GLASGOW COLLEGE UESTC

Final Exam

Calculus II (UESTC 1003)

Date: 2nd September 2020
Time: 09:30-11:30am

Attempt all PARTS. Total 100 marks

Use one answer sheet for each of the questions in this exam.
Show all work on the answer sheet.

Make sure that your University of Glasgow and UESTC Student Identification Numbers are on all answer sheets.

All graphs should be clearly labelled and sufficiently large so that all elements are easy to read.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

Continued overleaf

Q1 Suppose there is a surface expressed by the function $f(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

(a) Find all critical points of f; [8]

(b) Find the tangent planes and normal lines of the surface at the point $(1,1)$; [10]

(c) Find the local and absolute extreme values of f. [12]

Q2 Evaluate the following four integrals:

(a) $\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3}\, dydx$; [10]

(b) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dydx$; [10]

(c) $\iiint_D \, dV$,
where D is the solid between the sphere $\rho = \cos\phi$ and the hemisphere $\rho = 2, z \geq 0$. [10]

(d) Set up the iterated integral for $\iiint_D \, f(r,\theta,z)dzrdrd\theta$ in cylindrical coordinates over the region D, here D is the right circular cylinder whose base is the circle $r = 2\sin\theta$ in the xy-plane and whose top lies in the plane $z = 4 - y$. [10]

Q3 (a) Show that the following differential form is exact:

$$\sin y \cos x\, dx + \cos y \sin x\, dy + dz.$$ [10]

(b) Then evaluate the line integral:

$$\int_{(1,0,0)}^{(0,1,1)} \sin y \cos x\, dx + \cos y \sin x\, dy + dz.$$ [10]

Q4 Set C to be the boundary of a square D in the plane. Show that the value of the following line integral

$$\oint_C xy^2 dx + (x^2 y + 2x) dy$$

depends only on the area of D and not on its location in the plane. [10]

End of question paper