ME424 Project 1 Report
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1. Wind Power Forcast

solution：

(a) Data:

From the attachment "project1_1.ipynb" below, we import the data of the second column in the .csv file.

```
import numpy as np
winddata = np.loadtxt("winddata.csv", delimiter=",", skiprows=1, usecols=1)
```

(b) Identification:

Assuming that for the k-th wind speed data, it is correlated with the (k-n)-th to (k-1)-th data. Then the equation can be written as below:

$$y(k) = a_1 y(k-n) + a_2 y(k-n+1) + \cdots + a_n y(k-1) + v_n$$

In the matrix form:

$$[y(k-n) \quad y(k-n+1) \quad \cdots \quad y(k-1)] \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \hat{y}(n)$$

And if we expand the matrix and apply first 5000 wind speed data into the matrix, we will get:

$$\begin{bmatrix} y(1) & y(2) & \cdots & y(n-2) & y(n-1) \\ y(2) & y(3) & \cdots & y(n-1) & y(n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y(4999-n) & y(5000-n) & \cdots & y(4997) & y(4998) \\ y(5000-n) & y(5001-n) & \cdots & y(4998) & y(4999) \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} y(n) \\ y(n+1) \\ \vdots \\ y(4999) \\ y(5000) \end{bmatrix}$$

$$\text{where } H = \begin{bmatrix} y(1) & y(2) & \cdots & y(n-2) & y(n-1) \\ y(2) & y(3) & \cdots & y(n-1) & y(n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y(4999-n) & y(5000-n) & \cdots & y(4997) & y(4998) \\ y(5000-n) & y(5001-n) & \cdots & y(4998) & y(4999) \end{bmatrix}$$

$$\theta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix}$$

From n = 1 to n = 2, the error will be estimated into the formula:

$$err(n) = \sum \left\| y - H\hat{\theta}_{LS}^{(n)} \right\|$$

And then compare every err(n) to get the minimum one.
Since for every n = k, the amount of their samples may not be the same, for example, for n = 1, the amount of the samples is 4999, while for n = 2, it is 4998, it is necessary to let the first test data begin with 21-st wind speed data.
Step 1 of the code:
To define an AR model in a range of 1 to 20, and finally calculate the error and then put them into the array:

```
# define an AR model
def ARmodel():
    for n in range(1,21):
        H=np.zeros((5000-n,n))
        for row in range(0,5000-n):
            for col in range(0,n):
                H[row][col]=winddata[col+row]
        y=np.zeros((5000-n,1))
        for i in range(0,5000-n):
            y[i]=winddata[n+i]
        HT=np.conj(H).T

        if(np.linalg.det(HT.dot(H)) != 0):
            # alpha = (H.T*H)^(-1)*H.T*y to get the parameter
            alpha=((np.linalg.inv(HT.dot(H))).dot(HT)).dot(y)
            # calculate the least suqare predicted data
            theta=H.dot(alpha)
            # print(theta)
            # calculate the error
            for t in range(20,4999):
                error[n-1] = error[n-1]+(y[t-n]-theta[t-n])**2
        else:
            alpha=9999*np.ones((n,1))
            for t in range(20,4999):
                error[n-1] = error[n-1]+(y[t-n]-theta[t-n])**2
```

Step 2:
Get the minimum error, and also return the index of the error to get its "n", where n = index + 1.

```
# get the index of the minimum data in the 1-d matrix
def findMin_1d(mat):
    min=mat[0]
    min_index=0
    for i in range (0,len(mat)):
        if mat[i] < min:
            min = mat[i]
            min_index = i
    return min_index
```

