Glasgow College, UESTC

Signals and Systems —Semester 2, 2017 - 2018

Final Exam

10:00—12:00, Friday, 6th, July, 2018

<u>Notice:</u> Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.

All questions are compulsory. There is a maximum of 100 marks in total.

The following table is for grader only:

Question	**1**	**2**	**3**	**4**	**5**	**6**	**Total**	**Grader**
Score								

Score

Question 1 (5x4=20 points) Following multi-choice questions. Write down one correct answer in the blank.

(1) $\int_{-5}^{5} u_1(1-t)\cos(2\pi t)dt + \int_{-5}^{5} u_1(6-t)\sin(2\pi t)dt = ($).

(a) -1 **(b)** 0

(c) 1 **(d)** 3

(2) If an Linear Time Invariable(LTI) system has the response $y_1[n]$ to the input $x_1[n]$ as in Figure 1-1 shows, the response of this LTI systems to x[n] is ( ).

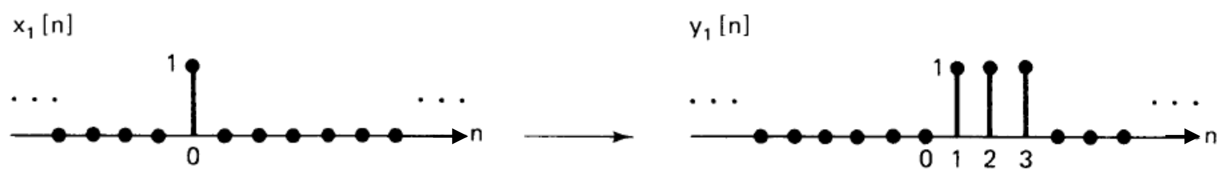


Figure 1-1

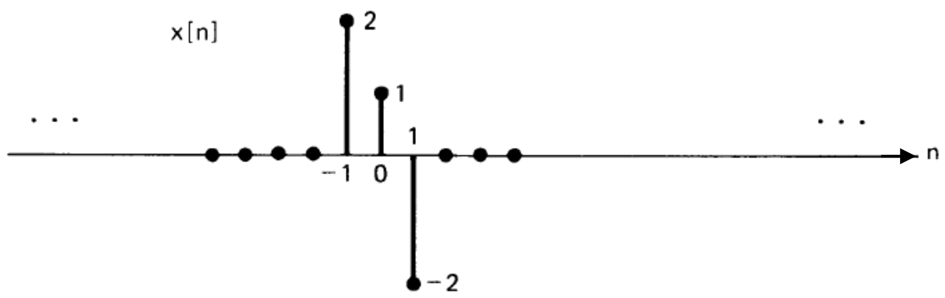


Figure 1-2

(a) $y[n] = \{1,2,0,1,2\}, n = 0,1,2,3,4$ **(b)** $y[n] = \{2,3,2,0,-1\}, n = -1,0,1,2,3$

(c) $y[n] = \{2, 3,1,-1,-2\}, n = -1,0,1,2,3$ **(d)** $y[n] = \{2, 3,1,-1,-2\}, n = 0,1,2,3,4$

(3) Determine which of the following statements concerning LTI systems is false. ( ).

(a) If h(t) is the impulse response of an LTI system and h(t) is periodic and nonzero, the system is unstable.

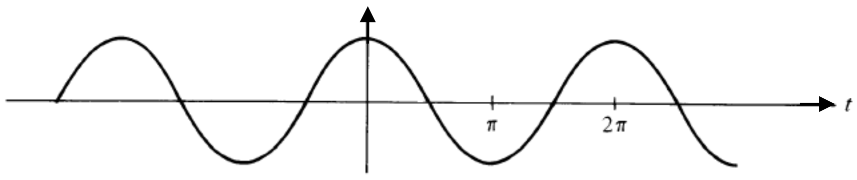
(b) If a discrete-time LTI system has an impulse response h[n] of finite duration, the system is stable.

(c) If an LTI system is causal, it is stable.

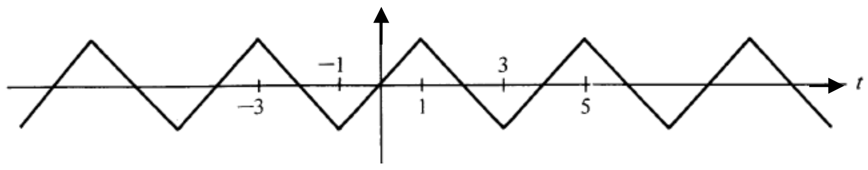
(d) A continuous-time LTI system is stable if and only if its impulse response h(t) is absolutely integrable.

(4) Determine which Fourier series coefficient of the following periodic signals is purely imaginary and only has odd harmonic components. ( ).

