## Introduction to Machine Learning Analysis and Modeling

Multivariate EDAN

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### End to end process

Recall ML workflow is a sequence of steps to build and deploy a model that solves a problem using data.

## The pipeline

:BEAMER<sub>env</sub>: block :END:

Ingestion & Preprocessing	Analysis	Modeling	Deployment
Definition	EDA	Selection	Tuning
Data Collection	Feature Engineering	Training	Deployment
Cleaning		<b>Evaluation</b>	Monitoring

#### ML Workflow Graph



Figure: ML workflow steps rendered as a flowchart



### Exploratory Data Analysis

Exploratory Data Analysis (EDA) in the context of machine learning (ML) refers to the systematic process of examining and visualizing the structure, patterns, anomalies, and relationships within a dataset before applying machine learning algorithms. The goal is to gain intuition and insight about the data to inform: Understand the distribution of each feature (e.g., normality, skewness, outliers).

Assess relationships between input features and the target variable (e.g., correlation, mutual information).

#### Exploratory Data Analysis in Pandas

#### Pandas tools:

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'.info()' Structure Overview

df.info()

- Displays a concise summary of the DataFrame:
  - Number of non-null values per column
  - Data types of each column
  - Memory usage
  - Total number of rows and columns

#### Pandas example

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1000 entries, 0 to 999
Data columns (total 5 columns):

## Pandas '.describe()'

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#### '.describe()' Summary Statistics

df.describe()

- Returns summary statistics for numeric columns:
  - 'count', 'mean', 'std', 'min', '25%', '50%', '75%', 'max'

	age	income
count	950.000000	1000.000000
mean	35.5	60000.0
std	10.0	15000.0
min	18.0	20000.0
25%	28.0	50000.0
50%	35.0	60000.0
75%	42.0	70000.0
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#### Non-numeric results

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For non-numeric columns:

