x votes for alternative 2 upon observing signal m , then all types $x' > x$ vote for 2 upon observing the same signal. Similarly, if type x' votes for 1 after observing signal m , then all types $x < x'$ vote for 1 after observing the same signal. Furthermore, (by SMLRP) agents place more weight on higher states when they observe a higher signal. Therefore, we might expect voting behavior to be monotone in the signal: If an agent with preference type x votes for 2 upon receiving signal m , then he also votes for 2 upon receiving signal $m' > m$. The following definition of ordered cutpoints captures these two types of monotonicity.

DEFINITION 1. A strategy σ is characterized by ordered cutpoints if there are preference types $\{(x_m^1, x_m^2)\}_{m \in \mathcal{M}}$ with the following properties: $\in \mathcal{M}x_m^1 \leq x_m^2$ for every m ; $1 > x_1^1 \geq \dots \geq x_M^1 > -1$ for every $j \in \{1, 2\}$; and $\sigma_1(x, m) = 1$ if $x < x_m^1$, $\sigma_2(x, m) = 1$ if $x > x_m^2$ and $\sigma_A(x, m) = 1$ if $x_m^1 < x < x_m^2$.

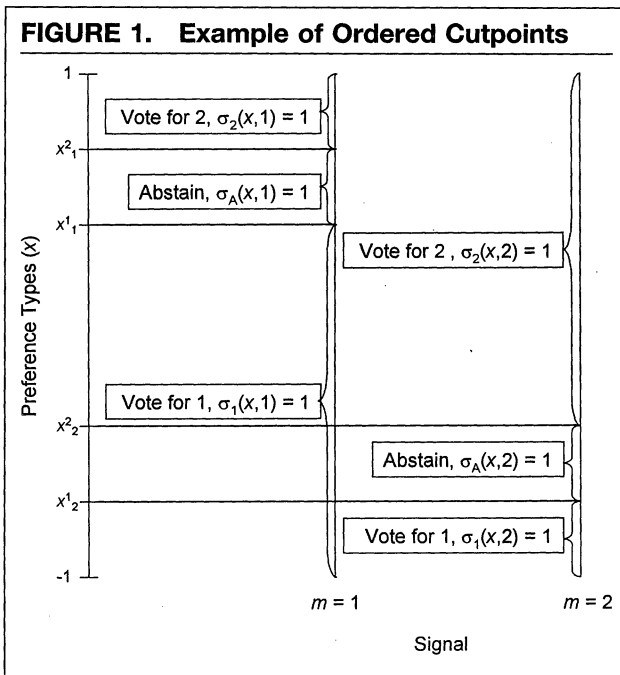
Observe that if a strategy profile is characterized by ordered cutpoints, then voters of type (x, m) with $x < x_M^1$ vote for alternative 1 regardless of their private signal, whereas those for whom $x > x_1^2$ vote for alternative 2 regardless of their private signal. We say that voter types with $x \in (x_M^1, x_1^2)$ take informative action.

In the following we consider symmetric Nash equilibria. The first proposition demonstrates that every symmetric Nash equilibrium can be characterized by ordered cutpoints.

PROPOSITION 1. Suppose assumptions 1–5 hold. Then a symmetric equilibrium exists, and any such equilibrium can be characterized by ordered cutpoints.

Figure 1 illustrates a symmetric strategy profile in a model with two informative signals that is consistent with ordered cutpoints.

The fact that the cutpoints are strictly ordered together with SMLRP implies that in any symmetric equilibrium the expected vote share for alternative 1 will be strictly decreasing in the state, while the expected vote share for alternative 2 will be strictly increasing. This is demonstrated by the following proposition. We include the proof in the text because it is short and because we use the monotonicity of the vote shares repeatedly below.



PROPOSITION 2. In a symmetric equilibrium $t_1(s)$ is strictly decreasing in s , and $t_2(s)$ is strictly increasing in s .

Proof. Since a symmetric equilibrium can be characterized by ordered cutpoints, we can rewrite 2 as follows:

$$t_1(s) = \sum_{m \in \mathcal{M}} p(m|s) F(x_m^1), \quad (8)$$

and

$$t_2(s) = \sum_{m \in \mathcal{M}} p(m|s) (1 - F(x_m^2)). \quad (9)$$

Since x_m^j is strictly decreasing in m for every $j \in \{1, 2\}$, it follows from a standard property of SMLRP that $t_1(s)$ is strictly decreasing, and $t_2(s)$ is strictly increasing. *Q.E.D.*

The fact that every symmetric equilibrium is characterized by ordered cutpoints does not imply that there are always some voters who abstain. Consider the case in which there are two states $S = \{0, 1\}$ and two messages $\mathcal{M} = \{1, 2\}$. Suppose each voter receives perfect information about state s ; message 1 implies the state is 0; and message 2 implies the state is 1. Thus, every voter knows the true state and, except for one type, has a strictly dominant strategy to vote for one of the two alternatives. In particular, voters get no additional information by conditioning on the event a vote is pivotal for either alternative, that is, $\beta(s|piv_1, m) = \beta(s|piv_2, m)$ for every s and m , and hence it must be the case that $x_m^1 = x_m^2$ for all m .

More generally, a signal $m \in \mathcal{M}$ is perfectly informative if there is a unique state s_m in which the signal can be received, that is, $p(m|s) > 0$ if and only if $s = s_m$. The following proposition demonstrates that if a message is not perfectly informative, then a strictly positive fraction of preference types will abstain.

PROPOSITION 3. Suppose assumptions 1–5 hold. Suppose σ is an equilibrium strategy, and let (x_m^j) , $j = 1, 2$ be the corresponding cutpoints. If m is not perfectly informative, then $x_m^2 - x_m^1 > 0$.

To gain an intuition for proposition 3, consider a (not perfectly informed) voter who is indifferent between voting for alternative 2 and abstaining. This implies that, conditional on the event that $n_1 = n_2$, the voter is indifferent between the two alternatives. Such a voter strictly prefers abstaining to voting for alternative 1. To see this note that voting for 1 instead of abstaining only makes a difference in the event that $n_2 = n_1 + 1$. Because the expected vote share of alternative 2 is strictly increasing in s , the probability distribution over states conditional on $n_2 = n_1 + 1$ puts more weight on states that are favorable to alternative 2.⁹ Thus, alternative 2 is even more desirable in the event that $n_2 = n_1 + 1$ than in the event $n_2 = n_1$, and the voter strictly prefers to abstain rather than vote for alternative 1. As a consequence, there is an interval of preference types who prefer abstaining to voting for either alternative.¹⁰

EQUILIBRIUM CHARACTERIZATION FOR LARGE POPULATIONS

Proposition 3 does not exclude the possibility that the expected fraction of voters who choose to abstain in equilibrium converges to zero as the population goes to infinity. To conclude that our model predicts significant abstention, we need a stronger result: We need to demonstrate that the fraction of voters who abstain stays bounded away from zero as the expected population size becomes very large.

In this section, we analyze equilibrium voting behavior for large electorates. We do so by considering a sequence of populations in which the expected size of the population grows to infinity. Recall that v denotes the expected number of agents. In the following we consider a sequence of populations $(v_k)_{k=1}^\infty$ in which $v_k \rightarrow \infty$. Along the sequence we fix the information structure defined by $g(s)$, $p(m|s)$, and $F(x)$. We use the subscript k to indicate the dependence of the equilibrium on the parameter v_k . Thus, for example, σ_k denotes a voting equilibrium for a population with expected size v_k .

Preliminary Results

In this section we demonstrate three technical lemmas that will allow us to prove our main results on abstention and information aggregation. The main results of this section (lemmas 2 and 3) are concerned with the

⁹ Technically, the distribution over states conditional on $n_2 = n_1 + 1$ first-order stochastically dominates the probability distribution over states conditional on $n_2 = n_1$.

¹⁰ Proposition 3 demonstrates that every symmetric equilibrium (under the stated assumptions) leads to abstention. Note that this relies on the assumption that the number of voters is uncertain. If, instead, the number of voters is common knowledge, then there are also equilibria in which no abstention occurs. In large elections, however, it seems unreasonable to assume that every voter can exactly predict the number of other voters.

distribution over states conditional on a vote being pivotal. Since every voter behaves *as if* a vote is pivotal, characterizing this distribution is essential for our analysis. Basically, the results together imply that, conditional on a vote being pivotal, the probability distribution over states must take the following simple form.

- (1) In the limit, voters can put positive probability on at most two states s^1 and s^2 (lemma 2).
- (2) Consider an election in which each agent knows the state and votes for her preferred alternative. Then, s^1 is the largest state in which alternative 1 wins, and s^2 is the smallest state in which alternative 2 wins (lemma 2).
- (3) While uncertainty is limited to two states, it does not disappear completely, that is, voters assign nonnegligible probability to both states in the limit (lemma 3). This result requires the further assumption that not too many agents have perfectly informative signals.

The three items above should serve as a nontechnical summary of the results of this subsection. Readers not interested in the technical details may skip to the next subsection.

We define the ratio of expected vote shares for the two alternatives given state $s \in S$ and profile σ as:

$$\rho_k(s) = \frac{t_{1k}(s)}{t_{2k}(s)}. \quad (10)$$

Lemma 1 says that in large elections the probability that a vote is pivotal for alternative 2 ($n_1 = n_2$) differs from the probability a vote is pivotal for alternative 1 ($n_1 = n_2 + 1$) by a factor that is approximately equal to $\sqrt{\rho_k(s)}$. Lemma 1 was first proven by Myerson (1998).

LEMMA 1. *Suppose assumptions 1–3 hold.*

1. *There is an $\varepsilon > 0$ such that $t_{1k}(s) > \varepsilon$ and $t_{2k}(s) > \varepsilon$ for all k .*
2. *As $k \rightarrow \infty$, $|Pr_k\{\text{piv}_2|s\}/Pr_k\{\text{piv}_1|s\} - \sqrt{\rho_k(s)}| \rightarrow 0$ for every $s \in S$.*

The first part of the lemma follows immediately from assumption 1. The second part follows from a calculation of limits using the Poisson distribution. We give a proof in the Appendix. An alternative proof can be found in Myerson (1998). Lemma 1 allows us to approximate the relative likelihood of the pivot probabilities by $\sqrt{\rho_k(s)}$. Recall that by proposition 2 $t_{1k}(s)$ is decreasing in s and $t_{2k}(s)$ is increasing in s . Hence, $\sqrt{\rho_k(s)}$ is decreasing in s . Therefore, a consequence of lemma 1 is that the likelihood ratio of piv_1 and piv_2 is decreasing in s .