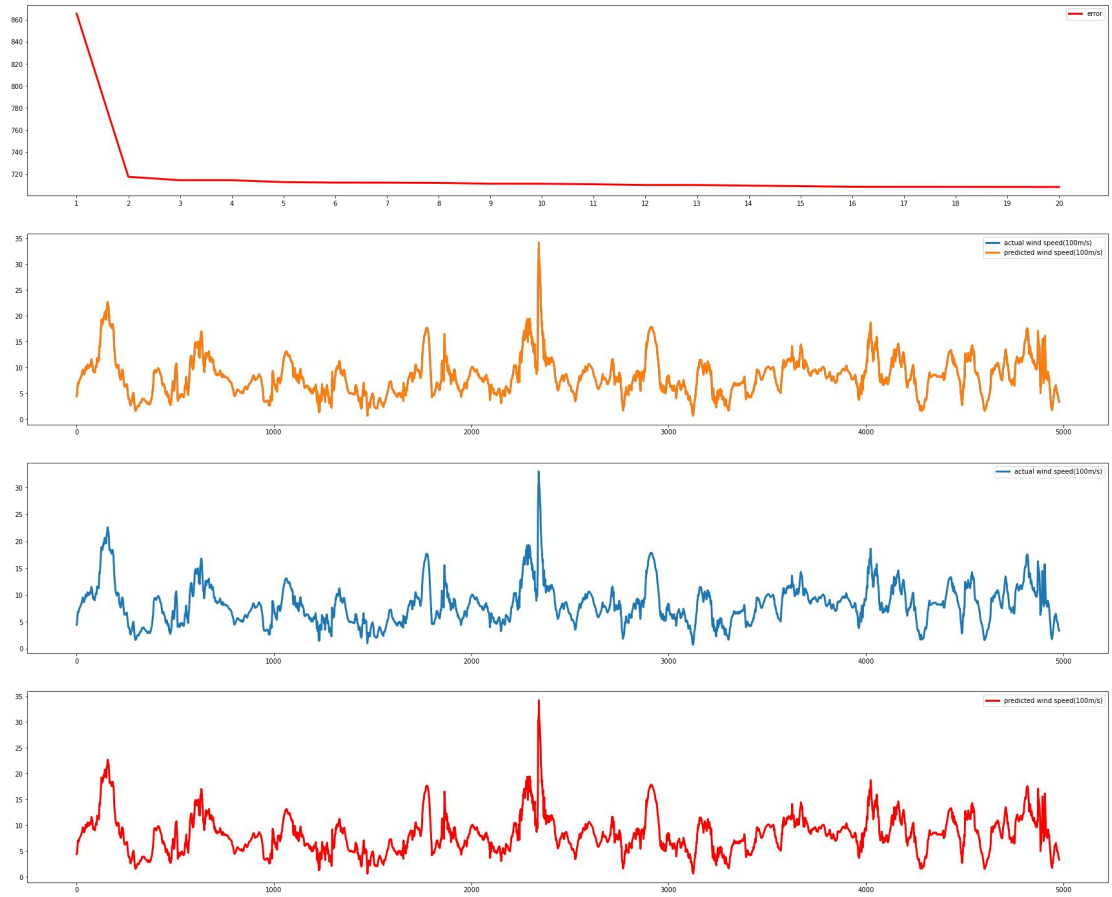
```
ARmodel()


print(error)
min_item = min(error)
print(min_item)
index=findMin_1d(error)
print(index+1)
num=index+1
```

Step 3:
Get “n” and do the AR model again with the specific “n”, where n = 20.

```
H=np.zeros((5000-num,num))
for row in range(0,5000-num):
    for col in range(0,num):
            H[row][col]=winddata[col+row]
y=np.zeros((5000-num,1))
for i in range(0,5000-num):
    y[i]=winddata[num+i]
HT=np.conj(H).T
if(np.linalg.det(HT.dot(H)) != 0):
    alpha=((np.linalg.inv(HT.dot(H))).dot(HT)).dot(y)
yAR=H.dot(alpha)
print(alpha)
```