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```
df.describe(include='object')
```

• Shows: 'count', 'unique', 'top', 'freq'

# Pandas Tool Summary

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Method	Purpose	Applies To
'df.info()'	Structure & metadata	All columns
<pre>'df.describe()'</pre>	Descriptive stats (summary)	Numeric by default

### Univariate analysis Visualize

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#### Look at your data

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- Histogram + KDE  $\rightarrow$  quick skew/kurtosis check.
- Q-Q Plot → best for tail behavior.
- ullet Boxplot o highlights symmetry and outliers.

[Live Code 2]

### Univariate Analysis Tests

#### Tests for Skewness and Kurtosis

- D'Agostino's  $K^2$  Test: Combines measures of skewness and kurtosis.
  - Based on transformations of the sample skewness and kurtosis.
  - Null Hypothesis: The data is normally distributed.
  - Available in 'scipy.stats.normaltest'.
  - JarqueBera Test:
    - Specifically evaluates skewness and excess kurtosis against a normal distribution.
    - Null Hypothesis: Data is normally distributed.

## Summary Univariate Analysis

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#### Interpretation

- Low p-value (< 0.05): Reject null  $\rightarrow$  evidence of non-normal skew/kurtosis.
- High p-value (0.05): Fail to reject null  $\rightarrow$  no evidence of non-normality.

#### Motivation for Multivariate FDA

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- Univariate EDA is insufficient for understanding dependencies and structure in multivariate data.
- Multivariate EDA focuses on relationships, redundancy, and conditional structure across features.
- Goal: Identify informative, redundant, or interacting features.

#### Joint and Marginal Distributions

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- Let  $X = (X_1, X_2, \dots, X_d) \in \mathbb{R}^d$  be a random vector.
- The joint distribution  $P_X$  describes full probabilistic structure.
- The marginal distribution of a feature  $X_i$  is obtained by integrating out all other variables.
- Understanding joint vs. marginal behavior is central to multivariate EDA.

### Statistical Dependence

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• Two variables X and Y are independent if:

$$P_{X,Y}(x,y) = P_X(x)P_Y(y)$$

- EDA seeks to discover dependencies between variables.
- Classical tools: covariance, correlation but these are limited to linear dependence.

#### Mutual Information

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• Mutual Information (MI) is a nonparametric measure of dependence:

$$I(X; Y) = \int \int p(x, y) \log \left( \frac{p(x, y)}{p(x)p(y)} \right) dxdy$$

•  $I(X; Y) = 0 \text{ iff } X \perp Y.$ 

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• Captures all kinds of dependence not just linear.

## Connection to KL Divergence

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Mutual Information is a special case of the Kullback-Leibler divergence:

$$I(X;Y) = D_{\mathrm{KL}}(P_{X,Y} || P_X \otimes P_Y)$$

- It measures how far the joint distribution is from the product of the marginals.
- Interpreted as: How surprising is the joint distribution, compared to independence?

#### Why It Matters in EDA

- Helps detect feature redundancy or relevance.
- Basis for feature selection and structure learning.
- Multivariate visualizations (pair plots, heatmaps, etc.) are motivated by mathematical notions of dependence.

[Live Code]

Feature Engineering

# What is KL Divergence?

KL Divergence is a measure of how one probability distribution Q differs from a reference distribution P.

- It is not symmetric:  $D_{KI}(P \parallel Q) \neq D_{KI}(Q \parallel P)$
- KL divergence is always non-negative:  $D_{KI}(P \parallel Q) \geq 0$
- $D_{KL}(P \parallel Q) = 0$  if and only if P = Q almost everywhere

## Mathematical Definition (Discrete)

Let P and Q be probability mass functions over a finite or countable set  $\mathcal{X}$ .

$$D_{KL}(P \parallel Q) = \sum_{x \in \mathcal{X}} P(x) \log \frac{P(x)}{Q(x)}$$
 (1)

- The log is typically taken to base 2 (bits) or base e (nats)
- Requires Q(x) > 0 wherever P(x) > 0

#### Interpretation

- Measures the expected number of extra bits needed to code samples from P using a code optimized for Q
- It is the relative entropy of P with respect to Q

Feature Engineering

# Mathematical Definition (Continuous)

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Let p(x) and q(x) be probability density functions over a domain  $\mathcal{X} \subseteq \mathbb{R}^n$ :

$$D_{KL}(P \parallel Q) = \int_{\mathcal{X}} p(x) \log \frac{p(x)}{q(x)} dx$$
 (2)

• Again, the divergence is zero iff p(x) = q(x) almost everywhere

#### Practical Calculation

Given empirical data samples  $x_1, \ldots, x_n \sim P$ , estimate KL divergence:

- Use histograms or kernel density estimators (KDE) to estimate p(x). q(x)
- Approximate:

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$$\hat{D}_{KL}(P \parallel Q) = \frac{1}{n} \sum_{i=1}^{n} \log \frac{p(x_i)}{q(x_i)}$$
(3)

Common in variational inference and mutual information estimation

#### KL Divergence vs. Other Measures

Measure	Symmetric	Interpretable
KL Divergence		Code inefficiency
Jensen-Shannon		Interpolated KL
Mutual Information		Redundancy

Feature Engineering

### Summary of KL Divergence

- KL divergence quantifies divergence from a reference distribution
- Central to many ML methods: variational inference, GANs, language modeling
- Not symmetric, not a true metric
- Requires careful estimation for continuous variables

# What is Feature Engineering?

- Feature engineering is the process of transforming raw data into meaningful input features for machine learning models.
- It involves:

- Creating new features
- Modifying existing ones
- Selecting the most relevant subset
- The goal is to enhance model performance by exposing the most useful signal in the data.

### Why is Feature Engineering Important?

- Quality of features often outweighs choice of algorithm.
- Poor features = poor model performance, regardless of the model used.
- Good features can:

- Improve accuracy
- Speed up training
- Reduce overfitting
- Make models interpretable

# Common Types of Feature Engineering

- Normalization/Scaling: StandardScaler, MinMaxScaler
- Encoding: One-hot, Label encoding
- Discretization/Binning

- Polynomial Features: Capture interactions
- Date/Time decomposition: Day, month, weekday, etc.
- Log transformations: For skewed distributions

#### Feature Selection and Extraction

- Feature Selection: Identify and keep the most relevant variables.
  - RFE (Recursive Feature Elimination):
    - Iteratively builds a model and removes the least important feature.
    - Works with any estimator that exposes 'coef\_' or 'feature<sub>importances\_</sub>'.
- Feature Extraction: Derive new features from raw data.
  - t-SNE (t-distributed Stochastic Neighbor Embedding):
    - A nonlinear dimensionality reduction technique.
    - Preserves local structure; useful for visualizing high-dimensional data.
  - UMAP (Uniform Manifold Approximation and Projection):
    - Similar to t-SNE but faster and better preserves global structure.
    - Based on topological and geometric foundations.

#### Best Practices and Guidelines

- Understand the data context and business goals.
- Visualize feature distributions and relationships.
- Watch out for data leakage.
- Use cross-validation to evaluate engineered features.

### Summary of Feature Engineering

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- Feature engineering is essential for successful modeling.
- Methods like RFE, t-SNE, and UMAP help in selection and dimensionality reduction.
- Combining domain knowledge with statistics is key.