GLASGOW COLLEGE UESTC

Exam paper

Linear Algebra and Space Analytic Geometry (UESTC1001)

Date: 21st, June, 2021
Time: 14:30-16:30

Attempt all PARTS. Total 100 marks

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Q1 **Fill in the blanks.** [20]

(a) Suppose A is a 3-by-3 matrix， $|A|=-2, |B|=3$, $|3adj(A)B^{-1}|=$________. [4]

(b) Let $\theta = \frac{\pi}{6}$, $A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then $A^6 x =$________. [4]

(c) If the inverse of A is $A^{-1} = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$, then A=__________. [4]

(d) Suppose $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & x \end{bmatrix}$ is similar to $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & y \end{bmatrix}$, then x=____, y=____. [4]

(e) Let $A = \begin{bmatrix} 4 & 0 & 2 \\ -1 & 1 & 0 \\ 2 & -2 & 6 \end{bmatrix}$, then Rank(2$I$-$A$)=__________. [4]

Q2 **Computations – Solve the following problems.** [35]

(a) Let $A = \begin{bmatrix} 0 & x & y & z \\ x & 1 & 0 & 0 \\ y & 0 & 1 & 0 \\ z & 0 & 0 & 1 \end{bmatrix}$, calculate det$A$. [10]

(b) Let A be the matrix of the quadratic form

$$9x_1^2 + 7x_2^2 + 11x_3^2 - 8x_1x_2 + 8x_1x_3$$

where the eigenvalues of A are 3, 9, and 15. Find an **orthogonal** matrix P such that the change of variable $x = Py$ transforms $x^T Ax$ into a quadratic form with no cross-product term. <u>Give P and the new quadratic form</u>. [15]

(c) Let $A = \begin{bmatrix} 2 & -1 & 3 & -1 & 1 & -5 \\ -1 & 4 & 2 & 11 & -3 & 10 \\ 1 & 1 & 3 & 4 & 2 & -7 \end{bmatrix}$. Find a basis for the null space and a basis for the column space of A. [10]

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Q3 **Discussions – Answer the following questions.** [25]

(a) Let $A = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & a & 0 \end{bmatrix}, \alpha = \begin{bmatrix} b \\ c \\ 1 \end{bmatrix}$, if α is an eigenvector of A corresponds to eigenvalues -2, what are the values for a, b and c? [10]

(b) Let $v_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$. Is b in $span\{v_1, v_2\}$? If not, find the orthogonal projection of b onto $span\{v_1, v_2\}$. [15]

Q4 **Proofs – Prove the following statements.** [20]

(a) Suppose $v_1, \ldots, v_m$ is linearly independent in V, $w \in V$, and $v_1 + w, \ldots, v_m + w$ is linearly dependent. Prove that $w \in \text{span}\{v_1, \ldots, v_m\}$. [10]

(b) If A is a real symmetric matrix, and $A^2=0$, prove that $A=0$. [10]

End of question paper