Step 4: draw the diagram project 1-1 requires: Plot err versus n

```
yreal = np.zeros((5000-num,1))
# yAR = np.zeros((4978,1))
# ypast = np.zeros((num,1))
for i in range(0,5000-num):
    yreal[i] = winddata[i+num]
t=range(0,5000-num)
# for i in range(0,4978):
#     for k in range(i,i+17):
#         ypast = winddata[i:i+20:1]
#     yAR[i][0]=ypast.T.dot(alpha);
import matplotlib.pyplot as plt
# print(yAR)
plt.figure(figsize=(30,25))
plt.subplot(411)
n=range(1,21)
plt.plot(n,error,'r',linewidth=3)  #error versus n, where we can find out that n=20 is the best one
plt.xticks(n)
plt.legend(["error"])
plt.subplot(412)
plt.plot(t,yreal,linewidth=3)
plt.plot(t,yAR,linewidth=3)
plt.legend(["actual wind speed(100m/s)","predicted wind speed(100m/s)"])
plt.subplot(413)
plt.plot(t,yreal,linewidth=3)
plt.legend(["actual wind speed(100m/s)"])
plt.subplot(414)
plt.plot(t,yAR,'r',linewidth=3)
plt.legend(["predicted wind speed(100m/s)"])
```

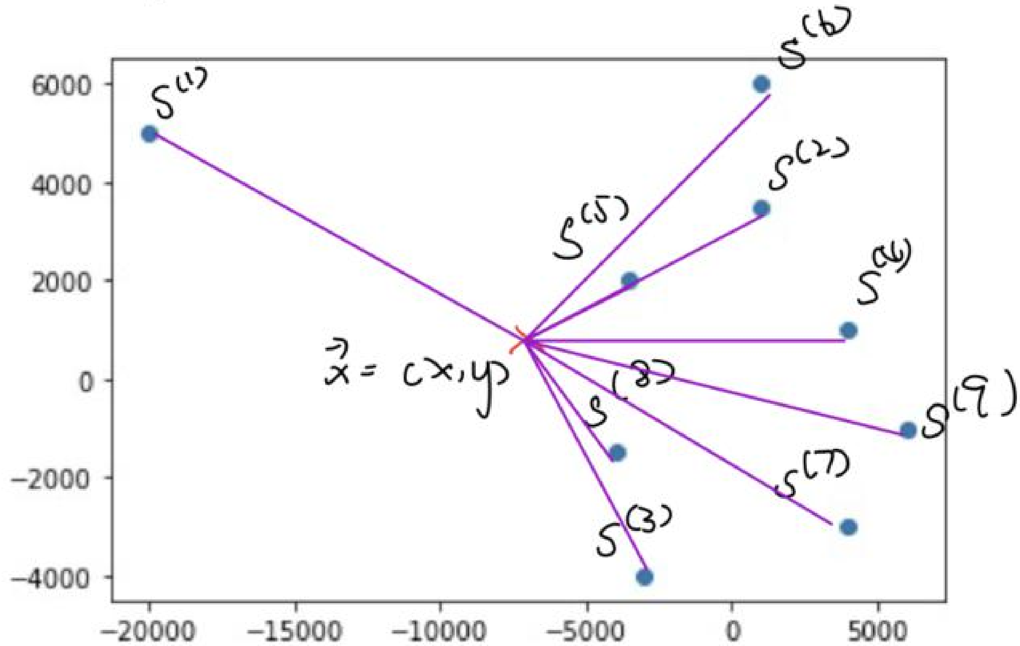

error
actual wind speed(100m/s)
predicted wind speed(100m/s)
actual wind speed(100m/s)
predicted wind speed(100m/s)

2. E911

(a)

The problem can be see as a nonlinear least square problem, where $|x| \le 3000, |y| \le 3000, |\tau| \le 5000$

See the figure below:



$$C(t_i - \tau) = \left\| s^{(i)} - x \right\| + v_i, i = 1,2,\cdots,9$$

Let $C(t_i - \tau) = d_i, i = 1,2,\cdots,9$

then we can choose a cost function:

$$J(x) = \sum_{i=1}^{9} (d_i - \left\| s^{(i)} - x \right\|), i = 1,2,\cdots,9$$

Through the definition of the least square, we would find $J(x)_{\min}$ and

$$x = \arg_{x \in R^2}(J(x)_{\min})$$

(2) There are two methods to calculate the least square location：

A. Convert the nonlinear least square problem into a linear one.

B. Solve the nonlinear least square problem by iteration applying python minimize method.

A: Linear optimization by the least square:

