GLASGOW COLLEGE UESTC

Exam paper

Digital Signal Processing (UESTC 4005)

Date: 1, July, 2021

Time: 9:30-11:30

Attempt all PARTS. Total 100 marks

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

DATA/FORMULAE SHEET IS PROVIDED AT THE END OF PAPER

Reference table 1:Parameters of fix windows

Type of Window	Main lobe width	Relative sidelobe level	Minimum stopband attenuation	Transition bandwidth
Rectangular	4π/(2M+1)	13.3dB	20.9dB	0.92π/M
Hann	8π/(2M+1)	31.5dB	43.9dB	3.11π/M
Hamming	8π/(2M+1)	42.7dB	54.5dB	3.32π/M
Blackman	12π/(2M+1)	58.1dB	75.3dB	5.56π/M

$$\textit{Hann}:\ w[n] = \frac{1}{2}\left[1+\cos\left(\frac{\pi n}{M}\right)\right], \quad -M \le n \le M,$$

$$\textit{Hamming}:\ w[n] = 0.54 + 0.46\cos\left(\frac{\pi n}{M}\right), \quad -M \le n \le M,$$

$$\textit{Blackman}:\ w[n] = 0.42 + 0.5\cos\left(\frac{\pi n}{M}\right) + 0.08\cos\left(\frac{2\pi n}{M}\right), \quad -M \le n \le M.$$

Continued overleaf

Q1 Given a causal system with the transfer function being $H(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}}$.

(1) Calculate its difference equation. [4]

(2) Sketch the zero-pole plot of the system, then determine whether the system is stable. [6]

(3) Determine its impulse response. [6]

(4) Sketch a canonic direct realization of the system. [4]

Q2 Computation:

(1) Consider a length-9 sequence x[n] = {2, -3, -1, 0, -4, 3, 1, 2, 4}, $-2 \le n \le 6$. The z-transform X(z) of x[n] is sampled at N points $\omega_k = 2\pi k/N$, $0 \le k \le N-1$, on the unit circle yielding the frequency samples:

$$\tilde{X}[k] = X(z)|_{z=e^{j2\pi k/N}}, \quad k = 0,\ 1,\ \cdots,\ N-1$$

Determine the periodic sequence $\tilde{x}[n]$ whose discrete Fourier series coefficients are given by $\tilde{X}[k]$ as

(a) N = 4 [10]

(b) N = 11 [10]

respectively (without evaluating $\tilde{X}[k]$).

(2) Let H(z) be a lowpass filter with the magnitude response as shown in figure Q2, and its impulse response h[n] is real.

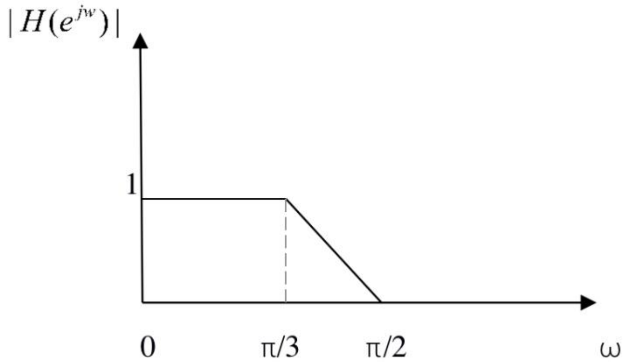

Figure Q2

Sketch the magnitude responses of the following systems.

(a) H(-z)+H(z) [5]

(b) $H(z^2)$ [5]

(c) $H(-z^2)$ [5]

(d) 1-H(z) [5]

Continued overleaf

Q3 Use the windowed Fourier series method to design a causal low-pass FIR digital filter with the following specifications:

Pass band edges f_p:	6kHz
Stop band edges f_s:	8kHz
Pass band ripple α_p:	1dB
Stop band attenuation α_s:	43dB
Sampling frequency F_T:	20kHz

Requirements:

(1) Write down the impulse response of the ideal lowpass filter. [6]

(2) Choose a fixed window which can make the filter be the least order to perform the design. [4]

(3) Based on the window chosen in (1), try to determine the order of the filter. [4]

(4) Write down the impulse response of the filter. [6]

Q4 A continuous time signal $x_c(t)$ with FT being $X_c(j\Omega)$ as shown in Figure Q4.1 is sampled at sampling interval $T = 2\pi/\Omega_0$ to get a sequence $x[n] = x_c(nT)$.

(1) for $|\omega| < \pi$, plot the DTFT $X(e^{j\omega})$ of x[n]. [6]

(2) If we want to recover $x_c(t)$ from x[n] as shown in Figure Q4.2, mark the parameters of the analog filter of $H_r(j\Omega)$ with the narrowest bandwidth of passband. Assume the ideal filter can be used. [6]

(3) Is it possible to recover $x_c(t)$ from x[n] for other values of T in the case of under sampling? If your answer is positive, please determine the range of T (in term of Ω_0). [8]

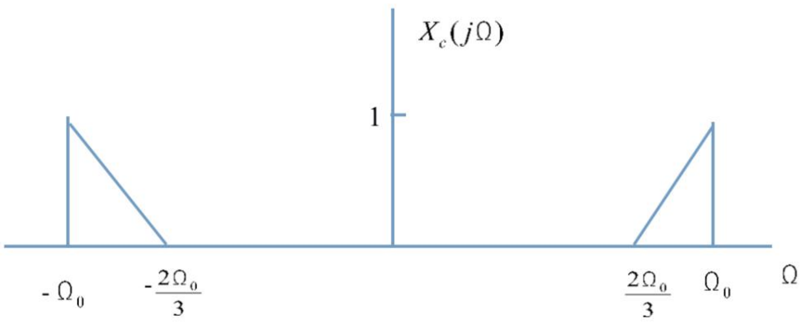

Figure Q4.1

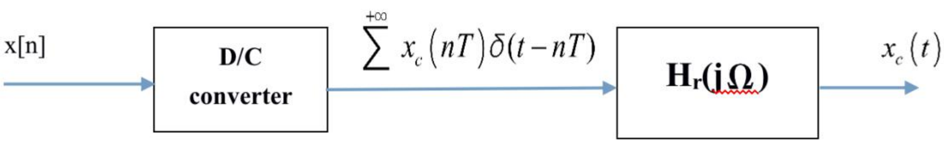

Figure Q4.2

End of question paper