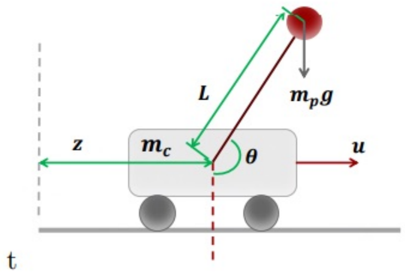
ME424 Project #2. SUN Yinghan. Total: 40 pts



$m_c = 10\,kg$

$m_p = 1\,kg$

$g = 9.81\ m/s^2$

$L = 0.5\,m$

$$\ddot{z} = \frac{1}{m_c + m_p \sin^2\theta}\left[u + m_p \sin\theta\left(L\dot{\theta}^2 + g\cos\theta\right)\right]$$

$$\ddot{\theta} = \frac{1}{L(m_c + m_p \sin^2\theta)}\left[-u\cos\theta - m_p L\dot{\theta}^2\cos\theta\sin\theta - (m_c+m_p) g\sin\theta\right]$$

1.
1 pts

$$\dot{x} = \begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \frac{1}{m_c + m_p \sin^2\theta}\left[u + m_p \sin\theta\left(L\dot{\theta} + g\cos\theta\right)\right] \\ \frac{1}{L(m_c + m_p \sin^2\theta)}\left[-u\cos\theta - m_p L\dot{\theta}^2\cos\theta\sin\theta - (m_c+m_p) g\sin\theta\right] \end{bmatrix}$$

2.
1 pts

$$\dot{x}_\theta = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \frac{1}{L(m_c + m_p \sin^2\theta)}\left[-u\cos\theta - m_p L\dot{\theta}^2\cos\theta\sin\theta - (m_c+m_p) g\sin\theta\right] \end{bmatrix}$$

3.
3 pts

$$\dot{x}_\theta = f(x_\theta, u) \approx \hat{A}\cdot\Delta x_\theta + \hat{B}\,\Delta u + f(\hat{x}_\theta, \hat{u})$$

$$\hat{A} = \left[\frac{\partial f_\theta(x_\theta,u)}{\partial x_\theta}\right]_{x_\theta=\hat{x}_\theta,\ u=\hat{u}} = \begin{bmatrix} \frac{\partial f_{\theta 1}(x_\theta,u)}{\partial x_{\theta 1}} & \frac{\partial f_{\theta 1}(x_\theta,u)}{\partial x_{\theta 2}} \\ \frac{\partial f_{\theta 2}(x_\theta,u)}{\partial x_{\theta 1}} & \frac{\partial f_{\theta 2}(x_\theta,u)}{\partial x_{\theta 2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 21.582 & 0 \end{bmatrix}$$

$$\hat{B} = \left[\frac{\partial f_\theta(x_\theta,u)}{\partial u}\right]_{x_\theta=\hat{x}_\theta,\ u=\hat{u}} = \begin{bmatrix} \frac{\partial f_{\theta 1}(x_\theta,u)}{\partial u} \\ \frac{\partial f_{\theta 2}(x_\theta,u)}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

$$\Delta x_\theta = x_\theta - \hat{x}_\theta = \begin{bmatrix} \theta - \pi \\ \dot{\theta} \end{bmatrix} \qquad \Delta u = u - \hat{u} = u.$$

$$f(\hat{x}_\theta,\hat{u}) = \left[f(x_\theta,u)\right]_{x_\theta=\hat{x}_\theta,\, u=\hat{u}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\dot{x}_\theta = \begin{bmatrix} 0 & 1 \\ 21.582 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}-\pi \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} u$$

4. $\tilde{x}_\theta = \Delta x_{\theta,k+1} = x_{\theta,k+1} - \hat{x} = \begin{bmatrix} \theta_{k+1}-\pi \\ \dot{\theta}_{k+1} \end{bmatrix}$

3 pts

$$A = I + \hat{A}\cdot T = \begin{bmatrix} 1 & 0.005 \\ 0.10791 & 1 \end{bmatrix}$$

$$B = \hat{B}\cdot T = \begin{bmatrix} 0 \\ 0.001 \end{bmatrix}$$

$$\begin{bmatrix} \theta_{k+1}-\pi \\ \dot{\theta}_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0.005 \\ 0.10791 & 1 \end{bmatrix} \begin{bmatrix} \theta_k-\pi \\ \dot{\theta}_k \end{bmatrix} + \begin{bmatrix} 0 \\ 0.001 \end{bmatrix} u_k$$

$$y_k = \theta_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_k \\ \dot{\theta}_k \end{bmatrix}$$

5. (a) 2pts.

7 pts

(b) 5 pts

correct program: 3'

2 set of initial conditions 2'.

4pts

6. (a) Step 1: Similarity transform: find P, such that $x(k) = P\bar{x}(k)$. and $\bar{x}(k)$ dynamics is in controllable canonical form.

10pts

① identify characteristic polynomial of A:

$$\Delta_A(\lambda) = \det(\lambda I - A) = \lambda^2 - 2\lambda + 0.99946$$

0.5 pts

② Since $\Delta_{\bar{A}}(\lambda) = \Delta_A(\lambda)$, by controllable canonical form,

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ -0.99946 & 2 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$ 0.5pts

③ compute controllability matrix M_c and $\bar{M}_c$.

$$M_c = [B \;\; AB] = \begin{bmatrix} 0 & 5\times10^{-6} \\ 0.001 & 0.001 \end{bmatrix}$$

$$\bar{M}_c = [\bar{B} \;\; \bar{A}\bar{B}] = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$$ 0.5pts

④ $P = M_c \bar{M}_c^{-1} = \begin{bmatrix} 5\times10^{-6} & 0 \\ -0.001 & 0.001 \end{bmatrix}$ 0.5pts

Step 2: find $\bar{K}$ to assign desired eigenvalues for $\bar{A}$ and $\bar{B}$,

①. Since $T = 0.005$, we have

$$z_1 = e^{(-2+j)T} = e^{-0.01+0.005j}$$

$$z_2 = e^{(-2-j)T} = e^{-0.01-0.005j}$$ 0.5pts

② desired closed-loop characteristic polynomial.

$$\Delta_{desired}(\lambda) = (\lambda - e^{-0.01+0.005j})(\lambda - e^{-0.01-0.005j})$$

$$= \lambda^2 - 1.9801\lambda + 0.9802$$ 0.5pts

③ $\alpha_0^* = 0.9802$. $\quad \alpha_1^* = -1.9801$.

$\alpha_0 = 0.99946 \qquad \alpha_1 = -2$

$$\left.\begin{aligned} K_1 &= \alpha_0^* - \alpha_0 = -0.01926 \\ K_2 &= \alpha_1^* - \alpha_1 = 0.0199 \end{aligned}\right. \Rightarrow \bar{K} = [-0.01926 \;\; 0.0199]$$ 0.5pts

④ $K = \bar{K}P^{-1} = [128 \;\; 19.9]$ 0.5pts

(b) 6pts Implementation 4pts

Visualization & results 2pts

7. ① $K = [448 \quad 21.5]$ 2pts

6pts

② 4pts simulation & results 1pts

discussion 3pts

8. (a) $L = \tilde{K}^T = \begin{bmatrix} 0.0881 \\ 0.5143 \end{bmatrix}$ 4pts

9pts

(b) 5pts Implementation 3pts

simulation & results 2pts