Neuroprothetik Exercise 4
Hodgkin & Huxley Model

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1. Time constants and steady state values

Derive the relationship between the rate equations for α_x, β_x and the time constant τ_x and steady state value x_∞ with $x \in \{m, n, h\}$. To gain these two variables bring the gating ODEs into the following form:

$$\frac{dx}{dt} = \frac{1}{\tau_x}(x_\infty - x)$$

Plot τ_x and x_∞ against the voltage V $\in$ [-100 mV, 100 mV] at $6.3\,°C$ and $28\,°C$. Explain what you can read from these plots.

2. Hodgkin & Huxley Neuron Model

Change the LIF model from the previous exercise to a HH type model.

To do this, remove the fixed spiking threshold and reset value as the HH model will spike on its own. Next, implement a function that calculates the ionic current density i_{ion}. For example:

$$[i\_ion, gate] = hh\_current(V, dt, last\_gate)$$

Use the exponential-euler solver which was implemented in the previous exercise to solve the gating ODEs.

2.1. Experiments

Run the model for 100 ms ($\Delta t = 0.01$ ms) with the following settings

1. At $6.3\,°C$ induce a stair of five 5 ms long rectangular current pulses with a gap of 10 ms and the amplitudes $1\,\text{uA}/cm^2$, $2\,\text{uA}/cm^2$, $3\,\text{uA}/cm^2$, $4\,\text{uA}/cm^2$, $5\,\text{uA}/cm^2$

2. At $28\,°C$ induce a stair of five 5 ms long rectangular current pulses with a gap of 10 ms and the amplitudes $2\,\text{uA}/cm^2$, $4\,\text{uA}/cm^2$, $8\,\text{uA}/cm^2$, $16\,\text{uA}/cm^2$, $32\,\text{uA}/cm^2$

Create the following Plots of the results from the two experiments:

1. Plot the membrane potential over time.

2. Plot the gating constants m, n, h over time

3. Plot the current densities i_{Na}, i_K over time.

4. Plot the current densities i_{Na}, i_K, i_L, over the membrane potential (phase plot).

2.2. Analysis of the Results

Answer the following questions by interpreting the results. For some answers, it might be helpful to replot the graphs in different scales.

- Describe the differences between the results at $6.3\,°\mathrm{C}$ and $28\,°\mathrm{C}$

- How is an action potential generated and what is the role of the different currents and gating variables.

- Explain why consecutive action potentials decrease in amplitude (at at $28\,°\mathrm{C}$)

- How can you interpret the phase plot.

- Explain the differences between the LIF and the HH model, why would you implement one or the other.

A. Equations and Constants

All units are given as in the original publication. Hodgkin and Huxley obviously had no idea about SI units but if you use all units as given and ms for the time axis, all numbers can be used without conversions.

General equations:

$$\frac{dV}{dt} = \frac{1}{c}(-i_{ion} + i_{stimulus})$$
$$i_{ion} = i_{Na} + i_K + i_L$$
$$V = V_i - V_e - V_{rest}$$

Ionic currents:

$$i_{Na} = \bar{g}_{Na}m^3h(V - V_{Na}) \qquad i_K = \bar{g}_K n^4(V - V_K)$$
$$\frac{dm}{dt} = [\alpha_m(1 - m) - \beta_m m]k \qquad \frac{dn}{dt} = [\alpha_n(1 - n) - \beta_n n]k$$
$$\frac{dh}{dt} = [\alpha_h(1 - h) - \beta_h h]k \qquad i_L = \bar{g}_L(V - V_L)$$

Temperature correction (T in °C):

$$k = 3^{0.1(T-6.3)}$$

Rate equations (V in mV):

$$\alpha_m = \frac{2.5 - 0.1V}{e^{(2.5-0.1V)} - 1} \qquad \beta_m = 4e^{-V/18}$$
$$\alpha_n = \frac{0.1 - 0.01V}{e^{(1-0.1V)} - 1} \qquad \beta_n = 0.125e^{-V/80}$$
$$\alpha_h = 0.07e^{-V/20} \qquad \beta_h = \frac{1}{e^{(3-0.1V)} + 1}$$

Constants

Conductances in mS/cm^2		
$\bar{g}_{Na} = 120$	$\bar{g}_K = 36$	$\bar{g}_L = 0.3$
Resting potentials in mV		

$V_{Na} = 115$	$V_K = -12$	$V_L = 10.6$	$V_{rest} = -70$
Other constants			
$c = 1\,\mu F/cm^2$			