BWSI - UAV Linear Algebra Hw

1. If $\{x_1, x_2 \ldots x_n\}$ is a solution to a linear system of equations, it is also a solution to a new modified system of linear equation when one of the original equations is replaced by a new version that's equal to the sum of the old equations because the addition of the two equations are equal to the solution

Problem 2 ✓

$$x - 2y + 4z = 12$$
$$2x + y - z = 4$$

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & 1 & -1 \end{bmatrix}$$

a. $\{x(a), y(a), z(a)\}$

$$x = a$$
$$y = -4.5a + 14$$

$z = 2x + y - 4$

$z = 2a + (-4.5a + 14) - 4$

$z = -2.5a + 10$

$x - 2y + 4(2x + y - 4) = 12$

$x - 2y + 8x + 4y - 16 = 12$

$2y = -9x + 28$

$y = -4.5 + 14$

$$\begin{pmatrix} x(a) \\ y(a) \\ z(a) \end{pmatrix} = \begin{pmatrix} a \\ -4.5a + 14 \\ -2.5a + 10 \end{pmatrix}$$

b. $2(x - 2y + 4z = 12) = 2x - 4y + 8z = 24$

~~$2x$~~ $+ y - z = 4$

$-5y + 9z = 20$

c. $x + y + z = 20$

d. $x - 2y + 4z = 12$

$2x + y - z = 4$

$z = 5$

$x - 2y + 20 = 12$

$x - 2y = -8$

$x = -8 + 2y$

$x = -8 + 2(5)$

$x = 2$

$2x + y - 5 = 4$

$2x + y = 9$

$2(-8 + 2y) + y = 9$

$5y = 25$

$y = 5$

Problem 3 ✓

1 Sometimes
2 Never
3 Never
4 Sometimes
5 Never
6 Sometimes

Problem 4 ✓

$$\begin{aligned} x + y + 2z &= 1 \\ 2x + y - z &= 4 \\ x - y + z &= 0 \end{aligned} = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 1 & -1 & 4 \\ 1 & -1 & 1 & 0 \end{array}\right] \; R2 - 2R1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -1 & -5 & 2 \\ 1 & -1 & 1 & 0 \end{array}\right] R3 - R1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & -2 & -1 & -1 \end{array}\right] R2 \cdot -1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & -2 \\ 0 & -2 & -1 & -1 \end{array}\right] R1 - R2 \quad \left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & -2 & -1 & -1 \end{array}\right] R3 + 2R2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 9 & -5 \end{array}\right] R3/9 \quad \left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -5/9 \end{array}\right] R1 + 3R3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -5/9 \end{array}\right] R2 - 5R3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 7/9 \\ 0 & 0 & 1 & -5/9 \end{array}\right]$$

Problem 5

$v_1 = [1, 2, 3, 4)$ $\quad v_3 = [2, -1, 0, 4]$

$v_2 = [-1, 0, 2, -3)$ $\quad v_4 = [2, 1, 5, 5]$

$$\begin{bmatrix} 1 & -1 & 2 & 2 \\ 2 & 0 & -1 & 1 \\ 3 & 2 & 0 & 5 \\ 4 & -3 & 4 & 5 \end{bmatrix}$$

a. $\dim(\text{span}\{v_1, v_2, v_3, v_4\}) = \mathbb{R}^3$

↳ work on problem 6 to check if it is linear independence

Problem 6

$v_1 = [1, 2, 3, 4]$ $v_2 = [-1, 0, 2, -3]$ $v_3 = [2, -1, 0, 4]$

$v_4 = [2, 1, 5, 5]$

$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ $\sqrt{\sum_1^n |u_i|^2} = \sqrt{1^2 + 2^2 + 3^2 + 4^2} = \sqrt{30}$ $\frac{\sqrt{30}}{30} \cdot 1 = \frac{\sqrt{30}}{30}$ $\frac{\sqrt{30}}{30} \cdot 2 = \frac{\sqrt{30}}{15}$

$\frac{\sqrt{30}}{30} \cdot 3 = \frac{\sqrt{30}}{10}$ $\frac{\sqrt{30}}{30} \cdot 4 = \frac{2\sqrt{30}}{15}$

$\vec{e}_1 = \begin{bmatrix} \frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{15} \\ \frac{\sqrt{30}}{10} \\ \frac{2\sqrt{30}}{15} \end{bmatrix}$

$\frac{\vec{v} \cdot \vec{u}}{|u|^2} \cdot \vec{u}$ $[-1\ 0\ 2\ -3] \cdot [1\ 2\ 3\ 4] = -1 + 0 + 6 - 12 = -7$

$v_2 = [-1, 0, 2, -3]$ $\frac{-7}{(\sqrt{30})^2} \cdot [1, 2, 3, 4] =$

$\vec{u}_2 = \vec{v}_2 - \text{proj}_{\vec{u}_1}(v_2)$ $\left[\frac{-7}{30}, \frac{-7}{15}, \frac{-7}{10}, \frac{-14}{15}\right]$

$(-1, 0, 2, -3) - \left(\frac{-7}{30}, \frac{-7}{15}, \frac{-7}{10}, \frac{-14}{15}\right) =$

$\left(\frac{-23}{30}, \frac{7}{15}, \frac{27}{10}, \frac{-31}{15}\right)$

$\vec{e}_2 = \frac{\vec{u}_2}{|\vec{u}_2|} = \frac{\left(\frac{-23}{30}, \frac{7}{15}, \frac{27}{10}, \frac{-31}{15}\right)}{\sqrt{\left(\frac{-23}{30}\right)^2 + \left(\frac{7}{15}\right)^2 + \left(\frac{27}{10}\right)^2 + \left(\frac{-31}{15}\right)^2}} = \frac{\left(\frac{-23}{30}, \frac{7}{15}, \frac{27}{10}, \frac{-31}{15}\right)}{\frac{\sqrt{11130}}{30}}$

