TD N° 1

MTH 2302 D / Gr 2
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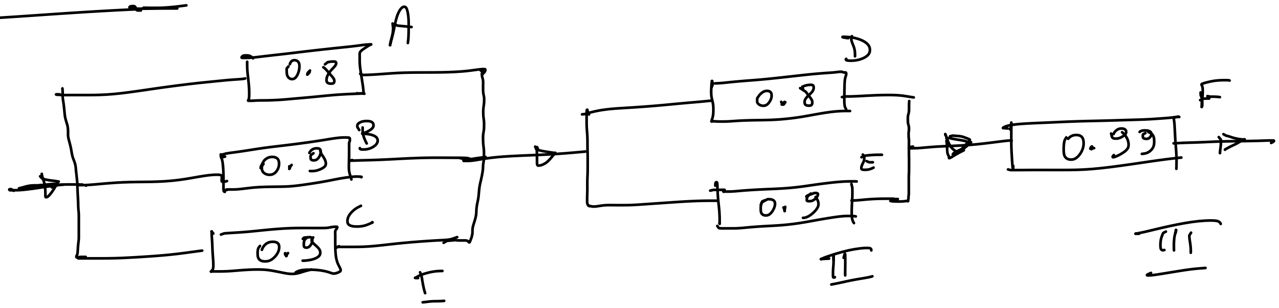
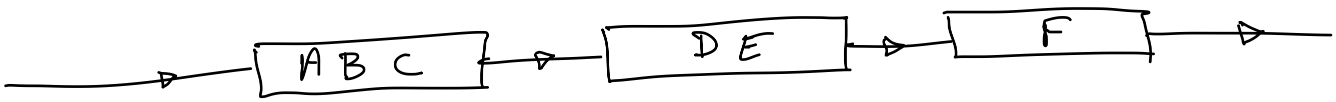
Exercice 1.26



Système équivalent :



Probabilité que le montage fonctionne.

$ABC = A \cup B \cup C$

$$\mathbb{P}(ABC) = \mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C)$$

on on a indépendance (voir énoncé)

$$\Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A) \times \mathbb{P}(B)$$
$$\mathbb{P}(A \cap C) = \mathbb{P}(A) \times \mathbb{P}(C)$$
$$\mathbb{P}(B \cap C) = \mathbb{P}(B) \times \mathbb{P}(C)$$
$$\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A)\,\mathbb{P}(B)\,\mathbb{P}(C)$$

$$\mathbb{P}(ABC) = 0.998$$

$$DE = D \cup E \Rightarrow \mathbb{P}(DE) = \mathbb{P}(D) + \mathbb{P}(E) - \mathbb{P}(D \cap E)$$

$$\stackrel{\perp\!\!\!\perp}{=} \mathbb{P}(D) + \mathbb{P}(E) - \mathbb{P}(D) \times \mathbb{P}(E)$$

$$= 0.98$$

$$\mathbb{P}(\text{" Le syst fonct "}) = \mathbb{P}\big((ABC) \cap (DE) \cap F\big)$$

$$\stackrel{\perp\!\!\!\perp}{=} \mathbb{P}(ABC) \times \mathbb{P}(DE) \times \mathbb{P}(F)$$

$$= 96.83\,\%.$$

Exercice 2

Soit X = " Elle atteint le point X "

On cherche $\mathbb{P}(X)$

Soit ; AB = " Elle emprunte le chemin AB "

AC = " Elle " " AC "

AD = " " " " AD "

AE = " " " " AE "

AB, AC, AD et AE forment une partition de Ω.

$\mathbb{P}(X) = ?$ Règle des Proba totales !

$$\mathbb{P}(X) = \mathbb{P}(AB \cap X) + \mathbb{P}(AC \cap X) + \mathbb{P}(AD \cap X) + \mathbb{P}(AE \cap X)$$

Rappel

$$\boxed{\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A \mid B)\,\mathbb{P}(B)}$$

$$\begin{aligned}\mathbb{P}(X) &= \mathbb{P}(X \mid AB)\,\mathbb{P}(AB) + \mathbb{P}(X \mid AC)\,\mathbb{P}(AC) \\ &\quad + \mathbb{P}(X \mid AD)\,\mathbb{P}(AD) + \mathbb{P}(X \mid AE)\,\mathbb{P}(AE) \\ &= \frac{1}{3} \times \frac{1}{4} + 1 \times \frac{1}{4} + 1 \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} = 0.683\end{aligned}$$

