Shortest Path in Directed Acyclic Graph

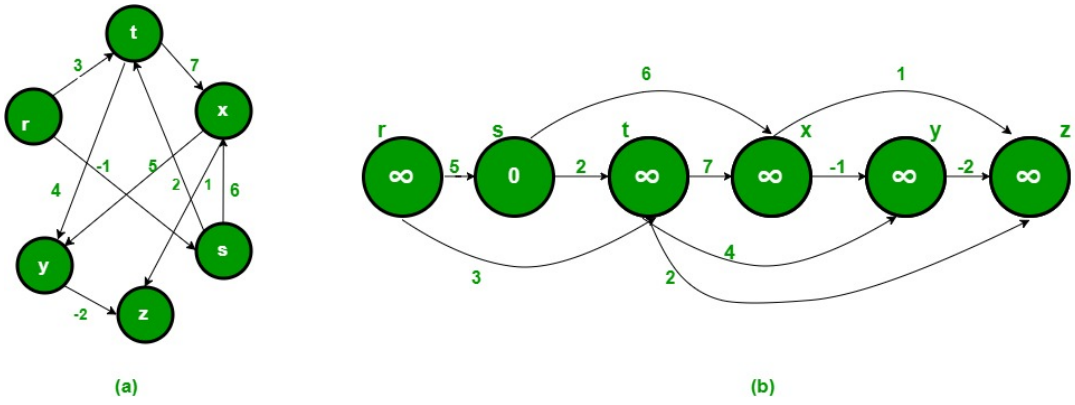
Given a Weighted Directed Ac graph, find the shortest paths from given source to all other vert

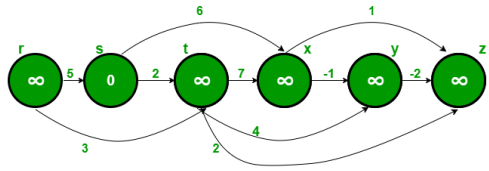
Recommended: Please solve it on "*PRACTICE*" first, before moving on to the solution.

For a general weighted graph, we can calculate single source shortest distances in O(VE) time using Bellman–Ford Algorithm. For a graph with no negative weights, we can do better and calculate single source shortest distances in O(E + VLogV) time using Dijkstra's algorithm. Can we do even better for Directed Acyclic Graph (DAG)? We can calculate single source shortest distances in O(V+E) time for DAGs. The idea is to use Topological Sorting.

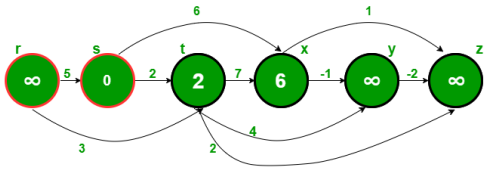
We initialize distances to all vertices as infinite and distance to source as 0, then we find a topological sorting of the graph. Topological Sorting of a graph represents a linear ordering of the graph (See below, figure (b) is a linear representation of figure (a)). Once we have topological order (or linear representation), we one by one process all vertices in topological order. For every vertex being processed, we update distances of its adjacent using distance of current vertex.

Following figure is taken from this source. It shows step by step process of finding shortest paths.

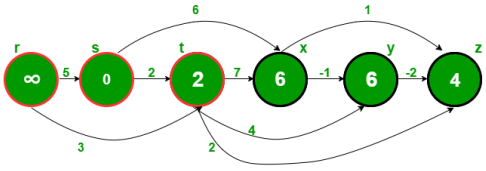




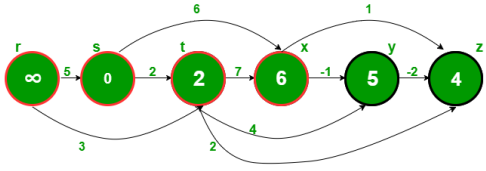
(c)



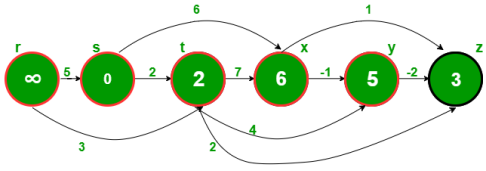
(d)



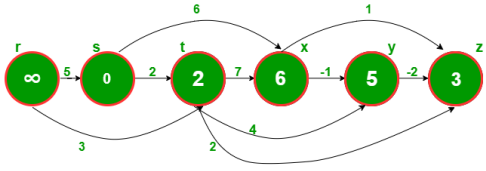
(e)



(f)



(g)



(h)

Following is complete algorithm for finding shortest distances.

