UESTC Student ID_____ UoG Student ID_____ Course Title_____ Lecturer_____

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**Glasgow College, UESTC**

**Calculus I —Semester 1, 2018 – 2019**

**Final Exam**

**10:00—12:00, AM, Tuesday, January 8, 2019**

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**Notice: Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.**

**All questions are compulsory. There are 6 questions and a maximum of 100 marks in total.**

**The following table is for grader only:**

| Question | 1 | 2 | 3 | 4 | 5 | 6 | Total | Grader |
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**Question 1 (16 points)**

**Find the following limits:**

**(a)** $\lim_{x\to 0}\dfrac{x-\tan x}{x^3}$; **(4 points)**

**(b)** $\lim_{x\to+\infty}\dfrac{\frac{p}{2}-\tan^{-1}x}{\sin\frac{1}{x}}$; **(4 points)**

**(c)** $\lim_{t\to 0}\left(\dfrac{1}{\sin t}-\dfrac{1}{t}\right)$; **(4 points)**

**(d)** $\lim_{x\to 0^+} x(\ln x)^2$. **(4 points)**

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**Question 2 (8 points)**

**Find the surface area of the cone frustum generated by revolving the line segment** $y=(x/2)+(1/2),1£x£3,$ **about the** $y$ **-axis.**

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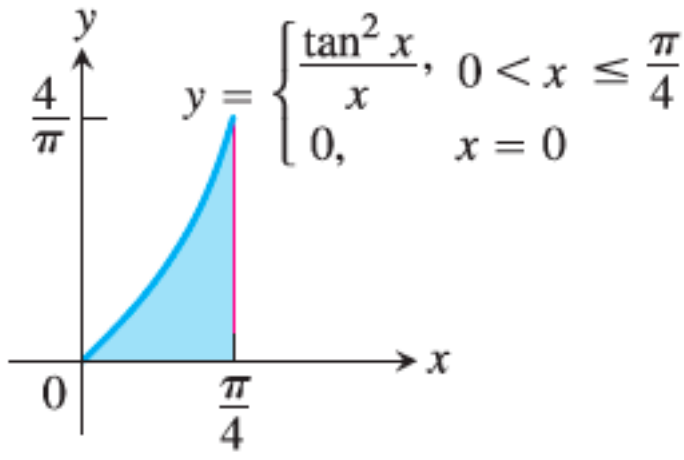
**Question 3 (10 points)**

**Let**

$$f(x)=\begin{cases}(\tan x)^2/x, 0<x\le p/4\\ 0, x=0.\end{cases}$$

**(1) Show that** $xf(x)=(\tan x)^2, 0£x£p/4$. **(3 points)**

**(2) Find the volume of the solid generated by revolving the shaded region about the** $y$**-axis in the accompanying figure. (7 points)**



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**Question 4 (36 points)**
**Evaluate the following integrals:**

**(a)** $\int \frac{dx}{\sqrt{x}\left(1+x\right)}$; **(9 points)**

**(b)** $\int \frac{x}{1+\cos x}dx$; **(9 points)**

**(c)** $\int \cos^3 2x \sin 2x\, dx$; **(9 points)**

**(d)** $\int \frac{-2x+4}{(x^2+1)(x-1)^2}dx$. **(9 points)**

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**Question 5 (10 points)**
**The following function is called Error function:**

$$f(x)=\int_0^x \frac{2e^{-t^2}}{\sqrt{p}}dt,$$

**which has important applications in probability and statistics. Please explore the convergence of**

$$\int_0^{\infty} \frac{2e^{-t^2}}{\sqrt{p}}dt.$$

**If it converges, please show the rough bounds of its value.**

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**Question 6 (20 points)**

**(1) The following system is called Initial Value Problem (IVP):**

$$P(x)y''(x)+Q(x)y'(x)+R(x)y(x)=G(x)$$
$$y(x_0)=y_0, y'(x_0)=y_1.$$

**It can be proved that if $P,Q,R,G$ are continuous throughout an open interval $I$, then there exists one and only one function satisfying the above system. Solve the following Initial Value Problem:**

$$y''-2y'+2y=0, y(0)=0, y'(0)=2.$$ **(10 points)**

**(2) The following system is called Boundary Value Problem (BVP):**

$$P(x)y''(x)+Q(x)y'(x)+R(x)y(x)=G(x)$$
$$y(x_1)=y_1, y(x_2)=y_2.$$

**Unlike the result in question (1), Boundary Value Problem may exist one solution, or more than one solution, or does not exist any solutions. Please list <u>three examples</u> to explain the above results in detail. (10 points)**