Equations: Joint spatial modeling of relative activity and disease processes

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$$
\begin{aligned}
y_i &\sim \text{Bernoulli}(\pi_i) \\
\text{logit}(\pi_i) &= \alpha_0 + \eta_i \\
\begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{\eta}^* \end{bmatrix} &\sim \mathcal{N}\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \right)
\end{aligned}
$$

$$\Sigma_{11}^{(ij)} = \sigma^2 K(\boldsymbol{s}_i, \boldsymbol{s}_j) = \sigma^2 \exp\left(-\frac{||\boldsymbol{s}_i - \boldsymbol{s}_j||^2}{2\phi^2}\right)$$

$$\Sigma_{22}^{(ij)} = \sigma^2 K(\boldsymbol{s}_i^*, \boldsymbol{s}_j^*) = \sigma^2 \exp\left(-\frac{||\boldsymbol{s}_i^* - \boldsymbol{s}_j^*||^2}{2\phi^2}\right)$$

$$\Sigma_{12}^{(ij)} = \sigma^2 K(\boldsymbol{s}_i, \boldsymbol{s}_j^*) = \sigma^2 \exp\left(-\frac{||\boldsymbol{s}_i - \boldsymbol{s}_j^*||^2}{2\phi^2}\right)$$

$$
\begin{aligned}
c_i &\sim \mathrm{NB}(\xi, p_i) \\
\mathrm{logit}(p_i) &= \boldsymbol{x}_i^T \boldsymbol{\beta} + \alpha \eta_i^*
\end{aligned}
$$

$$\boldsymbol{\eta}^*|\boldsymbol{\eta} \sim \mathcal{N}(\boldsymbol{\mu}_{2|1}, \boldsymbol{\Sigma}_{2|1})$$

$$
\begin{aligned}
\boldsymbol{\Sigma}_{2|1} &= \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12} \\
\boldsymbol{\mu}_{2|1} &= \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\eta}
\end{aligned}
$$