GLASGOW COLLEGE UESTC

Exam paper

Calculus I (UESTC 1002)

Date:5th, Jan.2022
Time: 9:30-11:30

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.
Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Q1 The region R in the first quadrant is bounded above by the curve $y = x^{-\frac{1}{4}}$, on the left by the line $x = \frac{1}{16}$, and below by the line $y = 1$. Find the volume of the solid generated by revolving R about the x-axis. [15]

Q2 Find the following integrals (if they exist); prove otherwise.

(a) $\int \left(\ln x + \frac{1}{x} \right) e^x dx$; [10]

(b) $\int \frac{1}{x(x^2+1)^2} dx$; [10]

(c) $\int \frac{x}{\sqrt{4x-3}} dx$; [10]

(d) $\int_0^{\frac{1}{2}} \frac{x \, \mathrm{si}^{-1} x}{\sqrt{1-x^2}} dx$; [10]

(e) $\int_{-1}^1 \frac{e^{\frac{1}{x}}}{x^2} dx$. [10]

Q3 Let $f(x)$ be a continuous function on $[0,1]$.

(a) Prove that
$\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$; [10]

(b) By using the formula in part (a), evaluate the following definite integral:
$\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$. [10]

Q4 Assume that function $f(x)$ is defined on $(-\infty, \infty)$ and satisfies
$$f'(x) - e^x = \int_0^x \left(1 + f(t) \right) dt.$$
Find $f(x)$. [15]

End of question paper