$$c^2(t_i - \tau)^2 = (x_i - x)^2 + (y_i - y)^2$$

for $i = n-1$

$$x_n^2 + x^2 - 2x_n x + y_n^2 + y^2 - 2y_n y = c^2 t_n^2 + c^2\tau^2 - 2c^2 t_n \tau$$

for $i = n-1$

$$x_{n\text{-}1}^2 + x^2 - 2x_{n\text{-}1} x + y_{n\text{-}1}^2 + y^2 - 2y_{n\text{-}1} y = c^2 t_{n\text{-}1}^2 + c^2\tau^2 - 2c^2 t_{n\text{-}1} \tau$$

The first equation subtracts with the second one:

$$2(x_{n\text{-}1} - x_n)x + 2(y_{n\text{-}1} - y_n)y - 2c^2(t_{n-1} - t_n)\tau = x_{n\text{-}1}^2 - x_n^2 + y_{n\text{-}1}^2 - y_n^2 - c^2(t_{n-1}^2 - t_n^2)$$

Then we can transform it into a matrix form and expand it:

$$\begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) & -2c^2(t_1 - t_2) \\ 2(x_2 - x_3) & 2(y_2 - y_3) & -2c^2(t_1 - t_2) \\ \vdots & \vdots & \vdots \\ 2(x_8 - x_9) & 2(y_8 - y_9) & -2c^2(t_1 - t_2) \end{bmatrix} \begin{bmatrix} x \\ y \\ \tau \end{bmatrix} = \begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 - c^2(t_1^2 - t_2^2) \\ \vdots \\ x_8^2 - x_9^2 + y_8^2 - y_9^2 - c^2(t_8^2 - t_9^2) \end{bmatrix}$$

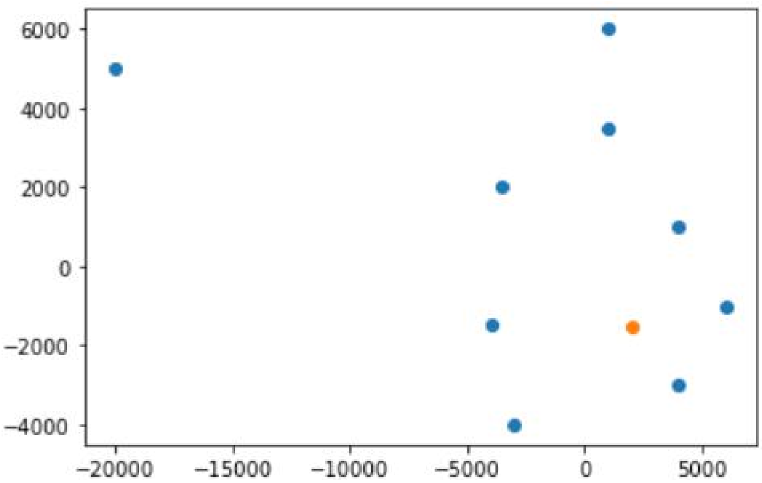
$$H = \begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) & -2c^2(t_1 - t_2) \\ 2(x_2 - x_3) & 2(y_2 - y_3) & -2c^2(t_1 - t_2) \\ \vdots & \vdots & \vdots \\ 2(x_8 - x_9) & 2(y_8 - y_9) & -2c^2(t_1 - t_2) \end{bmatrix}$$

$$\theta = \begin{bmatrix} x \\ y \\ \tau \end{bmatrix}$$

$$y = \begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 - c^2(t_1^2 - t_2^2) \\ \vdots \\ x_8^2 - x_9^2 + y_8^2 - y_9^2 - c^2(t_8^2 - t_9^2) \end{bmatrix}$$

