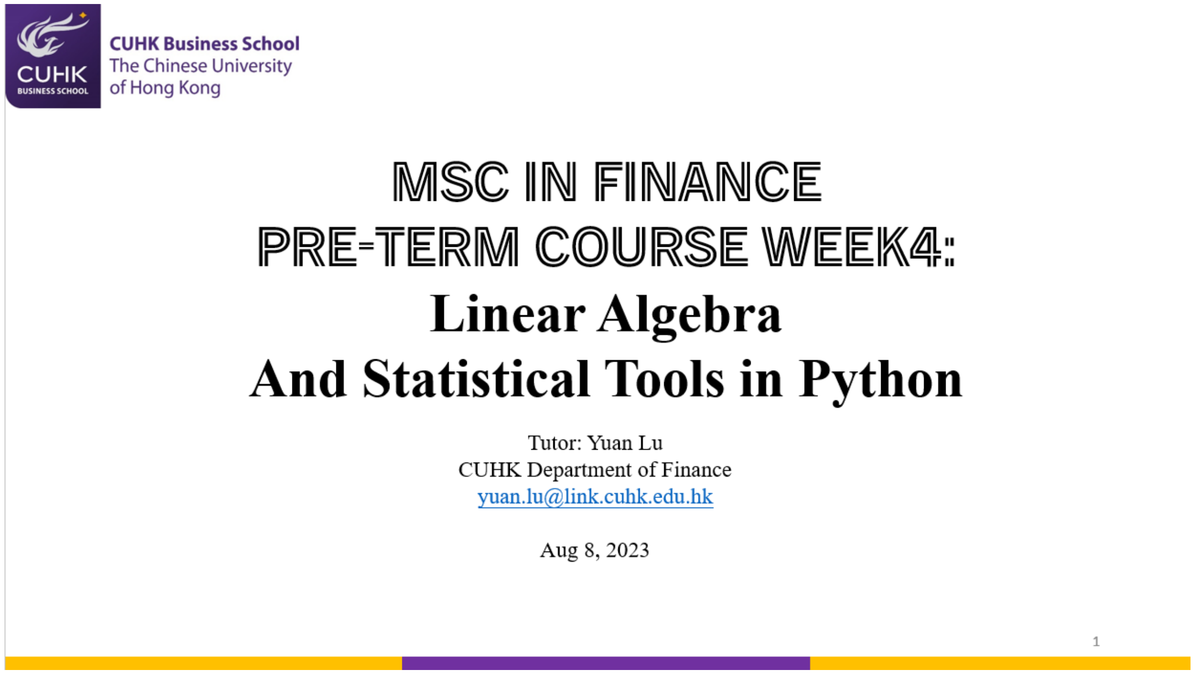



# 1 Agenda

- Review of Linear Algebra
- Linear Regressions
  - Statsmodels
  - Scikit-learn
- Portfolio Optimization
  - Monte-Carlo Simulation
  - Scipy Optimize Module

# 2 Review of Linear Algebra

Linear algebra is essential for machine learning. This Jupyter notebook reviews some of the basic concepts and operations in lienar algebra. In addition, we also provide the Python codes for various operations in linear algebra.

## 2.1 Import Related Libraries

In [6]:

```
import pandas as pd
import numpy as np
```

executed in 1.39s, finished 18:10:11 2023-08-08

## 2.2 Scalars, Vectors and Matrices

1. scalars are constants.

2. row vector is a row of scalars, column vector is a column of scalars
3. matrix is a collection of many row vectors (or column vectors)
4. scalar is a special case of vector, vector is a special case of matrix

In [7]:

```python
# This is a scalar
a = 2
```

executed in 14ms, finished 18:10:11 2023-08-08

In [8]:

```python
# This is a row vector
# It is important to do the double brakets
b = np.array([[1,2,5]])
display(b.shape,b)
```

executed in 14ms, finished 18:10:11 2023-08-08

```
(1, 3)

array([[1, 2, 5]])
```

In [9]:

```python
# This is a column vector
c = np.array([[1],[2],[5]])
display(c.shape,c)
```

executed in 14ms, finished 18:10:11 2023-08-08

```
(3, 1)

array([[1],
       [2],
       [5]])
```

In [10]:

```python
# It is important to note that b and c are 2-dimensional arrays in NumPy.
# If you want to create one-dimensional arrays, do this
b1 = np.array([1,2,5])
display(b1.shape,b1)
```

executed in 13ms, finished 18:10:11 2023-08-08

```
(3,)

array([1, 2, 5])
```

In [11]:
```
# You can convert 1-dimensional array to column vector or row vector
# using reshape
b2 = b1.reshape([1,3])
display(b2.shape,b2)

c2 = b1.reshape([3,1])
display(c2.shape,c2)
```
executed in 14ms, finished 18:10:11 2023-08-08

```
(1, 3)

array([[1, 2, 5]])

(3, 1)

array([[1],
       [2],
       [5]])
```

In [12]:
```
# You can also convert column or row vector to 1-dimensional array
b3 = b2.flatten()
display(b3.shape,b3)
c3 = c2.flatten()
display(c3.shape,c3)
```
executed in 14ms, finished 18:10:11 2023-08-08

```
(3,)

array([1, 2, 5])

(3,)

array([1, 2, 5])
```

In [13]:
```
# This is a 3x4 matrix
A = np.array([[1,2,5,-1],[2,0,4,6],[3,5,2,1]])
display(A.shape,A)
```
executed in 14ms, finished 18:10:11 2023-08-08

```
(3, 4)

array([[ 1,  2,  5, -1],
       [ 2,  0,  4,  6],
       [ 3,  5,  2,  1]])
```

## 2.3 Transpose

1. Transpose of a row vector is a column vector, transpose of a column vector is a row vector
2. Transpose of a matrix is a new matrix whose rows are the columns of the original matrix

$b = [1, 2, 5]$

In [14]:

```
c = b.T
display(c.shape,c)
```

executed in 14ms, finished 18:10:11 2023-08-08

```
(3, 1)

array([[1],
       [2],
       [5]])
```

$$A = \begin{bmatrix} 1 & 2 & 5 & -1 \\ 2 & 0 & 4 & 6 \\ 3 & 5 & 2 & 1 \end{bmatrix}$$

In [15]:

```
A1 = A.T
display(A1.shape,A1)
```

executed in 13ms, finished 18:10:11 2023-08-08

```
(4, 3)

array([[ 1,  2,  3],
       [ 2,  0,  5],
       [ 5,  4,  2],
       [-1,  6,  1]])
```

## 2.4 Some Special Matrices

- $0_n$ is an $n$-vector of zeros

In [16]:

```
zero_vec = np.zeros([4,1])
print(zero_vec)
```

executed in 14ms, finished 18:10:11 2023-08-08

```
[[0.]
 [0.]
 [0.]
 [0.]]
```

- $1_n$ is an $n$-vector of ones

In [17]:

```
one_vec = np.ones([4,1])
print(one_vec)
```

executed in 15ms, finished 18:10:11 2023-08-08

```
[[1.]
 [1.]
 [1.]
 [1.]]
```

- $I_n$ is an $n \times n$ identity matrix (only ones on the diagonal and zeros elsewhere)

In [18]:
```
idtymat = np.eye(5)
print(idtymat)
```
executed in 14ms, finished 18:10:11 2023-08-08

```
[[1. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0.]
 [0. 0. 1. 0. 0.]
 [0. 0. 0. 1. 0.]
 [0. 0. 0. 0. 1.]]
```

## 2.5 Additions and Subtractions

1. Additions and subtractions of two vectors and matrices are done on an element-by-element basis
2. It means the two vectors (or two matrices) must be of same dimension
3. However, numpy allows broadcasting, which allows you to add a column (or row) vector to a matrix.

In [19]:
```
b1 = np.array([[2,3,1]])
b2 = np.array([[4,2,0]])
b3 = b1+b2
print(b3)
```
executed in 14ms, finished 18:10:11 2023-08-08

```
[[6 5 1]]
```

In [20]:
```
A1 = np.array([[2,3,5],[1,2,3],[7,6,4],[5,2,1]])
A2 = np.array([[1,2,3],[7,8,9],[4,5,6],[2,4,2]])
A3 = A1+A2
print(A3)
```
executed in 14ms, finished 18:10:11 2023-08-08

```
[[ 3  5  8]
 [ 8 10 12]
 [11 11 10]
 [ 7  6  3]]
```

In [21]:

```
# Here is an example of array broadcasting
display(b1,A1,A1+b1)
```

executed in 15ms, finished 18:10:11 2023-08-08

```
array([[2, 3, 1]])

array([[2, 3, 5],
       [1, 2, 3],
       [7, 6, 4],
       [5, 2, 1]])

array([[4, 6, 6],
       [3, 5, 4],
       [9, 9, 5],
       [7, 5, 2]])
```

## 2.6 Dot Product and Matrix Multiplication

1. We can multiply two vectors together with dot product $a' b = \sum_{i=1}^{n} a_i b_i$.
2. We can multiply a matrix with a vector If $A$ is $m \times n$ and $b$ is $n \times 1$, then $c = Ab$ is an $m \times 1$ vector, with $c_i = \sum_{j=1}^{n} a_{i,j} b_j, \quad i = 1, \ldots, m$.
3. We can multiply two matrices together If $A$ is $m \times n$ and $B$ is $n \times p$, then $C = AB$ is an $m \times p$ matrix, with $c_{i,j} = \sum_{k=1}^{n} a_{i,k} b_{k,j}, \quad i = 1, \ldots, m, \; j = 1, \ldots, p$.
4. Matrix multiplication can only be performed if the number of columns of the left matrix is the same as the number of rows of the right matrix.

