GLASGOW COLLEGE UESTC

Final Exam

Calculus II (UESTC 1003)

Date: 2021 June 25th
Time: 0930-1130

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Continued overleaf

Q1

Let the function with two variables as

$$f(x,y)=\begin{cases}(x^2+y^2)\sin\dfrac{1}{x^2+y^2}, & x^2+y^2\neq 0\\ 0, & x^2+y^2=0.\end{cases}$$

(a) Calculate the partial derivatives $f_x(x,y)$ and $f_y(x,y)$;

[10]

(b) Is the function $f_x(x,y)$ continuous at $(0,0)$? Give your reasons in detail.

[5]

Q2

(a) Find the derivative of the surface $f(x,y)=x^2-xy+y^2$ at the point $P(-1,1)$ in the direction $v=2i+j$;

[10]

(b) Find the directions in which f increases most rapidly and decreases most rapidly at the point P.

[4]

(c) What are the directions of zero change in f at P.

[2]

(d) Write down the tangent plane and normal line of the surface f at P.

[4]

Continued overleaf

Q3

Evaluate the following integrals:

(a) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$;

[10]

(b) $\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y)\, dydx$;

[10]

(c) Rewrite the following integral as an equivalent iterated integral in cylindrical coordinates:

$$\int_{-1}^{1} \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \int_0^{x^2+y^2} f(x,y,z)\, dzdydx \,;$$

[10]

(d) Find the volume of the region which is cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$;

[10]

(e) Calculate the triple integral $\iiint_D (x^2 + y^2) dV$, here the domain D is cut from the solid sphere $\rho \le 1$ by the cone $\phi = \pi/3$.

[10]

Q4

(a) Write down the tangent form of the Green's Theorem for the vector field $F = Mi + Nj$ and curve C. Make sure that the conditions on F and C are clear.

[5]

(b) Show that the outward flux of the position vector field $F = xi + yj$ across any closed curve to which Green's Theorem applies is twice the area of the region enclosed by the curve.

[10]

End of question paper