Glasgow College, UESTC

Electromagnetic Field and Microwave Technology —Semester 2, 2017 - 2018

Final Exam

14:30-16:30, 9th July, 2018

Notice: Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the

All questions are compulsory. There are 6 questions and a maximum of 100 marks in total.

The following table is for grader only:

space given below on the answer sheets.

Question	1	2	3	4	5	Total	Grader
Score							

Score

Question1 (2x13=26 points) Select the best answer among the choices for each question

- 1. The divergence of magnetic fields is ______.
 - **A.** 0 B. current
- C. charge
- D. potential
- 2. If in a lossless medium, the permittivity is ε , and the permeability is μ , the wave speed can be expressed as _____.

- **A.** μ/ε **B.** ε/μ **C.** $\sqrt{\varepsilon/\mu}$ **D.** $1/\sqrt{\varepsilon\mu}$
- 3. If the permittivity and permeability of a medium with zero magnetization are ε and μ , respectively, and the magnetic flux density in the medium is $3\vec{e}_x + 1\vec{e}_y$, the magnetic field intensity can be written as ______.

A.
$$3\mu e_{x} + \mu e_{y}$$

B.
$$3\varepsilon e^{0} + \varepsilon e^{0}$$

C.
$$(3e^{1} + e^{1})/(3e^{1} + e^{1})$$

A.
$$3\mu e_x^y + \mu e_y^y$$
 B. $3\varepsilon e_x^y + \varepsilon e_y^y$ C. $(3e_x^y + e_y^y)/\mu$ D. $(3e_x^y + e_y^y)/\varepsilon$

4. The dynamic scalar electric potential is introduced based on the equation_____.

$$\mathbf{A.} \quad \nabla \cdot \overset{\mathcal{V}}{D} = 0$$

A.
$$\nabla \cdot \vec{b} = 0$$
 B. $\nabla \times \vec{E} = -\frac{\partial \vec{b}}{\partial t}$ **C.** $\nabla \times \vec{b} = 0$ **D.** $\nabla \cdot \vec{b} = \rho$

$$\mathbf{C.} \ \nabla \times \overset{\mathcal{V}}{D} = 0$$

$$\mathbf{D.} \ \nabla \cdot \overset{\mathcal{V}}{D} = \mu$$

5. In a dielectric body whose permittivity is ε , the polarization vector is $\stackrel{\begin{subarray}{c}}{P}$ and the unit normal vector of the dielectric body's boundary is $\stackrel{U}{e_n}$ (pointing from inwards to outwards), the volume density of polarization charge is ______.

$$\mathbf{A} \cdot - \mathbf{e} \cdot \mathbf{P}$$

B.
$$-\nabla \cdot \overset{\mathcal{C}}{P}$$

A.
$$-\mathcal{E}_n \cdot P$$
 B. $-\nabla \cdot P$ **C.** $\varepsilon \mathcal{E}_n \nabla \times P$ **D.** $-\varepsilon \nabla \cdot P$

$$\mathbf{D.} - \varepsilon \nabla \cdot \mathbf{A}$$

6. The integral form of Faraday's law of electromagnetic induction is _____.

A.
$$\int_{C}^{\mathbf{T}} \mathbf{B} \cdot d\mathbf{l} = \int_{S}^{\mathbf{T}} \left(\mathbf{J} + \frac{\partial E}{\partial t} \right) \cdot d\mathbf{S}$$
B.
$$\int_{S}^{\mathbf{T}} \cdot d\mathbf{S} = -\int_{V} \frac{\partial \rho}{\partial t} dv$$
C.
$$\int_{C}^{\mathbf{T}} \mathbf{E} \cdot d\mathbf{l} = -\int_{S}^{\mathbf{D}} \frac{\partial h}{\partial t} \cdot d\mathbf{S}$$
D.
$$\int_{C}^{\mathbf{T}} \mathbf{E} \cdot d\mathbf{l} = -\int_{S}^{\mathbf{D}} \frac{\partial h}{\partial t} \cdot d\mathbf{S}$$