Let $x(s)$ denote the preference type who is indifferent between the two alternatives in state s . Thus, $x(s)$ is defined by the equation

$$v(x(s), s) = 0. \quad (11)$$

Assumption 1 implies that $x(s)$ is well defined for every $s \in S$ and strictly decreasing in s . All voters with

preference types $x < x(1)$ prefer alternative 1 in every state and hence have a strictly dominant strategy to vote for 1. Similarly, voters with preference types $x > x(0)$ always vote for alternative 2. If there are many voters, and if $F(x(1)) > 1/2$ ($F(x(0)) < 1/2$), then with high probability the majority of voters prefer alternative 1 (alternative 2), irrespective of their information, and hence alternative 1 (alternative 2) must win the election with high probability in every state. The following assumption rules out these cases.

ASSUMPTION 6. $F(x(0)) > 1/2$, $F(x(1)) < 1/2$.

We define s^1 to be the largest state in which 1 is elected if all voters know the state, that is,

$$s^1 = \max\{s : F(x(s)) \geq 1/2\}.$$

Similarly, we define s^2 to be the smallest state in which 2 is elected if all voters know the state, that is,

$$s^2 = \min\{s : F(x(s)) < 1/2\}.$$

Clearly, $s^1 < s^2$, and (s^1, s^2) is a pair of consecutive states. Also note that s^1, s^2 are well defined if assumption 6 holds.

The following lemma says that in a voting equilibrium with a large electorate beliefs over states conditional on a voter being pivotal are concentrated on s^1 and s^2 .

LEMMA 2. *Suppose assumptions 1–6 hold. Consider a sequence of voting equilibria. Then $\beta_k(s^1|\text{piv}) + \beta_k(s^2|\text{piv}) \rightarrow 1$ as $k \rightarrow \infty$. Also, for $j = 1, 2$, $\beta_k(s^1|\text{piv}_j) + \beta_k(s^2|\text{piv}_j) \rightarrow 1$ as $k \rightarrow \infty$.*

First, observe that in a large election, conditional on a vote being pivotal, voters must put almost all probability weight on those states in which the expected vote shares for each alternative are closest to being tied. Second, note that the set of states for which the election is closest to being tied consists either of a unique state or a pair of consecutive states. Suppose the set is $\{s', s''\}$. We need to show that in fact $\{s', s''\} = \{s^1, s^2\}$. Suppose that $s^1 > s'$, and $s^1 > s''$. The fact that all beliefs are concentrated on the set $\{s', s''\}$ implies that $t_{1k}(s) > F(x(s^1)) > 1/2$ for $s \leq s^1$, since all voters with preference types $x > x(s^1)$ must vote for alternative 1, irrespective of their private information in states $s \leq s^1$. But now recall that the vote share of alternative 1 is decreasing in s , and hence $1/2 < t_{1k}(s^1) < t_{1k}(s'')$; therefore, it is more likely that the election is tied in state s^1 than in either s' or s'' , and we have a contradiction.

In the following lemma we refine the characterization of the limit support of the distribution $\beta_k(s|\text{piv})$: We give conditions under which both s^1 and s^2 have positive probability conditional on a voter being pivotal. Let

$$\bar{M} = \{m : p(m, s^1)p(m, s^2) = 0 \text{ and } p(m, s^1) + p(m, s^2) > 0\}$$

denote the set of signals that allow an agent to distinguish states s^1 and s^2 with certainty. In other words, an

agent who receives a signal $m \in \bar{M}$ and knows that $s \in \{s^1, s^2\}$ is perfectly informed. The expression $(F(x(s^1)) - F(x(s^2))) \sum_{m \in \bar{M}} p(m|s)$ denotes the expected fraction of voters who receive signals in \bar{M} in state s and who have state-dependent preferences over the states in $\{s^1, s^2\}$. We call such voters perfectly informed swing voters.

Assumption 7 requires that $F(x(s^2)) < 1/2 < F(x(s^1))$ and puts a bound on the fraction of perfectly informed swing voters.

ASSUMPTION 7. $2|1/2 - F(x(s^i))| > (F(x(s^1)) - F(x(s^2))) \sum_{m \in \bar{M}} p(m|s)$ for all s and $i = 1, 2$.

Assumption 7 is satisfied if there are no perfectly informative signals (i.e., \bar{M} is empty) and $F(x(s^2)) < 1/2 < F(x(s^1))$. Even if some agents receive perfectly informative signals, assumption 7 is satisfied as long as $F(x^1)$ and $F(x^2)$ are approximately symmetric around $1/2$: If $F(x^1) = 1 - F(x^2)$, then assumption 7 holds as long as there is a strictly positive probability that agents receive a signal that does not perfectly discriminate between s^1 and s^2 .

The next lemma uses assumption 7 to show that, conditional on a vote being pivotal, an agent believes both states s^1 and s^2 occur with probability bounded away from zero.

LEMMA 3. Suppose assumptions 1–7 hold. Then there is an $\epsilon > 0$ and a $k' < \infty$ such that $\beta_k(s^1|\text{piv}) \geq \epsilon$, $\beta_k(s^2|\text{piv}) \geq \epsilon$ for all $k \geq k'$.

To understand why assumption 7 is needed in lemma 3, consider the following example.

Example. Suppose there are two states, $s = 1, 2$. Further suppose that $F(x(1)) = 0.75$, that is, the probability that a randomly drawn voter prefers 1 in state 1 is 0.75, and $F(x(2)) = 0.4$, that is, the probability that a randomly drawn voter prefers 2 in state 2 is 0.6. Thus, $2|0.5 - F(x(2))| = 0.2$, and $F(x(1)) - F(x(2)) = .35$.

First consider the case in which all voters know the state. Clearly, this implies that in a large electorate alternative 1 gets almost 75% of the vote in state 1 and about 40% of the vote in state 2. Conditional on a vote being pivotal, the probability of state 2 converges to one in this case. Also note that assumption 7 is violated in this case, since $.35 > .2$.

Now assume that some voters do not know s , that is, do not get a perfectly informative signal. In a large electorate, if there is a small fraction of uninformed voters, then these voters must behave as if the state is $s = 2$. Thus, an uninformed voter will vote for alternative 1 with probability 0.4 and for alternative 2 with probability 0.6. If q is the probability that a voter is informed, then the expected vote share for alternative 1 in state 1 is now

$$t_1(s = 1) = q \cdot 0.75 + (1 - q) \cdot 0.4,$$

and the expected vote share for alternative 2 in state 2 is

$$t_2(s = 2) = q \cdot 0.6 + (1 - q) \cdot 0.6 = 0.6.$$

Thus, the described equilibrium strategies and the resulting limit distribution over states conditional on a vote being pivotal is valid as long as

$$q \cdot 0.75 + (1 - q) \cdot 0.4 > 0.6$$

or

$$q > \frac{0.2}{0.35}.$$

For $q < 0.2/.35$, assumption 7 is satisfied, and the lemma shows that both states $s = 1$ and $s = 2$ must have strictly positive probability in the limit distribution over states conditional on a vote being pivotal.

Full Information Equivalence

We now give conditions under which elections satisfy full information equivalence. Full information equivalence is satisfied if the election outcome under asymmetric information converges in probability to the election outcome that would occur if all the voters knew the true state and voted for their preferred alternative. This result extends the results in Feddersen and Pesendorfer (1996, 1997). Recall that $x(s)$ denotes the preference type that is indifferent between the two alternatives in state s . Furthermore, s^1 denotes the largest state in which the expected fraction of voters who prefer alternative 1 is greater than or equal to $1/2$. Hence, $F(x(s^1)) \geq 1/2$. In the following proposition we assume that this inequality is strict.¹¹

PROPOSITION 4. Suppose assumptions 1–5 hold, and $F(x(s^1)) > 1/2$. Then, as $k \rightarrow \infty$, the probability that alternative 1 is elected converges to 1 for $s \leq s^1$ and to 0 for $s > s^1$.

To provide an intuition for the result we sketch the proof for the case in which there are two states ($s \in \{0, 1\}$) and no agent is perfectly informed.

Sketch of Proof: Since no agent is perfectly informed we may apply lemma 3 and conclude that, conditional on a vote being pivotal, the probability of state s must stay bounded away from zero for both states $s = 0$ and $s = 1$. Hence, a voter's private signal changes her beliefs about the state conditional on a vote being pivotal even as the expected size of the electorate grows to infinity. Consequently, the fraction of voters who change their vote as a function of their signal stays bounded away from zero. This implies that the expected vote share for alternative 1 is at least ϵ larger in state 0 than in state 1, that is, $t_{1k}(0) > t_{1k}(1) + \epsilon$; similarly, the expected vote share of alternative 2 is at least ϵ larger in state 1 than in state 0, that is, $t_{2k}(1) > t_{2k}(0) + \epsilon$. Thus, it follows that

$$t_{1k}(0) - t_{2k}(0) > t_{1k}(1) - t_{2k}(1) + 2\epsilon.$$

¹¹ If there is no state in which the expected vote shares are exactly equal under full information, the result would still hold. We excluded this case here because it would require a different proof. The reason is that in this case the fraction of voters who use their private information may converge to zero. See Feddersen and Pesendorfer (1997) for such an argument.

Observe that lemma 3 implies that $t_{1k}(0) - t_{2k}(0) > 0 > t_{1k}(1) - t_{2k}(1)$. To see this assume to the contrary that $t_{1k}(0) - t_{2k}(0) \leq 0$. We know that $t_{1k}(1) - t_{2k}(1) + 2\varepsilon < t_{1k}(0) - t_{2k}(0) \leq 0$. But this in turn implies that it is much more likely a vote is pivotal in state 0 than in state 1. Therefore, in a large electorate, a voter must put almost zero probability on being in state 1 conditional on a vote being pivotal—a contradiction to lemma 3. Similarly, lemma 3 implies that $|t_{1k}(s) - t_{2k}(s)|$ stays bounded away from zero for $s \in \{0, 1\}$.¹² Thus, there is an $\eta > 0$ such that

$$t_{1k}(0) - t_{2k}(0) > \eta > -\eta > t_{1k}(1) - t_{2k}(1),$$

and hence the law of large numbers implies that for large k , alternative 1 is elected with probability close to one in state 0, and alternative 2 is elected with probability close to one in state 1. *Q.E.D.*

Abstention

In this section we give conditions under which the fraction of voters who abstain stays bounded away from zero as $k \rightarrow \infty$.

PROPOSITION 5. *Suppose assumptions 1–7 hold. Then there is an $\alpha > 0$ and a k' such that the expected fraction of voters who abstain in equilibrium is larger than α for all $k > k'$.*

To provide an intuition for proposition 5, observe that lemma 3 says that voters are uncertain about the state conditional on being pivotal even in the limit. This in turn allows us to conclude that the probability distribution over states conditional on a tie (the event piv_2) is bounded away from the probability distribution conditional on alternative 2 being ahead by one vote (the event piv_1). More precisely, there is an $\varepsilon' > 0$ such that

$$\beta_k(s^1|piv_2) < \beta_k(s^1|piv_1) - \varepsilon',$$

for all k . But this is enough to allow us to make the argument given in proposition 3 for an interval of preference type with length uniformly bounded away from zero, and hence proposition 5 follows.