1) Initialize dist[] = {INF, INF, ….} and dist[s] = 0 where s is the source vertex.

2) Create a toplogical order of all vertices.

3) Do following for every vertex u in topological order.

……Do following for every adjacent vertex v of u

……if (dist[v] > dist[u] + weight(u, v))

……dist[v] = dist[u] + weight(u, v)

C++

```
// C++ program to find single source shortest paths for Directed Acyclic Graphs
#include<iostream>
#include <bits/stdc++.h>
#define INF INT_MAX
using namespace std;

// Graph is represented using adjacency list. Every node of adjacency list
// contains vertex number of the vertex to which edge connects. It also
// contains weight of the edge
class AdjListNode
{
    int v;
    int weight;
public:
    AdjListNode(int _v, int _w)  { v = _v;  weight = _w;}
    int getV()       {  return v;  }
    int getWeight()  {  return weight; }
};

// Class to represent a graph using adjacency list representation
class Graph
{
```

```
    int V;    // No. of vertices'

    // Pointer to an array containing adjacency lists
    list<AdjListNode> *adj;

    // A function used by shortestPath
    void topologicalSortUtil(int v, bool visited[], stack<int> &Stack);
public:
    Graph(int V);   // Constructor

    // function to add an edge to graph
    void addEdge(int u, int v, int weight);

    // Finds shortest paths from given source vertex
    void shortestPath(int s);
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<AdjListNode>[V];
}

void Graph::addEdge(int u, int v, int weight)
{
    AdjListNode node(v, weight);
    adj[u].push_back(node); // Add v to u's list
}

// A recursive function used by shortestPath. See below link for details
// https://www.geeksforgeeks.org/topological-sorting/
void Graph::topologicalSortUtil(int v, bool visited[], stack<int> &Stack)
{
    // Mark the current node as visited
    visited[v] = true;

    // Recur for all the vertices adjacent to this vertex
    list<AdjListNode>::iterator i;
    for (i = adj[v].begin(); i != adj[v].end(); ++i)
    {
        AdjListNode node = *i;
        if (!visited[node.getV()])
            topologicalSortUtil(node.getV(), visited, Stack);
    }

    // Push current vertex to stack which stores topological sort
    Stack.push(v);
}

// The function to find shortest paths from given vertex. It uses recursive
// topologicalSortUtil() to get topological sorting of given graph.
void Graph::shortestPath(int s)
{
    stack<int> Stack;
    int dist[V];

    // Mark all the vertices as not visited
    bool *visited = new bool[V];
    for (int i = 0; i < V; i++)
        visited[i] = false;

    // Call the recursive helper function to store Topological Sort
    // starting from all vertices one by one
    for (int i = 0; i < V; i++)
```

```
            if (visited[i] == false)
                topologicalSortUtil(i, visited, Stack);

        // Initialize distances to all vertices as infinite and distance
        // to source as 0
        for (int i = 0; i < V; i++)
            dist[i] = INF;
        dist[s] = 0;

        // Process vertices in topological order
        while (Stack.empty() == false)
        {
            // Get the next vertex from topological order
            int u = Stack.top();
            Stack.pop();

            // Update distances of all adjacent vertices
            list<AdjListNode>::iterator i;
            if (dist[u] != INF)
            {
              for (i = adj[u].begin(); i != adj[u].end(); ++i)
                 if (dist[i->getV()] > dist[u] + i->getWeight())
                    dist[i->getV()] = dist[u] + i->getWeight();
            }
        }

        // Print the calculated shortest distances
        for (int i = 0; i < V; i++)
            (dist[i] == INF)? cout << "INF ": cout << dist[i] << " ";
    }

    // Driver program to test above functions
    int main()
    {
        // Create a graph given in the above diagram.  Here vertex numbers are
        // 0, 1, 2, 3, 4, 5 with following mappings:
        // 0=r, 1=s, 2=t, 3=x, 4=y, 5=z
        Graph g(6);
        g.addEdge(0, 1, 5);
        g.addEdge(0, 2, 3);
        g.addEdge(1, 3, 6);
        g.addEdge(1, 2, 2);
        g.addEdge(2, 4, 4);
        g.addEdge(2, 5, 2);
        g.addEdge(2, 3, 7);
        g.addEdge(3, 4, -1);
        g.addEdge(4, 5, -2);

        int s = 1;
        cout << "Following are shortest distances from source " << s <<" n";
        g.shortestPath(s);

        return 0;
    }
```