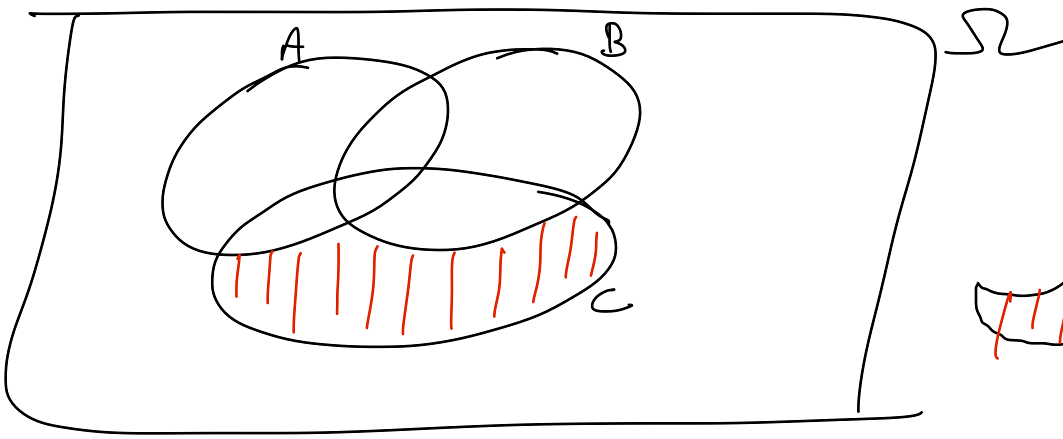
Exercice 3

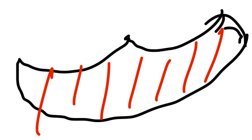
* D = " Il choisit une des 3 opt° "

= " Il choisit au moins une des 3 "

$= A \cup B \cup C \Rightarrow \mathbb{P}(D) = \mathbb{P}(A \cup B \cup C) = 0.95$

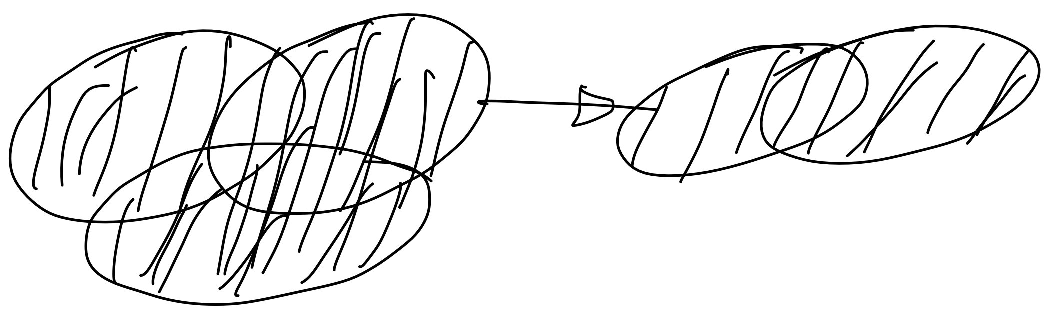
* E = " Il choisit la radio seulement "





$$\mathbb{P}(E) = \mathbb{P}(A \cup B \cup C) - \mathbb{P}(A \cup B)$$

$$= 0.95 - 0.80 = 0.15$$



* F = " Il ne choisit aucune des options "

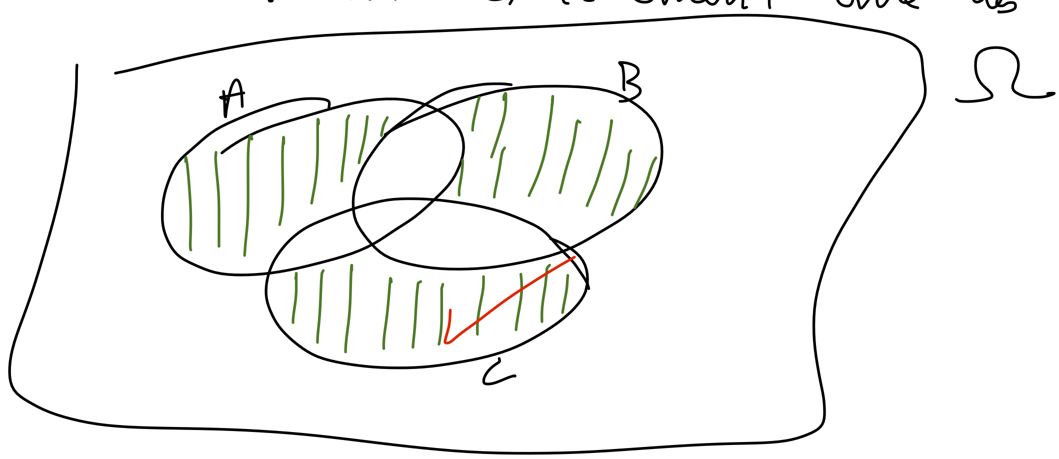
$\overline{F}$ = " Il choisit au moins une des 3 "