The previous proposition demonstrated that a strictly positive fraction of the electorate will always abstain, but it did not provide a sense of how large that fraction may be. The next proposition provides a bound on the fraction of agents who abstain in equilibrium. Define a *critical state* to be $s^* \in \arg \min_s |F(x(s)) - 1/2|$, that is, a state in which the fraction of the electorate that prefers alternative 1 to alternative 2 is closest to 1/2. In the example on the previous page, the critical state is $s = 0$. It follows from the definition of s^1 and s^2 that $s^* \in \{s^1, s^2\}$.

We now show that the fraction of the electorate that abstains in a large election goes to zero as the fraction

of the electorate that prefers alternative 1 in state s^* goes to 1/2.

PROPOSITION 6. *Suppose assumptions 1–6 hold. For every $\varepsilon > 0$ there is an $\eta > 0$ such that $|F(x(s^*)) - 1/2| \leq \eta$ implies that $\limsup_k (1 - t_{1k}(s) - t_{2k}(s)) \leq \varepsilon$ for all $s \in S$, that is, the expected fraction of voters who abstain is bounded above by ε for sufficiently large k .*

A key step in the proof is to show that if the fraction of the electorate that prefers alternative 1 to alternative 2 in the critical state is very close to 1/2, then the equilibrium vote share of each alternative must be close to 1/2 in both of the states, s^1 and s^2 , in which a vote is most likely to be pivotal. But in that case a single vote provides very little information about the true state, since voters are expected to vote for either alternative with close to equal probability in both s^1 and s^2 . Hence, conditioning on the event piv_1 provides very similar information to conditioning on the event piv_2 , and a small fraction of the electorate abstains.¹³

As a corollary this implies that if the state space is “fine,” that is, if the utility variation between any pair of consecutive states is small, then the level of abstention is small.

COROLLARY 1. *Suppose assumptions 1–6 hold. Then, for every $\varepsilon > 0$ there is an v and a k' such that if $\max_x |v(x, s^2) - v(x, s^1)| < v$, then the expected fraction of voters who abstain is less than ε for all $k > k'$.*

We can use propositions 1 and 5 to relate the level of abstention to the “aggregate” level of information.

Example. Consider the bridge example from the introduction and suppose voters are uncertain about the true cost of the bridge. Specifically, let the true cost of the bridge be a continuous random variable, $c \in [0, 1]$, that is drawn by nature according to some probability distribution. Suppose that there are K television stations who each do an independent investigation into the cost of the bridge, and each televises a news report only if it finds that the bridge is likely to be very expensive. The probability that a television station reports the bridge is expensive is a strictly increasing function of the true cost. Each voter watches at most one television news report, but some do not watch at all. The probability a voter watches a report is q . Let s_k denote the expected cost of the bridge conditional on k out of K television stations reporting that the bridge is expensive. Finally, suppose that the expected payoff of agents given k reports is given by

$$E\{v(x, t)|k\} = v(x, s_k),$$

¹³ Assumption 7 implies that there cannot be a state in which the expected vote shares are exactly equal under full information. In this case the fraction of voters who abstain converges to zero. We excluded it here because it would require a different proof. The reason is that in this case the fraction of voters who use their private information may converge to zero. Therefore, a more delicate argument is needed to establish full information equivalence (see Feddersen and Pesendorfer 1997), which in turn is needed to prove proposition 5.

¹² If there is a state for which $|t_{1k}(s) - t_{2k}(s)| \rightarrow 0$, then voters must believe that this state has occurred with probability converging to one conditional on being pivotal, which again is in contradiction to lemma 3.

where $v(x, s_k)$ satisfies assumption 1. In words, the utility difference between building and not building the bridge is strictly decreasing in relation to the number of reports that the bridge is expensive.

This example can be represented in our model as a finite state space model in which the state space is $S = \{s_0, \dots, s_K\}$, and the probability a voter observes report $m \in \{\emptyset, l, h\}$ in state s_k is

$$p(m|s_k) = \begin{cases} \frac{qk}{K} & \text{if } m = h; \\ q\left(1 - \frac{k}{K}\right) & \text{if } m = l; \\ (1 - q) & \text{if } m = \emptyset. \end{cases}$$

When $K = 1$ there are two types of agents, those who are perfectly informed and those who are uninformed. When $K > 1$ there are no perfectly informed agents. When $K \rightarrow \infty$ the state space approximates a continuous state space $[0, 1]$.

Clearly, if the number of independent investigations by television stations is large, then $s_{k+1} - s_k$ is small. Thus, many independent reports lead to better aggregate information and a smaller fraction of the population abstaining in equilibrium.

INFORMATION AND PARTICIPATION

In this section we use a simplified version of our model to demonstrate how private information combined with preference diversity can result in two seemingly contradictory comparative statics: More informed voters participate with higher probability than less informed voters; yet, increasing the fraction of the electorate that is better informed results in increased abstention.

Assume there are two states, $s = 0, 1$, and

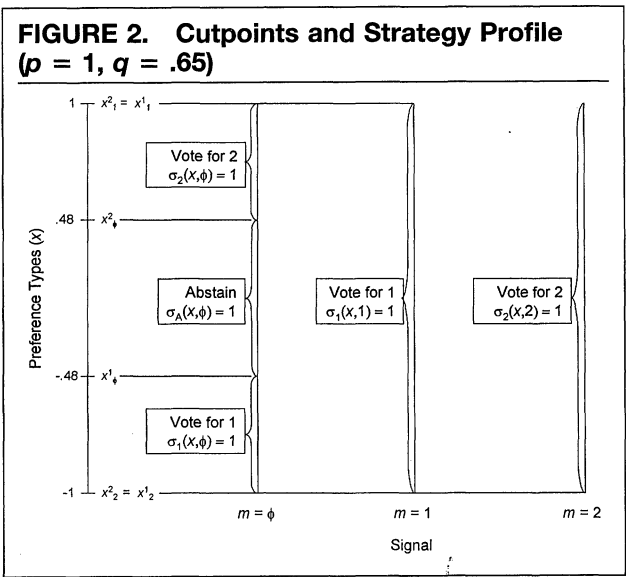
$$v(x, s) = \begin{cases} x - 1 & \text{if } s = 0; \\ x + 1 & \text{if } s = 1. \end{cases} \tag{12}$$

Thus, in state 0 all voter types strictly prefer alternative 1 to alternative 2, while in state 2 preferences are reversed. The agents' prior assigns probability 1/2 to each state. Agents are distributed uniformly on $[-1, 1]$. There are three signals: $\mathcal{M} = \{\emptyset, 1, 2\}$. Signals 1 and 2 are informative, while \emptyset provides no information. Voters receive an informative signal with probability q . The conditional probability of observing signal $m \in \mathcal{M}$ given state s is $p(m|s)$, where:

$$p(m|s) = \begin{cases} pq & \text{if } m = s + 1; \\ (1 - p)q & \text{if } m \neq s + 1 \text{ and } m \neq \emptyset; \\ (1 - q) & \text{if } m = \emptyset; \end{cases}$$

and $p > 1/2$.

An equilibrium in this model is characterized by the vector of cutpoints at which x_m^j is the voter type who is indifferent between voting for alternative j and abstaining conditional upon observing signal m .



Perfectly Informative Signals

Suppose the signal $m \in \{1, 2\}$ is perfectly informative, that is, $p = 1$. In this example the only variable is the fraction of the electorate that is perfectly informed (q). This assumption, combined with the symmetry of the setting, greatly simplifies the system of equations and permits an analytical solution. Figure 2 illustrates the cutpoints for the case when $q = .65$. As noted above, informed agents have perfect information and always vote for alternative 1 when they observe signal 1 and alternative 2 when they observe signal 2. The uninformed agents are partitioned into three intervals of preference types: those who always vote for alternative 1 ($x \in (-1, -.48)$), those who abstain ($x \in (-.48, .48)$), and those who always vote for alternative 2 ($x \in (.48, 1)$).

Figure 3 plots abstention as a function of the probability an agent is perfectly informed. Since perfectly informed voters never abstain, one might imagine that increasing the fraction of such voters in the electorate would automatically reduce abstention. Yet, as the fraction of informed voters grows, so does the informativeness of the election result. As a consequence, the uninformed voters become more willing to abstain. When the fraction of the informed electorate is small, this equilibrium effect dominates, and abstention increases. When the fraction of the informed electorate is large, the additional abstention of the uninformed is outweighed by the additional participation of the informed.

Noisy Signals

Now suppose that those who are informed observe a noisy signal, that is, $p \in (.5, 1)$. There are three cutpoints that must be calculated, one for each signal. The reason is that the inference of agents who receive signals $m \in \{1, 2\}$ now also depends on the pivotal event. In this section we present numerical examples because we are unable to solve the system analytically.

