GLASGOW COLLEGE UESTC

Exam Paper

Elements of Information Theory (UESTC) (UESTC3021)

DATE: 6th Jan. 2020
Time: 18:30-20:30

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Continued overleaf

Q1 Let joint probabilities $\{p(X = x, Y = y)\}$ of information source X and information destination Y be given by the following *Table for Q1*

Table for Q1

X \ Y	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

Find

(a) The entropy $H(X)$ of X and the entropy $H(Y)$ of Y. [5]

(b) The conditional entropy $H(X|Y)$ of X, when we have the knowledge of Y, and the conditional entropy $H(Y|X)$ of Y, when we have the knowledge of X. [5]

(c) The joint entropy $H(X,Y)$. [5]

(d) The entropy difference $H(Y) - H(Y|X)$. [2]

(e) The mutual information $I(X;Y)$ between the information source X and the information destination Y. [3]

(f) Draw a Venn diagram for the quantities in (a) through (e). [5]

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Q2 Consider a random variable X which takes 6 values $\{A, B, C, D, E, F\}$ with probabilities $\{0.5, 0.25, 0.1, 0.05, 0.05, 0.05\}$

(a) Construct a binary Huffman code for this random variable. What is its average length? [7]

(b) Construct a quaternary Huffman code for this random variable, i.e., a code over an alphabet of four symbols (call them a, b, c and d). What is the average length of this code? [7]

(c) One way to construct a binary code for the random variable is to start with a quaternary code, and convert the symbols into binary using the mapping a→00, b→01, c→10 and d→11. What is the average length of the binary code for the above random variable constructed by this process? [4]

(d) For any random variable X, let L_H be the average length of the binary Huffman code for the random variable, and let L_{QB} be the average length of the code constructed by first building a quaternary Huffman code and converting it to binary. Show that

$$L_H \leq L_{QB} < L_H + 2$$ [4]

(e) The lower bound in (d) is tight. Give an example where the code constructed by converting an optimal quaternary code is also the optimal binary code. [3]

Continued overleaf

Q3 Consider a channel with binary inputs that has both erasures and errors. Let the probability of error be ε and the probability of erasure be α, so the channel is shown as *Figure Q3*:

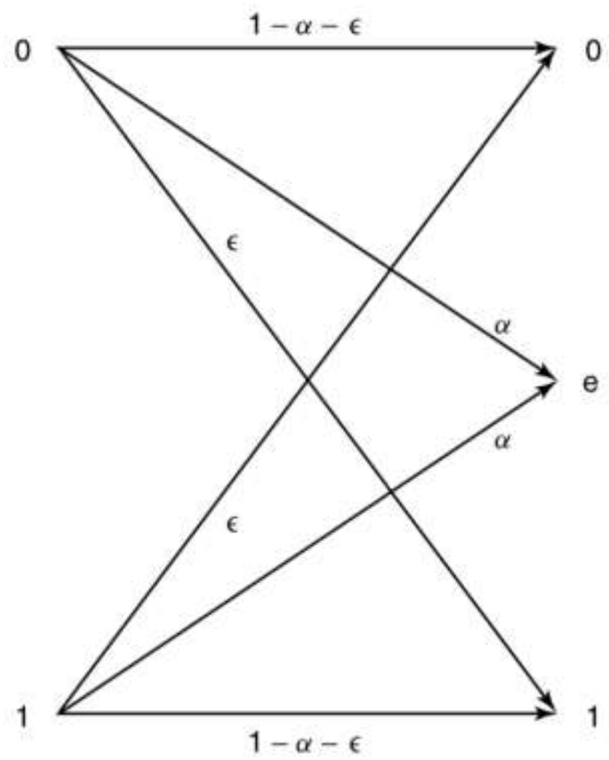

Figure Q3.

(a) Find the capacity of this channel. [10]

(b) Specialize to the case of the binary symmetric channel ($\alpha = 0$). [5]

(c) Specialize to the case of the binary erasure channel ($\epsilon = 0$). [5]

(d) Explain the usage of capacity for both binary symmetric channel and binary erasure channel, with application example in practical communications engineering or communications systems, respectively. [5]

Continued overleaf

Q4 Assume information rate for a designed signal over telephone line is 5.6×10^{4} bit/second, and the noise power spectrum density $N_0 = 5\times10^{-6}$ mW/Hz, the bandwidth of telephone line is assumed to be *F* Hz, the input power for the signal is *P* (W).

(a) If *F* = 4 kHz, what is the minimum input power *P* (W) to allow the probability of error at the receiver to be made arbitrarily small" rather than zero-error? [6]

(b) If $F \to \infty$, what is the minimum input power *P* (W) to allow the probability of error at the receiver to be made arbitrarily small" rather than zero-error? [6]

(c) Explain the implication of channel coding theorem on improving the zero-error transmission performance in digital communications systems, and present simple implementation example by repetition code like (3,1) repetition code with one information bit and two check bits. And what is the average error probability for the repetition codes of length 3 over a binary symmetric channel with the error probability 0.01, i.e., the transition probability from 1 to 0 or 0 to 1 equals to 0.1. [7]

(d) Consider a (7,4) Hamming codes over binary symmetric channel with the error probability 0.01, i.e., the transition probability from 1 to 0 or 0 to 1 equals to 0.01, what is average error probability? [6]

End of question paper