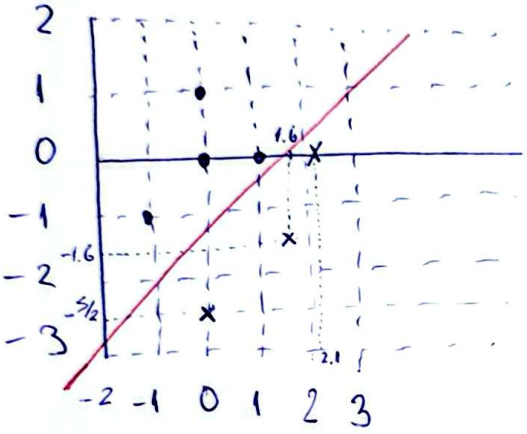
1η Σειρά Ασκήσεων Δεδηγιάννου Ευστράτιος

1) Category A: (0,0), (0,1), (1,0), (-1,-1) → •
Category B: (2.1,0), (0,-2.5), (1.6,-1.6) → x



There is a clear line seperating the two categories therefore they are linearly seperable

Perceptron Design:

Vector $X = [x_1, x_2]$

Weights $W = [w_0, w_1, w_2] = [0.5, 0.5, 0.5]$

Target Output: Cat A → $t = +1$ & Cat B → $t = -1$

Perceptron Output: $y = \text{sign}(W_0 + w_1 x_1 + w_2 x_2)$ (net)

Learning Rate: $\eta = 1$

Samples with bias input 1:

	sample	t	net	sign(net)	if y≠t w'=w+t·sample
A:	[1,0,0]	t=+1	0.5	1	✓
	[1,0,1]	t=+1	1	1	✓
	[1,1,0]	t=+1	1	1	✓
	[+1,-1,-1]	t=+1	-0.5	-1	w'=[1.5,-0.5,-0.5]
B:	[1,2.1,0]	t=-1	0.45	1	w'=[0.5,-2.6,-0.5]
	[1,0,-2.5]	t=-1	+1.75	+1	w'=[-0.5,-2.6,2]
	[1,1.6,-1.6]	t=-1	0.5	1	✓

After Epoch 1 : $W = [-0.5, -2.6, 2]$

Continuing w/ the 2nd Epoch:

A: [0,0] t=1 y=-0.5 sign=-1 ≠1 => w'=[0.5,-2.6,2]
[0,1] t=1 y=2.5 sign=1 => w'=w
[1,0] t=1 y=-2.1 sign=-1 ≠1 => w'=[1.5,-1.6,2]
[-1,-1] t=1 y=1.1 sign=1 => w'=w

B: [2.1,0] t=-1 y=-1.86 sign=-1 => w'=w
[0,-2.5] t=-1 y=-3.5 sign=-1 => w'=w
[1.6,-1.6] t=-1 y=-4.26 sign=-1 => w'=w

After Epoch 2 : $W = [1.5, -1.6, 2]$

Epoch	w_0	w_1	w_2	
0	0.5	0.5	0.5	
1	-0.5	-2.6	2	
2	1.5	-1.6	2	← final

2) Epoch 1

Sample	x_0	x_1	x_2	t	y (net)	t - y	w_0	w_1	w_2
Init	-	-	-	-	-	-	0.5	0.5	0.5
(0, 0)	1	0	0	1	0.5	0.5	1	0.5	0.5
(0, 1)	1	0	1	1	1.5	-0.5	0.5	0.5	0
(1, 0)	1	1	0	1	1	0	0.5	0.5	0
(-1, -1)	1	-1	-1	1	0	1	1.5	-0.5	-1
(2.1, 0)	1	2.1	0	-1	1.5+2.1(-0.5) = 0.45	-1.45	0.05	-3.545	-1
(0, -2.5)	1	0	-2.5	-1	0.05+(-2.5)(-1) = 2.55	-3.55	-3.5	-3.545	7.875
(1.6, -1.6)	1	1.6	-1.6	-1	-3.5+1.6(-3.545) + +(-1.6)(+7.875) = = -3,5 - 5.672 - 12.6 = = -21.772	20.772	17.272	29.961	-25.335

↳ init for epoch 2

Epoch 2:

same sample

x_0	x_1	x_2	t	y (net)	t - y	w_0	w_1	w_2
				17.272	-16.272	1	29.961	-25.335
				1 - 25.335 = -24.335	25.335	26.335	29.961	0
				26.335 + 29.961 · 1 = 56.296	-55.296	-28.961	-25.335	0
				-28.961 + (-25.335)(1) = = -28.961 + 25.335 = -3.626	4.626	-24.335	-29.961	-4.626
				-24.335 + 2.1(-29.961) = = -24.335 - 62.9181 = = -87.2531	86.2531	61.9181	151.2575	-4.626
				61.9181 + (-2.5)(-4.626) = = 61.9181 + 11.565 = 73.4831	-74.4831	-12.565	151.2575	182.5418
				-12.565 + 1.6 * 151.2575 + 182.5418(-1.6) = = -12.565 + 242.012 - 292.067 = = -62.62	61.62	49.055	249.8495	84.94198

Final $W = [49.055, 249.8495, 84.94198]$

3) From the 2D graph in Ex.1 we can directly assume that a straight line that intercepts the x-axis at (1.6, 0) and y-axis at (0, -3) is capable of seperating the two categories.

Slope m: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3}{-1.6} = 1.875$

from (1.6, 0) → $y - 0 = 1.875(x - 1.6)$

$\Rightarrow \boxed{y = 1.875x - 3}$

Step 2 → Decision Func

$f(x_1, x_2) = w_0 + w_1 x_1 + w_2 x_2 = 0$

$x_2 = 1.875x_1 - 3 \rightsquigarrow x_2 - 1.875x + 3 = 0 \Rightarrow w_0 = 3$

$w_1 = -1.875$

$w_2 = 1$

Step 3 → Perceptron Weights

$[w_0, w_1, w_2] = [3, -1.875, 1]$

Step 4 → D. Func

$f(x_1, x_2) = 3 - 1.875x_1 + x_2$

4) Activation function (Weighted sum):

$a = 0.2 \cdot 0.5 + 0.7 \cdot 1.5 + 0.5(-1) = 0.1 + 1.05 - 0.5 = 0.65$

Output (Perceptron, step func): if $a > 0$

output = 1

else

output = 0

Sine $a = 0.65 > 0 \Rightarrow$ output = 1