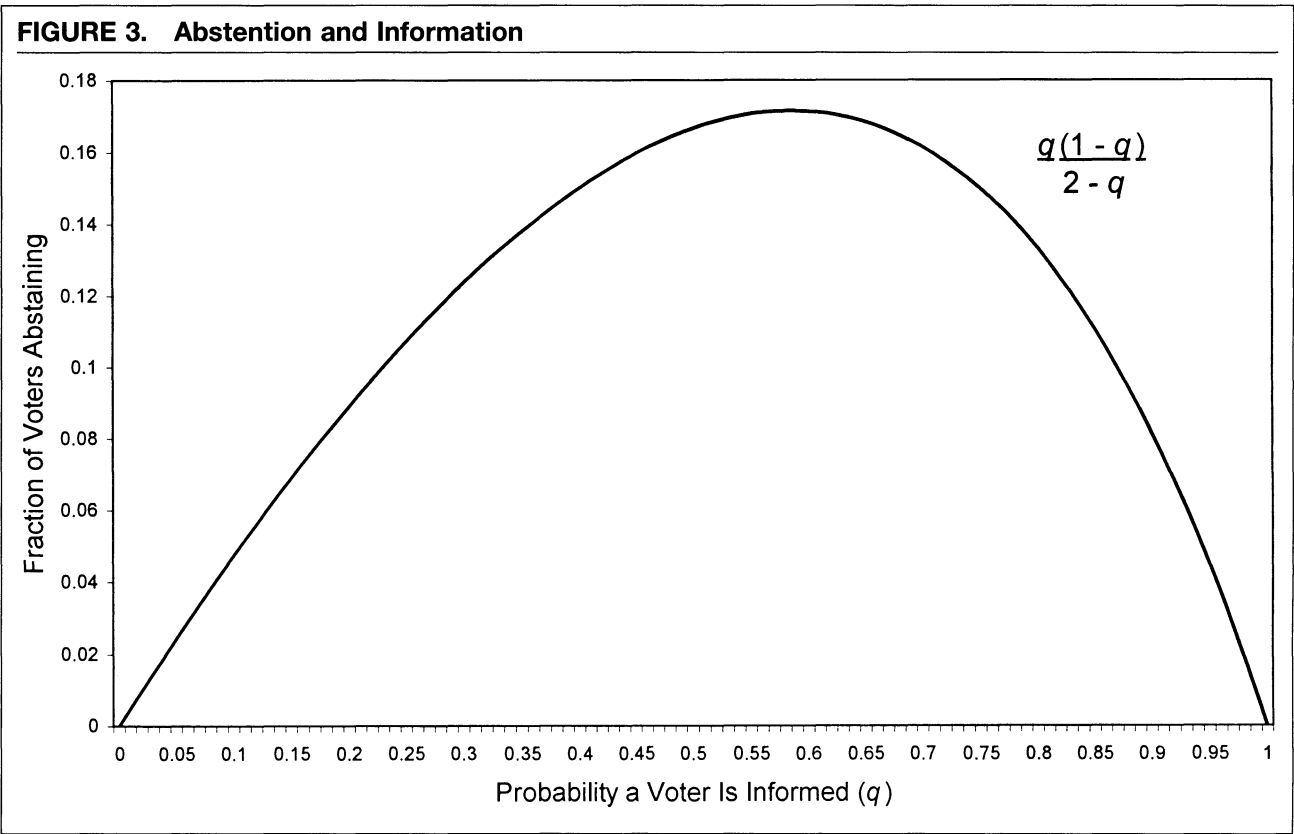
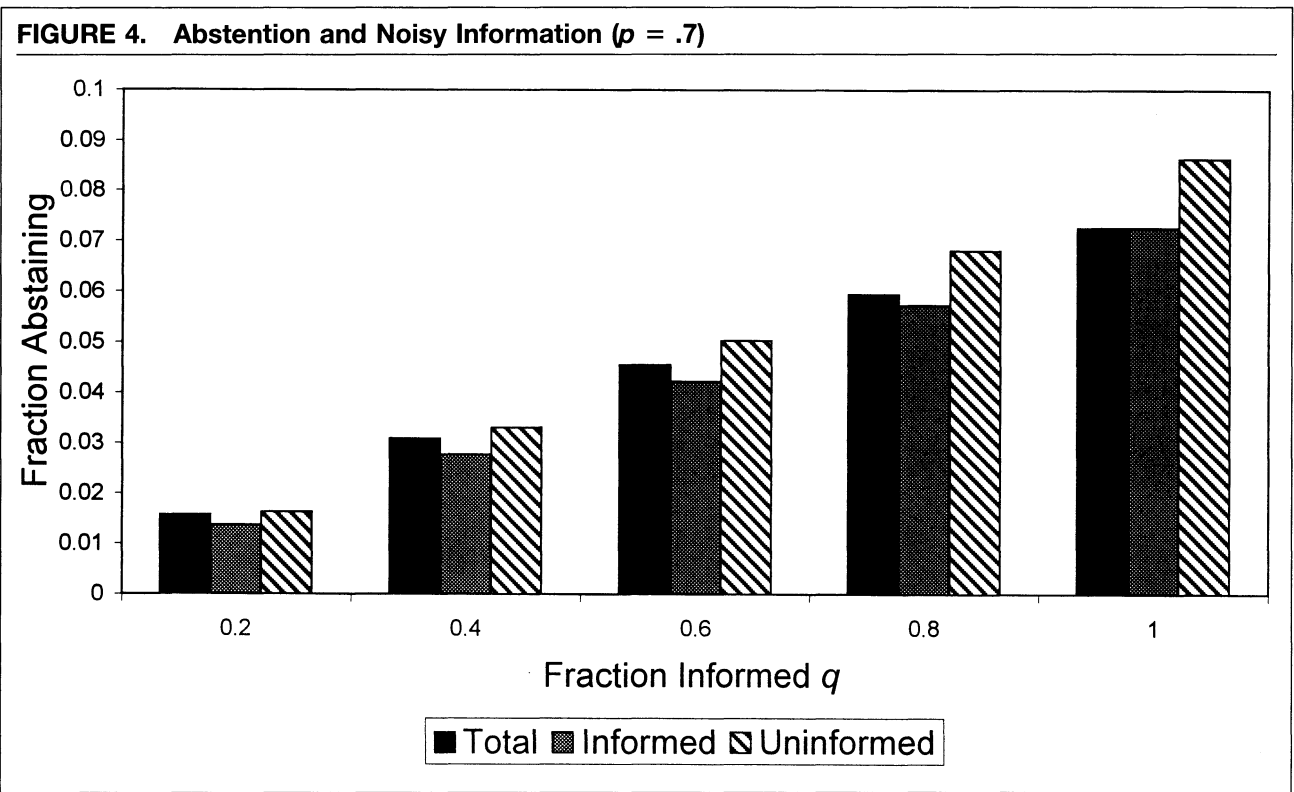
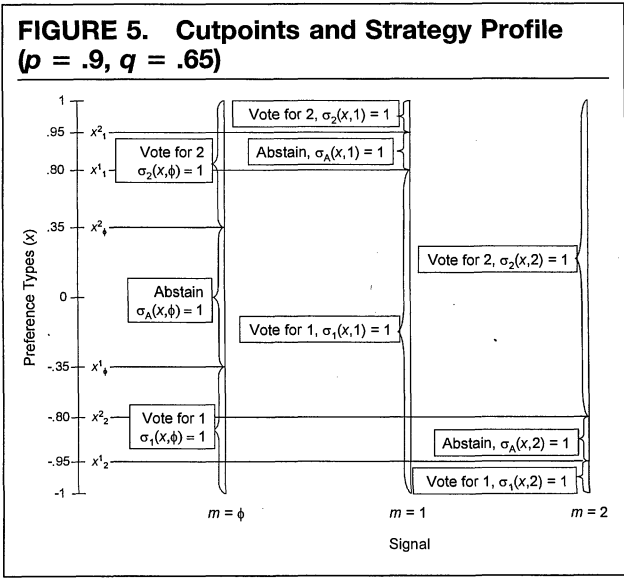


Figure 4 demonstrates the relationship between abstention and information when agents are not perfectly informed. Three consequences of increasing the fraction of informed agents stand out. First, abstention by informed voters is always strictly lower than abstention by the uninformed. Second, increasing the fraction of the electorate that is informed always results in higher levels of abstention by both the informed and unin-





formed. Third, increasing the fraction informed always results in a larger increase in abstention among the uninformed than among the informed.¹⁴

Figure 5 shows the cutpoints and strategy profile for one particular example in which $p = .9$ and $q = .65$. To understand the effect of noise on the equilibrium strategy profile, it is useful to compare the strategy profile in Figure 5 with the profile in Figure 2. The introduction of noise reduces the fraction of the uninformed who abstain, but it creates two new categories of voter: informed types who vote independent of their signal and informed abstainers. In the model with noise, voters with types in the interval $(-1, -.95)$ always vote for alternative 1, while those in the interval $(.95, 1)$ always vote for alternative 2. Voters whose type is in the interval $(-.95, -.80)$ vote for alternative 1 unless they are informed and observe signal 2, in which case they abstain. Similarly, those with types in the interval $(.80, .95)$ vote for alternative 2 unless they are informed and observe the signal 1, in which case they abstain.

The fact that some informed voters abstain as a function of their signal means that in any given election the expected median voter differs from the expected population median. The expected median preference type in this section is $x = 0$. We say that the voting electorate is biased in state s if the expected median voter's preference type is not equal to zero.

The strategy profile in Figure 5 implies that the expected median voter's preference type in state 0 is strictly less than zero, while in state 1 the expected

median's preference type is strictly greater than zero. In this example the expected median participant has a preference type of $-.06$ in state 0, while in state 1 it is $.06$. Since the election satisfies full information equivalence, alternative 1 always wins in state 0, while alternative 2 always wins in state 1.¹⁵ Thus, in this example the voting electorate is always biased toward the winner.¹⁶

Although the voting electorate is always biased toward the winner, the outcome is not biased. To define bias of the election outcome, we compare the election outcome in a model in which abstention is permitted to the election outcome in a model in which abstention is not permitted. Since the election without abstention satisfies full information equivalence, it follows that the election outcome in the model with abstention is unbiased whenever it satisfies full information equivalence.¹⁷

The fact that turnout is biased in favor of the supporters of the winning alternative creates the possibility of the correlation between, for example, turnout of the lower classes and left-wing policy outcomes. Turnout has no independent effect on policy outcomes, however. Rather, it is the state that determines both. In an empirical study, Hill, Leighley, and Hinton-Andersson (1995) find a positive relationship between lower-class voter turnout and welfare policy.¹⁸ As we have argued, this correlation is consistent with our model. If, for example, economic conditions constitute the state variable, then both turnout and policy choice may be a function of economic conditions.¹⁹ A regression of policy choice on turnout will discover a positive correlation, but there is no direct relationship between those variables.

BIAS AND ABSTENTION

Another stylized fact about participation in elections is that those on the Left participate less frequently than those on the Right (Lijphart 1997). The following example demonstrates that a biased distribution of information in our model can lead to an electorate that is always biased toward the more informed end of the ideological spectrum. While the bias in the electorate may be dramatic, this does not result in biased outcomes. Indeed, in this example the biased distribution of information results in strictly higher probabilities the election will satisfy full information equivalence than does an unbiased distribution of information.

¹⁴ The reader may wonder if there is a theorem that might be proved to the effect that those with better information always participate more frequently and that abstention among the uninformed always increases faster as the electorate as a whole becomes more informed. While this pattern appears to be robust for a variety of examples, it is possible to construct a counterexample in which those with better information abstain with higher probability than those with worse information. The example requires the introduction of considerable asymmetry in preferences. Details are available upon request from the authors.

¹⁵ Feddersen and Pesendorfer (1997) demonstrate full information equivalence in a model without abstention. It is also straightforward to adapt the arguments of proposition 4 to the case without abstention.
¹⁶ In general, since the states need not be equally likely, it is also the case that the ex-ante (prior to the realization of the state) expected median participant need not be equal to the population median type.
¹⁷ Adapting the argument in the proof of proposition 4 to the case in which abstention is not permitted is straightforward. For a proof in a related setting, see Feddersen and Pesendorfer (1997).
¹⁸ See Ringquist, Hill, Leighley, and Hinton-Andersson (1997) as well.
¹⁹ For example, during a recession everyone may be more disposed to support government spending.