B.
$$\int_{S} \int_{V}^{T} \cdot dS = -\int_{V} \frac{\partial \rho}{\partial t} dv$$

$$\mathbf{C.} \quad \mathbf{N}_{C}^{\mathbf{r}} \cdot d\mathbf{l}^{\mathbf{r}} = -\mathbf{N}_{S}^{\mathbf{\partial B}} \cdot d\mathbf{l}$$

D.
$$\iint_{C} \vec{E} \cdot d\vec{l} = - \iint_{C} \frac{\partial \vec{H}}{\partial t} \cdot d\vec{k}$$

7. Consider the two media shown in Figure 1. The permeability in media 1 and 2 are μ_1 and μ_2 , respectively. Assume the surface current density on the interface of the two media is J_s . On the interface of the two media, the general boundary conditions for the magnetic vector potential $\overset{1}{A}$ are ______

A.
$$A_1 = A_2$$
, $e_n \cdot (A_1 - A_2) = 0$

A.
$$A_1 = A_2$$
, $e_n \cdot (A_1 - A_2) = 0$ **B.** $A_1 - A_2 = 0$, $e_t \cdot (\mu_1 A_1 - \mu_2 A_2) = 0$

C.
$$\stackrel{\mathsf{p}}{e}_n \cdot \left(\stackrel{\mathsf{p}}{A}_1 - \stackrel{\mathsf{p}}{A}_2 \right) = 0, \qquad \stackrel{\mathsf{p}}{e}_n \times \left(\stackrel{\mathsf{p}}{A}_1 - \stackrel{\mathsf{p}}{A}_2 \right) = 0$$

D.
$$A_1 - A_2 = 0$$
,

$$\mathbf{C.} \quad \stackrel{\boldsymbol{\rho}}{\boldsymbol{e}_n} \cdot \left(\stackrel{\boldsymbol{\nu}}{\boldsymbol{A}_1} - \stackrel{\boldsymbol{\nu}}{\boldsymbol{A}_2} \right) = 0, \qquad \stackrel{\boldsymbol{\rho}}{\boldsymbol{e}_n} \times \left(\stackrel{\boldsymbol{\nu}}{\boldsymbol{A}_1} - \stackrel{\boldsymbol{\nu}}{\boldsymbol{A}_2} \right) = \stackrel{\boldsymbol{\nu}}{\boldsymbol{J}_S} \qquad \quad \mathbf{D.} \quad \stackrel{\boldsymbol{\rho}}{\boldsymbol{A}_1} - \stackrel{\boldsymbol{\rho}}{\boldsymbol{A}_2} = 0, \qquad e_n \times \left(\frac{1}{\mu_1} \nabla \times \boldsymbol{A}_1 - \frac{1}{\mu_2} \nabla \times \boldsymbol{A}_2 \right) = \stackrel{\boldsymbol{\rho}}{\boldsymbol{J}_S}$$

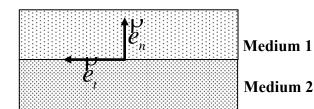


Figure 1

8. For dynamic scalar electric potential V and vector magnetic potential A, The Lorentz gauge is ______

A.
$$\nabla \times \stackrel{\mathbf{r}}{A} = -\mu\varepsilon \frac{\partial V}{\partial t}$$
 B. $\nabla \times \stackrel{\mathbf{r}}{A} = -\frac{1}{\mu\varepsilon} \frac{\partial V}{\partial t}$

C.
$$\nabla \times \stackrel{\mathbf{r}}{A} = -\frac{1}{\mu \varepsilon} \nabla \frac{\partial V}{\partial t}$$
 D. $\nabla \cdot \stackrel{\mathbf{r}}{A} = -\mu u$

- **A.** $\nabla \times \stackrel{\mathbf{r}}{A} = -\mu \varepsilon \frac{\partial V}{\partial t}$ **B.** $\nabla \times \stackrel{\mathbf{r}}{A} = -\frac{1}{\mu \varepsilon} \frac{\partial V}{\partial t}$ **C.** $\nabla \times \stackrel{\mathbf{r}}{A} = -\frac{1}{\mu \varepsilon} \nabla \frac{\partial V}{\partial t}$ **D.** $\nabla \cdot \stackrel{\mathbf{r}}{A} = -\mu \varepsilon \frac{\partial V}{\partial t}$
- 9. A uniform plane wave is propagating in a lossy medium whose permittivity and conductivity are ε and σ , respectively. If the angular frequency of the uniform plane wave is ω , the lossy medium is a low-loss dielectric when ______.