$= D$

$\mathbb{P}(F) = 1 - \mathbb{P}(\overline{F}) = 1 - \mathbb{P}(D)$

$= 1 - 0.95 = 0.05$

* G = " Il choisit exactement une des 3 options "



G = " Exactement A " ou " Exactement B "

ou " Exactement C "

$\mathbb{P}(\text{" Exactement A"}) = \mathbb{P}(A \cup B \cup C) - \mathbb{P}(B \cup C) = 0.05$

$\mathbb{P}(\text{" Exactement B "}) = \mathbb{P}(A \cup B \cup C) - \mathbb{P}(A \cup C) = 0.10$

$\mathbb{P}(\text{" Exactement C"}) = \mathbb{P}(E) = 0.15$

$\mathbb{P}(G) = \mathbb{P}(\text{Exactement A}) + \mathbb{P}(\text{EX B}) + \mathbb{P}(\text{EX C})$

$= 0.30$

Car Evénements deux à deux disjoints

Exercice 4

$\{F, M, E\}$ = Partition

$\{A, B, C\}$ = Partition

* $\mathbb{P}(B \mid E) = \dfrac{\mathbb{P}(B \cap E)}{\mathbb{P}(E)}$

$\mathbb{P}(E) = ?$ Règle des Proba totales.

$$\mathbb{P}(E) = \mathbb{P}(E \cap A) + \mathbb{P}(E \cap B) + \mathbb{P}(E \cap C)$$
$$= 0.02 + 0.08 + 0.1 = 0.2$$

$$\mathbb{P}(B \mid E) = \frac{0,08}{0.2} = 0.4$$

* $\mathbb{P}(M \mid C) = \dfrac{\mathbb{P}(M \cap C)}{\mathbb{P}(C)}$ 0,15

$\mathbb{P}(C) = ?$ Règle des Proba totales avec la Partition $\{F, M, E\}$

$$\mathbb{P}(C) = \mathbb{P}(C \cap F) + \mathbb{P}(C \cap M) + \mathbb{P}(C \cap E) = 0.35$$

$$\mathbb{P}(M \mid C) = \frac{0.15}{0.35} \simeq 42.86\,\%.$$

* $\mathbb{P}(A \mid M) = \dfrac{\mathbb{P}(A \cap M)}{\mathbb{P}(M)} = \dfrac{0.13}{0.4} = 32.5\,\%$

* $\mathbb{P}(M \mid A) = \dfrac{\mathbb{P}(M \cap A)}{\mathbb{P}(A)}$ Bayes $\rightarrow \dfrac{\mathbb{P}(A \mid M)\,\mathbb{P}(M)}{\mathbb{P}(A)}$

$$= \frac{0.13}{0.25} = 0.52$$

$$* \ \mathbb{P}(M \cap B \mid C) = \mathbb{P}((M \cap B) \mid C)$$

$$= \frac{\mathbb{P}((M \cap B) \cap C)}{\mathbb{P}(C)}$$

$$= \frac{\mathbb{P}(M \cap B \cap C)}{\mathbb{P}(C)}$$

$$= \frac{\mathbb{P}(M \cap \phi)}{\mathbb{P}(C)} = \frac{\mathbb{P}(\phi)}{\mathbb{P}(C)}$$

$$= 0$$

$$* \ \mathbb{P}(F \cup M \mid C) = \frac{\mathbb{P}((F \cup M) \cap C)}{\mathbb{P}(C)}$$

$$= \frac{\mathbb{P}((F \cup M) \cap C)}{0.35}$$

$$(F \cup M) \cap C = (F \cap C) \cup (M \cap C)$$

$$\mathbb{P}((F \cup M) \cap C) = \mathbb{P}((F \cap C) \cup (M \cap C))$$

$$= \mathbb{P}(F \cap C) + \mathbb{P}(M \cap C)$$

$$- \mathbb{P}(\underbrace{(F \cap C) \cap (M \cap C)}_{\phi}) \to 0$$

$$= 0.10 + 0.15$$

$$= 0.25$$

$$\mathbb{P}(F \cup M \mid C) = \frac{0.25}{0.35} \simeq 71.43\%$$

Exercice 5

A = "La 1ere Hors d'usage"

