GLASGOW COLLEGE UESTC

Exam paper

Calculus I (UESTC 1002)

Date: 5th Jan. 2020
Time: 09:30-11:30am

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.
Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Q1 Find the following limits and integrals (if they exist):

(a) $\lim\limits_{x\to 0} \dfrac{5-5\cos x}{e^x - x - 1}$ [5]

(b) $\lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}}}{x}$ [5]

(c) $\int_0^{1/2} x(1-4x^2)^{10}\,dx$ [10]

(d) $\int \cos^4(3x)\,dx$ [10]

(e) $\int x^2 \tan^{-1} x\,dx$ [10]

(f) $\int_{-1}^{1} \dfrac{1}{x^3}\sin\dfrac{1}{x^2}\,dx$ [10]

Q2 Determine a curve through the point $(1,1)$ whose length from $x = 1$ to $x = 2$ is

$$L = \int_1^2 \sqrt{1 + \frac{1}{x^2}}\,dx.\,[10]$$

Q3 Let $f(x) = x^{2/3}$:

(a) Find the length of the curve determined by f from $x = 0$ to $x = 8$. [10]
(b) Find the volume of the solid generated by revolving the region between the y–axis and the curve $f(x) = x^{2/3}, 0 \le x \le 8$, about y–axis. [10]

Q4   Suppose that $y = e^{2x} + (1+x)e^x$ is a particular solution of the following differential equation with constant coefficients:

$$y'' + ay' + by = ce^x.$$

(a) Determine the constants a, b and c. [10]
(b) Find the general solution of this differential equation. [10]