TABLE 1. Participation and a Biased Distribution of Information		
Profile	Bias	No Bias ($q = .5$)
x_ϕ^1	-.8	-1/3
x_ϕ^2	-.2	1/3
$t_1(0)$.6	2/3
$t_2(0)$.1	1/6
$t_1(1)$.1	1/6
$t_2(1)$.6	2/3
Abstention	.3	1/6

For purposes of exposition, call those with preference types below 0 leftists and those above 0 rightists. Assume the same structure as in the example on pages 388–9 above with the proviso that all rightists observe the perfectly informative signal, while leftists receive the uninformative signal. Thus, the distribution of information is maximally biased toward those on the Right. It is clear that any abstention will occur only among leftists, which guarantees that, independent of the state, the voting electorate is biased to the Right. Because the rightists all vote for 1 when they observe signal 1 and vote for 2 otherwise, it only remains to determine the behavior of the leftists.

The first column of Table 1 gives the cutpoints, vote shares, and total level of abstention for this example. The total level of abstention is 30%, but abstention only occurs among leftists. Indeed, 60% of the leftists abstain. Nevertheless, by looking at the expected vote shares in each state, it is clear that alternative 1 wins in a landslide in state 0, as does alternative 2 in state 1, and the expected margin of victory is the same in each state. Thus, the abstention may be concentrated on one end of the ideological spectrum without biasing the outcome of the election. It should be noted that in this example voters have different preferences but all prefer alternative 1 in state 1 and alternative 2 in state 2.²⁰

To get a sense of the effect that a biased distribution of information has on equilibrium behavior, suppose $q = .5$, $p = 1$ (those informed have perfect information), and there is no bias in the distribution of information. In this case the fraction of the electorate that is informed is the same as in the above example. The difference is that a leftist is as likely to receive an informative signal as a rightist. The cutpoints and expected vote shares for each state in this case are presented in column 2 of Table 1.

Abstention is lower with the unbiased distribution of information. While both the biased and the unbiased electorate choose alternative 1 in state 1 and alternative 2 in state 2 with high probability, in large finite elections the biased distribution of information actually results in lower probabilities of error than the unbiased distribution. This follows from the fact that all the informed voters vote correctly in each setting. The uninformed voters only introduce noise; thus, the fact

that abstention is higher under the biased distribution of information marginally improves the performance of the electoral mechanism.

CONCLUSION

In the first section we demonstrated that abstention due to asymmetric information is robust to preference diversity and variable information environments. In the second section we presented a set of examples that demonstrate the range of phenomena consistent with the model. In particular, we demonstrated that a biased electorate may be caused by a biased distribution of information without creating a biased outcome. The fact that a large range of behavior may be supported as equilibrium in some information environment complicates the task of using this model as a predictive tool. Nevertheless, the normative result that elections work well as information aggregation mechanisms even in the face of what would appear to be a systematically biased electorate and large-scale abstention is robust.

It should be noted that we do not claim the outcomes of recent U.S. elections were unbiased, that is, would remain unchanged if all those eligible to vote had participated. Rather, we argue that correlations between outcomes and patterns of participation do not constitute evidence in favor of biased outcomes. Our examples show that such correlation may be generated in a model in which outcomes are always unbiased.

One objection to our model is that computations of best responses are beyond the capabilities of actual voters. In our environment, voting optimally requires correctly calculating two probability distributions over the state space, updating these distributions to include private information, and then deciding how to vote or whether to abstain. An objection commonly raised against mathematical models of human behavior is that the calculations are too difficult. The standard response is that mathematical models do not claim to capture the actual thought process of decision makers but aspire simply to predict behavior. The examples presented above suggest that our model is capable of predicting some of the stylized facts that appear puzzling in the context of decision-theoretic models. Whether our framework is better than existing models is an open empirical question beyond the scope of this article. Recent work by Ladha, Miller, and Oppenheimer (1996) and McKelvey and Palfrey (1998) shows, however, that the equilibrium predictions of strategic voting models work remarkably well in laboratory experiments.

APPENDIX

PROPOSITION 1. Suppose assumptions 1–5 hold. Then a symmetric equilibrium exists, and any such equilibrium can be characterized by ordered cutpoints.

Proof. Existence is straightforward. For example, the existence proof in Myerson (1997) applies.

Since there is a strictly positive probability that any player is the only agent, it follows that the probability a vote is pivotal for 2 is always strictly positive. Therefore, by assumption

²⁰ It is a simple matter to generalize the example to include voters who always prefer one alternative or the other, independent of the state.

tion 1, there is an $\varepsilon > 0$ such that all types $x \in [1 - \varepsilon, 1]$ must vote for 2, independent of their signal m . By assumption 3, the probability that a type is in the interval $[1 - \varepsilon, 1]$ is strictly positive, and hence $t_2(s)$ is strictly positive in any equilibrium. This in turn implies a strictly positive probability that there are no votes for 1 and one vote for 2. Hence, the probability that a voter is pivotal for 1 is strictly positive. Therefore, $\beta(s|piv_j, m)$, $j = 1, 2$, and $\beta(s|piv, m)$ are well defined in equilibrium.

Fix a symmetric profile σ and let $E(v(x, s)|piv_j, m)$ denote the expectation of $v(x, s)$ with respect to $\beta(\cdot|piv_j, m)$. A voter of type (x, m) prefers to vote for 2 rather than abstain if

$$E\{v(x, s)|piv_2, m\} = \sum_s v(x, s)\beta(s|piv_2, m) > 0; \quad (13)$$

s/he prefers to vote for 1 rather than abstain if

$$E\{v(x, s)|piv_1, m\} = \sum_s v(x, s)\beta(s|piv_1, m) < 0, \quad (14)$$

and s/he prefers to vote for 2 rather than 1 if

$$E\{v(x, s)|piv, m\} = \sum_s v(x, s)\beta(s|piv, m) > 0. \quad (15)$$

By assumption 1, $v(x, s)$ is strictly increasing in x . Thus, if it is a best response for type x to vote for 2, then for every type $x' > x$ the unique best response must be to vote for 2. Similarly, if it is a best response for type x to vote for 1, then the unique best response for every type $x' < x$ must be to vote for 1. It follows that any voting equilibrium can be characterized by cutpoints, such that for any $m \in \mathcal{M}$ $x_m^1 \leq x_m^2$, and all types with $x < x_m^1$ vote for 1 when they receive signal m , and all types with $x > x_m^2$ vote for 2 if they receive signal m . Types in the interval (x_m^1, x_m^2) abstain.

Observe that voters with types x_m^j are either indifferent between voting for j and abstaining (if (x_m^1, x_m^2) is nonempty) or indifferent between voting for 1 and 2 (if $x_m^1 = x_m^2$). As a consequence, either

$$E\{v(x_m^j, s)|piv_j, m\} = 0,$$

or

$$E\{v(x_m^j, s)|piv, m\} = 0.$$

To show that cutpoints are ordered, observe that for $m > m'$, such that $m, m' \neq \emptyset$, SMLRP implies that for all x

$$E\{v(x, s)|piv_j, m\} > E\{v(x, s)|piv_j, m'\},$$

and

$$E\{v(x, s)|piv, m\} > E\{v(x, s)|piv, m'\}.$$

Therefore, it must be the case that $1 > x_1^j > \dots > x_M^j > -1$ for $j = 1, 2$. Q.E.D.

PROPOSITION 3. Suppose assumptions 1–5 hold. Suppose σ is an equilibrium strategy, and let (x_m^j) , $j = 1, 2$ be the corresponding cutpoints. If m is not perfectly informative, then $x_m^2 - x_m^1 > 0$.

Proof. Equations 3 and 4 imply that

$$\begin{aligned} \frac{\Pr\{piv_1|s\}}{\Pr\{piv_2|s\}} &= \frac{\sum_{x=0}^{\infty} \frac{[vt_1(s)]^x [vt_2(s)]^{x+1}}{x!(x+1)!}}{\sum_{x=0}^{\infty} \frac{[vt_1(s)]^x [vt_2(s)]^x}{x!x!}} \\ &= L(vt_1(s), vt_2(s)). \end{aligned}$$

Below (lemma A2) we demonstrate that

$$\frac{\partial}{\partial v} L(v, \omega) < 0, \quad \frac{\partial}{\partial \omega} L(v, \omega) > 0.$$

Therefore, since t_1 is strictly decreasing in s and t_2 is strictly increasing in s , it follows that $L(vt_1(s), vt_2(s))$ is strictly increasing in s .

Let $\beta(s|Y)$ denote the probability of state s conditional on the event Y . We can interpret $\beta(s|piv_j, m)$ as the distribution that is achieved by updating $\beta(s|m) = g(s)p(m|s)/\sum_m g(s)p(m|s)$ with the signal $piv_j \in \{piv_1, piv_2\}$, which satisfies SMLRP since $L(t_1(s), t_2(s))$ is strictly increasing in s . If m is not perfectly informative, then it is the case that $p(m|s)$ is nonzero for two or more states. Then, by a standard property of SMLRP (see Milgrom 1981), it follows that $\beta(s|piv_1, m)$ strictly first-order stochastically dominates $\beta(s|piv_2, m)$. Since v is strictly increasing in s , this allows us to conclude that

$$E\{v(x, s)|piv_1, m\} > E\{v(x, s)|piv_2, m\}. \quad (16)$$

Note that piv is the union of the events piv_1 and piv_2 , which both occur with strictly positive probability, and therefore

$$E\{v(x, s)|piv_1, m\} > E\{v(x, s)|piv, m\} > E\{v(x, s)|piv_2, m\}. \quad (17)$$

Consider a voter (\hat{x}, m) who is indifferent between voting for 1 and voting for 2, that is,

$$E\{v(\hat{x}, s)|piv, m\} = 0.$$

Such a voter exists by assumption 1. By inequality 17 this voter strictly prefers to abstain, since conditional on piv_1 (a vote for 1 is decisive) the voter strictly prefers alternative 2, and conditional on piv_2 (a vote for 2 is decisive) the voter strictly prefers alternative 1. Since v is continuous, it follows that there is an interval of voters who strictly prefer to abstain whenever m is not perfectly informative. Q.E.D.

For the subsequent lemmas we need to introduce the following notation. The modified Bessel function $I_v(x)$ is defined as

$$I_v(x) = \left(\frac{1}{2}x\right)^v \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}x^2\right)^k}{k!(v+k)!}. \quad (18)$$

Note the following facts (see Olver 1970, 376):

$$I'_0(x) = I_1(x);$$

$$I'_1(x) = I_0(x) - \frac{1}{x}I_0(x);$$

$$I'_2(x) = I_1(x) - \frac{2}{x}I_1(x);$$

$$I_0(0) = 1, I_1(0) = 0, I_2(0) = 0;$$

$$I_0(x) - I_2(x) = \frac{2}{x}I_1(x). \quad (19)$$

Also note that from equation 9.7.1. in Olver (1970, 377) it follows that, for $v \in \{0, 1, 2\}$,

$$\lim_{x \rightarrow \infty} \frac{e^x}{\sqrt{2\pi x} I_v(x)} = 1. \quad (20)$$

Clearly, the first two equations of 19, together with the initial conditions $I_0(0) = 1$, $I_1(0) = 0$, imply that for $x > 0$

$$I_0(x) > I_1(x), \quad (21)$$

since $I_1'(x) - I_0'(x) < I_0(x) - I_1(x)$, and hence $I_0(x) - I_1(x)$ cannot change sign (or be equal to zero) for $x > 0$.