In [22]:

```
a = np.array([[3],[2],[1]])
b = np.array([[4],[5],[6]])
c = np.dot(a.T,b)
display(a.T,b,c)
```

executed in 14ms, finished 18:10:11 2023-08-08

```
array([[3, 2, 1]])

array([[4],
       [5],
       [6]])

array([[28]])
```

In [23]:

```
A = np.array([[2,3,1],[3,2,7]])
c = A.dot(b)
display(A,b,c)
```

executed in 12ms, finished 18:10:11 2023-08-08

```
array([[2, 3, 1],
       [3, 2, 7]])

array([[4],
       [5],
       [6]])

array([[29],
       [64]])
```

In [24]:

```
# An alternative way of doing the above
A = np.array([[2,3,1],[3,2,7]])
c = np.matmul(A,b)
display(A,b,c)
```

executed in 14ms, finished 18:10:11 2023-08-08

```
array([[2, 3, 1],
       [3, 2, 7]])

array([[4],
       [5],
       [6]])

array([[29],
       [64]])
```

In [25]:

```
B = np.array([[3,2,1,4],[4,3,2,7],[3,0,-1,2]])
C = np.matmul(A,B)
display(A,B,C)
```

executed in 15ms, finished 18:10:11 2023-08-08

```
array([[2, 3, 1],
       [3, 2, 7]])

array([[ 3,  2,  1,  4],
       [ 4,  3,  2,  7],
       [ 3,  0, -1,  2]])

array([[21, 13,  7, 31],
       [38, 12,  0, 40]])
```

## 2.7 Matrix Inverse

1. Given an $n \times n$ square matrix $A$, we find a matrix $B$ such that $AB = I_n$, we call $B = A^{-1}$.
2. Note that $AA^{-1} = I_n$ but $A^{-1}A = I_n$ also holds.

In [26]:
```python
A = np.array([[3,2,1,4],[4,3,2,7],[3,0,-1,2],[2,1,5,4]])
B = np.linalg.inv(A)
display(A,B)
```
executed in 45ms, finished 18:10:11 2023-08-08

```
array([[ 3,  2,  1,  4],
       [ 4,  3,  2,  7],
       [ 3,  0, -1,  2],
       [ 2,  1,  5,  4]])
```

```
array([[ 0.91176471, -0.64705882,  0.20588235,  0.11764706],
       [ 1.38235294, -0.52941176, -0.55882353, -0.17647059],
       [ 0.32352941, -0.29411765, -0.08823529,  0.23529412],
       [-1.20588235,  0.82352941,  0.14705882, -0.05882353]])
```

In [27]:
```python
np.round(B,4)
```
executed in 14ms, finished 18:10:11 2023-08-08

Out[27]:
```
array([[ 0.9118, -0.6471,  0.2059,  0.1176],
       [ 1.3824, -0.5294, -0.5588, -0.1765],
       [ 0.3235, -0.2941, -0.0882,  0.2353],
       [-1.2059,  0.8235,  0.1471, -0.0588]])
```

$AB = I_n$

In [28]:
```python
C = np.matmul(A,B)
np.round(C,4)
```
executed in 60ms, finished 18:10:11 2023-08-08

Out[28]:
```
array([[ 1.,  0.,  0., -0.],
       [ 0.,  1.,  0.,  0.],
       [ 0., -0.,  1., -0.],
       [ 0.,  0.,  0.,  1.]])
```

$BA = I_n$

In [29]:
```python
C1 = np.matmul(B,A)
np.round(C1,4)
```
executed in 15ms, finished 18:10:11 2023-08-08

Out[29]:
```
array([[ 1.,  0.,  0.,  0.],
       [-0.,  1.,  0.,  0.],
       [ 0.,  0.,  1.,  0.],
       [-0.,  0., -0.,  1.]])
```

## 2.8 Solving a Linear System of Equations

1. Matrix inverse allows us to solve a system of linear equations.
2. If we want to solve:

$$\begin{aligned} a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 &= b_1, \\ a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 &= b_2, \\ a_{3,1}x_2 + a_{3,2}x_2 + a_{3,3}x_3 &= b_3, \end{aligned}$$

we can simply do $x = A^{-1}b$ if $A$ is invertible.

In [30]:
```
A = np.array([[2,3,4],[5,-1,2],[2,3,6]])
b = np.array([[2],[4],[6]])
x = np.matmul(np.linalg.inv(A),b)
display(A,b,x)
```
executed in 15ms, finished 18:10:11 2023-08-08

```
array([[ 2,  3,  4],
       [ 5, -1,  2],
       [ 2,  3,  6]])

array([[2],
       [4],
       [6]])

array([[-0.35294118],
       [-1.76470588],
       [ 2.        ]])
```

$Ax = b$

In [31]:
```
np.matmul(A,x)
```
executed in 14ms, finished 18:10:11 2023-08-08

Out[31]:
```
array([[2.],
       [4.],
       [6.]])
```

## 2.9 Trace and Determinant

1. Trace of a matrix is the sum of its diagonal elements.
2. Determinant of a $2 \times 2$ matrix
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
is $|A| = ad - bc$.
3. For general $n \times n$ matrix, see [Wikipedia (https://en.wikipedia.org/wiki/Determinant)](https://en.wikipedia.org/wiki/Determinant) for definition of its determinant

In [32]:
```
t = np.trace(A)
d = np.linalg.det(A)
display(A,t,d)
```
executed in 15ms, finished 18:10:11 2023-08-08

```
array([[ 2,  3,  4],
       [ 5, -1,  2],
       [ 2,  3,  6]])

7

-34.00000000000001
```

## 2.10 Positive Definite Matrix

1. A symmetric $n \times n$ matrix $A$ is positive definite if $x' Ax > 0$ for any non-zero $n$-vector $x$.
2. An example of positive definite matrix is covariance matrix (or correlation matrix).
3. For a positive definite matrix, its determinant is positive.

In [33]:
```
A = np.array([[3,2,1],[2,4,-1],[1,-1,5]])
A
```
executed in 10ms, finished 18:10:12 2023-08-08

Out[33]:
```
array([[ 3,  2,  1],
       [ 2,  4, -1],
       [ 1, -1,  5]])
```

In [34]:
```
# Use a vector x, and compute x'Ax, check if it is positive
x = np.array([[2],[3],[-7]])
np.dot(x.T,A.dot(x)).item()
```
executed in 15ms, finished 18:10:12 2023-08-08

Out[34]:
```
331
```

In [35]:
```
np.linalg.det(A)
```
executed in 14ms, finished 18:10:12 2023-08-08

Out[35]:
```
29.000000000000018
```

# 3 Linear Regressions

Based on Python library: Statsmodels and Sklearn.

Examples from the website: [https://blog.quantinsti.com/linear-regression-market-data-python-r/ (https://blog.quantinsti.com/linear-regression-market-data-python-r/)](https://blog.quantinsti.com/linear-regression-market-data-python-r/)

In [36]:
```
### Import the required libraries

import numpy as np
import pandas as pd
import datetime
import matplotlib.pyplot as plt

## To use statsmodels for linear regression
import statsmodels.formula.api as smf

## To use sklearn for linear regression
from sklearn.linear_model import LinearRegression
```
executed in 4.99s, finished 18:10:17 2023-08-08

We work with the historical returns of Coca-Cola (NYSE: KO), its competitor PepsiCo (NASDAQ: PEP), the US Dollar index (ICE: DX) and the SPDR S&P 500 ETF (NYSEARCA:

In [37]:

```python
# Read data from csv file, the file contains the log returns.
df=pd.read_csv("data_linear_reg.csv")
df
```

executed in 292ms, finished 18:10:17 2023-08-08

Out[37]:

| | Date | spy | ko | pep | usdx |
|---|---|---|---|---|---|
| **0** | 2020-08-04 | 0.003855 | 0.008388 | 0.005617 | -0.001284 |
| **1** | 2020-08-05 | 0.006192 | 0.011288 | -0.008914 | -0.005907 |
| **2** | 2020-08-06 | 0.006662 | 0.005491 | -0.002866 | -0.000431 |
| **3** | 2020-08-07 | 0.000718 | 0.006717 | 0.006456 | 0.006981 |
| **4** | 2020-08-10 | 0.002984 | -0.001675 | -0.005574 | 0.001497 |
| **...** | ... | ... | ... | ... | ... |
| **747** | 2023-07-25 | 0.002726 | -0.003368 | 0.002302 | 0.000000 |
| **748** | 2023-07-26 | 0.000154 | 0.012770 | 0.001253 | -0.004549 |
| **749** | 2023-07-27 | -0.006652 | -0.009722 | -0.016153 | 0.008685 |
| **750** | 2023-07-28 | 0.009743 | 0.000640 | 0.009397 | -0.001475 |
| **751** | 2023-07-31 | 0.001902 | -0.008842 | -0.015089 | 0.002359 |

752 rows × 5 columns

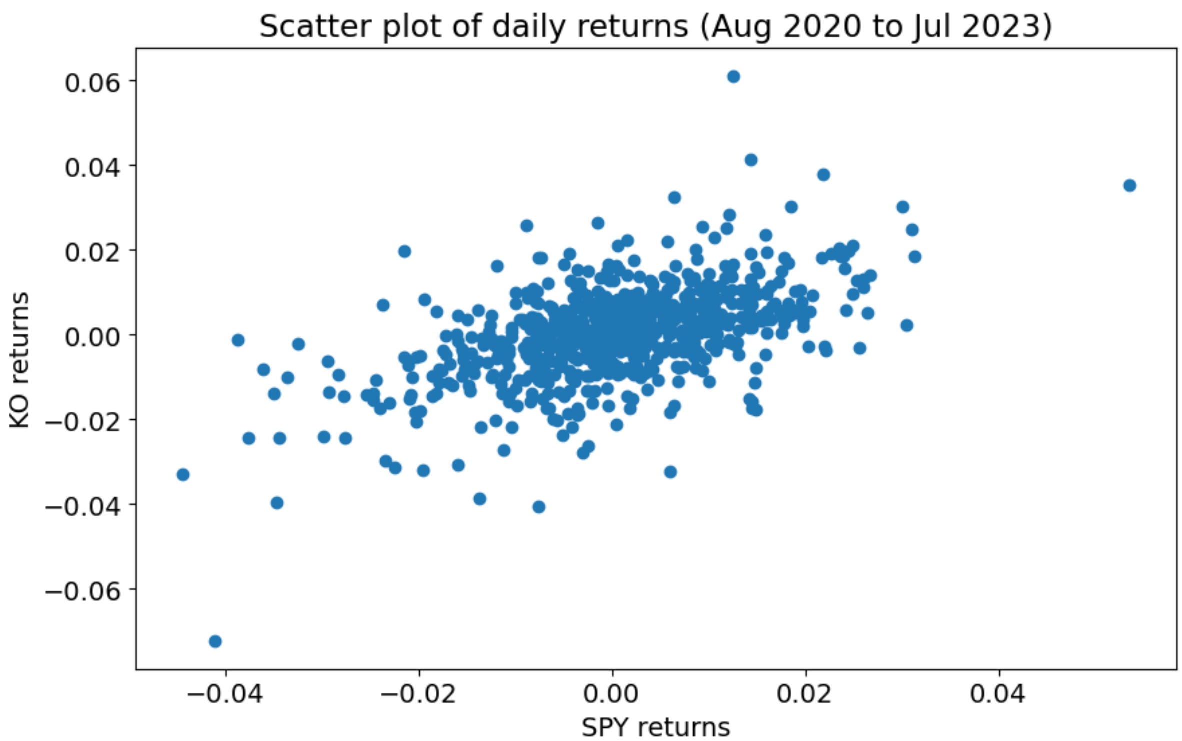
In [38]:

```python
# # Online Fetch data and calculate log returns. If you've read data from csv

# import yfinance as yf

# ######################################################
# ## Fetch data from yfinance
# ## 3-year daily data for Coca-Cola, SPY, Pepsi, and USD index

# end1 = datetime.date(2023, 8, 1)
# start1 = end1 - pd.Timedelta(days = 365 * 3)

# ko_df = yf.download("KO", start = start1, end = end1, progress = False)
# spy_df = yf.download("SPY", start = start1, end = end1, progress = False)
# pep_df = yf.download("PEP", start = start1, end = end1, progress = False)
# usdx_df = yf.download("DX-Y.NYB", start = start1, end = end1, progress = Fa

# ######################################################
# ## Calculate log returns for the period based on Adj Close prices

# ko_df['ko'] = np.log(ko_df['Adj Close'] / ko_df['Adj Close'].shift(1))
# spy_df['spy'] = np.log(spy_df['Adj Close'] / spy_df['Adj Close'].shift(1))
# pep_df['pep'] = np.log(pep_df['Adj Close'] / pep_df['Adj Close'].shift(1))
# usdx_df['usdx'] = np.log(usdx_df['Adj Close'] / usdx_df['Adj Close'].shift(

# ######################################################
# ## Create a dataframe with X's (spy, pep, usdx) and Y (ko)

# df = pd.concat([spy_df['spy'], ko_df['ko'],
#                 pep_df['pep'], usdx_df['usdx']], axis = 1).dropna()

# # ## Save the csv file. Good practice to save data files after initial proc
# # df.to_csv("data_linear_reg.csv")

# ######################################################
```

executed in 15ms, finished 18:10:17 2023-08-08

We first create a scatter plot of the SPY and KO returns to better understand how they are related.

In [39]:
```
## A scatter plot of X (spy) and Y (ko) to examine the nature of their relati

plt.figure(figsize = (10, 6))
plt.rcParams.update({'font.size': 14})
plt.xlabel("SPY returns")
plt.ylabel("KO returns")
plt.title("Scatter plot of daily returns (Aug 2020 to Jul 2023)")
plt.scatter(df['spy'], df['ko'])
plt.show()
```
executed in 185ms, finished 18:10:17 2023-08-08




We also calculate correlations between different variables to analyze the strength of the linear relationships here.

In [40]:
```
### Calculate correlation between Xs and Y

df.corr()
```
executed in 14ms, finished 18:10:17 2023-08-08

Out[40]:

| | spy | ko | pep | usdx |
|---|---|---|---|---|
| **spy** | 1.000000 | 0.539683 | 0.543284 | -0.376342 |
| **ko** | 0.539683 | 1.000000 | 0.743223 | -0.230901 |
| **pep** | 0.543284 | 0.743223 | 1.000000 | -0.175518 |
| **usdx** | -0.376342 | -0.230901 | -0.175518 | 1.000000 |

## 3.1 Statsmodels

In [41]:

```
### Fit a simple linear regression model to the data using statsmodels

### Create an instance of the class OLS
slr_sm_model = smf.ols('ko ~ spy', data=df)

### Fit the model (statsmodels calculates beta_0 and beta_1 here)
slr_sm_model_ko = slr_sm_model.fit()

### Summarize the model

print(slr_sm_model_ko.summary())

param_slr = slr_sm_model_ko.params
```

executed in 46ms, finished 18:10:17 2023-08-08

```
                            OLS Regression Results
==============================================================================
=
Dep. Variable:                     ko   R-squared:                       0.29
1
Model:                            OLS   Adj. R-squared:                  0.29
0
Method:                 Least Squares   F-statistic:                     308.
2
Date:                Tue, 08 Aug 2023   Prob (F-statistic):          4.61e-5
8
Time:                        18:10:17   Log-Likelihood:                 2458.
4
No. Observations:                 752   AIC:                            -491
3.
Df Residuals:                     750   BIC:                            -490
4.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
=
                 coef    std err          t      P>|t|      [0.025      0.97
5]
------------------------------------------------------------------------------
-
Intercept      0.0003      0.000      0.745      0.457      -0.000       0.00
1
spy            0.5152      0.029     17.556      0.000       0.458       0.57
3
==============================================================================
=
Omnibus:                       72.813   Durbin-Watson:                   1.89
2
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              455.25
8
Skew:                          -0.066   Prob(JB):                     1.39e-9
9
Kurtosis:                       6.810   Cond. No.                         87.
3
==============================================================================
=

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correc
tly specified.
```

In [42]:
```python
### Print the parameter estimates of the simple linear regression model

print("==================================================================")
print("The intercept in the statsmodels regression model is", \
      np.round(param_slr.Intercept, 4))
print("The slope in the statsmodels regression model is", \
      np.round(param_slr.spy, 4))
print("==================================================================")
```

executed in 14ms, finished 18:10:17 2023-08-08

```
====================================================================
The intercept in the statsmodels regression model is 0.0003
The slope in the statsmodels regression model is 0.5152
====================================================================
```

