GLASGOW COLLEGE UESTC

Degrees of MEng, BEng, MSc and BSc in Engineering

Stochastic Signal Analysis (UESTC3024)

Date: 29th Dec. 2019

Time: 14:30-16:30

Attempt all PARTS

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

DATA/FORMULAE SHEET IS PROVIDED AT THE END OF PAPER

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

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Q1. In a circuit, the output voltage Y is related to the input voltage X. It is known that Y = 6X + 2, and X can be seen as a random variable that has a Gaussian distribution in [0, 1].

(a) Determine the probability density function of Y. [6]

(b) Determine the characteristic function of Y. [4]

Q2. There are two signals received by a receiver, the arrival time of which are recorded as X and Y respectively. It is known that the joint probability density of X and Y is

$$f_{XY}(x,y) = xe^{-x(y+1)}u(x)u(y)$$

Determine the marginal probability density function $f_X(x)$ and $f_Y(y)$. [8]

Q3. Consider an equal distribution of independent semi-random binary transmission signals x(t), whose value is 0 or 1. The length of time interval is T0.

(a) Determine the expectation E[X(t)] [4]

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(b) Determine the autocorrelation function $R_X(t, t + \tau)$ [6]

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Q4. A radar transmits a signal W(t) = sin(2t). The resultant echo signal is X(t) = Asin(2t + Φ), where Φ is a random variable uniformly distributed in [0, 2π], and A may be a constant, a time function or a random variable.

(a) Determine the expectation and autocorrelation function of X(t). Note that there is no need to work out the value of $E[A^2]$. [4]

(b) Determine the time average of X(t). [4]

(c) On what conditions about A, X(t) would this be an ergodic process? [6]

Q5. Assume a random process $\mathrm{X}(t) = sgn(A)\cos(\omega_0 t + \theta)$, where ω_0 is a constant, A and θ are random variables independent of each other, and A has a Gaussian distribution with a mean of 0 and a variance of 1. θ has a uniform distribution in $[0,2\pi)$, $sgn()$ is a symbolic function. [10]

(a) Determine the mean and the autocorrelation function of $\mathrm{X}(t)$ [3]

(b) Determine the generalized ergodic properties of $\mathrm{X}(t)$ [7]

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End of question paper

Q6. It is known the power spectral density of the stationary random signal X(t) is

$$S_x(\omega) = \frac{4}{\omega^2 + 4}$$

The center frequency of X(t) is ω_0, and $\omega_0 = 0$. X(t) passes through a system with frequency response of $H(\omega) = \frac{1}{1 + j\omega}$ and output Y(t).

(a) Determine the mean and average power of Y(t); [8]

(b) Determine the equivalent noise bandwidth of the system; [5]

(c) Determine the rectangular equivalent bandwidth of Y(t). [3]

Q7. A known zero-mean stationary Gaussian noise X(t), has power spectral density as shown in Figure Q7.

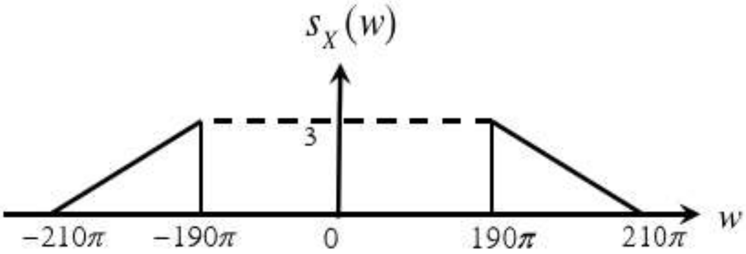

Figure Q7

(a) Sketch the power spectrum graph of $S_i(\omega)$ or $S_q(\omega)$, and determine $R_i(\tau)$ and $R_q(\tau)$, the autocorrelation function of the in-phase and quadrature components. [8]

(b) Determine $f_{iq}(i, q;\ t, t)$, the joint density function of in-phase and quadrature components. [6]

Formulae sheet

Q8. The synchronous geophone is shown in Figure Q8. The input $X(t)$ is narrow-band stationary noise, and its autocorrelation function is $R_X(\tau) = e^{-\beta|\tau|}\cos\omega_0\tau, \beta << \omega_0$. The other input is $Y(t) = A\cos(\omega_0 t + \theta)$, where A and ω_0 are constants, θ is a random variable uniformly distributed in $(0, 2\pi)$, and is independent of $X(t)$. The cut-off frequency of low-pass filter is ω_0.

(a) Determine the expectation and autocorrelation function of $Y(t)$ [4]

(b) Determine the expectation and autocorrelation function of $M(t)$ [6]

(c) Determine the power spectral density of $M(t)$ and $Z(t)$ [4]

(d) Determine the autocorrelation function and average power of $Z(t)$ [4]

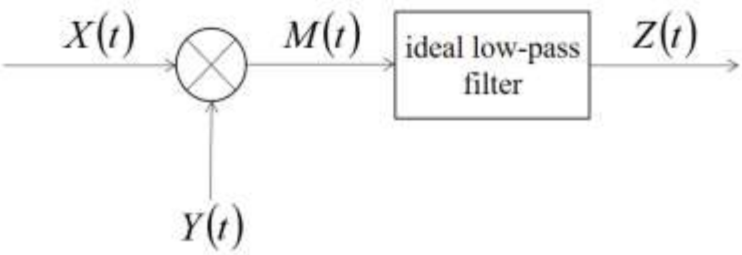

Figure Q8

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Selected Fourier transforms

$f(t)$	$\mathcal{F}\{f(t)\}$
$\sin(\omega_0 t)$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$e^{-at}u(t)$	$\frac{1}{a+j\omega},\ a>0$
$e^{-a\|t\|}$	$\frac{2a}{\omega^2+a^2},\ a>0$
$\frac{W}{\pi}Sa(Wt)=\frac{\sin(Wt)}{\pi t}$	$F(j\omega)=\begin{cases}1.\|\omega\|<W\\0,\|\omega\|>W\end{cases}$

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