GLASGOW COLLEGE UESTC

Exam Paper

Elements of Information Theory (UESTC) (UESTC3021)

DATE：Jan 6th，2022
Time：1430-1630

Attempt all PARTS. Total 100 marks

- **Use one answer sheet for each of the questions in this exam.**
- **Show all work on the answer sheet.**
- **Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.**
- **An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.**
- **All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.**
- **The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.**

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Q1 Entropy H can be exploited as a measure of correlation. Two information sources X_1 and X_2 are identically distributed, but not necessarily independent. $H(X_2|X_1)$ represents the conditional entropy of X_2 given X_1. Let

$$\rho = 1 - \frac{H(X_2 \mid X_1)}{H(X_1)}$$

represent a measure of correlation between the information sources X_1 and X_2.

(a) The mutual information is represented by $I(X_2; X_1)$. Show that

$$\rho = \frac{I(X_2; X_1)}{H(X_1)}$$ [8]

(b) Show $0 \le \rho \le 1$ [7]

(c) When is $\rho = 0$? [4]

(d) When is $\rho = 1$? [6]

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Q2 Given a following single-symbol information source $X = \begin{pmatrix} x_1 & x_2 & x_3 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$, its expanded source of order two is denoted as X^2, which needs to be encoded by the following prefix codes.

(a) Find a binary Huffman code for X^2, its expected code length and its coding efficiency. [8]

(b) Find a binary Shannon code for X^2, its expected code length and its coding efficiency. [9]

(c) Given an arbitrary information source $Y = \begin{pmatrix} y_1 & y_2 & \cdots & y_n \\ p(y_1) & p(y_2) & \cdots & p(y_n) \end{pmatrix}$, demonstrate that by using the binary Shannon code, the expected code length $\overline{K}_Y$ satisfies the inequalities of $H(Y) \le \overline{K}_Y < H(Y) + 1$ [6]

(d) Discuss the reason why the coding efficiency of a Shannon code is low. [2]

Continued overleaf

Q3 Consider a 26-key typewriter.

(a) If pushing a key results in printing the associated letter, what is the capacity C in bits? [8]

(b) Now suppose that pushing a key results in printing that letter or the next (with equal probability). Thus A → A or B, . . . ,Z → Z or A. What is the capacity? [10]

(c) What is the highest rate code with block length one that you can find that achieves zero probability of error for the channel in part (b). [7]

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Q4 Consider the following block codes.

(a) Is the (5,1) repetition code a linear block code? If it is, write its generator matrix and all of codewords. Can the (5,1) repetition code detect and decode the error transmission simultaneously? Why? What is the minimum distance of the (5,1) repetition code? [6]

(b) Is the (5,4) even parity code a linear block code? If it is, write its generator matrix. When the info bit sequence is $\boldsymbol{m} = (m_0, m_1, m_2, m_3) = (1010)$, calculate the codeword after encoding. Can the (5,4) even parity code detect and decode the error transmission simultaneously? What is the minimum distance of (5,4) even parity code? [6]

(c) Consider the Binary Symmetric Channel (BSC) shown in Figure 4(c), where the crossover probability is denoted by p. Compute the average decoding error probability using the (5,1) repetition code to pass through the BSC channel. [5]

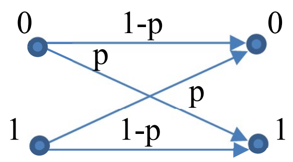

Figure 4(c) Binary symmetric channel

(d) Consider the following linear block code, which is generated by the rules below

$$x_1 = u_1$$

$$x_2 = u_2$$

$$x_3 = u_3$$

$$x_4 = u_1 \oplus u_2$$

$$x_5 = u_1 \oplus u_3$$

$$x_6 = u_2 \oplus u_3$$

$$x_7 = u_1 \oplus u_2 \oplus u_3$$

Write down the generator matrix and parity check matrix for the code. Write out a decoding table for the code, assuming a BSC with crossover probability $p < \frac{1}{2}$. What is the minimum Hamming distance of the code? [8]

End of question paper