Java

```
// Java program to find single source shortest paths in Directed Acyclic Graphs
import java.io.*;
import java.util.*;
```

```
class ShortestPath
{
    static final int INF=Integer.MAX_VALUE;
    class AdjListNode
    {
        private int v;
        private int weight;
        AdjListNode(int _v, int _w) { v = _v;  weight = _w; }
        int getV() { return v; }
        int getWeight()  { return weight; }
    }

    // Class to represent graph as an adjcency list of
    // nodes of type AdjListNode
    class Graph
    {
        private int V;
        private LinkedList<AdjListNode>adj[];
        Graph(int v)
        {
            V=v;
            adj = new LinkedList[V];
            for (int i=0; i<v; ++i)
                adj[i] = new LinkedList<AdjListNode>();
        }
        void addEdge(int u, int v, int weight)
        {
            AdjListNode node = new AdjListNode(v,weight);
            adj[u].add(node);// Add v to u's list
        }

        // A recursive function used by shortestPath.
        // See below link for details
        void topologicalSortUtil(int v, Boolean visited[], Stack stack)
        {
            // Mark the current node as visited.
            visited[v] = true;
            Integer i;

            // Recur for all the vertices adjacent to this vertex
            Iterator<AdjListNode> it = adj[v].iterator();
            while (it.hasNext())
            {
                AdjListNode node =it.next();
                if (!visited[node.getV()])
                    topologicalSortUtil(node.getV(), visited, stack);
            }
            // Push current vertex to stack which stores result
            stack.push(new Integer(v));
        }

        // The function to find shortest paths from given vertex. It
        // uses recursive topologicalSortUtil() to get topological
        // sorting of given graph.
        void shortestPath(int s)
        {
            Stack stack = new Stack();
            int dist[] = new int[V];

            // Mark all the vertices as not visited
            Boolean visited[] = new Boolean[V];
            for (int i = 0; i < V; i++)
                visited[i] = false;
```

```
            // Call the recursive helper function to store Topological
            // Sort starting from all vertices one by one
            for (int i = 0; i < V; i++)
                if (visited[i] == false)
                    topologicalSortUtil(i, visited, stack);

            // Initialize distances to all vertices as infinite and
            // distance to source as 0
            for (int i = 0; i < V; i++)
                dist[i] = INF;
            dist[s] = 0;

            // Process vertices in topological order
            while (stack.empty() == false)
            {
                // Get the next vertex from topological order
                int u = (int)stack.pop();

                // Update distances of all adjacent vertices
                Iterator<AdjListNode> it;
                if (dist[u] != INF)
                {
                    it = adj[u].iterator();
                    while (it.hasNext())
                    {
                        AdjListNode i= it.next();
                        if (dist[i.getV()] > dist[u] + i.getWeight())
                            dist[i.getV()] = dist[u] + i.getWeight();
                    }
                }
            }

            // Print the calculated shortest distances
            for (int i = 0; i < V; i++)
            {
                if (dist[i] == INF)
                    System.out.print( "INF ");
                else
                    System.out.print( dist[i] + " ");
            }
        }
    }

    // Method to create a new graph instance through an object
    // of ShortestPath class.
    Graph newGraph(int number)
    {
        return new Graph(number);
    }

    public static void main(String args[])
    {
        // Create a graph given in the above diagram.  Here vertex
        // numbers are 0, 1, 2, 3, 4, 5 with following mappings:
        // 0=r, 1=s, 2=t, 3=x, 4=y, 5=z
        ShortestPath t = new ShortestPath();
        Graph g = t.newGraph(6);
        g.addEdge(0, 1, 5);
        g.addEdge(0, 2, 3);
        g.addEdge(1, 3, 6);
        g.addEdge(1, 2, 2);
        g.addEdge(2, 4, 4);
        g.addEdge(2, 5, 2);
```

```
        g.addEdge(2, 3, 7);
        g.addEdge(3, 4, -1);
        g.addEdge(4, 5, -2);

        int s = 1;
        System.out.println("Following are shortest distances "+
                           "from source " + s );
        g.shortestPath(s);
    }
}
//This code is contributed by Aakash Hasija
```

