Introduction to the 2-Channel Quadrature Mirror Filter (QMF)

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***Abstract*—Quadrature Mirror Filters (QMF's) are one of the most important filter banks in the signal-processing world. They are widely used today in applications ranging from audio and voice processing to image processing and communications. This paper will go through the background on QMF's, how they work, and the types of challenges faced when implementing them into a design.**

***Index Terms*—QMF, speech and image processing, communications**

I. Introduction

THE QMF-method is a way of multi-rate signal processing, and is considered to be a near-perfect reconstruction filter bank. Designed in the late 1970s, it serves as a method to filter cellular audio, MP3s, videos, MPEG audio, and images, among many other things.

While most common implementations of the QMF focus on 2-channel filtering, some applications use an exponential 2-channel version of the QMF. For example, MP3 encoding is similar to a binary tree. One channel is high frequencies, and one channel is lower. Because the human ear is not as sensitive to higher frequencies, the high channel is usually done with processing after a few iterations, whereas the lower frequencies may be split and processed many more times. Although the QMF may not perfectly reconstruct every sample in this case, it saves a lot of computation and time without sacrificing too much signal fidelity in the long run.

This paper will focus mainly on the design of the general, 2-channel QMF as well as obstacles in its design.

II. Design and Analysis

A. Overview

A general QMF begins by taking a discrete signal and splitting it into N sub-band signals. Each of these signals goes through an analysis filter $H_k(z)$, a sequence of decimation and interpolation, and then finally a synthesis filter $F_k(z)$. The output of each sub-band is then summed to create the output of the filter. Fig. 1 shows an example of a signal, $x(n)$, being split into sub-bands, and then going through processing, with its resultant signal being $\hat{x}$(n), which is just a shifted version of the original signal. Extra processing can be done at the node(s) between decimation and interpolation. For the remainder of this paper, a 2-channel QMF will be referenced.

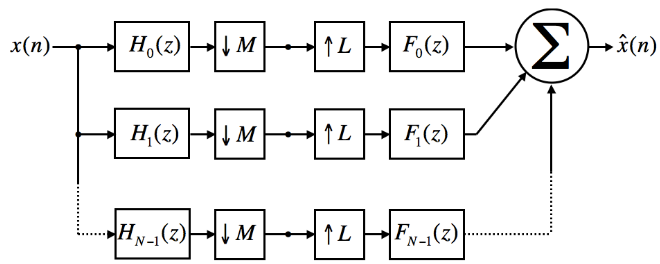

Fig. 1. Generalized N-channel QMF bank

B. Stage 1 – Analysis

The analysis filters in the 2-channel QMF bank are designed to be linear phase on a symmetry constraint, such that:

$$H_1(z) = H_0(-z). \tag{1a}$$

In the time domain, this is represented by:

$$h_1(n) = (-1)^n h_0(n). \tag{1b}$$

The types of filters for $H_0(z)$ and $H_1(z)$ are exact opposites. If $H_0(z)$ is a low-pass filter, with cutoff frequency at π/2, then $H_1(z)$ will be a symmetrical high-pass filter, rolling off at π/2 as well. This is shown in fig. 2. The first of the analysis filters, $H_0(z)$, defines the order, N, of the entire QMF. Subsequently, the group delay of the analysis and synthesis system is $N – 1$. In declaring the synthesis filters in this way, any aliasing and phase distortion can be minimized.

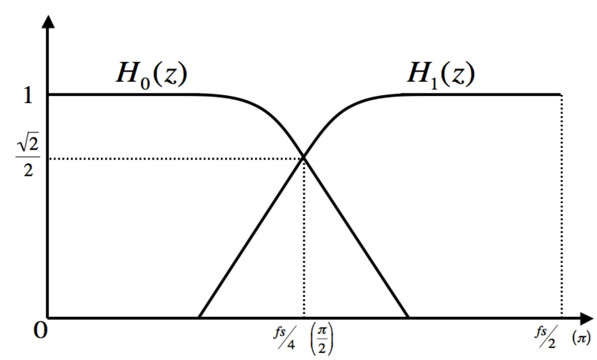

Fig. 2. Example magnitude responses for $H_0(z)$ and $H_1(z)$

C. Stage 2 – Decimation and Interpolation

Decimation is a method in which only every M^{th} sample is extracted, getting rid of everything in between. Interpolation is the opposite, where the samples are spaced out by multiples of L, and the extra space is replaced with zeros. Fig. 2 shows a sample signal being decimated, with $M = 2$, and Fig. 3a and 3b show a signal being interpolated by $L = 2$.

