GLASGOW COLLEGE UESTC

Exam Paper

Stochastic Signal Analysis (UESTC3024)

Date: 4^{th} Jan. 2019
Time: 09:30-11:30 am

Attempt all PARTS

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown.

DATA/FORMULAE SHEET IS PROVIDED AT THE END OF PAPER

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

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Q1. In a circuit, the output voltage Y is related to the input voltage X. It is known that Y=5X+1, and X can be seen as a random variable uniformly distributed in [0, 1]. Determine the probability density function of Y. [10]

Q2. There are two signals received by a receiver, the arrival time of which are recorded as *X* and *Y* respectively. It is known that the joint probability density of *X* and *Y* is

$$f_{XY}(x,y) = A\sin(x+y), \quad 0 \ll x,y \ll \frac{\pi}{2}$$

(a) Determine the value of *A*. [4]

(b) Determine the expectation of *X*, *Y*. [6]

Q3. When discussing additive signal interference, assume the stationary random signal $W(t) = A\cos(\omega t)$ is interfered by the random signal $Z(t) = B\sin(\omega t)$, where ω is determined, A and B are independent Gaussian variables, and $E[A] = E[B] = 0$, $E[A^2] = E[B^2] = \sigma^2$. The signal after interference is $X(t) = W(t)+Z(t)$. Determine the expectation and autocorrelation function of X(t). [10]

Q4. A radar transmits a signal $W(t) = \cos(\omega t)$, then the echo signal is $X(t) = A\cos(\omega t + \Phi)$, where Φ is a random variable uniformly distributed in $[0, 2\pi]$, and A may be a constant, a time function or a random variable.

(a) Determine the expectation and autocorrelation function of X(t), notice that there is NO need to work out the value of $E[A^2]$. [5]

(b) Determine the time average of X(t). [5]

(c) On what conditions about A, X(t) would be ergodic process? [6]

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Q5. Consider Figure Q5, below. The random process X(t) acts on a series system with impulse responses $h_1(t)$ and $h_2(t)$. X(t) is a stationary process, and $h_1(t)$ and $h_2(t)$ are Linear time-invariant systems (LTI systems). Determine the cross-correlation function of $Y_1(t)$ and $Y_2(t)$ expressed by the autocorrelation functions of $h_1(t)$, $h_2(t)$, and X(t). [7]

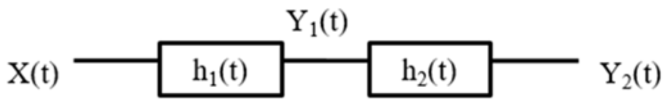

Figure Q5.

Q6. The power spectral density of a stationary random signal X(t) is

$$S_X(\omega) = \frac{4}{\omega^2 + 4}$$

The center frequency of X(t) is ω_0, and $\omega_0 = 0$. X(t) pass through a system with a frequency response of $H(\omega) = \frac{1}{1 + j\omega}$ and output Y(t).

(a) Determine the mean and average power of Y(t). [8]

(b) Determine the equivalent noise bandwidth of the system. [5]

(c) Determine the rectangular equivalent bandwidth of Y(t). [3]

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Q7. A known zero-mean stationary Gaussian noise X(t), has power spectral density as shown in Figure Q7.

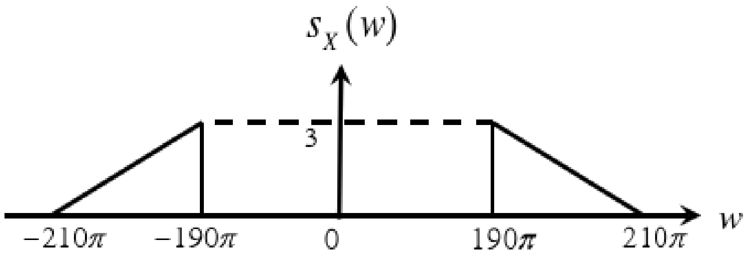

Figure Q7.

(a) Sketch the power spectrum graph of $S_i(\omega)$ or $S_q(\omega)$, and determine $R_i(\tau)$ and $R_q(\tau)$, the autocorrelation function of the in-phase and quadrature components. [10]

(b) Determine $f_{iq}(i, q;\ t, t)$, the joint density function of in-phase and quadrature components. [6]

Q8. Let the transfer function of the circuit be

$$H(j\omega) = \frac{1}{1 + j\omega}$$

and the input voltage be $X(t) = \cos(2\omega t + \Phi)$, where Φ is a random variable uniformly distributed on $[0, 2\pi]$. Determine the autocorrelation function of the output voltage Y(t). [15]

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Selected Fourier transforms

$f(t)$	$\mathcal{F}\{f(t)\}$
$\sin(\omega_0 t)$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$e^{-at}u(t)$	$\frac{1}{a+j\omega},\ a>0$
$e^{-a\lvert t\rvert}$	$\frac{2a}{\omega^2+a^2},\ a>0$
$\frac{W}{\pi}Sa(Wt)=\frac{\sin(Wt)}{\pi t}$	$F(j\omega)=\begin{cases}1.\lvert\omega\rvert<W\\0,\lvert\omega\rvert>W\end{cases}$

End of question paper