Glasgow College, UESTC

Linear Algebra and Space Analytic Geometry —Semester 2, 2019 - 2020

Final Exam

09:30-11:30AM, 1st September 2020

<u>Notice:</u> Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.

All questions are compulsory. There are 4 questions and a maximum of 100 marks in total.

The following table is for grader only:

Question	1	2	3	4	Total	Grader
Score						

Score

Question1 (4x5=20 points) Fill in the blanks.

1.1 Suppose $A^k = 0$ for some $k > 0$, then $(I - A)^{-1} =$ ____________________.

1.2 Suppose A is a 3×3 matrix and det$A$=$a \neq 0$, then det(adj A) =__________.

1.3 The reduced echelon form of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$ is ________________.

1.4 Suppose A is a 2×2 matrix, $\{u, v\}$ is a linearly independent set in R^2, and $Au = 0, Av = 2u + v \neq 0$, then the eigenvalues of A are _____________.

1.5 The column vectors of $A = \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix}$ are linearly ________________ (choose from dependent/independent to fill the blank).

Score

Question2 (30 points) Calculations

2.1 (10 points) $\begin{vmatrix} x & -1 & 1 & x-1 \\ x & -1 & x+1 & -1 \\ x & x-1 & 1 & -1 \\ x & -1 & 1 & -1 \end{vmatrix} = 0,$ **find x.**

2.2 (10 points) **Make a change of variable $x = Py$, that transforms the following quadratic form $x_1^2 + 10x_1x_2 + x_2^2$ into a quadratic form with no cross-product term. Give P and the new quadratic form.**

2.3 (10 points) **Determining whether the following system of equations is consistent for all b_1, b_2, b_3:**

$$2x_1 - 4x_2 - 2x_3 = b_1$$
$$-5x_1 + x_2 + x_3 = b_2$$
$$7x_1 - 5x_2 - 3x_3 = b_3$$

Score

Question3 (25 points) Calculations

3.1 (13 points) **Find the orthogonal projection of b on to the subspace spanned by $\{v_1, v_2\}$, where**

$$b = \begin{bmatrix}1\\1\\0\\1\end{bmatrix}, v_1 = \begin{bmatrix}-1\\3\\1\\1\end{bmatrix} \text{ and } v_2 = \begin{bmatrix}6\\-8\\-2\\-4\end{bmatrix}.$$

3.2 (12 points) **Find a basis for Nul$A$ and Col$A$, respectively, where** $A = \begin{bmatrix}1 & 2 & 0 & 2\\2 & 4 & -1 & 7\\1 & 2 & -2 & 8\end{bmatrix}$.

Score

Question4 (25 points) Proofs and Discussions

4.1 (12 points) Assuming matrix *A* is similar to matrix *B*, and matrix *C* is similar to matrix *D*. Prove that matrix $\begin{bmatrix} A & 0 \\ 0 & C \end{bmatrix}$ is similar to matrix $\begin{bmatrix} B & 0 \\ 0 & D \end{bmatrix}$, where the 0s denote zero matrices with appropriate sizes.

4.2 (13 points) The set $\{v_1, v_2, v_3, v_4\}$ is linearly independent, is $\{v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4 + v_1\}$ linearly independent? Why?