GLASGOW COLLEGE UESTC

Exam paper

Stochastic Signal Analysis (UESTC 3024)

Date: 11th Jan. 2021
Time: 14:00pm - 16:00pm

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.
For Multiple Choice Questions, use the dedicated answer sheet provided.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

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Q1 Let random signal $\{X(t) = A + B\cos t, t \in R\}$, where A and B are two statistically independent random variables. Known that $A \sim N(0,1), B \sim N(0,1)$. Please find

(a) The probability density function (PDF) of $X(t)$ and the two-dimensional PDF of $X(t)$ and $X(s)$ [15]

(b) Let $t_1 = 0$ and $t_2 = \frac{\pi}{2}$, determine the covariance matrix of $X(t)$ [10]

Q2 If the random signal $X(t) = a\cos(\omega_0 t + \theta) + N(t)$, where a and ω_0 are constants, and the random variable $\theta \sim U[0, 2\pi)$. The Gaussian white noise $N(t)$ satisfies: $N(t) \sim N(0,1)$, and is independent with r.v. θ. Please

(a) Find the mean of random signal $X(t)$ [8]

(b) Find the correlation function $R_x(t_1, t_2)$ of the random signal $X(t)$ [8]

(c) Show that whether or not the $X(t)$ is wide-sense stationary [2]

(d) Find the power spectrum of random signal $X(t)$ [7]

Q3 $X(t)$ is a stationary random process having an autocorrelation function

$$R_X(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| \leq T \\ 0, & |\tau| > T \end{cases}$$

If $X(t)$ is put through the RC circuit shown in Figure 1. Please find

(a) The power spectral density of the output signal $Y(t)$ [10]

(b) The power spectral density of $Q(t) = Y(t) - X(t)$ [15]

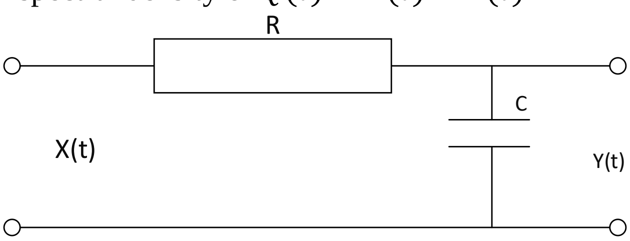

Figure 1

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Q4 Suppose $h(t) = e^{-bt}u(t), b > 0$ is the impulse response of an LTI system. The input signal $X(t)$ is a stationary zero-mean Gaussian signal and its correlation function is $R_X(\tau) = \sigma_X^2 e^{-a|\tau|}, (a > 0, a \neq b)$. Please find

(a) the power spectral density and correlation function of the output signal $Y(t)$ [8]

(b) the one-dimensional PDF of $Y(t)$ [10]

(c) $P\{Y(t) \geq 0\}$ [7]

End of question paper