Glasgow College, UESTC

Calculus II —Semester 2, 2017 - 2018

Final Exam

9:30—11:30, AM, Monday, July 9, 2018

Notice: **Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.**

All questions are compulsory. There are 7 questions and a maximum of 100 marks in total.

The following table is for grader only:

Question	1	2	3	4	5	6	7	Total	Grader
Score									

Score

Question 1 (15 points)

The derivative of function $f(x,y,z)$ **at point** P **is greatest in the direction** $\vec{u}=\vec{i}+\vec{j}-\vec{k}$. **In this direction, the value of the derivative is** $2\sqrt{3}$. **Then,**

(a) what is ∇f **at** P**? (8 points)**

(b) what is the derivative of f **at** P **in the direction of** $\vec{v}=\vec{i}+\vec{j}$**? (7 points)**

Score

Question 2 (15 points)

Let $f(x,y)=e^{2y-x}$,

(a) find the total differential df **of** $f(x,y)$ **at the origin** $(0,0)$**; (10 points)**

(b) find the linearization $L(x,y)$ **of** $f(x,y)$ **at the origin. (5 points)**

Score

Question 3 (20 points)

Find the following two double integrals:

(a) $\iint\limits_D \frac{\sin x}{x}\,dx\,dy$**, where the region** D **is bounded by the line** $y = x$ **and the parabola** $y = x^2$ **in the** xy**–plane; (10 points)**

(b) $\iint\limits_D e^{-(x^2+y^2)}\,dx\,dy$**, where the region** D **is the disk** $x^2 + y^2 \le R^2$ **in the** xy**–plane. (10 points)**

Score

Question 4 (15 points)

Convert the following integral to iterated integral in polar coordinates:

$$\int_0^1 \int_{1-x}^{\sqrt{1-x^2}} f(x^2+y^2)\,dy\,dx.$$

Score

Question 5 (15 points)

Convert the following triple integral to iterated integral in cylindrical coordinates, but do not evaluate the result:

$$\iiint_R f(x,y,z)\,dV,$$

where R **is the space region bounded by the cone** $z=\sqrt{x^2+y^2}$ **, the cylinder** $x^2+y^2=2x$ **and the place** $z=0$ **.**

Score

Question 6 (10 points)

Find the mass of a thin wire lying along the curve $\vec{r}(t)=\left(\sqrt{2}\right)t\vec{i}+\left(\sqrt{2}\right)t\vec{j}+\left(4-t^2\right)\vec{k}, 0\le t\le 1,$ **if the density function of this wire is** $\delta=3t.$

Score

Question 7 (10 points)

Find the circulation of the vector field $\vec{F} = (x^2 - y)\vec{i} + 4z\vec{j} + x^2\vec{k}$ **around the curve** C **in which the plane** $z = 2$ **meets the cone** $z = \sqrt{x^2 + y^2}$, **counterclockwise as viewed from above.**