As the results above, the model for KO's returns can be expressed as

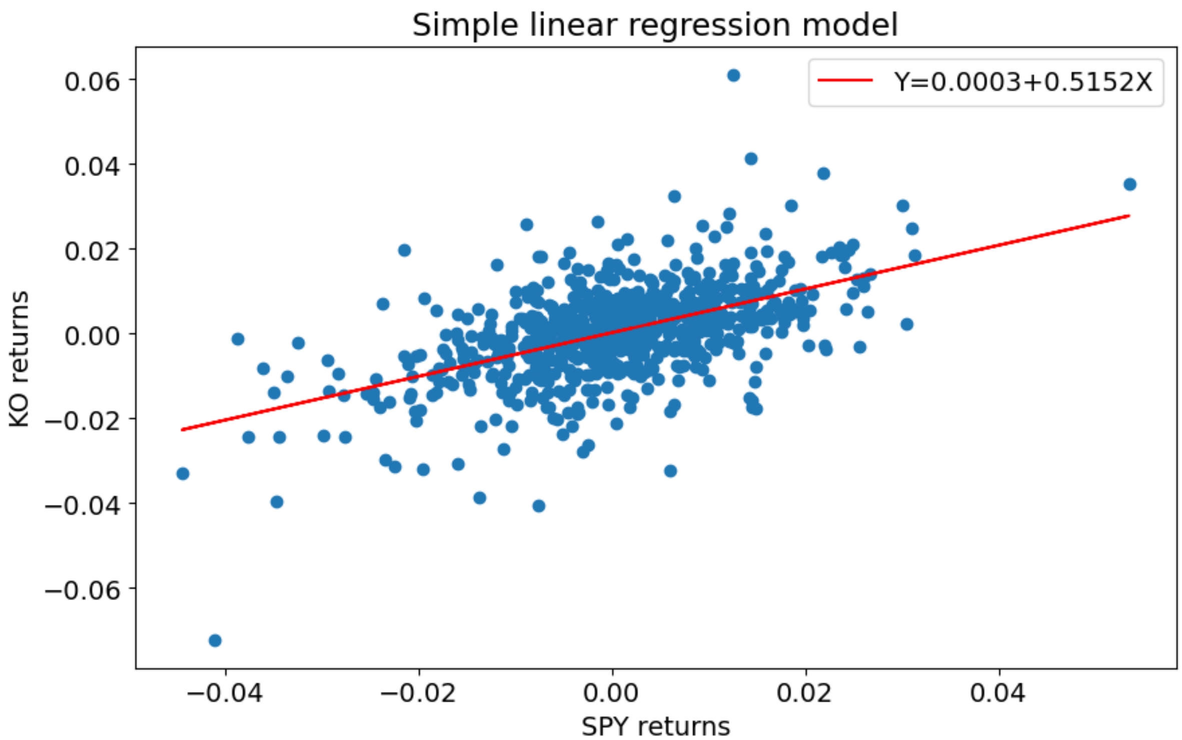
$$\hat{Y}_i = 0.0003 + 0.5152X_i$$

The caret mark or ^ above the $Y_i$ indicates that it is the fitted (or predicted) value of KO's returns as opposed to the observed returns. We obtain it by computing the RHS of the above equation.

We plot the best fit line (i.e. the regression line) for the data set as shown below.

In [43]:
```python
### Linear regression plot of X (spy) and Y (ko)
plt.figure(figsize = (10, 6))
plt.rcParams.update({'font.size': 14})
plt.xlabel("SPY returns")
plt.ylabel("KO returns")
plt.title("Simple linear regression model")
plt.scatter(df['spy'],df['ko'])
plt.plot(df['spy'], param_slr.Intercept+param_slr.spy * df['spy'],
         label='Y={:.4f}+{:.4f}X'.format(param_slr.Intercept, param_slr.spy),
         color='red')
plt.legend()
plt.show()
```

executed in 153ms, finished 18:10:17 2023-08-08




For multiple linear regression, we use SPY, PEP and DX returns as explanatory variables to explain KO returns.

In [44]:

```
### Fit a multiple linear regression model to the data using statsmodels

### Create an instance of the class OLS
mlr_sm_model = smf.ols('ko ~ spy + pep + usdx', data=df)

### Fit the model (statsmodels calculates beta_0, beta_1, beta_2, beta_3 here
mlr_sm_model_ko = mlr_sm_model.fit()

### Summarize the model

print(mlr_sm_model_ko.summary())
```

executed in 30ms, finished 18:10:17 2023-08-08

```
                            OLS Regression Results
==============================================================================
=
Dep. Variable:                     ko   R-squared:                       0.58
1
Model:                            OLS   Adj. R-squared:                  0.57
9
Method:                 Least Squares   F-statistic:                     345.
9
Date:                Tue, 08 Aug 2023   Prob (F-statistic):          7.91e-14
1
Time:                        18:10:17   Log-Likelihood:                 2656.
1
No. Observations:                 752   AIC:                            -530
4.
Df Residuals:                     748   BIC:                            -528
6.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
=
                 coef    std err          t      P>|t|      [0.025      0.97
5]
------------------------------------------------------------------------------
-
Intercept   8.604e-05      0.000      0.332      0.740      -0.000       0.00
1
spy            0.1635      0.029      5.715      0.000       0.107       0.22
0
pep            0.6692      0.029     22.713      0.000       0.611       0.72
7
usdx          -0.1285      0.061     -2.112      0.035      -0.248      -0.00
9
==============================================================================
=
Omnibus:                      196.858   Durbin-Watson:                   1.97
0
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1903.01
9
Skew:                           0.884   Prob(JB):                         0.0
0
Kurtosis:                      10.590   Cond. No.                         23
9.
==============================================================================
=

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correc
tly specified.
```

In [45]:
```python
### Print the parameter estimates of the muliple linear regression model

param_mlr = mlr_sm_model_ko.params

print("=====================================================================")
print("The intercept and slopes in the statsmodels regression model are")
print("\n")
print(param_mlr)
print("=====================================================================")
```

executed in 14ms, finished 18:10:17 2023-08-08

```
=====================================================================
The intercept and slopes in the statsmodels regression model are


Intercept    0.000086
spy          0.163520
pep          0.669178
usdx        -0.128472
dtype: float64
=====================================================================
```

As per the results above, the model for KO's returns can be expressed as

$$\hat{Y}_i = 0.0001 + 0.1635X_{1,i} + 0.6692X_{2,i} - 0.1285X_{3,i}$$

The caret mark above the $Y_i$ indicates that it is the fitted (or predicted) value of KO's returns based on the historical returns on SPY, PEP, and USDX.

## 3.2 Scikit-Learn

We can also use the scikit-learn library for regressional analysis. Unlike statsmodels, here, we don't have the option of a full summary table with a detailed output.

Notice also how the features and output specified are done so in the .fit() routine (with features first and then the output variable). In statsmodels, we specified the parameters (in reverse order) when we instantiated the OLS class.

The coefficients obtained as you see here and in multiple regression, are the same with both libraries.

In [46]:

```
### Create an instance of the class LinearRegression()
slr_skl_model = LinearRegression()

### Fit the model (sklearn calculates beta_0 and beta_1 here)

X = df['spy'].values.reshape(-1, 1)
slr_skl_model_ko = slr_skl_model.fit(X, df['ko'])

print("The intercept in the sklearn regression result is", np.round(slr_skl_r
print("The slope in the sklearn regression model is", np.round(slr_skl_model_
```

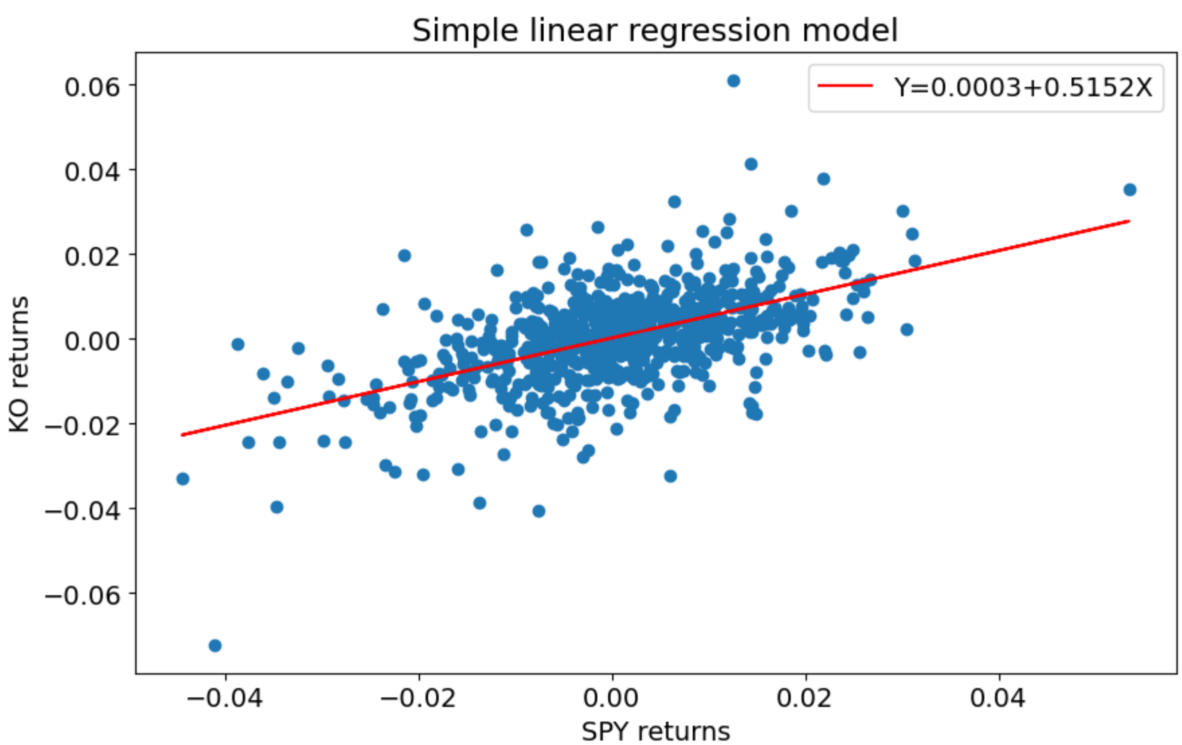
executed in 15ms, finished 18:10:17 2023-08-08

```
The intercept in the sklearn regression result is 0.0003
The slope in the sklearn regression model is 0.5152
```

