Glasgow College, UESTC

Linear Algebra and Space Analytic Geometry —Semester 2, 2017 - 2018

Final Exam

1000:12:00, 12th July, 2018

Notice: Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.

All questions are compulsory. There are 4 questions and a maximum of 100 marks in total.

The following table is for grader only:

Question	1	2	3	4	Total	Grader
Score						

Score

Question1 (4x5=20 points) Fill in the blanks.

1.1 Let A **be a** 3×3 **matrix, and have eigenvalues** $1,-1,2,$ **then** $\det(A^3-2A^{-1})=$________.

1.2 Let $A=\begin{bmatrix}1 & 2 & -2\\ 0 & t & 3\\ 0 & -1 & 1\end{bmatrix}$, B **be a** 3×3 **nonzero matrix, and** $AB=0$, **then** $t=$__________.

1.3 If $AB=0,(B\neq 0)$, **then the columns of** A **are linearly** _______________ **(choose one from dependent/independent to fill in the blank).**

1.4 Let $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}, C=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, **and** $ABC=I$, **then** $B^{-1}=$____________.

1.5 Suppose $a=(1,1,1)^T, A=aa^T$, **then** $A^4=$_____________.

Score

Question2 (10x4=40 points) Calculations

2.1. Write down the following system of equations as a matrix and reduce to find the parametric vector form.

$$\begin{cases} x_1+3x_2+x_3=1\\ -4x_1-9x_2+2x_3=-1\\ -3x_2-6x_3=-3\end{cases}$$

2.2. Let $A=\begin{bmatrix} 1 & -2 & 2 \\ -2 & -2 & 4 \\ 2 & 4 & -2 \end{bmatrix}$**, find an orthogonal matrix** Q**, such that** $Q^T AQ$ **is a diagonal matrix.**

2.3 Determine the value of a**, such that the three vectors** $v_1=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2=\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, v_3=\begin{bmatrix} 2 \\ 3 \\ a \end{bmatrix}$ **are linearly dependent. Then express** v_3 **as a linear combination of** v_1 **and** v_2**.**

2.4 Find the determinant $\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix}$.

Score

Question3 (10x2=20 points)

3.1 Let $S=\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\-1\end{bmatrix}\right\}$**. Find the orthogonal projection of** $y=\begin{bmatrix}1\\2\\1\end{bmatrix}$ **onto span*S*.**

3.2 Let $\begin{bmatrix}1&1&-1\\0&2&2\\1&-1&0\end{bmatrix}X+\begin{bmatrix}0&1\\1&0\\4&3\end{bmatrix}=\begin{bmatrix}1&-1\\1&1\\2&1\end{bmatrix}$**, find matrix X.**

Score

Question4 (10x2=20 points)

4.1 Let vectors v_1, v_2, v_3 **form a linearly independent set. Find the relation between** a, b **and** c **such that** $av_1 - v_2, bv_2 - v_3, cv_3 - v_1$ **are linearly dependent.**

4.2 Let $P: x_1 + 2x_2 - x_3 = 0$ **be a plane and** $L: span\left\{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\right\} + \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$ **be a line in** R^3**.**

a) Does L **intersect with** P**? If yes find the intersection point.**

b) Is L **orthogonal to** P**?**