Python

```
# Python program to find single source shortest paths
# for Directed Acyclic Graphs Complexity :OV(V+E)
from collections import defaultdict

# Graph is represented using adjacency list. Every
# node of adjacency list contains vertex number of
# the vertex to which edge connects. It also contains
# weight of the edge
class Graph:
    def __init__(self,vertices):

        self.V = vertices # No. of vertices

        # dictionary containing adjacency List
        self.graph = defaultdict(list)

    # function to add an edge to graph
    def addEdge(self,u,v,w):
        self.graph[u].append((v,w))


    # A recursive function used by shortestPath
    def topologicalSortUtil(self,v,visited,stack):

        # Mark the current node as visited.
        visited[v] = True

        # Recur for all the vertices adjacent to this vertex
        if v in self.graph.keys():
            for node,weight in self.graph[v]:
                if visited[node] == False:
                    self.topologicalSortUtil(node,visited,stack)

        # Push current vertex to stack which stores topological sort
        stack.append(v)


    ''' The function to find shortest paths from given vertex.
        It uses recursive topologicalSortUtil() to get topological
        sorting of given graph.'''
    def shortestPath(self, s):

        # Mark all the vertices as not visited
        visited = [False]*self.V
        stack =[]
```

```
        # Call the recursive helper function to store Topological
        # Sort starting from source vertice
        for i in range(self.V):
            if visited[i] == False:
                self.topologicalSortUtil(s,visited,stack)

        # Initialize distances to all vertices as infinite and
        # distance to source as 0
        dist = [float("Inf")] * (self.V)
        dist[s] = 0

        # Process vertices in topological order
        while stack:

            # Get the next vertex from topological order
            i = stack.pop()

            # Update distances of all adjacent vertices
            for node,weight in self.graph[i]:
                if dist[node] > dist[i] + weight:
                    dist[node] = dist[i] + weight

        # Print the calculated shortest distances
        for i in range(self.V):
            print ("%d" %dist[i]) if dist[i] != float("Inf") else  "Inf" ,


g = Graph(6)
g.addEdge(0, 1, 5)
g.addEdge(0, 2, 3)
g.addEdge(1, 3, 6)
g.addEdge(1, 2, 2)
g.addEdge(2, 4, 4)
g.addEdge(2, 5, 2)
g.addEdge(2, 3, 7)
g.addEdge(3, 4, -1)
g.addEdge(4, 5, -2)

# source = 1
s = 1

print ("Following are shortest distances from source %d " % s)
g.shortestPath(s)

# This code is contributed by Neelam Yadav
```

C#

```
// C# program to find single source shortest
// paths in Directed Acyclic Graphs
using System;
using System.Collections.Generic;

public class ShortestPath
{
    static readonly int INF = int.MaxValue;
    class AdjListNode
    {
        public int v;
        public int weight;
        public AdjListNode(int _v, int _w) { v = _v; weight = _w; }
```

```
        public int getV() { return v; }
        public int getWeight() { return weight; }
    }

    // Class to represent graph as an adjcency list of
    // nodes of type AdjListNode
    class Graph
    {
        public int V;
        public List<AdjListNode>[]adj;
        public Graph(int v)
        {
            V = v;
            adj = new List<AdjListNode>[V];
            for (int i = 0; i < v; ++i)
                adj[i] = new List<AdjListNode>();
        }
        public void addEdge(int u, int v, int weight)
        {
            AdjListNode node = new AdjListNode(v,weight);
            adj[u].Add(node);// Add v to u's list
        }

        // A recursive function used by shortestPath.
        // See below link for details
        public void topologicalSortUtil(int v, Boolean []visited,
                                        Stack<int> stack)
        {
            // Mark the current node as visited.
            visited[v] = true;

            // Recur for all the vertices adjacent to this vertex
            foreach(AdjListNode it in adj[v])
            {
                AdjListNode node = it;
                if (!visited[node.getV()])
                    topologicalSortUtil(node.getV(), visited, stack);
            }

            // Push current vertex to stack which stores result
            stack.Push(v);
        }

        // The function to find shortest paths from given vertex. It
        // uses recursive topologicalSortUtil() to get topological
        // sorting of given graph.
        public void shortestPath(int s)
        {
            Stack<int> stack = new Stack<int>();
            int []dist = new int[V];

            // Mark all the vertices as not visited
        Boolean []visited = new Boolean[V];
            for (int i = 0; i < V; i++)
                visited[i] = false;

            // Call the recursive helper function to store Topological
            // Sort starting from all vertices one by one
            for (int i = 0; i < V; i++)
                if (visited[i] == false)
                    topologicalSortUtil(i, visited, stack);

            // Initialize distances to all vertices as infinite and
            // distance to source as 0
```

```
            for (int i = 0; i < V; i++)
                dist[i] = INF;
            dist[s] = 0;

            // Process vertices in topological order
            while (stack.Count != 0)
            {
                // Get the next vertex from topological order
                int u = (int)stack.Pop();

                // Update distances of all adjacent vertices
                if (dist[u] != INF)
                {
                    foreach(AdjListNode it in adj[u])
                    {
                        AdjListNode i= it;
                        if (dist[i.getV()] > dist[u] + i.getWeight())
                            dist[i.getV()] = dist[u] + i.getWeight();
                    }
                }
            }

            // Print the calculated shortest distances
            for (int i = 0; i < V; i++)
            {
                if (dist[i] == INF)
                    Console.Write( "INF ");
                else
                    Console.Write( dist[i] + " ");
            }
        }
    }

    // Method to create a new graph instance through an object
    // of ShortestPath class.
    Graph newGraph(int number)
    {
        return new Graph(number);
    }

    // Driver code
    public static void Main(String []args)
    {
        // Create a graph given in the above diagram. Here vertex
        // numbers are 0, 1, 2, 3, 4, 5 with following mappings:
        // 0=r, 1=s, 2=t, 3=x, 4=y, 5=z
        ShortestPath t = new ShortestPath();
        Graph g = t.newGraph(6);
        g.addEdge(0, 1, 5);
        g.addEdge(0, 2, 3);
        g.addEdge(1, 3, 6);
        g.addEdge(1, 2, 2);
        g.addEdge(2, 4, 4);
        g.addEdge(2, 5, 2);
        g.addEdge(2, 3, 7);
        g.addEdge(3, 4, -1);
        g.addEdge(4, 5, -2);

        int s = 1;
        Console.WriteLine("Following are shortest distances "+
                          "from source " + s );
        g.shortestPath(s);
    }
}
```

```
// This code is contributed by Rajput-Ji
```

Output:

```
Following are shortest distances from source 1
INF 0 2 6 5 3
```

Time Complexity: Time complexity of topological sorting is O(V+E). After finding topological order, the algorithm process all vertices and for every vertex, it runs a loop for all adjacent vertices. Total adjacent vertices in a graph is O(E). So the inner loop runs O(V+E) times. Therefore, overall time complexity of this algorithm is O(V+E).

References:
http://www.utdallas.edu/~sizheng/CS4349.d/l-notes.d/L17.pdf

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