GLASGOW COLLEGE UESTC

Exam

Stochastic Signal Analysis (UESTC3024)

Date: Jan.4th,2022
Time: 7:00-9:00pm

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Continued overleaf

Q1 Consider a random signal $\{X(t) = A + Bcost, t \in R\}$, where A and B are two statistically independent random variables. $A \sim N(0,1)$ and $B \sim N(0,1)$. Find:

(a) the mean and autocorrelation of the $X(t)$ [5]

(b) the one-dimensional and two-dimensional probability density function of $X(t)$ [10]

(c) If $\mathrm{t}_1 = 0$ and $\mathrm{t}_2 = \frac{\pi}{2}$, determine the characteristic function and the covariance matrix of $X(0)$ and $X(\frac{\pi}{2})$ [10]

Q2 Assume that $X(t)$ is the signal transmitted from a radar. After encountering a specific target, the signal returned to the radar receiver is weak and defined as $\alpha X(t - \tau_1)$, where α is a constant and $\alpha \ll 1$, and τ_1 is also a constant. Since the received signal is contaminated by noise, $N(t)$, the received signal is $Y(t) = \alpha X(t - \tau_1) + N(t)$. $X(t)$ and $N(t)$ are both independent zero-mean generalized stationary random signals. The autocorrelation functions of the $X(t)$ and $N(t)$ are $R_X(\tau)$ and $R_N(\tau)$. Find:

(a) the mean and autocorrelation of the $Y(t)$ [7]

(b) the cross-correlation of the $\mathrm{X}(t)$ and $Y(t)$ [5]

(c) determine whether or not $Y(t)$ is generalized stationary? Justify your answer. [5]

(d) determine whether or not $X(t)$ and $Y(t)$ are joint generalized stationary? Justify your answer [8]

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Q3 $X(t)$ is a stationary random process having an autocorrelation function

$$R_X(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T}, & |\tau| \le T \\ 0, & |\tau| > T \end{cases}$$

Assume $X(t)$ is the input signal to the RC circuit shown in Figure 1. Find:

(a) the power spectral density of the output signal $Y(t)$ [5]

(b) the power spectral density of $Q(t) = Y(t) - X(t)$ [7]

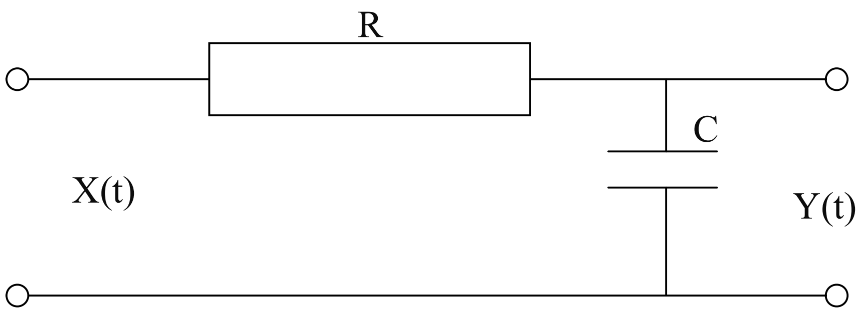

Figure 1

Assume $\mathrm{X}(t)$ is the input signal to the RL circuit shown in Figure 2. Please find:

(c) the power spectral density of the output signal $Y(t)$ [5]

(d) the power spectral density of $Q(t) = Y(t) - X(t)$ [8]

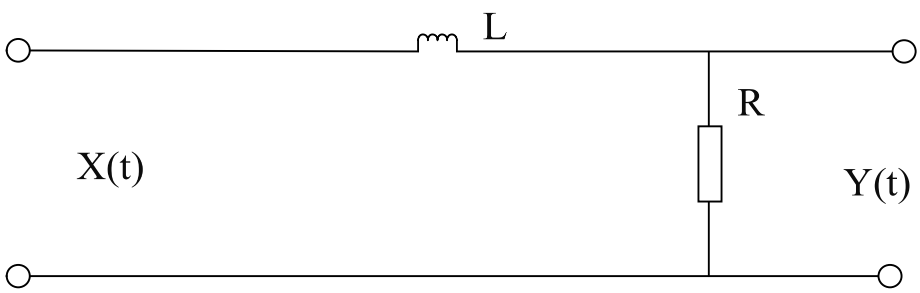

Figure 2

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Q4 Consider a narrow-band, stable, random process

$$X(t) = i(t) + cos\omega_0 t - q(t)sin\omega_0 t,$$

which has zero mean and PSD:

$$S_X(\omega) = \begin{cases} Pcos[\pi(\omega - \omega_0)/\Delta\omega], & |\omega - \omega_0| \leq \Delta\omega/2 \\ Pcos[\pi(\omega + \omega_0)/\Delta\omega], & |\omega + \omega_0| \leq \Delta\omega/2 \\ 0 & others \end{cases}$$

where $P, \Delta\omega$ and $\omega_0 \gg 0$ are all constants. Find:

(a) the average power of $X(t)$ [6]

(b) the power spectral density of $i(t)$ [7]

(c) the cross-correlation function $R_{iq}(\tau)$ and the PSD $S_{iq}(\omega)$ [7]

(d) whether $i(t)$ and $q(t)$ are orthogonal, unrelated or not? [5]

End of question paper