A.
$$\frac{\sigma}{\omega \varepsilon} >> 1$$

B.
$$\frac{\sigma}{\omega \varepsilon} \ll$$

c.
$$\frac{\sigma\omega}{\varepsilon}$$
 << 1

A.
$$\frac{\sigma}{\omega \varepsilon} >> 1$$
 B. $\frac{\sigma}{\omega \varepsilon} << 1$ C. $\frac{\sigma \omega}{\varepsilon} << 1$ D. $\frac{\sigma \omega}{\varepsilon} << 1$

10. The capacitance of a capacitor depends on_____

A. current and flux B. voltage and charge C. material property and voltage

D. material property and geometry

11. In a homogeneous medium whose permeability is μ , there is a volume current \hat{J} in region V. The magnetic vector potential at the observation point r is given by the formula

A.
$$\int_{V} \frac{\mu J}{4\pi |P-P|} dv$$
 B.
$$\int_{V} \frac{J}{4\pi \mu |P-P|} dv$$
 C.
$$\int_{V} \frac{\mu J}{2\pi |P-P|} dv$$
 D.
$$\int_{V} \frac{J}{2\pi \mu |P-P|} dv$$

- 12. The electric field intensity of a uniform plane wave is expressed as $\stackrel{1}{E}\binom{r}{r} = E_0 \stackrel{r}{a}_x e^{jkz}$, where E_0 is a real number, $\stackrel{\Gamma}{a}_x$ is the unit vector along x-direction, and k is the wavenumber. The uniform plane wave is propagating along ______. A. positive z direction **B.** negative z direction **C.** positive x direction **D.** negative x direction
- 13. The electric flux density and magnetic field intensity on a perfect conductor surface are D(r,t) and H(r,t), respectively. The unit vectors along the normal and tangential directions of the surface are $\stackrel{1}{e}_n$ and $\stackrel{1}{e}_t$, respectively. The charge density on the surface of the perfect conductor is____

A.
$$\stackrel{\mathbf{r}}{e}_{n} \times \stackrel{\mathbf{1}}{D} (\stackrel{\mathbf{r}}{r}, t)$$
 B. $\stackrel{\mathbf{r}}{e}_{t} \times \stackrel{\mathbf{1}}{H} (\stackrel{\mathbf{r}}{r}, t)$ C. $\stackrel{\mathbf{r}}{e}_{n} \cdot \stackrel{\mathbf{1}}{D} (\stackrel{\mathbf{r}}{r}, t)$ D. $\stackrel{\mathbf{r}}{e}_{n} \cdot \stackrel{\mathbf{1}}{H} (\stackrel{\mathbf{r}}{r}, t)$

B.
$$\stackrel{\mathbf{r}}{e_t} \times \stackrel{\mathbf{l}}{H} (\stackrel{\mathbf{r}}{r}, t)$$

C.
$$\stackrel{\mathbf{r}}{e}_n \cdot \stackrel{\mathbf{1}}{D} (\stackrel{\mathbf{r}}{r}, t)$$

D.
$$\stackrel{\mathbf{r}}{e}_{n} \cdot \stackrel{\mathbf{1}}{H} (\stackrel{\mathbf{r}}{r}, t)$$

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Question2 (14 points)

Consider the transmitting dipole antenna shown in Figure 2. The dipole antenna lies in the xz plane and its two infintely thin arms are alinged along the z-direction. The dipole antenna is excited by a time harmonic voltage source in between its two arms.

- 1. What is the direction of the current on the dipole antenna (4 points)?
- 2. At the observation point P that is far away from the dipole antenna, the transmitted wave can be considered as uniform plane wave. If the electric field intensity of the uniform plane wave can be expressed as $\stackrel{1}{E}(\stackrel{\Gamma}{r}) = E_0 \stackrel{\Gamma}{a_z} e^{-jkx}$, where k is the wavenumber and $\stackrel{\Gamma}{a_z}$ is the unit vector along z- direction, respectively. What are the propagation direction and the polarization of the uniform plane wave (6 points)?
- 3. At the observation point P, another dipole antenna is used to recieve the electroamgnetic wave transmitted by the transmitting antenna. For maximum transmission, which direction should the receiving dipole antenna be orinted to (4 points)?