In [47]:

```
### Linear regression plot of X (spy) and Y (ko)

plt.figure(figsize = (10, 6))
plt.rcParams.update({'font.size': 14})
plt.xlabel("SPY returns")
plt.ylabel("KO returns")
plt.title("Simple linear regression model")
plt.scatter(df['spy'], df['ko'])
plt.plot(X, slr_skl_model.predict(X), label='Y={:.4f}+{:.4f}X'.format(slr_skl
plt.legend()
plt.show()
```

executed in 138ms, finished 18:10:17 2023-08-08



In [48]:
```
### Print the parameter estimates of the simple linear regression model

print("================================================================")
print("The intercept in the sklearn regression result is", \
      np.round(slr_skl_model_ko.intercept_, 4))
print("The slope in the sklearn regression model is", \
      np.round(slr_skl_model_ko.coef_[0], 4))
print("================================================================")
```
executed in 14ms, finished 18:10:17 2023-08-08

```
====================================================================
The intercept in the sklearn regression result is 0.0003
The slope in the sklearn regression model is 0.5152
====================================================================
```

As the results above, the model for KO's returns can be expressed as

$$\hat{Y}_i = 0.0003 + 0.5152X_i$$

The regression estimations in sklearn are the same as those in statsmodels.

In [49]:
```
### Fit a multiple linear regression model to the data using sklearn

### Create an instance of the class LinearRegression()
mlr_skl_model = LinearRegression()

### Fit the model (sklearn calculates beta_0 and beta_1 here)

X = df[['spy', 'pep', 'usdx']]
y = df['ko']
mlr_skl_model_ko = mlr_skl_model.fit(X, y)
```
executed in 15ms, finished 18:10:17 2023-08-08

In [50]:
```
print("The intercept in the sklearn regression result is", \
      mlr_skl_model_ko.intercept_)
print("The slope in the sklearn regression model is", \
      mlr_skl_model_ko.coef_)
```
executed in 14ms, finished 18:10:18 2023-08-08

```
The intercept in the sklearn regression result is 8.603960244215318e-05
The slope in the sklearn regression model is [ 0.16352043  0.66917759 -0.1284
7178]
```

As per the results above, the model for KO's returns can be expressed as

$$\hat{Y}_i = 0.0001 + 0.1635X_{1,i} + 0.6692X_{2,i} - 0.1285X_{3,i}$$

The regression estimations in sklearn are the same as those in statsmodels.

# 4 Portfolio Optimization

- **Modern Portfolio Theory**
- Harry M. Markowitz (August 24, 1927 – June 22, 2023), Nobel-Winning Pioneer of Modern Portfolio Theory.

- Investors can optimize their portfolio by selecting a combination of assets that maximizes expected return for a given level of risk or minimizes risk for a given level of expected return.
- Diversify the portfolio across different assets and sectors, which helps to reduce the overall volatility and risk of the portfolio.

- Two key components: **expected return** and **risk** of a portfolio. By using Python libraries such as SciPy Optimize and the Monte Carlo Method, we can create a more efficient and accurate optimization process compared to traditional methods.
- The **SciPy Optimize**: a library that provides optimization algorithms for a wide range of optimization problems, including linear and nonlinear programming, global and local optimization, and optimization of differential equations.
- The **Monte Carlo Method**: a statistical method used to simulate the behavior of a system by generating a large number of random scenarios.

## 4.1 Import Packages and Read Data

In [51]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib as mpl
import seaborn as sns
```

executed in 247ms, finished 18:10:18 2023-08-08

In [52]:

```
# Read Data from csv file. Specify the first column as the index for the Data
adj_close=pd.read_csv('adj_close.csv',index_col=0)

symbols = adj_close.columns.tolist()  # ['AAPL','MSFT','AMZN','GOOGL'] #our s
num_assets = len(symbols)  #the number of assets

adj_close
```

executed in 292ms, finished 18:10:18 2023-08-08

Out[52]:

| | AAPL | AMZN | GOOGL | MSFT |
|---|---|---|---|---|
| **Date** | | | | |
| **2015-01-02** | 24.531769 | 15.426000 | 26.477501 | 40.620655 |
| **2015-01-05** | 23.840658 | 15.109500 | 25.973000 | 40.247131 |
| **2015-01-06** | 23.842915 | 14.764500 | 25.332001 | 39.656406 |
| **2015-01-07** | 24.177242 | 14.921000 | 25.257500 | 40.160248 |
| **2015-01-08** | 25.106186 | 15.023000 | 25.345501 | 41.341682 |
| **...** | ... | ... | ... | ... |
| **2019-12-24** | 69.421059 | 89.460503 | 67.221497 | 152.286896 |
| **2019-12-26** | 70.798401 | 93.438499 | 68.123497 | 153.535156 |
| **2019-12-27** | 70.771538 | 93.489998 | 67.732002 | 153.815781 |
| **2019-12-30** | 71.191574 | 92.344498 | 66.985497 | 152.490082 |
| **2019-12-31** | 71.711739 | 92.391998 | 66.969498 | 152.596542 |

1258 rows × 4 columns

In [53]:

```
# # Fetch Data online. Skip the part if you've read data from csv file
# from pandas_datareader import data as pdr
# import yfinance as yf

# yf.pdr_override()

# # symbols = ['AAPL','GOOG','TSLA','MSFT','META','AMZN']
# symbols = ['AAPL','MSFT','AMZN','GOOGL']
# num_assets = len(symbols)  #the number of assets

# data = pdr.get_data_yahoo(symbols,start="2015-01-01", end="2020-01-01")
# adj_close=data['Adj Close']
# adj_close
```

executed in 14ms, finished 18:10:18 2023-08-08

## 4.2 Calculate Mean and Variance of returns

- **Expected return** is the average return that an investor can expect to earn on an investment over a given period of time.
- **Arithmatic returns**

$$R_{i,t} = \frac{(adjClose_{i,t} - adjClose_{i,t-1})}{adjClose_{i,t-1}}$$

- **Log returns**

$$R_{i,t} = log(adjClose_{i,t}) - log(adjClose_{i,t-1})$$

In [54]:
```
# Calculate average returns for stocks as expected returns
def calc_returns(price_data, ret_type="arithmatic"):
    """
    Parameters
        price_data: price timeseries pd.DataFrame object.

        ret_type:   return calculation type. \"arithmatic\" or \"log\"

    Returns:
        returns timeseries pd.DataFrame object
    """

    if ret_type=="arithmatic":
        ret = price_data.pct_change().dropna()
    elif ret_type=="log":
        ret = np.log(price_data/price_data.shift()).dropna()
    else:
        raise ValueError("ret_type: return calculation type is not valid. use

    return(ret)
```

executed in 14ms, finished 18:10:18 2023-08-08

In [55]:
```
daily_ret = calc_returns(adj_close,  ret_type="log")
daily_ret
```

executed in 15ms, finished 18:10:18 2023-08-08

Out[55]:

| Date | AAPL | AMZN | GOOGL | MSFT |
|---|---|---|---|---|
| **2015-01-05** | -0.028577 | -0.020731 | -0.019238 | -0.009238 |
| **2015-01-06** | 0.000095 | -0.023098 | -0.024989 | -0.014786 |
| **2015-01-07** | 0.013925 | 0.010544 | -0.002945 | 0.012625 |
| **2015-01-08** | 0.037702 | 0.006813 | 0.003478 | 0.028994 |
| **2015-01-09** | 0.001072 | -0.011818 | -0.012286 | -0.008441 |
| **...** | ... | ... | ... | ... |
| **2019-12-24** | 0.000950 | -0.002116 | -0.004601 | -0.000191 |
| **2019-12-26** | 0.019646 | 0.043506 | 0.013329 | 0.008163 |
| **2019-12-27** | -0.000380 | 0.000551 | -0.005763 | 0.001826 |
| **2019-12-30** | 0.005918 | -0.012328 | -0.011083 | -0.008656 |
| **2019-12-31** | 0.007280 | 0.000514 | -0.000239 | 0.000698 |

1257 rows × 4 columns

After calculate the stock returns, we then comput the mean $\mu$ and covariance matrics.

In [56]:
```
def calc_returns_stats(returns):
    """
    Parameters
        returns: returns timeseries pd.DataFrame object

    Returns:
        mean_returns: Avereage of returns
        cov_matrix: returns Covariance matrix
    """
    mean_returns = returns.mean()
    cov_matrix = returns.cov()
    return (mean_returns, cov_matrix)
```
executed in 12ms, finished 18:10:18 2023-08-08

In [57]:
```
mean_returns, cov_matrix = calc_returns_stats(daily_ret)
display(mean_returns, cov_matrix)
```
executed in 14ms, finished 18:10:18 2023-08-08

```
AAPL     0.000853
AMZN     0.001424
GOOGL    0.000738
MSFT     0.001053
dtype: float64
```

| | AAPL | AMZN | GOOGL | MSFT |
|---|---|---|---|---|
| **AAPL** | 0.000246 | 0.000142 | 0.000122 | 0.000132 |
| **AMZN** | 0.000142 | 0.000333 | 0.000176 | 0.000167 |
| **GOOGL** | 0.000122 | 0.000176 | 0.000221 | 0.000144 |
| **MSFT** | 0.000132 | 0.000167 | 0.000144 | 0.000215 |

### 4.2.1 Portfolio Mean and Variance

1. Suppose $R_t$ is an $n$-vector of returns of $n$ assets at time $t$.
2. Let $\mu = E[R_t]$ and $\Sigma = \text{Var}[R_t]$.
3. Let $w$ be an $n$-vector of weights in the $n$ assets.
4. **Portfolio mean** $\mu_P$ :

$$\mu_P = w_0\mu_0 + w_1\mu_1 + w_2\mu_2 + \ldots + w_n\mu_n = w^T\mu = \sum_{i=1}^{n} w_i\mu_i$$

5. **Portfolio variance** $\sigma_P^2$ :

$$\sigma_P^2 = w^T\Sigma w = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

In [58]:
```python
def portfolio(weights, mean_returns, cov_matrix):

    portfolio_return = np.dot(weights.reshape(1,-1), mean_returns.values.resh
    portfolio_var = np.dot(np.dot(weights.reshape(1,-1), cov_matrix.values),

    return (portfolio_return.item()*252,portfolio_var.item()*252)
```
executed in 14ms, finished 18:10:18 2023-08-08

Scenario 1 : If we hold each asset equally-weighted. `weights=[0.25,0.25,0.25,0.25]`

In [59]:
```python
weights=np.array([1/len(symbols) for i in range(len(symbols))])
returns,variances=portfolio(weights, mean_returns, cov_matrix)

rf=0.01 # assume annualized risk-free interest rate is 1%
SharpeRatio=(returns-rf)/np.sqrt(variances)

print(f'The Expected Return of the Portfolio is {np.round(returns*100,2)}% ar
```
executed in 15ms, finished 18:10:18 2023-08-08

```
The Expected Return of the Portfolio is 25.63% and Variance is 4.38%.
 When risk-free interest rate is 1.0%, the Sharpe Ratio of the Portfolio is
1.18
```

Scenario 2 : If we hold each asset by `weights=[0.1,0.2,0.3,0.4]`

In [60]:
```python
weights=np.array([0.1,0.2,0.3,0.3])
returns,variances=portfolio(weights, mean_returns, cov_matrix)

rf=0.01 # assume annualized risk-free interest rate is 1%
SharpeRatio=(returns-rf)/np.sqrt(variances)

print(f'The Expected Return of the Portfolio is {np.round(returns*100,2)}% ar
```
executed in 15ms, finished 18:10:18 2023-08-08

```
The Expected Return of the Portfolio is 22.87% and Variance is 3.61%.
 When risk-free interest rate is 1.0%, the Sharpe Ratio of the Portfolio is
1.15
```

Scenario n : If we hold each asset by $weights = [w_1, w_2, w_3, w_4]$

- How to determine the **optimal weights**?
  - Maximize expected return while obtaining a certain level of risk.
  - Minimize risk (variance) while obtaining a certain level of return.

1. Monte Carlo Simulations

- Generate random portfolios by assigning random weights to each asset.

2. Scipy.Optimize Module

- Utilize the minimize method in the module to "minimum variance portfolio" and "maximum Sharpe ratio" optimization problem.

# 4.3 Caluculate Optimal Weights

### 4.3.1 Monte Carlo Method

- The **Monte Carlo Method** is based on the idea of simulating the behavior of a system by generating a large number of random scenarios. The Monte Carlo Method is used in MPT to generate random portfolios and calculate their expected return and risk.

- Steps for implementing the Monte Carlo Method in MPT:
  - Generate random portfolios by assigning weights to each asset.
  - Calculate expected return and risk for each portfolio.
  - Identify the portfolio with maximum return and minimum risk.
  - Repeat the process multiple times for robustness.

- **Advantages** of using the Monte Carlo Method in MPT: Handles non-linear problems and portfolios with many assets. Can estimate optimal solutions with uncertain or unknown return and risk.
- **Considerations** : The Monte Carlo Method is computationally expensive and time-consuming, requiring a large number of simulations. It does not guarantee global optima but can provide a good approximation.

In [61]:

```
rf = 0.01 # assume annualized risk-free rate is 1%

# list containers to store random portfolios
p_ret = []
p_vol = []
p_weights = []


# Simulation times
num_portfolios = 50000
```

executed in 13ms, finished 18:10:18 2023-08-08

In [62]:
```python
for i in range(num_portfolios):

    # Form random portfolio: Assign random weights to assets
    weights = np.random.random(num_assets) # Random floats in the half-open i
    weights = weights/np.sum(weights)

    # Calculate portfolio returns and variance
    porfolio_ret, porfolio_var = portfolio(weights, mean_returns, cov_matrix)

    # Append the random portfolio into our simulation list
    p_weights.append(weights)
    p_ret.append(porfolio_ret)
    p_vol.append(porfolio_var)


# Store the simulation results into DataFrame
data = {'Returns':p_ret, 'Variances':p_vol}
for counter, symbol in enumerate(symbols):
    data[symbol+' weight'] = [w[counter] for w in p_weights]

portfolios = pd.DataFrame(data)