Where $\hat{\theta}_{LS} = (H^T H)^{-1} H^T y$, and the code is below:

```
#Linear method
H = np.zeros((8,3))
for i in range(0,8):
    H[i][0] = 2*(x[i]-x[i+1])
for i in range(0,8):
    H[i][1] = 2*(y[i]-y[i+1])
for i in range(0,8):
    H[i][2] = -2*0.3*0.3*(t[i]-t[i+1])
# print(H)
Y = np.zeros((8,1))
for i in range(0,8):
    Y[i] = (x[i]**2-x[i+1]**2)+(y[i]**2-y[i+1]**2)-0.3*0.3*(t[i]**2-t[i+1]**2)
HT=H.T
print(Y)
# print(H.T)
theta=((np.linalg.inv(HT.dot(H))).dot(HT)).dot(Y)
alpha=H.dot(theta)
inv1 = np.linalg.inv(HT.dot(H))
theta = (inv1.dot(HT)).dot(Y)
print(alpha)
print()
print(theta)

posx=np.array([1999.512])
posy=np.array([-1506.86])
plt.scatter(x, y)
plt.scatter(posx, posy)
```



B. Nonlinear method:

In this method, what it matters is the code.

In this code, I choose the method of Basinhoping

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import minimize
from scipy.optimize import basinhopping
```

Since Basinhoping can help me find the global variable instead of the local extreme value.

Define the function of the error:

```
def func(s):
    func = 0
    for i in range(0,9):
        func = func + 0.3*(t[0] - s[2]) - np.sqrt((x[i]-s[0])*(x[i]-s[0]) + (y[i]-s[1])*(y[i]-s[1]))
    return func
```

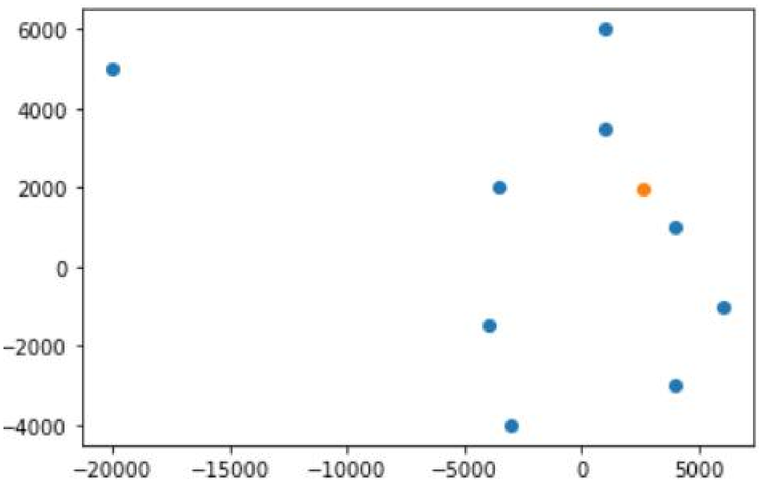
It is used to help me get the answer bounded:

```
class MyBounds:
    def __init__(self, xmax=[3000,3000,5000], xmin=[-3000,-3000,-5000] ):
        self.xmax = np.array(xmax)
        self.xmin = np.array(xmin)
    def __call__(self, **kwargs):
        x = kwargs["x_new"]
        tmax = bool(np.all(x <= self.xmax))
        tmin = bool(np.all(x >= self.xmin))
        return tmax and tmin
```

Find the minimum:

```
# cons = con()
x0 = np.array((1500,1500,2500))
# minLocation = minimize(fun(), x0, method='SLSQP', constraints=cons)
# minLocation = minimize(fun(), x0, method='trust-constr', constraints=cons)
mybounds = MyBounds()
# globalLocation = basinhopping(func,x0,niter=10000, minimizer_kwargs={"method":"L-BFGS-B"}, accept_test=mybounds)
globalLocation = basinhopping(func,x0,niter=1000, minimizer_kwargs={"method":"BFGS"}, accept_test=mybounds)
# print(minLocation)
print()
print(globalLocation)

posx=np.array([2601.01])
posy=np.array([1976.92])

plt.scatter(posx, posy)
```

```
x: array([2601.32769628, 1977.62175314, 4175.72331985])
```



To sum up, the nonlinear method really requires the accuracy and the iteration times. So it is better to use the linear method.

$x = 1999.5121$

$y = -1506.8679$

$\tau = 2970.4610$