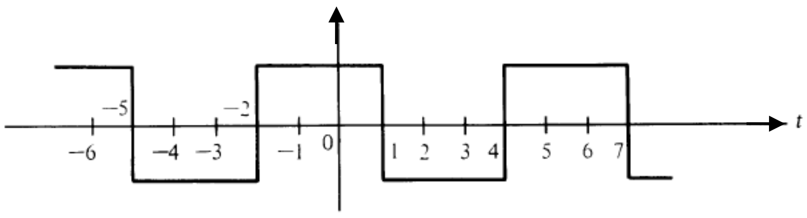
(a)



(b)



(c)



(d) $x(t)$ is periodic with period 4, which expression is

$$x(t)=\begin{cases}\sin \pi t, & 0\le t\le 2\\ 0, & 2<t<4\end{cases}$$

(5) How many different signals of which Laplace transform have the form of ( ).

$$\frac{(s\text{-}1)}{(s+2)(s+3)(s^2+s+1)}$$

(a) 2 **(b)** 3
(c) 4 **(d)** 5

Score

Question 2 (20 points) Electrical circuits are widely used to implemented continuous-time filtering operations. One of the simplest examples of such a circuit is first-order RC circuit depicted in Figure 2-1, where the source voltage $v_s(t)$ is the system input. This circuit can be used to perform either a lowpass or highpass filtering operation, depending upon what we take as the output. In this case, we take the capacitor voltage $v_c(t)$ as the output.

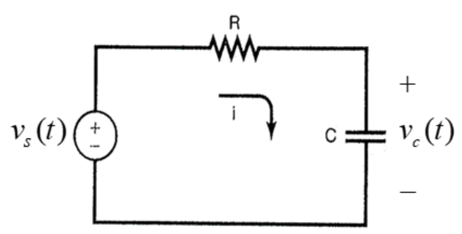


Figure 2-1

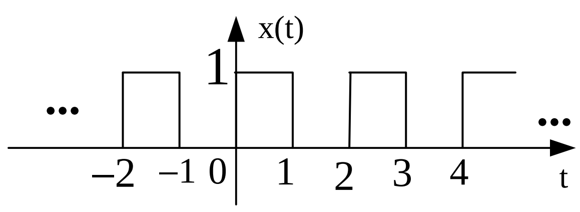


Figure 2-2

(a) Please model this system using the linear constant-coefficient differential equation (3 points).

(b) Please determine the frequency response $H(j\omega)$ of the RC circuit. Determine the expression of the magnitude and phase of $H(j\omega)$ and sketch them.(8 points)

(c) Is the system a highpass or a lowpass filter? (2 points).

(d) Let $x(t)$ shown in Figure 2-2 to be the input signal $v_s(t)$, let $y(t)$ to be the output signal $v_c(t)$,please determine the expression of the output response $y(t)$.(7 points)

Score

Question 3 (20 points) Consider the communication system shown in Figure 3-1. The carried signals are $m(t)=d(t)=Cos(\omega_c t)$,the frequency response of Filter 1 is $H_1(j\omega)$, the frequency response of Filter 2 is $H_2(j\omega)$.The Fourier transform of $x(t)$ is given by Figure 3-2. The Fourier transform of s(t) is given by Figure 3-3.

(a) Sketch the Fourier transform of $r_1(t)$, $r_2(t)$, $r_3(t)$, $r_4(t)$, and $y(t)$.（15 points）

(b) Please optimize the Filter 2 to let the output response of the whole communnication system to be the original input signal $x(t)$, sketch the optimized frequency response of Filter 2. (5 points)