portfolios['Sharpe Ratio']=(portfolios['Returns']-rf)/np.sqrt(portfolios['Var

portfolios
```

executed in 1.00s, finished 18:10:19 2023-08-08

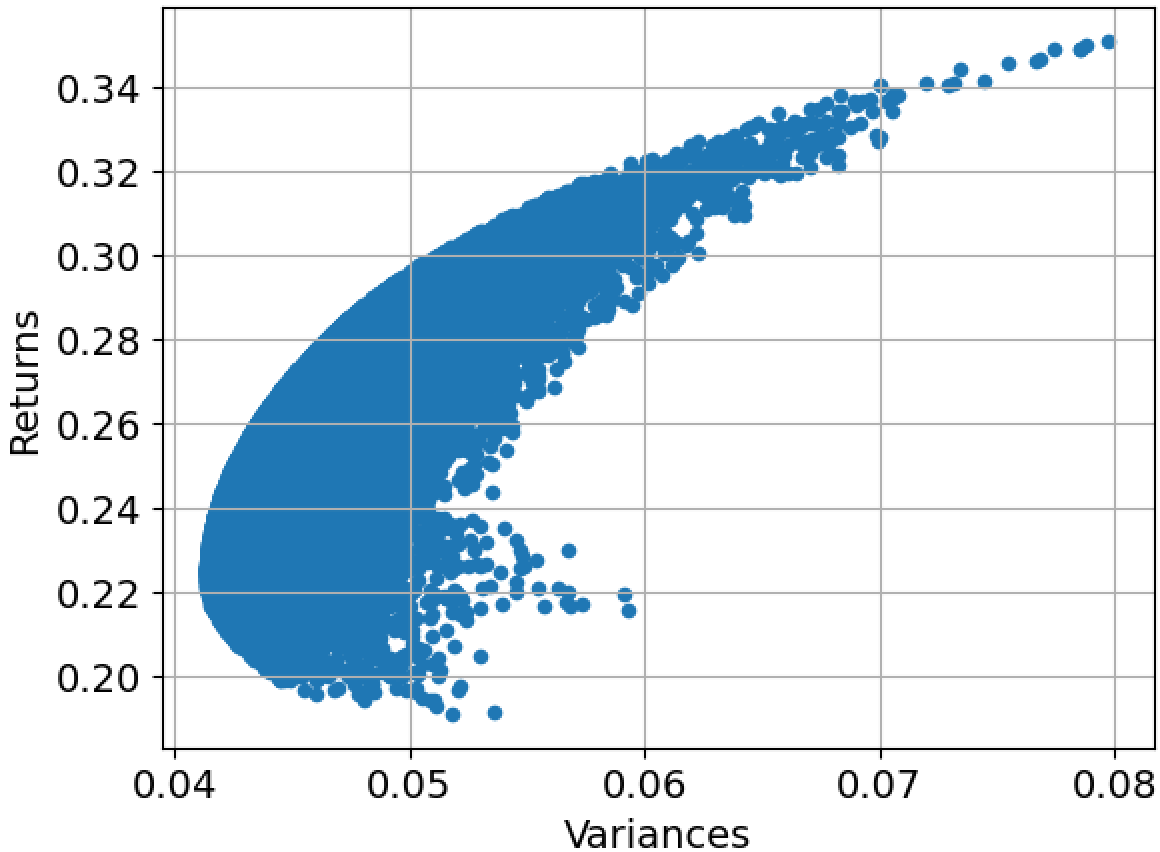
Out[62]:

| | Returns | Variances | AAPL weight | AMZN weight | GOOGL weight | MSFT weight | Sharpe Ratio |
|---|---|---|---|---|---|---|---|
| **0** | 0.270471 | 0.049097 | 0.201829 | 0.417636 | 0.299745 | 0.080790 | 1.175529 |
| **1** | 0.278706 | 0.050028 | 0.269485 | 0.457012 | 0.199442 | 0.074061 | 1.201349 |
| **2** | 0.273466 | 0.046194 | 0.296916 | 0.342263 | 0.112808 | 0.248012 | 1.225835 |
| **3** | 0.229954 | 0.042898 | 0.284858 | 0.140974 | 0.431763 | 0.142405 | 1.061976 |
| **4** | 0.293511 | 0.051516 | 0.264086 | 0.499224 | 0.065947 | 0.170743 | 1.249105 |
| **...** | ... | ... | ... | ... | ... | ... | ... |
| **49995** | 0.255847 | 0.044209 | 0.263428 | 0.266816 | 0.267237 | 0.202519 | 1.169259 |
| **49996** | 0.270552 | 0.046915 | 0.391147 | 0.347354 | 0.095799 | 0.165700 | 1.202927 |
| **49997** | 0.234473 | 0.046153 | 0.601115 | 0.137436 | 0.170071 | 0.091377 | 1.044877 |
| **49998** | 0.304593 | 0.053522 | 0.040979 | 0.487886 | 0.054316 | 0.416818 | 1.273373 |
| **49999** | 0.249286 | 0.043407 | 0.294467 | 0.225480 | 0.281548 | 0.198504 | 1.148522 |

50000 rows × 7 columns

In [63]:
```
portfolios.plot.scatter(x='Variances', y='Returns', grid=True)
```
executed in 231ms, finished 18:10:19 2023-08-08

Out[63]: <AxesSubplot:xlabel='Variances', ylabel='Returns'>




#### 4.3.1.1 Global Minimum Variance Portfolio

In [64]:
```
portfolios[portfolios['Variances']==portfolios['Variances'].min()]
```
executed in 15ms, finished 18:10:19 2023-08-08

Out[64]:

| | Returns | Variances | AAPL weight | AMZN weight | GOOGL weight | MSFT weight | Sharpe Ratio |
|---|---|---|---|---|---|---|---|
| **30485** | 0.223594 | 0.041358 | 0.311076 | 0.006748 | 0.337059 | 0.345117 | 1.050294 |

In [65]:
```
min_var_port = portfolios.iloc[portfolios['Variances'].idxmin()]
min_var_port
```
executed in 14ms, finished 18:10:19 2023-08-08

Out[65]:
```
Returns          0.223594
Variances        0.041358
AAPL weight      0.311076
AMZN weight      0.006748
GOOGL weight     0.337059
MSFT weight      0.345117
Sharpe Ratio     1.050294
Name: 30485, dtype: float64
```

In [66]:

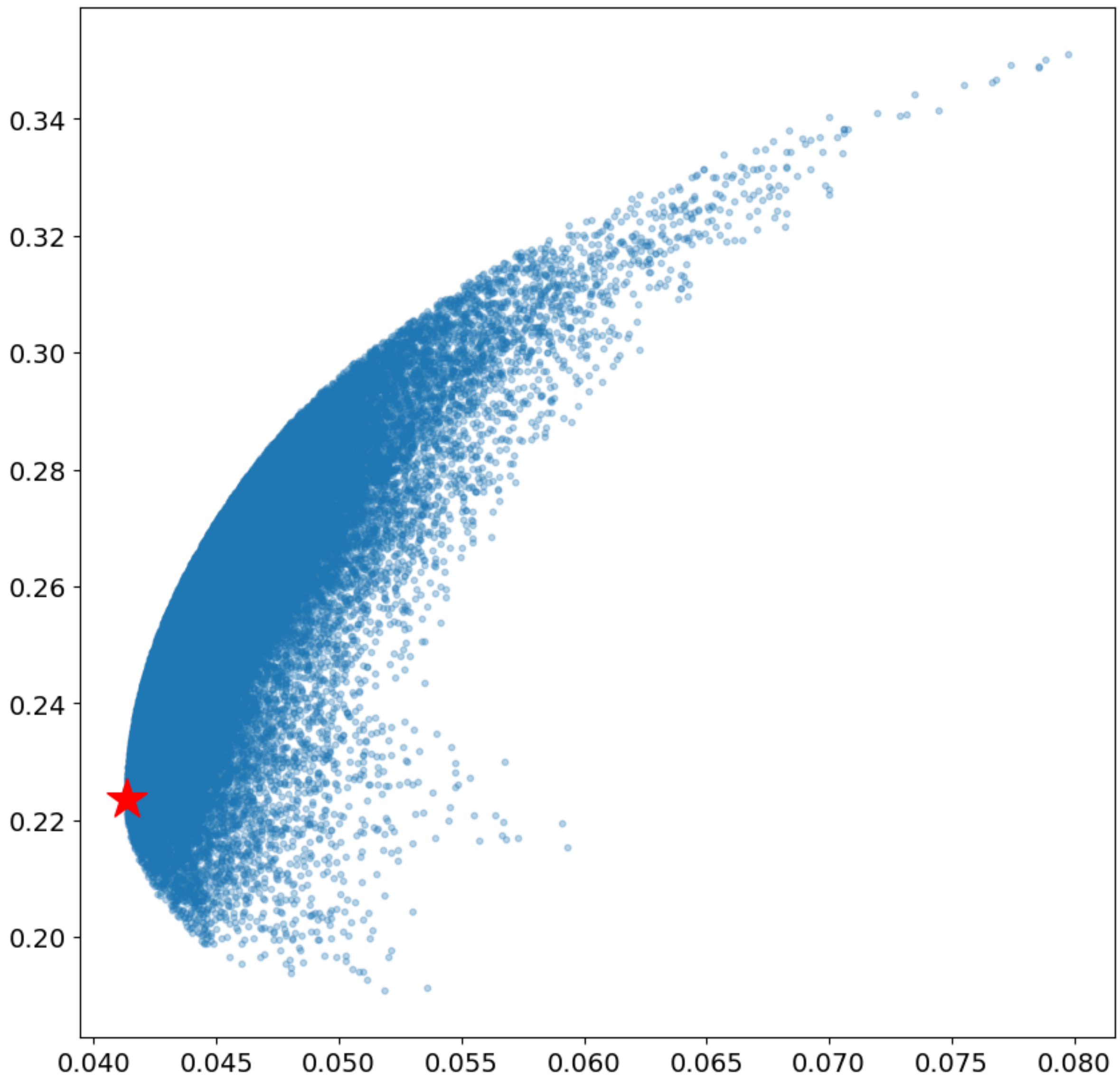
```
# Output Global-Minimum Variance portfolio
print("\nPortfolio with Minimum-Variance:\n")

print(f"Annual Sharpe Ratio: {round(min_var_port['Sharpe Ratio'],3)} | Annual
for index,symbol in enumerate(symbols):
    print(f'{symbol}:\t% {round(min_var_port[index+2]*100,2)}')
```

executed in 13ms, finished 18:10:19 2023-08-08

```

Portfolio with Minimum-Variance:

Annual Sharpe Ratio: 1.05 | Annual Return: % 22.36 | Annual Volatility: % 4.1
36

AAPL:	% 31.11
AMZN:	% 0.67
GOOGL:	% 33.71
MSFT:	% 34.51
```

In [67]:
```
# plotting the minimum volatility portfolio
plt.subplots(figsize=[10,10])
plt.scatter(portfolios['Variances'], portfolios['Returns'],marker='o', s=10,
plt.scatter(min_var_port[1], min_var_port[0], color='r', marker='*', s=500)
```
executed in 214ms, finished 18:10:20 2023-08-08

Out[67]: <matplotlib.collections.PathCollection at 0x12f7236e460>




#### 4.3.1.2 Optimal Risky Portfolio (Maximum-Sharpe Ratio (Tangency) portfolio )

In [68]:

```
# Finding the optimal portfolio

optimal_risky_port = portfolios.iloc[((portfolios['Returns']-rf)/np.sqrt(port

optimal_risky_port
```

executed in 15ms, finished 18:10:20 2023-08-08

Out[68]:

```
Returns          0.309058
Variances        0.054155
AAPL weight      0.108440
AMZN weight      0.526302
GOOGL weight     0.000534
MSFT weight      0.364725
Sharpe Ratio     1.285098
Name: 29902, dtype: float64
```

In [69]:

```
# Output Maximum-Sharpe Ratio (Tangency) portfolio
print("\nPortfolio with Maximum-Sharpe Ratio:\n")

print(f"Annual Sharpe Ratio: {round(optimal_risky_port['Sharpe Ratio'],3)} |
for index,symbol in enumerate(symbols):
    print(f'{symbol}:\t% {round(optimal_risky_port[index+2]*100,2)}')
```

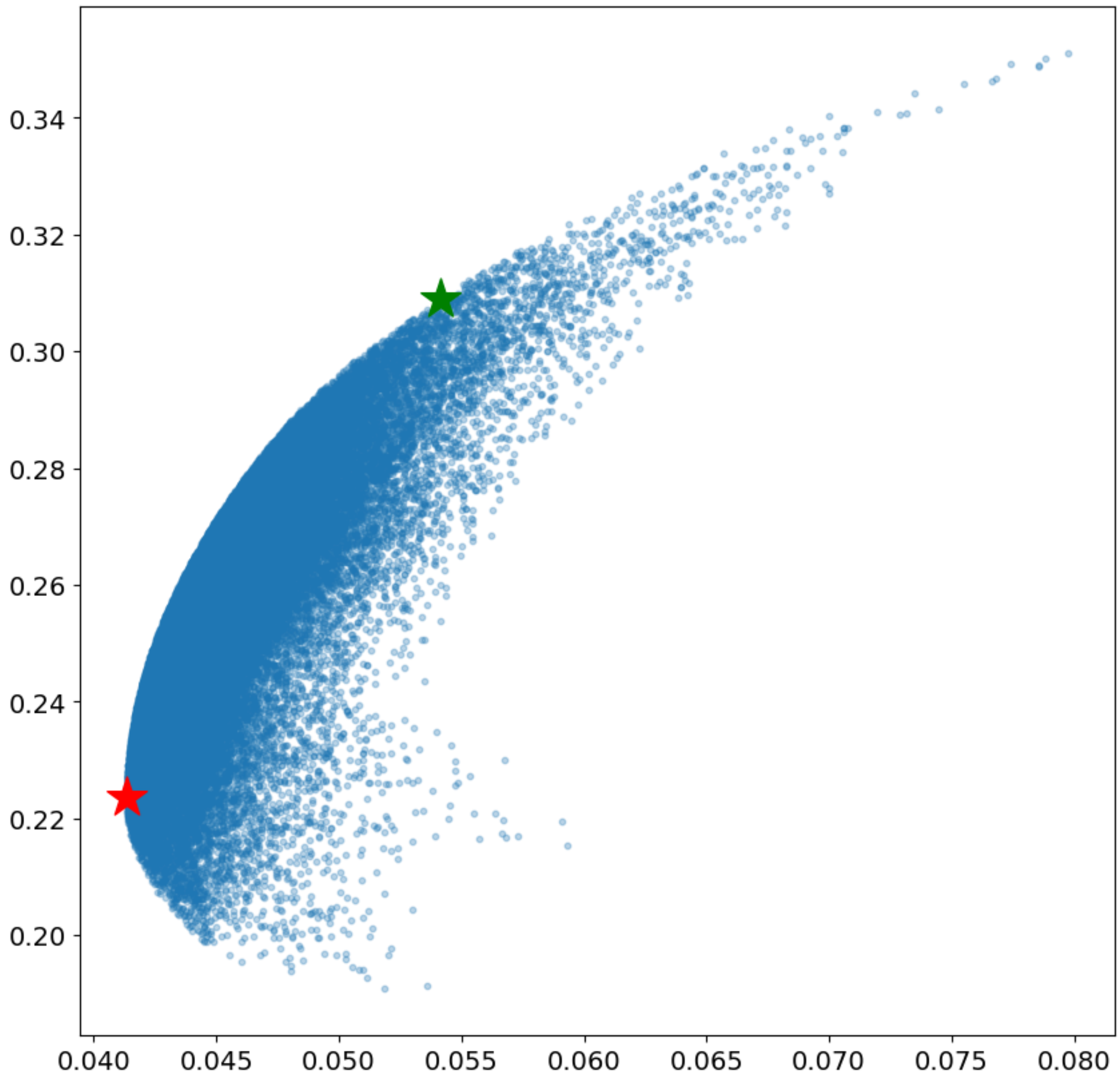
executed in 14ms, finished 18:10:20 2023-08-08

```
Portfolio with Maximum-Sharpe Ratio:

Annual Sharpe Ratio: 1.285 | Annual Return: % 30.91 | Annual Volatility: % 5.
415

AAPL:   % 10.84
AMZN:   % 52.63
GOOGL:  % 0.05
MSFT:   % 36.47
```

In [70]:
```
# Plotting optimal portfolio
plt.subplots(figsize=(10, 10))
plt.scatter(portfolios['Variances'], portfolios['Returns'],marker='o', s=10,
plt.scatter(min_var_port[1], min_var_port[0], color='r', marker='*', s=500)
plt.scatter(optimal_risky_port[1], optimal_risky_port[0], color='g', marker=
```

executed in 201ms, finished 18:10:20 2023-08-08

Out[70]: <matplotlib.collections.PathCollection at 0x12f723ec700>




### 4.3.2 Scipy Optimize Module

In [71]:
```
import scipy.optimize as opt
```

executed in 14ms, finished 18:10:20 2023-08-08

#### 4.3.2.1 Minimize Portfolio Variance

In [72]:

```python
# Objective Function: Minimize portfolio variance
def portfolio_variance(weights, mean_returns, cov_matrix):
    portfolio_return, portfolio_var = portfolio(weights, mean_returns, cov_ma
    return(portfolio_var)


def minimize_portfolio_variance(mean_returns, cov_matrix, risk_free_rate=0, w
    "This function finds the portfolio weights which minimize the portfolio v

    init_guess = np.array([1/len(mean_returns) for i in range(len(mean_return
    args = (mean_returns, cov_matrix)
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
    result = opt.minimize(fun=portfolio_variance,
                          x0=init_guess,
                          args=args,
                          method='SLSQP',
                          bounds=tuple(w_bounds for i in range(len(mean_retur
                          constraints=constraints,
                          )

    if result['success']:
        print(result['message'])
        min_var = result['fun']
        min_var_weights = result['x']
        min_var_return, min_var_variance= portfolio(min_var_weights, mean_ret

        min_var_sharpe = ((min_var_return-risk_free_rate)/np.sqrt(min_var_var
        return(min_var_sharpe, min_var_weights, min_var_return, min_var_varia
    else:
        print("Optimization operation was not succesfull!")
        print(result['message'])
        return(None)
```

executed in 15ms, finished 18:10:20 2023-08-08

In [73]:

```
# Call minimize function
min_var_sharpe, min_var_weights, min_var_return, min_var_variance = minimize_


# Output minimum volatility porfolio
print("\nPortfolio with Minimum-Variance:\n")

print(f"Annual Sharpe Ratio: {round(min_var_sharpe,3)} | Annual Return: % {ro
for index,symbol in enumerate(symbols):
    print(f'{symbol}:\t% {round(min_var_weights[index]*100,2)}')
```

executed in 14ms, finished 18:10:20 2023-08-08

```
Optimization terminated successfully

Portfolio with Minimum-Variance:

Annual Sharpe Ratio: 1.049 | Annual Return: % 22.34 | Annual Volatility: % 4.
135

AAPL:   % 31.05
AMZN:   % 0.95
GOOGL:  % 34.34
MSFT:   % 33.67
```

#### 4.3.2.2 Maximize Portfolio Sharpe Ratio

minimize negative portfolio Sharpe ratio

In [74]:

```
# Objective Function: Maximize portfolio Sharpe Ratio
def neg_sharpe_ratio(weights, mean_returns, cov_matrix, risk_free_rate=0):
    portfolio_return, portfolio_var = portfolio(weights, mean_returns, cov_ma
    sr = ((portfolio_return - risk_free_rate)/np.sqrt(portfolio_var))
    return(-sr)


def optimize_sharpe_ratio(mean_returns, cov_matrix, risk_free_rate=0, w_bounc
    "This function finds the portfolio weights which minimize the negative sh

    init_guess = np.array([1/len(mean_returns) for i in range(len(mean_returr

    args = (mean_returns, cov_matrix, risk_free_rate)
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
    result = opt.minimize(fun=neg_sharpe_ratio,
                          x0=init_guess,
                          args=args,
                          method='SLSQP',
                          bounds=tuple(w_bounds for i in range(len(mean_retur
                          constraints=constraints,
                          )

    if result['success']:
        print(result['message'])
        opt_sharpe = - result['fun']
        opt_weights = result['x']
        opt_return, opt_variance = portfolio(opt_weights, mean_returns, cov_r
        return(opt_sharpe, opt_weights, opt_return, opt_variance)
    else:
        print("Optimization operation was not succesfull!")
        print(result['message'])
        return(None)
```

executed in 15ms, finished 18:10:20 2023-08-08

In [75]:
```python
# Call minimize function to get max sharpe portfolio
opt_sharpe, opt_weights, opt_return, opt_variance = optimize_sharpe_ratio(mea

# Output maximum sharpe ratio porfolio
print("\nPortfolio with maximum sharpe ratio:\n")
print(f"Annual Sharpe Ratio: {round(opt_sharpe,3)} | Annual Return: % {round(

for index,symbol in enumerate(symbols):
    print(f'{symbol}:\t% {round(opt_weights[index]*100,3)}')
```

executed in 15ms, finished 18:10:20 2023-08-08

```
Optimization terminated successfully

Portfolio with maximum sharpe ratio:

Annual Sharpe Ratio: 1.286 | Annual Return: % 30.892 | Annual Volatility: %
5.406

AAPL:   % 8.746
AMZN:   % 51.314
GOOGL:  % 0.0
MSFT:   % 39.94
```

# 5 This Week

- Review of Linear Algebra
- Linear Regressions
  - Statsmodels
  - Scikit-learn
- Portfolio Optimization
  - Monte-Carlo Simulation
  - Scipy Optimize Module

# 6 Next Week

- Python Project: Web Scraping
- Course Review
- Q&A