GLASGOW COLLEGE UESTC

Exam paper

Calculus I (UESTC 1002)

Date: 6th Jan. 2021
Time: 09:30am - 11:30am

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.
Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Continued overleaf

Q1 Find the following limits and integrals (if they exist):

(a) $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x$ [5]

(b) $\lim_{x\to 0}\frac{\int_{\cos x}^{1} e^{-t^2}\,dt}{\sin^2 x}$ [5]

(c) $\int \sec^3 x\,dx$ [10]

(d) $\int_0^{\pi}\sqrt{\cos^2 x-\cos^4 x}\,dx$ [10]

(e) $\int \frac{1}{1+\sqrt[3]{x+2}}\,dx$ [10]

(f) $\int x\sin^{-1} x\,dx$ [10]

Q2 Suppose that x and y satisfy the following equation:

$$x=\int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt.$$

Show that $\frac{d^2y}{dx^2}$ is proportional to y and find the constant of proportionality. [10]

Q3 Let $f(x)=\frac{x^3}{12}+\frac{1}{x}$.

(a) Find the length of the curve determined by f from $x=1\ to\ x=4$. [10]

(b) Find the area of surface generated by revolving the above curve $f(x), 1\le x\le 4,$ about the x-axis. [10]

Q4 (a) Let $a>0$. Show that

$$y=\frac{1}{a}\int_0^x f(t)\sin(a(x-t))dt$$

is a solution of the following initial value problem

$$\frac{d^2y}{dx^2}+a^2y=f(x),\ \frac{dy}{dx}=0\ and\ y=0\ when\ x=0.$$ [10]

(b) If $f(x)=0$, find the general solution of $\frac{d^2y}{dx^2}+a^2y=0$. [10]

End of question paper