```
[[ 1999.51209958]
 [-1506.86794166]
 [ 2970.4610061 ]]
```

3. Robot Dynamics Simulation and Parameter Identification:

3. solution:

(a)

$$\tau = M(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta)$$

Let $c(\theta,\dot{\theta}) + g(\theta)$ is $h(\theta,\dot{\theta})$

$$\tau = M(\theta)\ddot{\theta} + h(\theta,\dot{\theta})$$

$$\ddot{\theta} = M^{-1}(\tau - h(\theta,\dot{\theta}))$$

All in all,

$$\tau = M(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta)$$

$$x_1 = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad x_2 = \dot{x}_1 = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Let $y = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$

then $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ M^{-1}[\tau - c(x_1,x_2) + g(x_1)] \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For a discret model:

$\Delta T = 0.001$

Therefore $x[k] = x(kT)$, then $x[k+1] = x[k] + \dot{x}(kT)\cdot \Delta T$.

$$x[k+1] = \begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} x_1[k] + x_2(kT)\cdot\Delta T \\ x_2[k] + M^{-1}\left[\tau(kT) - C(x_1(kT), x_2(kT)) + g(x_1(kT))\right]\cdot \Delta T \end{bmatrix}$$

$$= \begin{bmatrix} x_1[k] + 0.001\, x_2(kT) \\ x_2[k] + 0.001 M^{-1}\left[\tau(kT) - C(x_1(kT), x_2(kT)) + g(x_1(kT))\right] \end{bmatrix}$$

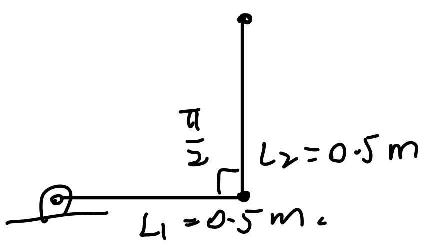
(b)

$$\tau = 0 \Rightarrow \begin{bmatrix} \tau_1 = 0 \\ \tau_2 = 0 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ M^{-1}\left[0 - C(x_1 + x_2) + g(x_1)\right] \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

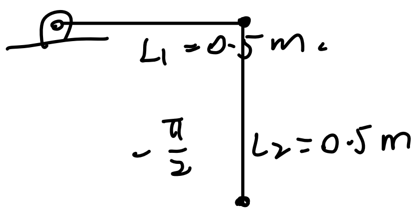
(i) $\theta_1 = 0$, $\theta_2 = \frac{\pi}{2}$



$g = 9.81\ m/s^2$

(i) $\theta_1 = 0$, $\theta_2 = \frac{-\pi}{2}$

$g = 9.81\ m/s^2$



see the code.

(c) $\tau = M(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta)$

$$M(\theta) = \begin{bmatrix} m_1 L_1^2 + m_2\left(L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2\right) & m_2\left(L_1L_2\cos\theta_2 + L_2^2\right) \\ m_2\left(L_1L_2\cos\theta_2 + L_2^2\right) & m_2 L_2^2 \end{bmatrix}$$

$$c(\theta,\dot{\theta}) = \begin{bmatrix} -m_2L_1L_2\sin\theta_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right) \\ m_2L_1L_2\dot{\theta}_1^2\sin\theta_2 \end{bmatrix}$$

$$g(\theta) = \begin{bmatrix} (m_1+m_2)L_1 g\cos\theta_1 + m_2 g L_2\cos(\theta_1+\theta_2) \\ m_2 g L_2\cos(\theta_1+\theta_2) \end{bmatrix}$$

In the question (a)

We know that $\ddot{\theta} = M(\theta)^{-1}\left[\tau - c(\theta,\dot{\theta}) + g(\theta)\right]$

Right now, θ and $\dot{\theta}$, $\ddot{\theta}$ are known

where we can get them in part (b) when $\tau = 0$.

We can have $\ddot{\theta} = [\ddot{\theta}_1 \ \ddot{\theta}_2]$ in a matrix form