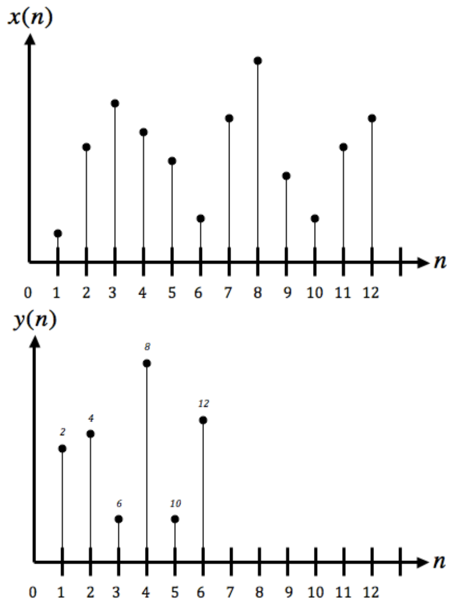

Fig. 3. Decimation example

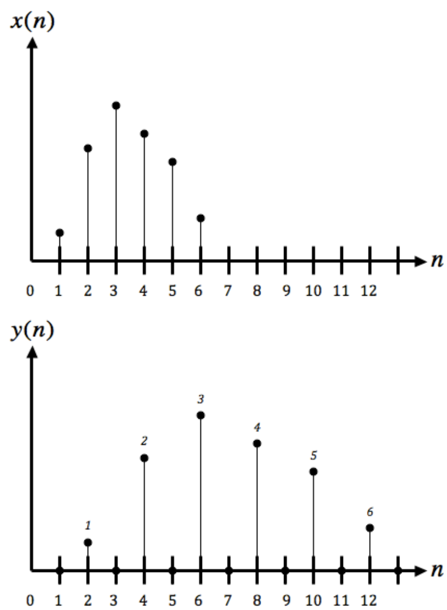

Fig. 4. Interpolation example

For the decimation and interpolation, respectively, the output is defined by

$$y_D(n) = x(M \times n) \qquad (2a)$$

$$y_I(n) = \begin{cases} x\left(\dfrac{n}{L}\right) & n \textit{ is multiple of } L \\ 0 & \textit{otherwise} \end{cases} . \qquad (2b)$$

D. Stage 3 – Synthesis

The final stage in QMF is the synthesis stage. To continue with the symmetrical filter design and minimize distortion, the synthesis stage is where aliasing cancellation occurs. These synthesis filters are designed on the basis of the initial analysis filter, $H_0(z)$. The synthesis filters, $F_0(z)$ and $F_1(z)$, are defined as follows:

$$F_0(z) = H_1(-z) = H_0(z) \qquad (3a)$$

$$F_1(z) = -H_1(z) = -H_0(-z). \qquad (3b)$$

The input-output Z-transform is then given by

$$\begin{aligned} \hat{X}(z) &= \frac{1}{2}\left[H_0(z)F_0(z) + H_1(z)F_1(z)\right]X(z) \\ &+ \frac{1}{2}\left[H_0(-z)F_0(z) - H_1(-z)F_1(z)\right]X(-z) \end{aligned} . \qquad (4a)$$

Simplifying this even more, the final output depends solely on $H_0(z)$, such that:

$$\begin{aligned} \hat{X}(z) &= \frac{1}{2}\left[H_0(z)F_0(z) + H_1(z)F_1(z)\right]X(z) \\ &= \frac{1}{2}\left[H_0^2(z) - H_0^2(-z)\right]X(z) \end{aligned} . \qquad (4b)$$

In the time domain, this output looks like:

$$x(n) = \sum_{i=0}^{N-1} \sum_{k=-\infty}^{\infty} x_i(k) f_i(n - kN). \qquad (5)$$

The outputs of $F_0(z)$ and $F_1(z)$ are summed to the output, $\hat{x}$(n) which is a perfectly reconstruct a scaled and delayed version of the input. Fig. 6 on the next page shows a working QMF with a simple cosine input. The output is essentially the input, just shifted by four samples.

III. Obstacles of Design

A. Alias Distortion

The onset of multi-rate processing, while speeding up the processing of a system and increasing efficiency, does in fact introduce more error in a system. Alias distortion is one of these problems. If a signal contains frequency content higher than the Nyquist sampling rate, that content is reflected down, causing aliasing, or frequency error, on the original signal. In the QMF, this is dealt with in the final stage of processing, known as the synthesis stage.

Although the signal is band-limited from previous filters, it does not attenuate all problem frequencies, so the synthesis filters effectively get rid of alias distortion.

B. *Amplitude Distortion*

Another common error that could possibly be introduced in a signal path is amplitude distortion. This will occur whenever the output of a stage or system of filters is not a linear representation of the amplitude of the input.

In some cases, it may be necessary to have a non-linear filtering stage, and then compensate for any amplitude distortion that may occur later on. In the QMF though, by choosing the analysis filters to be linear phase FIR filters, any possible alias distortion will be eliminated then. Through optimization, it is possible to then minimize any amplitude distortion in the system.

C. *Phase Distortion*

The last major common error found in multi-rate systems is phase distortion. This can occur whenever a filter's phase response is non-linear over the specified frequency spectrum. This is shown below, in fig. 5. One way to avoid this is to use only linear-phase FIR filters, but this is very computationally expensive. It turns out, however, that by mirroring $H_0(z)$ and $H_1(z)$, the resultant filters eliminate both aliasing and phase distortion. Therefore, by compensating for amplitude and alias distortion in the QMF, phase distortion will not be an issue.

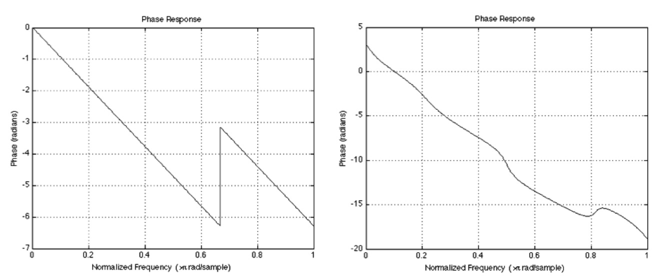

Fig. 5. Linear phase vs. non-linear, distorted phase

IV. CONCLUSION

This paper explored the basic design method as well as the different design obstacles introduced when building the filter bank. The 2-channel QMF is widely used in a variety of applications. It offers a novel way of processing a signal, whether it be an audio, video, or communication signal, through a filter bank, to quickly and efficiently reconstruct an scaled and delayed input signal. In some cases, multiple iterations can be called, such as audio encoding. Since the frequency scale is logarithmic, and the human ear is less sensitive to high frequencies, it is more beneficial to filter out the lower frequency spectrum more so than the higher frequencies.

Although the QMF and other multi-rate processing techniques can introduce different errors and distortions, such as aliasing, phase, and amplitude distortion, different design procedures can be done to minimize or completely eliminate the distortions within the system to minimize error and optimize the functionality of the filter bank.

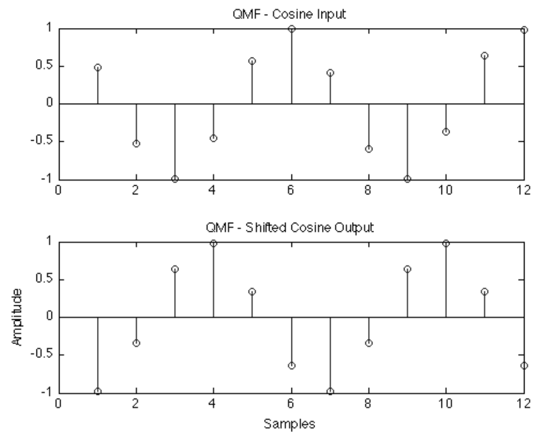

Fig. 6. Cosine input, cosine output, shifted by four samples

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