Similarly, the second and the third equations imply that $I_2'(x) - I_1'(x) < \frac{1}{x}(I_1(x) - I_2(x))$, and together with the initial conditions this again implies that $I_1(x) > I_2(x)$ for $x > 0$. Also note that

$$I_0(x) - I_2(x) = \frac{2}{x} I_1(x).$$

This, together with $I_0(x) - I_2(x) > I_0(x) - I_1(x)$ implies that

$$0 < \frac{I_0(x)}{I_1(x)} - 1 < \frac{I_0(x) - I_2(x)}{I_1(x)} = \frac{2}{x}.$$

Therefore, it follows that

$$\frac{I_0(x)}{I_1(x)} \rightarrow 1. \quad (22)$$

DEFINITION 2. Define the function

$$L(v, \omega) = \frac{\sum_{x=0}^{\infty} \frac{v^x \omega^{x+1}}{x!(x+1)!}}{\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!x!}}$$

for any $v > 0, \omega > 0$.

LEMMA A1. For $I_v(x)$ as defined in equation 18 we have that

1. $I_0(2\sqrt{v\omega}) = \sum_{x=0}^{\infty} \frac{v^x \omega^x}{(x!x!)};$
2. $I_1(2\sqrt{v\omega}) = (\sqrt{\omega}/\sqrt{v}) \sum_{x=0}^{\infty} \frac{v^x \omega^{x+1}}{(x!(x+1)!)};$
3. $L(v, \omega) = (\sqrt{\omega}/\sqrt{v}) I_1(2\sqrt{v\omega})/I_0(2\sqrt{v\omega}).$

Proof. To see part 1, note that

$$\begin{aligned} I_0(2\sqrt{v\omega}) &= 1 + \frac{(2\sqrt{v\omega})^2}{2^2(1!)^2} + \frac{(2\sqrt{v\omega})^4}{2^4(2!)^2} + \frac{(2\sqrt{v\omega})^6}{2^6(3!)^2} + \dots \\ &= 1 + \frac{(v\omega)^1}{(1!)^2} + \frac{(v\omega)^2}{(2!)^2} + \frac{(v\omega)^3}{(3!)^2} + \dots \\ &= \sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!x!}. \end{aligned}$$

To see part 2, observe that

$$\begin{aligned} \frac{\sqrt{\omega}}{\sqrt{v}} I_1(2\sqrt{v\omega}) &= \frac{\sqrt{\omega}}{\sqrt{v}} \left(\frac{(2\sqrt{v\omega})}{2} + \frac{(2\sqrt{v\omega})^3}{2^3 1!2!} + \frac{(2\sqrt{v\omega})^5}{2^5 2!3!} \right. \\ &\quad \left. + \dots \right) \\ &= \frac{\sqrt{\omega}}{\sqrt{v}} \left(\frac{(\sqrt{v\omega})}{1} + \frac{(\sqrt{v\omega})^3}{1!2!} + \frac{(\sqrt{v\omega})^5}{2!3!} + \dots \right) \\ &= \left(\frac{\omega}{1} + \frac{v\omega^2}{1!2!} + \frac{v^2\omega^3}{2!3!} + \dots \right) = \sum_{x=0}^{\infty} \frac{v^x \omega^{x+1}}{x!(x+1)!}. \end{aligned}$$

Part 3 now follows by taking ratios.

Q.E.D.

LEMMA A2. $\partial L(v, \omega)/\partial v < 0, \partial L(v, \omega)/\partial \omega > 0$.

Proof. From lemma A1 we know that

$$\begin{aligned} L(v, \omega) &= \frac{\sqrt{\omega} I_1(2\sqrt{v\omega})}{\sqrt{v} I_0(2\sqrt{v\omega})} \\ &= \frac{\sum_{x=0}^{\infty} \frac{v^x \omega^{x+1}}{x!(x+1)!}}{\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!x!}}. \end{aligned}$$

Therefore,

$$\frac{\partial}{\partial v} L(v, \omega) = \frac{\left[\left(\sum_{x=0}^{\infty} \frac{xv^{x-1}\omega^{x+1}}{x!(x+1)!} \right) \left(\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!x!} \right) - \left(\sum_{x=0}^{\infty} \frac{v^x \omega^{x+1}}{x!x!} \right) \left(\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!(x+1)!} \right) \right]}{\left(\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!x!} \right)^2}.$$

We need to show that the numerator is positive.

$$\begin{aligned} &\left(\sum_{x=0}^{\infty} \frac{v^{x-1}\omega^{x+1}}{(x-1)!(x+1)!} \right) \left(\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!x!} \right) - \left(\sum_{x=0}^{\infty} \frac{v^x \omega^{x+1}}{(x-1)!x!} \right) \\ &\quad \cdot \left(\sum_{x=0}^{\infty} \frac{v^x \omega^x}{x!(x+1)!} \right) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} \left(\frac{v^{x-1}\omega^{x+1}}{(x-1)!(x+1)!} \frac{v^y \omega^y}{y!y!} \right. \\ &\quad \left. - \frac{v^{x-1}\omega^x}{(x-1)!x!} \frac{v^y \omega^{y+1}}{y!(y+1)!} \right) = \sum_{x=0}^{\infty} \sum_{y \leq x} \frac{v^{x+y-1}\omega^{x+y+1}}{(x-1)!x!(y-1)!y!} \\ &\quad \cdot \left[\frac{1}{(x+1)y} - \frac{1}{(y+1)y} + \frac{1}{(y+1)x} - \frac{1}{(x+1)x} \right]. \end{aligned}$$

A straightforward calculation shows that

$$\begin{aligned} &\left[\frac{1}{(x+1)y} - \frac{1}{(y+1)y} + \frac{1}{(y+1)x} - \frac{1}{(x+1)x} \right] \\ &= \frac{2xy - x^2 - y^2}{(x+1)y(y+1)x} < 0. \end{aligned}$$

Therefore, we have established the first part of the lemma.

For the second part we use the following two properties of the Bessel functions (see Olver 1970, 377):

$$I_0'(x) = I_1(x),$$

$$I_1'(x) = I_0(x) - \frac{1}{x} I_1(x).$$

Using these two equations, a straightforward calculation yields

$$\frac{\partial L}{\partial \omega} = \frac{I_0(2\sqrt{v\omega})^2 - I_1(2\sqrt{v\omega})^2}{I_0(2\sqrt{v\omega})^2}.$$

Since $I_0(x) > I_1(x) > 0$ for $x > 0$ (equation 21), the result follows. Q.E.D.

LEMMA 1. Suppose assumptions 1–3 hold.

1. There is an $\varepsilon > 0$ such that $t_{1k}(s) > \varepsilon$, and $t_{2k}(s) > \varepsilon$ for all k .
2. As $k \rightarrow \infty, |Pr_k\{\text{piv}_2|s\}/Pr_k\{\text{piv}_1|s\} - \sqrt{\rho_k(s)}| \rightarrow 0$.

Proof. To see part 1, note that by assumption 1 there is an $\varepsilon > 0$ such that for all $x \in [-1, -1 + \varepsilon]$ an agent strictly prefers alternative 1 for all states s , and similarly for all $x \in$

$(1 - \varepsilon, 1]$ an agent strictly prefers alternative 2. This implies that $t_{jk}(s)$ is bounded away from zero for all k and all s .

To see part 2, note that by equation 3 in lemma A1

$$\begin{aligned} \frac{\Pr_k\{piv_2|s\}}{\Pr_k\{piv_1|s\}} &= \frac{\sqrt{v_k t_{1k}(s)} I_0(2v_k \sqrt{t_{1k}(s)t_{2k}(s)})}{\sqrt{v_k t_{2k}(s)} I_1(2v_k \sqrt{t_{1k}(s)t_{2k}(s)})} \\ &= \sqrt{\rho_k(s)} \frac{I_0(2v_k \sqrt{t_{1k}(s)t_{2k}(s)})}{I_1(2v_k \sqrt{t_{1k}(s)t_{2k}(s)})}. \end{aligned}$$

Now the result follows from equation 22.

Q.E.D.

Let

$$T_k = \{\arg \min_s |\sqrt{t_{1k}(s)} - \sqrt{t_{2k}(s)}|\} \quad (23)$$

be the states that minimize the difference between the square roots of the expected vote shares for each alternative. The next lemma shows that in a large election conditional on a vote being pivotal almost all probability weight must be on states in T_k . The lemma assumes that a positive fraction of agents takes informative action, that is, $(x_{1k}^j - x_{Mk}^j)$ stays bounded away from zero.

LEMMA A3. Suppose assumptions 1–6 hold and that $(x_{1k}^j - x_{Mk}^j) > \varepsilon > 0$ for $j = 1$ or $j = 2$ and all k . Then, as $k \rightarrow \infty$, $\sum_{s \in T_k} \beta_k(s|piv_j) \rightarrow 1$ for $j = 1, 2$, and $\sum_{s \in T_k} \beta_k(s|piv) \rightarrow 1$.

Proof. By the assumption of strict SMLRP and the fact that $(x_{1k}^j - x_{Mk}^j) > \varepsilon > 0$ for $j = 1$ or $j = 2$, it follows that $t_{1k}(s)$ is strictly decreasing, while $t_{2k}(s)$ is strictly increasing in s , with

$$t_{1k}(s) - t_{1k}(s') > \varepsilon' > 0$$

and

$$t_{2k}(s') - t_{2k}(s) > \varepsilon' > 0 \quad (24)$$

for $s' > s$ and for all k . Consider a convergent subsequence and let $t_j(s) = \lim_{k \rightarrow \infty} t_{jk}(s)$. Furthermore, let

$$T = \{\arg \min_s |\sqrt{t_1(s)} - \sqrt{t_2(s)}|\}. \quad (25)$$

It follows from 24 that T consists either of a singleton or a pair of consecutive states. We now show that agents must believe that, conditional on a vote being pivotal, the state is almost certainly in T .

