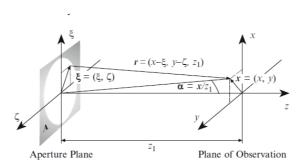


Scalar diffraction:

The Rayleigh-Sommerfeld diffraction integral describes the propagation of the amplitude V() in space through:

$$V(ec{x}) = rac{1}{i\lambda}\int\!\int_A V(ec{\xi})rac{e^{ikec{r}}}{ec{r}}dec{\xi}$$

where the geometry is described from:



By placing a focusing element with focal lens F in the aperture plane, we can further simply the integral through the Fresnel and Fraunhofer. In a Fresnel approximation, we assume that the observation plane is far from the aperture, and the r term can be approximated by z. In the case of a Fraunhofer approximation, we assume that the curvature term can also suffer the same approximation. As such, the integral takes the form of a Fourier transform of the product of the aperture function A with the incoming field at the aperture:

$$V(ec{lpha}) = rac{1}{i\lambda F}\int\int V(ec{\xi})A(ec{\xi})e^{-ikec{lpha}\cdotec{\xi}}dec{\xi} = \mathcal{F}_{ec{\xi}}\left(V(ec{\xi})A(ec{\xi})
ight)$$

In the case of a plane wave

$$V(ec{\xi}) = V_0 \exp(ikz)$$

Which is produced from a faraway point-like source propagating in free space. Then the field at the observation plane is given by the fourier transform of a constant multiplied by a circular aperture, which is the familiar Besinc(\alpha) function.

Atmospheric propagation

We construct the building blocks of atmospheric propagation by scaling in complexity. We evaluate the following EMF propagation cases:

- 1. EMF propagation without atmospheric effects.
- 2. EMF propagation with atmospheric effects.

We assume that each case's propagation distance is larger than the aperture.

$$\Delta z > rac{2D^2}{\lambda}$$

where z is the propagation distance, D is the aperture size and λ is the observed wavelength.

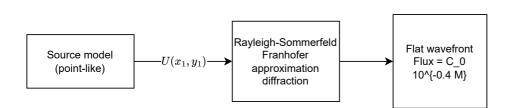
We first describe the propagation of the natural guide star up to the start of the atmosphere:

$$|I|^2 = |F(U(x_1,y_1))|^2 = |F(E_0\delta(x-x_0))|^2 = E_0e^{2\pi i x_0 f}$$

Which can be taken simply as a plane wavefront at the point where it meets the atmosphere. The incoming flux of the star can be taken from the magnitude of the observed star at the corresponding optical band at which the observation is made. It follows that the flux is:

$$Flux = C_0 10^{-0.4M} [ph/m^2/s]$$

M is the magnitude, and C_0 is the zero point for the optical band.



Imaging with a perfect telescope

We now know the field just before the aperture of the Telescope. The image formed by the telescope is given by the PSF (point-source function) of the telescope, which is simply the $|F()|^2$ of the aperture function of the telescope, A(x2,y2), where (x2,y2) are the coordinates at the aperture stop. Assuming a perfect circle with no central obstruction, the PSF takes a familiar shape:

$$PSF = |\int_{-\infty}^{\infty} A(-D/2,D/2)e^(2\pi i x f) dx|^2 \propto sinc^2(x)$$

which is simply the 1D diffraction pattern of a circular aperture. In the case of a 2D aperture, we get a Bessel function integral of the first kind:

$$I(heta) = \left[rac{2J_1(x)}{x}
ight]^2$$

From which the Airy disk falls (first zero of the function).

The optical response of the system is simply the convolution of the source distribution with the PSF:

$$I(.) = O(.) * PSF(.)$$

In the case of the point source star, this sets the star's flux to the telescope response.

We can also take the Fourier transform of the components and work with the Optical Transfer Function of the telescope. The translation between the two is given by the Wiener-Khinchine theorem:

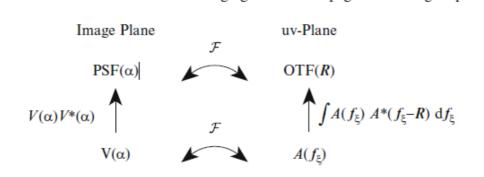
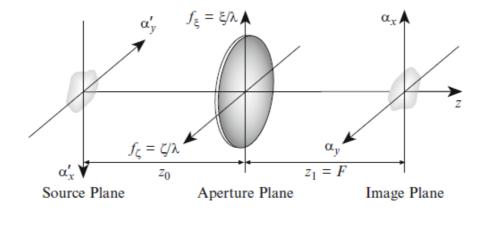


Fig. 3.3 Illustration of the autocorrelation theorem, called the Wiener–Khinchine theorem. The PSF being the modulus squared of the amplitude $V(\alpha)$ – and, thus, real and positive – implies that the OTF is the autocorrelation function of the aperture function $A(f_{\xi})$ of the optical system, which is the Fourier transform of the amplitude $V(\alpha)$



Scalar diffraction: The Rayleigh-Sommerfeld diffraction integral describes the propagation of the amplitude V() in space through: