GLASGOW COLLEGE UESTC

Exam paper

Calculus II (UESTC 1003)

Date: 30th June 2019
Time: 14:30-16:30pm

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

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Q1 There are four statements as follows, but only one is true. Which of the following statements is true? ( ) Explain in detail why each statement is true or false.

(A) The derivative of $f(x,y) = xe^y + \cos(xy)$ at the point $(2,0)$ in the direction $\vec{v} = 3\vec{i} - 4\vec{j}$ is 1. [4]

(B) If $f(x,y)$ is defined in an open region R of the xy-plane and its corresponding partial derivatives f_x and f_y are bounded on R, then $f(x,y)$ is continuous on R. [5]

(C) At every point (x_0, y_0) in the domain of a differentiable function $f(x,y)$, the gradient of f is not always normal to the level curve through (x_0, y_0). [3]

(D) If a function $f(x,y)$ has the same limit L along all straight lines approaching (x_0, y_0), then the limit $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ exists and equals to L. [3]

Q2 Find the critical point of

$$f(x,y) = xy + 2x - \ln(x^2 y)$$

in the open first quadrant and show that f takes on a minimum value there. [15]

Q3 Evaluate the following two double integrals:

(a) $\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{(x^2+y^2)^2} dydx;$ [10]

(b) $\int_0^{\sqrt{\pi/2}} \int_y^{\sqrt{\pi/2}} \sin(x^2) dxdy.$ [10]

Q4 The Gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt,$$

generalizes the factorial function from the nonnegative integers to any real numbers.

(a) Show that

$$\int_0^\infty e^{-y^2} dy = \frac{\sqrt{\pi}}{2}.$$ [8]

(b) From the result of part (a), find the value of $\Gamma(1/2)$. [7]

Q5 Express the following triple integral in cylindrical coordinates iteratedly:

$$\iiint_D f(r,\theta,z) dV,$$

where D is the space region bounded below by the plane $z = 0$, laterally by the circular

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cylinder $x^2+(y-1)^2=1$, and above by the paraboloid $z=x^2+y^2$. [10]

Q6 Show that the differential form $3x^2dx+\frac{z^2}{y}dy+2z\ln ydz$ is exact and evaluate the integral

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2dx+\frac{z^2}{y}dy+2z\ln ydz$$

over any path from $(1,1,1)$ to $(1,2,3)$. [15]

Q7 Find the outward flux of the vector field $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ across the boundary of the cylindrical can D which is cut from the solid cylinder $x^2+y^2\le 4$ by the planes $z=0$ and $z=1$. [10]

End of question paper