GLASGOW COLLEGE UESTC

Exam paper

Signal and System (UESTC 2026)

Date: 29th August 2020
Time: 09:30-11:30am

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

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Q1 The properties of the Fourier transform provide us with a significant amount of insight into the transform and into the relationship between the time-domain and frequency-domain descriptions of a signal. In addition, many of the properties are often useful in reducing the complexity of the evaluation of Fourier transforms or inverse transforms. Signal $x(t)$ is shown in Figure Q1, let $X(j\omega)$ denotes the Fourier transform of $x(t)$. Please perform all these calculations without explicitly evaluating $X(j\omega)$.

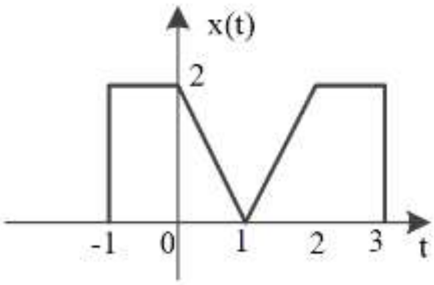

Figure Q1.

(a) Determine the phase spectrum of $X(j\omega)$. [5]

(b) Find $X(j0)$. [5]

(c) Find $\int_{-\infty}^{+\infty} X(j\omega)\ d\omega$. [5]

(d) Evaluate $\int_{-\infty}^{+\infty} X(j\omega) \dfrac{2\sin\omega}{\omega} e^{j2\omega}\ d\omega$. [10]

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Q2 Consider the system shown in Figure Q2 with $p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$. The input signal $x(t)$ is generated by $x(t)=x_0(t)\times\cos(\pi t)$ where the spectrum $X_0(j\omega)$ is given in Figure Q2.

(a) Sketch the spectrum $X(j\omega)$ of the input signal $x(t)$. [5]

(b) In order to recover $x(t)$ from the impulse train sampled signal $y(t)$, determine the maximum T that can be used in the system. [5]

(c) If $T=\frac{1}{4}$, sketch the spectrum of the signal $y(t)$. [5]

(d) If $T=\frac{1}{4}$, design a 4-th order low-pass filter $H(j\omega)=\frac{1/4}{(1+j\omega/a)^4}$ with $a>0$ to recover the signal $x(t)$ from the signal $y(t)$. Write out the magnitude response of this filter and determine the parameter a given the requirement that the magnitude response drops to $|H(j0)|/100$ at $\omega=4\pi$, that is $|H(j4\pi)|=|H(j0)|/100$. Find the phase spectrum of this 4-th order low-pass filter. [10]

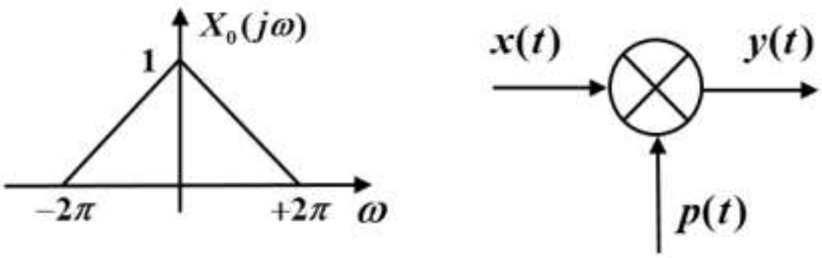

Figure Q2.

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Q3 Consider an LTI system with unit impulse response $h(t) = e^{-t}u(t) - e^{-2t}u(t)$.

(a) Determine the system function $H(s)$, and indicate the ROC of $H(s)$. [6]

(b) Is this system causal? Is this system stable? Clearly justify your answer. [6]

(c) If the input $x(t) = 1$, please compute the output. [5]

(d) If the input $x(t) = e^{-t}u(t)$, please compute the output. [8]

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Q4 For a discrete-time LTI system, it is known that the output signal is $y[n] = \left(-\frac{1}{2}\right)^n u[n]$ when the input signal is $x[n] = \delta[n] - (1/4)\delta[n-1]$.

(a) Recover the system function $H(z)$ from the above information, and find the poles and zeros plus the ROC of $H(z)$. [10]

(b) Determine the system's unit impulse response $h[n]$. Is the system causal or stable? [10]

(c) Compute the output of the system, if the input signal is $x[n] = \cos(\pi n)$. [5]

End of question paper