UESTC Student ID______________________ UOG Student ID______________________ Course Title______________________________ Lecturer__________________

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Glasgow College, UESTC

Linear Algebra and Space Analytic Geometry —Semester 2, 2018 - 2019

Final Exam

14:30-16:30, 2nd July, 2019

<u>Notice:</u> Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.

All questions are compulsory. There are 4 questions and a maximum of 100 marks in total.

The following table is for grader only:

Question	**1**	**2**	**3**	**4**	**Total**	**Grader**
Score						

Score

Question1 (4x5=20 points) Fill in the blanks.

1.1 Let A **be a** 3×3 **matrix, and** $\det A = 2$**, then** $\det(2A) =$ ________ **and** $\det A^{-1} =$ ______.

1.2 Let $A=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$**, and *ABC=I*, then** $B^{-1} =$__________.

1.3 If $AB = AC$ **implies** $B = C$**, then the columns of** A **are linearly** ____________________ **(choose one from dependent/independent to fill in the blank).**

1.4 Let $u=\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v=\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$**, then the distance between** u **and** v **is** ____________.

1.5 Suppose $a=(1,-1,1)^T, A = aa^T$**, then** $A^4 =$_____________.

Score

Question2 (50 points) Calculations

2.1. Let $A=\begin{bmatrix} 1 & -2 & 2 \\ -2 & -2 & 4 \\ 2 & 4 & -2 \end{bmatrix}$**. Compute all the eigenvalues and eigenvectors of *A*. (10 points)**

2.2. Let $A=\begin{bmatrix} -1 & 6 & 5 \\ 3 & -8 & -5 \\ 1 & -2 & -1 \\ 1 & -4 & -3 \end{bmatrix}$ **and** $b=\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$**. Find an orthogonal basis for Col*A*, and then find the best approximation of *b* in Col*A*. (15 points)**

2.3 Determine the value of *a*, such that the three vectors $v_1=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2=\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, v_3=\begin{bmatrix} 2 \\ 3 \\ a \end{bmatrix}$ **are linearly dependent. Then express** v_3 **as a linear combination of** v_1 **and** v_2**. (10 points)**

2.4 Make a change of variable that transforms the following quadratic form into a quadratic form with no cross-product term.

$f(x_1,x_2,x_3)=3x_1^2+3x_3^2+4x_1x_2+8x_1x_3+4x_2x_3$. **(15 points)**

Score

Question3 (10x3=30 points)

3.1 Suppose v_1, v_2, v_3 **are three vectors. Prove that** $v_1 - v_2, v_2 - v_3, v_3 - v_1$ **form a linearly dependent set.**

3.2 Show that $(A+B)^2 = A^2 + 2AB + B^2$ **if and only if** $AB = BA$**.**

3.3 Suppose there exists an invertible matrix *P*, such that $A = PBP^{-1}$**. Show that** $\det A = \det B$**.**