GLASGOW COLLEGE UESTC

Exam paper

Digital Signal Process (UESTC4005)

Date: 30th June 2019
Time: 09:30-11:30am

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

An electronic calculator may be used provided that it does not allow text storage or display, or graphical display.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Continued overleaf

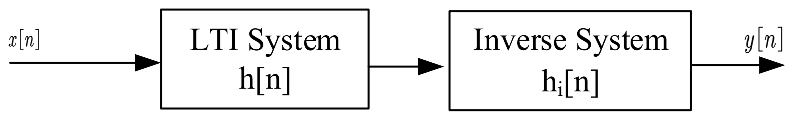
Q1 For the following systems specified by the input-output relationships, determine whether each of these systems is linear or not, and time invariant or not. Please give the reason.

(a) $y[n]=2x[n]+1$ [5]

(b) $y[n]=(x[n])^3$ [5]

(c) $y[n]=x[n]e^{j\pi/3n}$ [5]

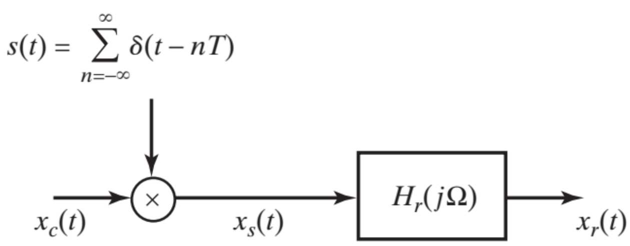
Q2 The cascade of an LTI system with its inverse system is shown below.



(a) The impulse response of the first system is $h[n]=\delta[n]+2\delta[n-1]$. Design the stable system $h_i[n]$, such that $h[n]\circledast h_i[n]=\delta[n]$ (specify the impulse response of the inverse system). Is this inverse system causal? [10]

(b) The impulse response of the first system is $h[n]=\delta[n]+a\delta[n-1]$. We want the inverse system $h_i[n]$ causal and stable. Please specify the requirements of a. [10]

Q3 The block diagram of the sampling and reconstruction system is shown as below



Assume the input signal is

$$x_c(t)=2\cos(100\pi t)+\cos(300\pi t) \qquad -\infty<t<\infty$$

The frequency response of the reconstruction filter is

$$H_r(j\Omega)=\begin{cases}1 & |\Omega|\le\pi/T\\ 0 & |\Omega|>\pi/T\end{cases}$$

(a) Determine the continuous time Fourier transform of $x_c(t)$, and sketch its spectrum [5]

(b) Assume that the sampling frequency is $f_s=1/T=500$ samples/second. Sketch the Fourier transform of the sampled signal $X_s(j\Omega)$ for the interval $-2\pi/T<\Omega<2\pi/T$. Write the expression of the reconstructed signal $x_r(t)$ [10]

(c) Assume the sampling frequency is $f_s=1/T=250$ samples/second, repeat part (b) [10]

Continued overleaf

Q4 Let x[n] be the sequence of length L=8, and h[n] be the impulse response of the system. The sequence x[n], and h[n] are given as:

$$x[n] = \cos\left(\frac{\pi}{2}n\right)\{u[n] - u[n-8]\} \qquad h[n] = [1, 2, -2, -1]$$

(a) Compute the time domain response of the system. [5]

(b) Let $X_N[k]$, $H_N[k]$ be the N point DFT of the sequence x[n] and impulse response h[n] respectively. If we choose N=8. Find the 2th point of the 8-point inverse DFT of [10]

$$Y_8[k] = X_8[k]H_8[k]$$

(c) Compare the result of (a) and (b) and explain the result [5]

Q5 A discrete time low pass filter is used to filter the continuous time signal with 3-dB cut-off frequency of $\Omega_c = 1kHz$. The sampling rate of $\Omega_s = 8k$ samples/sec.

(a) Design the discrete-time low-pass filter following the bilinear transformation method to a continuous time second order Butterworth analog filter. [15]

(b) Determine the difference equation of the resulting digital filter and draw the block diagram of the discrete time filter in [5]

(hint: the normalized transfer function of the second-order Butterworth analog filter is in the form of $H_a(s) = \dfrac{1}{(s/\Omega_c)^2 + \sqrt{2}(s/\Omega_c) + 1}$, and $\tan\left(\dfrac{\pi}{8}\right) = \sqrt{2} - 1$

End of question paper