Transmitting dipole antenna

Figure 2

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Question3 (20 points)

Consider the infinitely large parallel conducting plates shown in Figure 2. The two plates are separated by two layers of dielectric: medium 1, and medium 2. The permittivity of medium 1 is ε_1 and of medium 2 is ε_2 respectively. Also, the conductivity of medium 1 and medium 2 are $\sigma_1 = 0$ and $\sigma_2 = 0$. The width of medium 1 and medium 2 is d_1 and d_2 , respectively. The voltage between the two plates is U, as shown in Figure 2.

- 1. Find the free charge density on the two plates (10 points).
- 2. Find the electric field intensity in medium 1 and medium 2 (6 points).
- 3. Calculate the capacitance per unit area of the parallel plates (4 points).

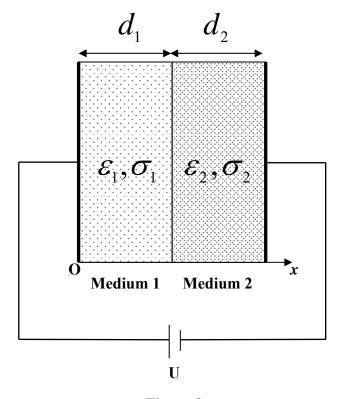


Figure 3

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Question4 (20 points)

In free space, a uniform plane wave is normally incident at the surface of a semi-infinite lossless medium. In the free space, the standing wave ratio of the total wave is 3, the wavelength in the lossless medium is 1/6 of the wavelength in free space, and the minimum of standing wave occurs at the boundary between free space and the medium.

- 1. Is the reflection coefficient at the interface between free spase and the lossless medium negative or positive (2 points);
- 2. Find the value of the reflection coefficient (6 points).
- 3. Find the relative permittivity and permeability of the medium (12 points).

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Question4 (20 points)

A uniform plane wave is propagating in free space. The magnetic field intensity of the wave can be expressed as $\overset{1}{H}=H_0\Big(\overset{\mathrm{V}}{e_y}+\overset{\mathrm{V}}{e_z}\Big)e^{-j(y-z)}.$ For this plane wave find an expression for:

- 1. The unit vector along the propagation direction of the plane wave (3 points);
- 2. The electric field intensity of the plane wave (4 points);
- 3. The wavenumber and period of the plane wave (4 points);
- 4. The average Poynting vector of the plane wave (3 points);

Moreover, Consider the perfect dielectric boundary shown in Figure 4. The angle between the boundary and y-axis is $\pi/4$. When the plane wave is incident on the boundary of a perfect conductor, find the expression of the electric field intensity of the reflected wave (6 points).

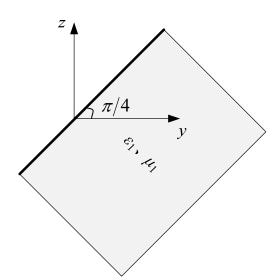


Figure 4 Boundary of a perfect dielectric

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Appendix

Cylindrical coordinate system

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\rho}) + \frac{1}{\rho} \frac{\partial A_{\phi}}{\partial \phi} + \frac{\partial A_{z}}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho} \begin{vmatrix} \mathbf{e}_{\rho} & \rho \mathbf{e}_{\phi} & \mathbf{e}_{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_{\rho} & \rho A_{\phi} & A_{z} \end{vmatrix}$$

$$\nabla^2 u = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \frac{\partial u}{\partial \rho}) + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2}$$

Spherical coordinate system

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \qquad \nabla \times \mathbf{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \mathbf{e}_r & r \mathbf{e}_\theta & r \sin \theta \mathbf{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \mathbf{e}_r & r\mathbf{e}_\theta & r\sin \theta \mathbf{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin \theta A_\phi \end{vmatrix}$$

$$\nabla^2 u = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2}$$