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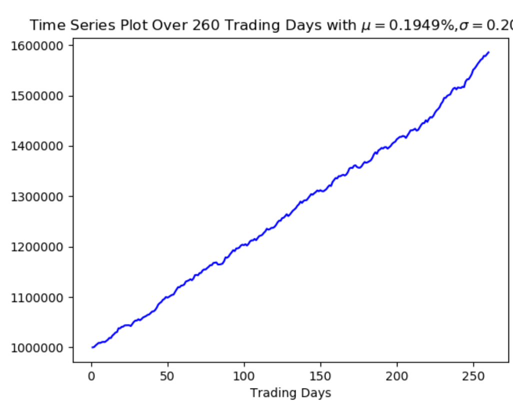
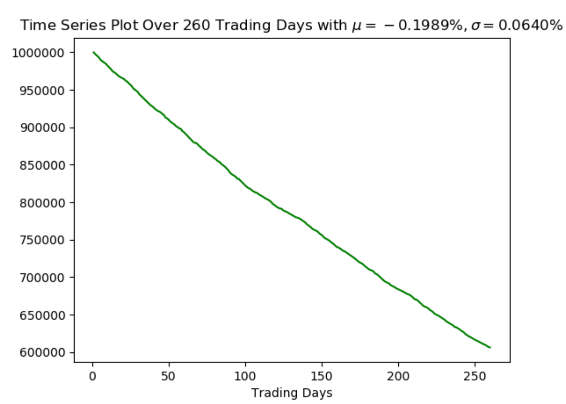
AMS 326
Report 4
4/20/2018

4.1

260 trading days ago, Mr. Poor invests \$1,000,000, with the change rate X, for his portfolio following a normal distribution. We wish to see how much money he makes (or loses), given the mean and standard deviation of the random variable X.
First, we generate 2 random numbers of the form $y_{i+1} = (y_i + y_{i-1}) \mod 1$, and perform a Box-Muller Transformation to convert them to numbers which follow a standard normal distribution. 260 of these numbers are kept, and then transformed to either $X \sim N(0.1949\%, 0.2018\%^2)$, or $X \sim N(-0.1989\%, 0.0640\%^2)$ depending on the part of them problem. This transform is done taking the standard normally distributed numbers, Z, and defining a new collection of numbers as $X_i = Z_i \cdot \sigma + \mu$.
After each day, Mr. Poor will gain or lose money. If he gains (when $s_i > 0$), he will have to pay a fee of 3.333%. If he loses money, or has no loss/return, he does not pay a fee, nor receive anything.
Let V_i denote the value on the i^{th} day. For day 1, the amount is 1,000,000. On each successive day, his new value is $V_{i+1} = V_i(1 + s_i)$. Once all of these values are calculated, for both distributions, we want to look at the total amount Mr. Poor has on the 260$^{\text{th}}$ day, and plot a time series plot for each day.
Below are the resulting time series plots, side-by-side:

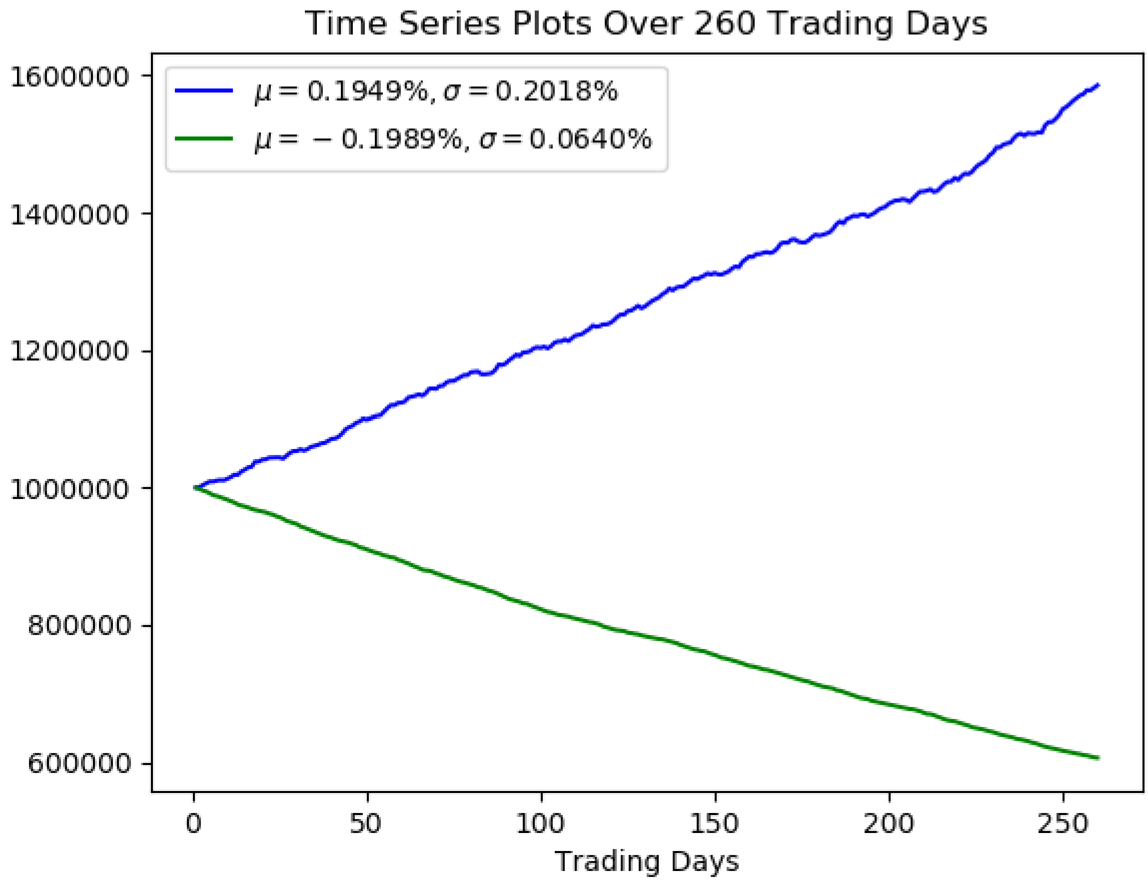


(a) $X \sim N(0.1949\%, 0.2018\%^2)$



(b) $X \sim N(-0.1989\%, 0.0640\%^2)$

Next is a plot of the two series together:



As expected, the line with positive μ is increasing, and the one with negative μ is decreasing. Furthermore, the second curve, with smaller standard deviation (and thus variance) σ has a change which appears more steady and smooth. On the other hand, the series with higher standard deviation has a more rough appearance with easier to spot changes in increase/decrease. Furthermore, we see that both, respective, have an increase/decrease of roughly \$600,000.

The final values are \$1,585,602.53 and \$606,221.19.

The code used for plotting and computations can be seen in the file, '326HW4-1.py'.

4.2

The second problem is a modification of Buffon's Needle, where instead of dropping needles inbetween parallel lines, we use equilateral triangles. $N = 300{,}000{,}000$ trials will be performed, and with this large number, we can compute the probability of a triangle falling on one of the lines. In the case of a triangle of radius $\frac{1}{2}$, with line spacing $= 1$, the probability of an equilateral triangle crossing over a line is equal to $\frac{3\sqrt{3}}{pi} \approx 0.82699$.
First, a centroid for a triangle is generated at the origin. The position of the top vertex directly along the y-axis is defined as $A = (0, \frac{l}{\sqrt{3}})$, with l, the side length equal to $\frac{1}{2}$. To generate the other two points of the triangle, we define the rotation matrix:

$$T_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

The bottom left point is positioned at $T_\theta \cdot A$, with $\theta = \frac{2\pi}{3}$. For the bottom right point, we do the same, but use $\theta = \frac{4\pi}{3}$ instead. With this generic triangle created, we can then start the simulation.
To do this, we first generate a center coordinate for the triangle to be shifted over by, and generate an arbitrary angle $\theta \in [0, 2\pi]$. Let $A' = T_\theta \cdot A$. Note that $A_x = 0$, so $A' =< A_y \sin\theta, A_y \cos\theta >$. The y components for B', C' are generated by performing $T_\theta \cdot A'$, with θ as the same set of values used to generate the original triangle. However, to minimize computation time, we only use the equation of the y component.
So, $B'_y = A'_x \sin\frac{2\pi}{3} + A'_y \cos\frac{2\pi}{3}$, and $C'_y = B'_y = A'_x \sin\frac{4\pi}{3} + A'_y \cos\frac{4\pi}{3}$. After transforming these points, they are then shifted by the center coordinate previously generated. If A'_y, B'_y, or C'_y is less than 0, or greater than 1, we know that the triangle has crossed a line and can increment a counter. This is done via a loop of range(N=300,000,000). We find this to be roughly 0.4348.
The code used for this can be seen in the file, 'MCTriangle.java'.

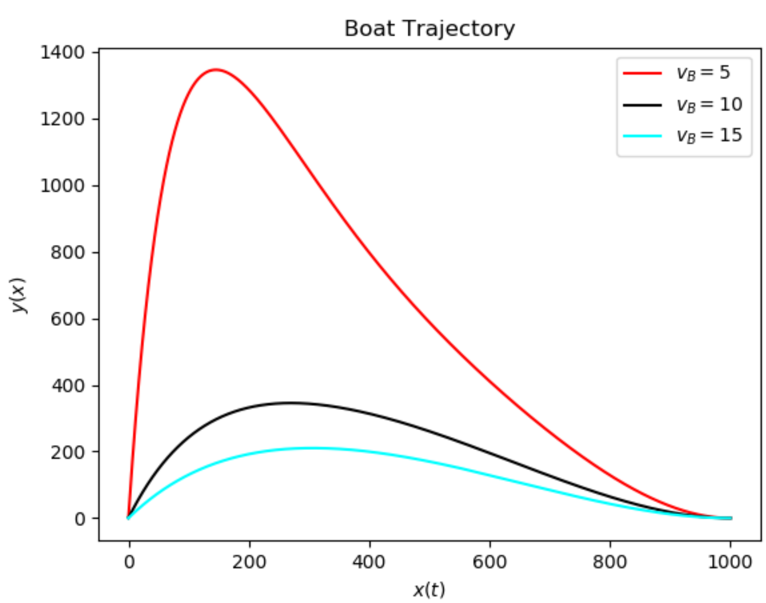
4.3

The third problem gives us a set of conditions for a boat traveling across a river, and asks us to compute its trajectory as its position along the x-axis changes. Wind must also be accounted for, and is represented as $w(x) = 4v_0(\frac{x}{a} - (\frac{x}{a})^2)$, where $(a, 0)$ denotes the starting position of the boat.
We model this as the system of ODEs defined as

$$y'(t) = w(x) - v_B \sin\theta = w(x) - v_B \frac{y}{\sqrt{x^2 + y^2}}$$

$$x'(t) = v_B \cos\theta = v_B \frac{x}{\sqrt{x^2 + y^2}}$$

A two dimensional Runge-Kutta method is implemented on this, with values $k_i,\ l_i$ defined as: $k_1 = f(t_i, x_i, y_i),\ l_1 = g(t_i, x_i, y_i)$
$k_2 = f(t_i + \frac{1}{2}\Delta t, x_i + \frac{1}{2}\Delta t k_1, y_i + \frac{1}{2}\Delta t l_1),\ l_2 = g(t_i + \frac{1}{2}\Delta t, x_i + \frac{1}{2}\Delta t k_1, y_i + \frac{1}{2}\Delta t l_1)$
$k_3 = f(t_i + \frac{1}{2}\Delta t, x_i + \frac{1}{2}\Delta t k_2, y_i + \frac{1}{2}\Delta t l_2),\ l_3 = g(t_i + \frac{1}{2}\Delta t, x_i + \frac{1}{2}\Delta t k_2, y_i + \frac{1}{2}\Delta t l_2)$
$k_4 = f(t_i + \Delta t, x_i + \Delta t k_3, y_i + \Delta t l_3),\ l_4 = g(t_i + \Delta t, x_i + \Delta t k_3, y_i + \Delta t l_3)$
$k = \frac{1}{6}(k_1 + 2(k_2 + k_3) + k_4),\ l = \frac{1}{6}(l_1 + 2(l_2 + l_3) + l_4)$
with $t_{i+1} = t_i + \Delta t,\ x_{i+1} = x_i + \Delta t k,\ y_{i+1} = y_i + \Delta t l$. This is iterated $n = a/h = 1000/0.001 = 1{,}000{,}000$ times. The initial conditions are $y(x = a) = 0$, $y(x = 0) = 0$, with $a = 1000$ and $v_0 = 10$. The Runge-Kutta method is performed for $v_B = 5, 10, 15$ giving maximum y values of 1345.87, 345.43, and 210.30 respectively. Following is a graph for these three v_B conditions.



The code used for plotting and computations can be seen in the file, '326HW4-3.py'.