The DickeyFuller Test for Stationarity

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1. What is Stationarity?

- A time series $\{y_t\}$ is strictly stationary if its statistical properties (like mean, variance, autocorrelation) do not change over time.
- Weak (covariance) stationarity:
  1. $E[y_t] = \mu$ (constant mean)
  2. $\mathrm{Var}(y_t) = \sigma^2$ (constant variance)
  3. $\mathrm{Cov}(y_t, y_{t+k})$ depends only on lag k, not on t.

2. What is a Unit Root?

- Consider the AR(1) model: -$y_t = \phi\, y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim \mathrm{IID}(0, \sigma^2)$.
- If $|\phi| < 1$, the process is stationary.
- If $\phi = 1$, there is a unit root, and $y_t = y_{t-1} + \varepsilon_t$ is a random walk $\rightarrow$ nonstationary.

What does IID mean?

- IID stands for independent and identically distributed
  - Each ε_t is drawn from the same distribution (e.g., Normal with mean 0, variance σ^2)
  - No dependence across time: each error term is independent of the others

3. Rewriting Using Differences

- Define the first difference: $\Delta y_t = y_t - y_{t-1}$.
- Then:
  - $\Delta y_t = (\phi - 1)\, y_{t-1} + \varepsilon_t = \delta\, y_{t-1} + \varepsilon_t,$

  where $\delta = \phi - 1$.
- Hypotheses:
  - H_0: $\delta = 0$ (i.e., $\phi = 1$, nonstationary).
  - H_1: $\delta < 0$ (i.e., $|\phi| < 1$, stationary).

4. DickeyFuller Test Statistic

- Estimate regression: $\Delta y_t = \delta\, y_{t-1} + \varepsilon_t$.
- Compute tstatistic: $t_\delta = \widehat{\delta}/\mathrm{SE}(\widehat{\delta})$.
- Compare t_δ to nonstandard critical values (e.g., from MacKinnon tables).

5. Augmented DickeyFuller (ADF)

- Add intercept and trend if needed:

$$\Delta y_t = \alpha + \beta\, t + \delta\, y_{t-1} + \sum_{i=1}^{p} \gamma_i\, \Delta y_{t-i} + \varepsilon_t$$

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- The additional lag terms control for higherorder correlation in residuals.