We can write the ratio of pivot probabilities conditional on states s and s' as follows:

$$\frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} = \frac{\sum_{x=0}^{\infty} \frac{e^{-v(t_{1k}(s') + t_{2k}(s'))} (v t_{2k}(s'))^x (v t_{1k}(s'))^x}{x! x!}}{\sum_{x=0}^{\infty} \frac{e^{-v(t_{1k}(s) + t_{2k}(s))} (v t_{2k}(s))^x (v t_{1k}(s))^x}{x! x!}}.$$

Using equation 1 in lemma A1, we rewrite the right-hand side to yield

$$\begin{aligned} \frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} &= \frac{e^{-v(t_{1k}(s') + t_{2k}(s'))} I_0(2v \sqrt{t_{1k}(s') t_{2k}(s')})}{e^{-v(t_{1k}(s) + t_{2k}(s))} I_0(2v \sqrt{t_{1k}(s) t_{2k}(s)})} \\ &= e^{v(t_{1k}(s) - t_{1k}(s') + t_{2k}(s) - t_{2k}(s'))} \frac{I_0(2v \sqrt{t_{1k}(s') t_{2k}(s')})}{I_0(2v \sqrt{t_{1k}(s) t_{2k}(s)})}. \end{aligned}$$

Note that from equation 20 it follows that

$$\begin{aligned} \frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} &\rightarrow e^{v_k(t_{1k}(s) - t_{1k}(s') + t_{2k}(s) - t_{2k}(s'))} \frac{\frac{e^{2v_k \sqrt{t_{1k}(s') t_{2k}(s')}}}{\sqrt{2\pi 2v_k \sqrt{t_{1k}(s') t_{2k}(s')}}}}{\frac{e^{2v_k \sqrt{t_{1k}(s) t_{2k}(s)}}}{\sqrt{2\pi 2v_k \sqrt{t_{1k}(s) t_{2k}(s)}}}} \\ &= \frac{\sqrt{t_{1k}(s) t_{2k}(s)}}{\sqrt{t_{1k}(s') t_{2k}(s')}} e^{v_k((\sqrt{t_{1k}(s)} - \sqrt{t_{2k}(s)})^2 - (\sqrt{t_{1k}(s')} - \sqrt{t_{2k}(s')})^2)}. \end{aligned} \quad (26)$$

Expression 26 and the fact that $t_{1k}(s)$ and $t_{2k}(s')$ are bounded away from zero for all s (lemma 1) allow us to conclude that

$$\frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} \rightarrow 0$$

if

$$(\sqrt{t_1(s')} - \sqrt{t_2(s')}) > (\sqrt{t_1(s)} - \sqrt{t_2(s)}). \quad (27)$$

Since 27 holds for any $s \in T$ and $s' \notin T$, it follows that

$$\frac{\sum_{s' \in T} \Pr_k\{piv_2|s'\}}{\sum_{s \in S} \Pr_k\{piv_2|s\}} \rightarrow 1. \quad (28)$$

Lemma 1 implies the ratio $\Pr_k\{piv_1|s'\}/\Pr_k\{piv_2|s\}$ is bounded away from zero and infinity since $t_{1k}(s)$ and $t_{2k}(s)$ are both bounded away from zero. Therefore, 28 implies that

$$\frac{\sum_{s' \in T} \Pr_k\{piv_1|s'\}}{\sum_{s \in S} \Pr_k\{piv_1|s\}} \rightarrow 1. \quad (29)$$

Together, 28 and 29 imply

$$\frac{\sum_{s \in T} \Pr_k\{piv|s\}}{\sum_{s \in S} \Pr_k\{piv|s\}} \rightarrow 1.$$

Since

$$\beta_k(s|piv_j) = \frac{g(s) \Pr_k\{piv_j|s'\}}{\sum_{s \in S} g(s) \Pr_k\{piv_j|s'\}},$$

the lemma follows.

Q.E.D.

LEMMA 2. Suppose assumptions 1–6 hold. Consider a sequence of voting equilibria. Then $\beta_k(s^1|piv) + \beta_k(s^2|piv) \rightarrow 1$ as $k \rightarrow \infty$. Also, for $j = 1, 2$, $\beta_k(s^1|piv_j) + \beta_k(s^2|piv_j) \rightarrow 1$ as $k \rightarrow \infty$.

Proof. Consider a sequence of voting equilibria (x_{mk}^j) , $j = 1, 2$. Consider a convergent subsequence of (x_{mk}^j) .

Step 1: Suppose $x_{1k}^j - x_{Mk}^j \rightarrow 0$ for $j = 1, 2$. Then $\beta_k(s^1|piv) \rightarrow 1$ as $k \rightarrow \infty$, and $F(x(s^1)) = 1/2$.

Proof. Since

$$E(v(x_{1k}^2, s)|piv_1, 1) = E(v(x_{Mk}^2, s)|piv_1, M)$$

and

$$E(v(x_{1k}^1, s)|piv_2, 1) = E(v(x_{Mk}^1, s)|piv_2, M),$$

it follows from the continuity of v that for $x \in [x_{Mk}^2, x_{1k}^2]$ and $x' \in [x_{Mk}^1, x_{1k}^1]$

$$E(v(x, s)|piv_1, 1) - E(v(x, s)|piv_1, M) \rightarrow 0,$$

and

$$E(v(x', s)|piv_2, 1) - E(v(x', s)|piv_2, M) \rightarrow 0. \quad (30)$$

Since the signal satisfies assumption 2, it follows that there exists an s, s' such that $\Pr_k(s|piv_1) \rightarrow 1$ and $\Pr_k(s'|piv_2) \rightarrow 1$. Now recall that piv_1 is the event $n_1 = n_2 + 1$, and piv_2 is the event $n_1 = n_2$. Hence, the difference is exactly one vote. Since $t_{jk}(s)$ is bounded away from zero, it follows that a vote is always a noisy signal of the state and hence contains limited information. Therefore, $\Pr_k(s'|piv_2) \rightarrow 1$ implies that $\Pr_k(s'|piv_1) \rightarrow 1$ and hence $s' = s$. (A straightforward algebraic argument can be made using equations 3 and 4.)

We now show that it must be the case that $F(x(s')) = 1/2$. Suppose this were false, and $F(x(s')) < 1/2$. Then the fraction of voters who prefer 2 in state s is less than $1/2 - \epsilon$ for some $\epsilon > 0$. Since $x'_{1k} - x'_{2k} \rightarrow 0$ for $j = 1, 2$, it follows that the fraction of voters who vote for 2 is less than $1/2 - \epsilon/2$ for sufficiently large k for all s . This in turn implies that the probability that $n_2 = n_1$ is maximized at $s = 1$, and since every state has strictly positive prior it follows that $s' = 1$. But this contradicts the assumption that $F(x(1)) > 1/2$ (assumption 6). An analogous argument can be made for the case in which $F(x(s')) > 1/2$.

Step 2: Suppose $\lim(x'_{1k} - x'_{2k}) > \epsilon > 0$ for $j = 1$ or $j = 2$. Then $\beta_k(s^1|piv) + \beta_k(s^2|piv) \rightarrow 1$ as $k \rightarrow \infty$.

Proof. By lemma A3 we know that $\sum_T \beta_k(s|piv) \rightarrow 1$. Thus, it is sufficient to show that $T \subseteq \{s^1, s^2\}$.

Consider a convergent subsequence, and let $t_j(s) = \lim_{k \rightarrow \infty} t_{jk}(s)$. From the definition of T and the monotonicity of $t(s)$ it follows that, if $t_1(s^1) \geq t_2(s^1)$ and $t_1(s^2) \leq t_2(s^2)$, then $T \subseteq \{s^1, s^2\}$.

Suppose, contrary to what we need to show, that

$$t_1(s^1) < t_2(s^2). \quad (31)$$

It follows that $\{s > s^1\} \cap T = \emptyset$, and hence

$$\beta_k(s > s^1|piv_1) \rightarrow 0. \quad (32)$$

Consider any m such that $p(m|s^1) > 0$. Expression 32 implies that a voter who receives signal m and conditions on the event that a vote is pivotal believes the probability of state $s > s^1$ is arbitrarily close to zero for large k . Thus, for all $\epsilon > 0$ there is a k' such that for $k > k'$

$$x'_{mk} > x(s^1) - \epsilon,$$

and hence

$$\liminf_k x'_{mk} \geq x(s^1).$$

But since $F(x(s^1)) > 1/2$, this in turn implies that $\liminf_k t_{1k}(s^1) - 1/2 \geq 0$, and hence $\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)} > 0$, which contradicts 31. An analogous argument can be used to establish $t_1(s^2) \leq t_2(s^2)$. *Q.E.D.*

LEMMA 3. Suppose assumptions 1–7 hold. Then there is an $\epsilon > 0$ and a $k' < \infty$ such that $\beta_k(s^1|piv) \geq \epsilon$, $\beta_k(s^2|piv) \geq \epsilon$ for all $k \geq k'$.

Proof. Suppose contrary to the lemma that there is a subsequence such that

$$\beta_k(s^1|piv) \rightarrow 0.$$

This implies that

$$\beta_k(s^2|piv) \rightarrow 1,$$

and hence $t_{1k}(s^2) \rightarrow F(x(s^2))$ and $t_{2k}(s^2) \rightarrow 1 - F(x(s^2))$. Moreover, since all voters who receive a noisy signal behave as if state 2 had occurred,

$$t_{1k}(s^1) \rightarrow F(x(s^2)) + (F(x(s^1)) - F(x(s^2))) \sum_{\bar{m}} p(m|s^1),$$

and

$$t_{2k}(s^1) \rightarrow 1 - F(x(s^2)) - (F(x(s^1)) - F(x(s^2))) \sum_{\bar{m}} p(m|s^1).$$

Let $\Delta = (F(x(s^1)) - F(x(s^2))) \sum p(m|s^1)$ and observe that

$$(\sqrt{F(x(s^2)) + \Delta} - \sqrt{1 - F(x(s^2)) - \Delta})^2 < (\sqrt{F(x(s^2))} - \sqrt{1 - F(x(s^2))})^2 \quad (33)$$

whenever $0 < \Delta < 1 - 2F(x(s^2))$.

Assumption 7 says that $0 \leq \Delta < 1 - 2F(x(s^2))$. Hence 33 holds with a strict inequality whenever $\Delta > 0$, and therefore we have that for sufficiently large k

$$(\sqrt{t_{1k}(s^1)} - \sqrt{t_{2k}(s^1)})^2 < (\sqrt{t_{1k}(s^2)} - \sqrt{t_{2k}(s^2)})^2. \quad (34)$$

To see that 34 also holds (for large k) when $\Delta = 0$, observe that in this case $t_{1k}(s^j) \rightarrow F(x(s^2)) < 1/2$ and $t_{2k}(s^j) \rightarrow 1 - F(x(s^2)) > 1/2$, since every agent behaves as if state s^2 had occurred. Thus we have that for large k

$$t_{1k}(s^2) < t_{1k}(s^1) < 1/2,$$

$$t_{2k}(s^2) > t_{2k}(s^1) > 1/2,$$

and 34 follows for large k .