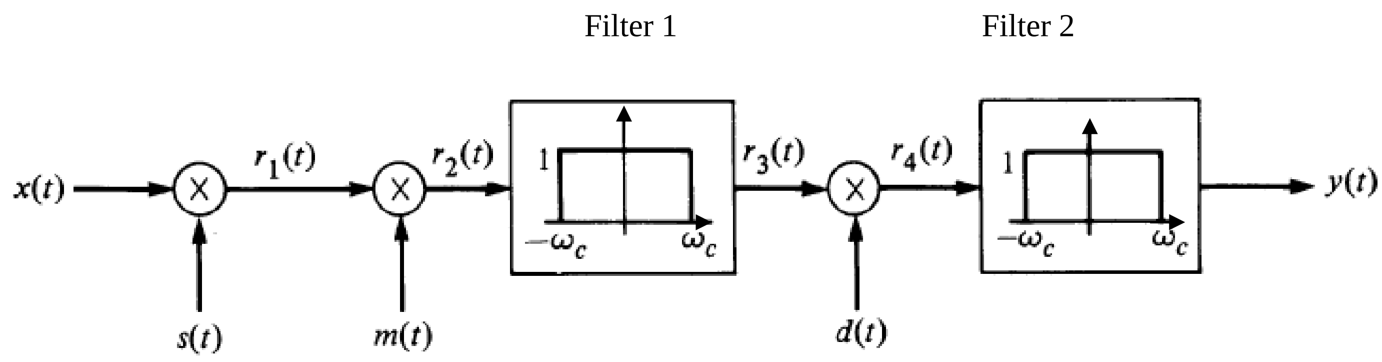


Figure 3-1

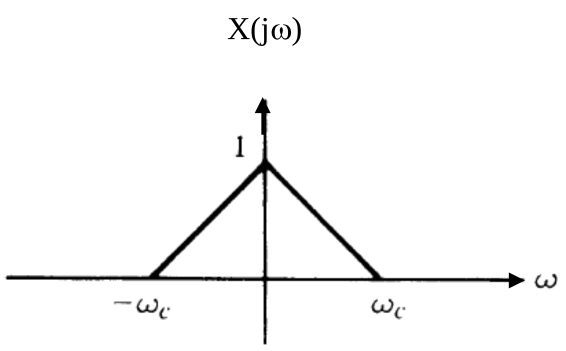


Figure 3-2

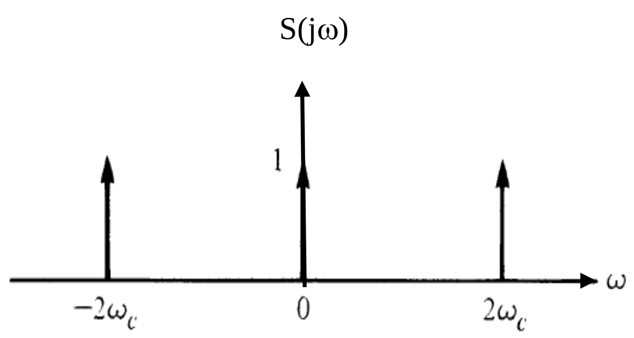


Figure 3-3

Score

Question 4 (10 points) Consider the sample and recover system in Figure 4, find the maximum value of *T* (denoted as *Tmax*), the range of *Wc*, such that $x_r(t) = x(t)$ in the following cases. *Where*

$x(t) \xleftrightarrow{FT} X(j\omega)$, $x_1(t) \xleftrightarrow{FT} X_1(j\omega)$, $x(t) \xleftrightarrow{FT} X(j\omega)$, $x_2(t) \xleftrightarrow{FT} X_2(j\omega)$

(a) Let $X(j\omega) = 0$ for $|\omega| > \text{Wmax}$. (4 points)

(b) Let $X_1(j\omega) = 0$ for $|\omega| > 2\text{Wmax}$, $x(t) = x_1(10t)$. (3 points)

(c) Let $X_1(j\omega) = 0$ for $|\omega| > 2\text{Wmax}$ and $X_2(j\omega) = 0$ for $|\omega| > \text{Wmax}$, $x(t) = x_1(t) * x_2(t)$. (3 points)

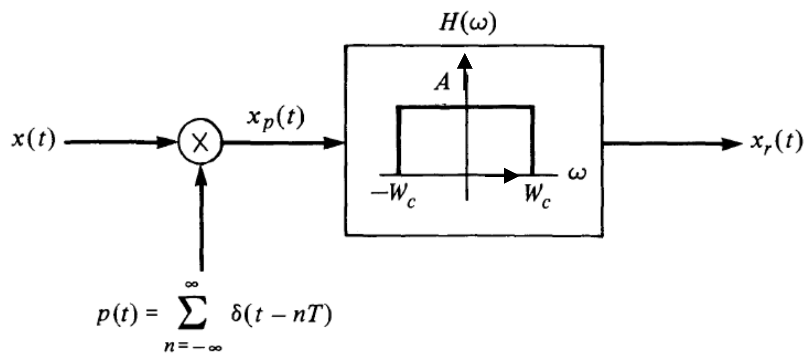


Figure 4

Score

Question 5 (15 points) We are given the following facts about a real system function *h(t)* with Laplace transform *H(s)*:

1. *H(s)* has exactly two poles.
2. *H(s)* has no zeros in the finite s-plane.
3. *H(s)* has a pole at $s=-1+j$.
4. $e^{2t}h(t)$ is not absolutely integrable.
5. $H(0)=8$.

Try to :

(a) Determine the system function of *H(s)* (5 points)

(b) Determine the differential equation relating the input *x(t)* and *y(t)*. (5 points)

(c) Draw a direct-form block-diagram representation of this system. (5 points)

Score

Question 6 (15 points) Consider the digital filter structure shown in Figure 5. x[n] is the input sequence, y[n] is the output sequence of the LTI system.

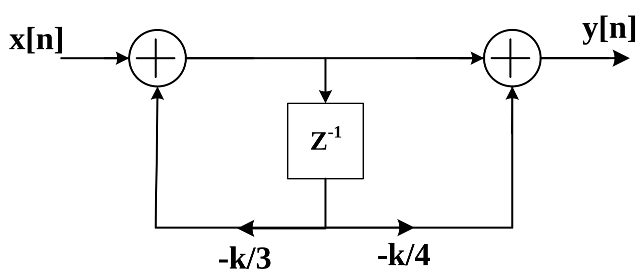


Figure 5

(a) Find *H(z)* for this causal filter. (5 points)

(b) Plot the pole-zero pattern and indicate the region of convergence. (5 points)

(c) For what values of the *k* is the system stable? (2 points)

(d) Determine *y[n]* if k=1 and $x[n]=(2/3)^n$ for all n. (3 points)