```
for i in range(0, 10000):
    u = np.zeros((2,1)) # zero torque control
    M = np.mat([[
        m1 * L1**2 + m2 * (L1**2 + 2 * L1 * L2 * np.cos(x_traj[i][1]) + L2**2),
        m2 * (L1 * L2 * np.cos(x_traj[i][1]) + L2**2)
    ], [m2 * (L1 * L2 * np.cos(x_traj[i][1]) + L2**2), m2 * L2**2]])
    c = np.array(
        [[-1 * m2 * L1 * L2 * np.sin(x_traj[i][1]) * (2 * x_traj[i][2] * x_traj[i][3] + x_traj[i][3]**2)],
         [m2 * L1 * L2 * x_traj[i][2]**2 * np.sin(x_traj[i][1])]])
    gtheta = np.array([[(m1 + m2) * L1 * g * np.cos(x_traj[i][0]) +
                        m2 * g * L2 * np.cos(x_traj[i][0] + x_traj[i][1])],
                       [m2 * g * L2 * np.cos(x_traj[i][0] + x_traj[i][1])]])
    a = (np.linalg.inv(M)).dot(u - c - gtheta)
    theta_acc[i][0] = a[0]
    theta_acc[i][1] = a[1]
```

theta_acc is the angle acceleration $[\ddot{\theta}_1 \ \ddot{\theta}_2]$.

Then substitute into the equation:

$\tau = M\ddot{\theta} + C(\theta, \dot{\theta}) + g(\theta)$ where m_1, m_2, L_1, L_2 are unknow

Under this condition, $J(m_1, m_2, L_1, L_2) = \sum \|\tau - 0\|$

$= \sum_{i=1}^{10000} \tau_i$ as θ_i alter.

What we need to do is minimze the $J(m_1, m_2, L_1, L_2)$

And return the arguments (m_1, m_2, L_1, L_2)

```
def error(x):
    error = 0
    u = np.zeros((2,1)) # zero torque control
    M = np.mat([[
        x[0] * x[2]**2 + x[1] * (x[2]**2 + 2 * x[2] * x[3] * np.cos(x_traj[i][1]) + x[3]**2),
        x[1] * (x[2] * x[3] * np.cos(x_traj[i][1]) + x[3]**2)
    ], [x[1] * (x[2] * x[3] * np.cos(x_traj[i][1]) + x[3]**2), x[1] * x[3]**2]])
    c = np.array(
        [[-1 * x[1] * x[2] * x[3] * np.sin(x_traj[i][1]) * (2 * x_traj[i][2] * x_traj[i][3] + x_traj[i][3]**2)],
         [x[1] * x[2] * x[3] * x_traj[i][2]**2 * np.sin(x_traj[i][1])]])
    gtheta = np.array([[(x[0] + x[1]) * x[2] * g * np.cos(x_traj[i][0]) +
                        x[1] * g * x[3] * np.cos(x_traj[i][0] + x_traj[i][1])],
                       [x[1] * g * x[3] * np.cos(x_traj[i][0] + x_traj[i][1])]])
    # a = M.dot(theta_acc)-(u - c - gtheta)
    for row in range(0,10000):
        a = M.dot(theta_acc[row][:].T)-(u - c - gtheta)
        # for col in range(0,2):
        error = error + a[0]+ a[1]
    return error
```

similarily, we still use basinhoping to calculate the global extreme value and return the parameter of m_1, m_2, L_1, L_2

```python
class MyBounds:
    def __init__(self, xmax=[10,10,10], xmin=[-10,-10,-10] ):
        self.xmax = np.array(xmax)
        self.xmin = np.array(xmin)
    def __call__(self, **kwargs):
        x = kwargs["x_new"]
        tmax = bool(np.all(x <= self.xmax))
        tmin = bool(np.all(x >= self.xmin))
        return tmax and tmin

mybounds = MyBounds()
x0 = np.array((0,0,0,0))
# mybounds = MyBounds()
globalLocation = basinhopping(error,x0,niter=1000, minimizer_kwargs={"method":"BFGS"}, accept_test=mybounds)
```