B = " La seconde " " "

T = " Temp $\geqslant 30°C$ "

S =

F =

M =

5.1 - S, F et M forment une partition de Ω

$\Rightarrow \Omega = S \cup F \cup M$

$* \; S = \bar{A} \cap \bar{B} \cap \bar{T}$

$* \; F = (A \cup B) \cap T$

$* \; M = \overline{(S \cup F)} = \overline{(\bar{A} \cap \bar{B} \cap \bar{T}) \cup ((A \cup B) \cap T)}$

$= \overline{(\bar{A} \cap \bar{B} \cap \bar{T})} \cap \left(\overline{(A \cup B) \cap T}\right)$

$= (A \cup B \cup T) \cap ((\bar{A} \cap \bar{B}) \cup \bar{T})$

$\Omega =$

5.3) $(A \cap \overline{B}) \cup (\overline{A} \cap B) = E$

$$\mathbb{P}(E) = \mathbb{P}(A \cap \overline{B}) + \mathbb{P}(\overline{A} \cap B) - \mathbb{P}\Big(\underbrace{(A \cap \overline{B}) \cap (\overline{A} \cap B)}_{\phi}\Big) \to 0$$

$$\overset{\perp\!\!\!\perp}{=} \mathbb{P}(A)\,\mathbb{P}(\overline{B}) + \mathbb{P}(\overline{A}) \times \mathbb{P}(B)$$

$$= 2.96\%$$

5.4) * $\mathbb{P}(S)$ où $S = \overline{A} \cap \overline{B} \cap \overline{T}$

$$\Rightarrow \mathbb{P}(S) = \mathbb{P}(\overline{A} \cap \overline{B} \cap \overline{T})$$

$$\overset{\perp\!\!\!\perp}{=} \mathbb{P}(\overline{A}) \times \mathbb{P}(\overline{B}) \times \mathbb{P}(\overline{T})$$

$$= 0.67914$$

* $\mathbb{P}(F)$ où $F = (A \cup B) \cap T$

$$= (A \cap T) \cup (B \cap T)$$

$$\mathbb{P}(F) = \mathbb{P}(A \cap T) + \mathbb{P}(B \cap T) - \mathbb{P}(A \cap T \cap B \cap T)$$

$$= \mathbb{P}(A) \times \mathbb{P}(T) + \mathbb{P}(B) \times \mathbb{P}(T) - \mathbb{P}(A \cap B \cap T)$$

$$= \quad '' \quad\quad '' \quad - \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(T)$$

$$= 0.894\%$$

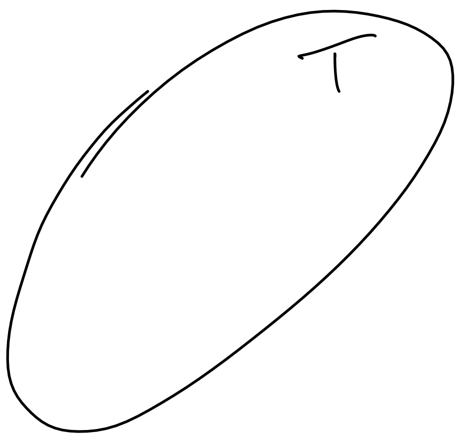
* $\{F, M, S\}$ Partition $\Rightarrow$ $\mathbb{P}(M) = 1 - \big(\mathbb{P}(S) + \mathbb{P}(F)\big)$

$$= 0.31192$$

$$(A \cap T) \cap (B \cap T)$$

$$= A \cap T \cap B \cap T = A \cap \underbrace{T \cap T}_{T} \cap B$$

$$= A \cap T \cap B$$



1.24

$$(M \cap B) \mid C$$

?? $M \cap (B \mid C) = M \cap \phi = \phi$

$B \mid C = \phi$??

On sait que $B \cap C = \phi$ (Partition ....)

$$\mathbb{P}(B \mid C) = \frac{\mathbb{P}(B \cap C)}{\mathbb{P}(C)} = \frac{0}{\mathbb{P}(C)} = 0$$

$\Rightarrow B \mid C = \phi$

Brouillon.

(ne pas considérer)

$\mathbb{P}(\quad) = 0$

$F \cup (M \mid C)$

$$\mathbb{P}(M \mid C) = \frac{\mathbb{P}(M \cap C)}{\mathbb{P}(C)} = \frac{0.15}{0.35}$$

$$\mathbb{P}(F \cup (M \mid C)) = \mathbb{P}(F) + \mathbb{P}(M \mid C) - \mathbb{P}(F \cap (M \mid C))$$

??