From the proof of lemma 2 we know that

$$\begin{aligned} & \frac{\Pr_k\{piv_j|s^1\}}{\Pr_k\{piv_j|s^2\}} \\ & \rightarrow \frac{\sqrt{t_{1k}(s^1)t_{2k}(s^1)}}{\sqrt{t_{1k}(s^2)t_{2k}(s^2)}} e^{vk((\sqrt{t_{1k}(s^2)} - \sqrt{t_{2k}(s^2)})^2 - (\sqrt{t_{1k}(s^1)} - \sqrt{t_{2k}(s^1)})^2)}. \end{aligned} \quad (35)$$

Inequality 34 then implies that for every $\epsilon > 0$ there is a k' , such that for $k > k'$

$$\beta_k(s^1|piv) \geq \frac{\sqrt{t_{1k}(s^1)t_{2k}(s^1)}}{\sqrt{t_{1k}(s^2)t_{2k}(s^2)}} \frac{g(s^1)}{g(s^2)} \beta_k(s^2|piv) - \epsilon.$$

Since $g(s)$ and $t_{jk}(s)$ are bounded away from zero for all s , this in turn contradicts $\beta_k(s^1|piv) \rightarrow 0$. (The argument for the case in which $\beta_k(s^2|piv) \rightarrow 0$ is analogous.) *Q.E.D.*

PROPOSITION 4. Suppose assumptions 1–5 hold, and $F(x(s^1)) > 1/2$. Then, as $k \rightarrow \infty$, the probability that alternative 1 is elected converges to 1 for $s \leq s^1$ and to 0 for $s > s^1$.

Proof. By step 1 of the proof of lemma 2 it follows from $F(x(s^1)) > 1/2$ that there is an $\epsilon > 0$ such that $(x'_{mk} - x'_{m'k}) > \epsilon$ for $m' > m$, $m, m' \neq 0$ and all k . As a consequence, there is an $\epsilon' > 0$ such that

$$|t_j(s) - t_j(s')| > \epsilon'. \quad (36)$$

Hence, t_1 is strictly decreasing in s with slope bounded away from zero, and t_2 is strictly increasing in s with slope bounded away from zero. By the proof of step 2 of lemma 2 we know that for any convergent subsequence

$$\lim(\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)}) \geq 0 \geq \lim(\sqrt{t_1(s^2)} - \sqrt{t_2(s^2)}).$$

If the above two inequalities are strict, then 36 together with the strong law of large numbers implies the theorem.

In the remainder of the proof we show that, indeed, both of these inequalities must be strict. To see this first note that 36 implies that at least one of the two inequalities is strict.

Thus, suppose that $\lim(\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)}) = 0$. Then it follows that $T_k = \{s^1\}$ for large k , and hence by lemma A3 $\beta(s^1|piv_j) \rightarrow 1$. This implies that $t_1(s^1) = F(x(s^1)) > 1/2$ (by assumption), which contradicts the hypothesis that $\lim(\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)}) = 0$. An analogous argument shows that the second inequality is strict. *Q.E.D.*

PROPOSITION 5. *Suppose assumptions 1–7 hold. Then there is an $\alpha > 0$ and a k' such that the expected fraction of voters who abstain in equilibrium is larger than α for all $k > k'$.*

Proof. Since assumption 7 holds, we can apply lemma 3 to establish that $\beta_k(s^1|piv) > \varepsilon > 0$ and $\beta_k(s^2|piv) > \varepsilon > 0$ for large k . By assumption 1 this in turn implies that $x_{1k}^j - x_{Mk}^j > \varepsilon > 0$ for $j = 1$ or $j = 2$ for large k . Thus, there exists an $\varepsilon' > 0$ such that $\rho_k(s) - \rho_k(s') \geq \varepsilon'$ for all $s < s'$. From lemma 1 we know that

$$\beta_k(s^1|piv_2) - \frac{g(s^1) \frac{\Pr_k(piv_1|s^1)}{\sqrt{\rho_k(s^1)}}}{g(s^1) \frac{\Pr_k(piv_1|s^1)}{\sqrt{\rho_k(s^1)}} + g(s^2) \frac{\Pr_k(piv_1|s^2)}{\sqrt{\rho_k(s^2)}}} \rightarrow 0,$$

and hence there is an k' and an ε'' such that

$$\beta_k(s^1|piv_2) < \beta_k(s^1|piv_1) - \varepsilon'' \quad (37)$$

for all $k > k'$. Consider an agent who receives a signal m with $p(m|s^1) > 0$ and $p(m|s^2) > 0$. Since 37 holds, it follows that there is an $\varepsilon''' > 0$ and a k' such that for $k > k'$

$$E\{v(x, s)|piv_1, m\} < E\{v(x, s)|piv_2, m\} - \varepsilon'''.$$

Since $x_{m,k}^1$ is given by $E\{v(x_{m,k}^1, s)|piv_1, m\} = 0$ and $x_{m,k}^2$ is given by $E\{v(x_{m,k}^2, s)|piv_2, m\} = 0$, the result now follows. *Q.E.D.*

PROPOSITION 6. *Suppose assumptions 1–6 hold. For every $\varepsilon > 0$ there is an $\eta > 0$ such that $|F(x(s^*)) - 1/2| \leq \eta$ implies that $\limsup_k(1 - t_{1k}(s) - t_{2k}(s)) \leq \varepsilon$ for all $s \in S$, that is, the expected fraction of voters who abstain is bounded above by ε for sufficiently large k .*

Proof.

Step 1: First we demonstrate that the fraction of players who abstain and who receive a signal that allows them to exclude a state in the set $\{s^1, s^2\}$ converges to zero. That is, we show that the fraction of voters who abstain and who receive a signal with the property $p(m|s^1)p(m|s^2) = 0$ converges to zero.

To prove Step 1, we distinguish two cases.

Case 1: $p(m|s^1) > 0$, and $p(m|s^2) = 0$, or $p(m|s^1) = 0$, and $p(m|s^2) > 0$. Suppose that $p(m|s^1) > 0$, and $p(m|s^2) = 0$. In this case receiving message m permits the agent to discriminate perfectly between states s^1 and s^2 . Now it follows that

$$E(v(x, s)|piv_1, m) - E(v(x, s)|piv_2, m) \rightarrow 0,$$

since

$$E(v(x, s)|piv_1, m) \rightarrow v(x, s^1),$$

and

$$E(v(x, s)|piv_2, m) \rightarrow v(x, s^1).$$

The case $p(m|s^1) = 0$ and $p(m|s^2) > 0$ is analogous.

Case 2. $p(m|s^1) = p(m|s^2) = 0$. In this case, receiving the message m tells the agent that the state is neither s^1 or s^2 . It follows from SMLRP that $p(m|s) = 0$ for any $s < s^1$ or for

any $s > s^2$. Let $p(m|s) = 0$ for any $s < s^1$ (the other case is analogous). Let $\bar{s} < s^1$ be the largest state, such that $p(\bar{s}|m) > 0$. Now it follows from $\rho(s)$ strictly decreasing in s that

$$E(v(x, s)|piv_1, m) \rightarrow v(x, \bar{s}),$$

$$E(v(x, s)|piv_2, m) \rightarrow v(x, \bar{s}),$$

and the fraction of types receiving message m that abstain goes to zero.

Step 2. In Step 2 we demonstrate that the fraction of voters who receive a signal with the property $p(m|s^1)p(m|s^2) > 0$ and who abstain can be made arbitrarily small if η is sufficiently small. These are voters whose signal does not allow them to exclude a state in the set $\{s^1, s^2\}$. To prove this we demonstrate that for small η the beliefs conditional on the event piv_1 are very close to the beliefs conditional on the event piv_2 .

Assume that the critical state is s^1 , that is, $s^1 \in \arg \min_s |F(x(s)) - 1/2|$. (The case in which $s^2 \in \arg \min_s |F(x(s)) - 1/2|$ is entirely analogous.)

Observe that $\liminf t_{2k}(s^1) \geq 1 - F(x(s^1))$, since every voter with type $x > x(s^1)$ prefers alternative 2 if the state is in the set $s \in \{s^1, s^2\}$, and by lemma 2 we know that $\beta_k(s^1|piv_j) + \beta_k(s^2|piv_j) \rightarrow 1$ as $k \rightarrow \infty$. Since alternative 1 wins the election in state s^1 with probability close to one (proposition 4), it follows that $t_{1k}(s^1) \geq t_{2k}(s^1)$. Since the expected vote shares must be less than or equal to 1 in each state, it must be the case that $t_{1k}(s^1) \leq F(x(s^1))$ for sufficiently large k , and we get the following inequality:

$$0 \geq \sqrt{t_{2k}(s^1)} - \sqrt{t_{1k}(s^1)} \geq \sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1))}.$$

Therefore,

$$1 \leq \liminf \sqrt{\rho_k(s^1)} \leq \limsup \sqrt{\rho_k(s^1)} \leq \frac{\sqrt{F(x(s^1))}}{\sqrt{1 - F(x(s^1))}}. \quad (38)$$

Furthermore, observe that since vote shares are monotone, it follows that

$$t_2(s^2) \geq 1 - F(x(s^1)).$$

By lemma 3 $\beta_k(s^1|piv_j) \geq \varepsilon$, $\beta_k(s^2|piv_j) \geq \varepsilon > 0$, and by lemma 2 this implies that

$$(\sqrt{t_{1k}(s^2)} - \sqrt{t_{2k}(s^2)}) - (\sqrt{t_{2k}(s^1)} - \sqrt{t_{1k}(s^1)}) \rightarrow 0.$$

As a result we get that

$$\begin{aligned} \liminf \sqrt{t_{1k}(s^2)} &\geq \sqrt{1 - F(x(s^1))} + \sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1))} \\ &= 2\sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1))} \end{aligned}$$

and that

$$\begin{aligned} 1 &\geq \limsup \sqrt{\rho_k(s^2)} \geq \liminf \sqrt{\rho_k(s^2)} \\ &\geq \frac{2\sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1))}}{\sqrt{1 - F(x(s^1))}}. \end{aligned} \quad (39)$$

Equations 38 and 39 imply that, for any $\varepsilon > 0$, there is an $\eta > 0$ such that for $F(x(s^1)) - 1/2 \leq \eta$ we have

$$1 \leq \liminf \sqrt{\rho_k(s^1)} \leq \limsup \sqrt{\rho_k(s^1)} \leq 1 + \varepsilon$$

and

$$1 \geq \limsup \sqrt{\rho_k(s^2)} \geq \liminf \sqrt{\rho_k(s^2)} \geq 1 - \varepsilon. \quad (40)$$

Lemma 1 and the fact that $p(m|s^1)p(m|s^2) > 0$ then implies that, for every ε' , there is an η such that $F(x(s^1)) - 1/2 \leq \eta$ implies that

$$\beta_k(s^1|piv_1, m) - \beta_k(s^1|piv_2, m) \leq \varepsilon'$$

for sufficiently large k , and therefore for every ε'' there is an η such that for $F(x(s^1)) - 1/2 \leq \eta$

$$E\{v(x, s)|piv_1, m\} - E\{v(x, s)|piv_2, m\} \leq \varepsilon'', \quad (41)$$

which yields the result.

Q.E.D.

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