$= \left[\left(\frac{-23}{30}\right)\left(\frac{\sqrt{11130}}{30}\right), \left(\frac{7}{15}\right)\left(\frac{\sqrt{11130}}{30}\right), \left(\frac{27}{10}\right)\left(\frac{\sqrt{11130}}{30}\right), \left(\frac{-31}{50}\right)\left(\frac{\sqrt{11130}}{30}\right)\right]$

$= \left[\frac{-23\sqrt{11130}}{11130}, \frac{\sqrt{11130}}{795}, \frac{27\sqrt{11130}}{3710}, \frac{-31\sqrt{11130}}{5565}\right]$

$v_3 = [2, -1, 0, 4]$ $[2, -1, 0, 4] \cdot [1, 2, 3, 4] =$
$2 + -2 + 0 + 16 = 16$

$\vec{u}_3 = \vec{v}_3 - \text{proj}_{u_1}(v_3) - \text{proj}_{u_2}(v_3)$ $\frac{16}{(\sqrt{30})^2} = \frac{16}{30} \cdot (1, 2, 3, 4)$

$[2, -1, 0, 4] - \left[\frac{2}{15}, \frac{16}{15}, \frac{8}{5}, \frac{32}{15}\right] - [-1, 0, -2, 3]$ $= \left[\frac{8}{15}, \frac{16}{15}, \frac{8}{5}, \frac{32}{15}\right]$

$= \left[\frac{7}{15}, \frac{-31}{15}, \frac{-18}{15}, \frac{-17}{15}\right]$

$[2, -1, 0, 4] + [-1\ 0\ 2\ -3] =$
$-2 + 0 + 0 - 12 = -14$

$e_3 = \frac{u_3}{|u_3|}$ $\sqrt{\left(\frac{7}{15}\right)^2 + \left(\frac{-31}{15}\right)^2 + \left(\frac{-18}{15}\right)^2 + \left(\frac{-17}{15}\right)^2}$ $\sqrt{-1^2 + 0^2 + 2^2 + -3^2} = \sqrt{14}$

$= \frac{\sqrt{1623}}{15}$

$\left[\frac{7\sqrt{1623}}{225}, \frac{-31\sqrt{1623}}{225}, \frac{8\sqrt{1623}}{75}, \frac{32\sqrt{1623}}{225}\right]$ $\frac{-14}{(\sqrt{14})^2} = -1$

$-1 \cdot [-1, 0, 2, -3] = [1, 0, -2, 3]$

$$\begin{bmatrix} \frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{15} \\ \frac{\sqrt{30}}{10} \\ \frac{2\sqrt{30}}{15} \end{bmatrix} \begin{bmatrix} \frac{-23\sqrt{11130}}{11130} \\ \frac{\sqrt{11130}}{795} \\ \frac{27\sqrt{11130}}{3710} \\ \frac{-31\sqrt{11130}}{5565} \end{bmatrix} \begin{bmatrix} \frac{7\sqrt{1623}}{225} \\ \frac{-31\sqrt{1623}}{225} \\ \frac{8\sqrt{1623}}{75} \\ \frac{32\sqrt{1623}}{225} \end{bmatrix}$$

b. $v = [4\ 2\ 10\ 10]$

$$\begin{bmatrix} \frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{15} \\ \frac{\sqrt{30}}{10} \\ \frac{2\sqrt{30}}{15} \end{bmatrix} \begin{bmatrix} \frac{-23\sqrt{11130}}{11130} \\ \frac{\sqrt{11130}}{795} \\ \frac{27\sqrt{11130}}{3710} \\ \frac{-31\sqrt{11130}}{5565} \end{bmatrix} \begin{bmatrix} \frac{7\sqrt{1623}}{225} \\ \frac{-31\sqrt{1623}}{225} \\ \frac{8\sqrt{1623}}{75} \\ \frac{32\sqrt{1623}}{225} \end{bmatrix}$$

Problem 7 ✓

(1, 0, 1)

$$a\begin{bmatrix}5\\-1\\2\end{bmatrix} + b\begin{bmatrix}2\\0\\1\end{bmatrix} + c\begin{bmatrix}1\\-1\\1\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

$5a + 2b + c = 1$

$c = -5a - 2b + 1$

$c = 5c - 2b + 1$

$c = 5c - 2(1+c) + 1$

$-4c = -1 - 2c$

$2c = 1$

$\boxed{c = \frac{1}{2}}$

$-1a - c = 0$

$c = -a$

$\frac{1}{2} = -a$

$\boxed{a = -\frac{1}{2}}$

$2a + b + c = 1$ (÷ −2)

$-a - \frac{b}{2} - \frac{c}{2} = \frac{-1}{2}$

$c - \frac{b}{2} - \frac{c}{2} = \frac{-1}{2}$

$2c - b - c = -1$

$-b + c = -1$

$b - c = 1$

$b = 1 + c$

$2a + b + c = 1$

$2\left(\frac{-1}{2}\right) + b + \frac{1}{2} = 1$

$-\frac{1}{2} + b = 1$

$\boxed{b = \frac{3}{2}}$

Problem 8

A = 10m/t
(x) 5w

B = 15m/t
(y) 3w

100m
250
25

9
10

$5x + 3y = 25$

$x = 2$
$y = 3$