Teoreme ale calculului propozitional clasic

$$(A1) \quad \varphi \rightarrow (\psi \rightarrow \varphi)$$

$$(A2) \quad (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$$

$$(A3) \quad (\neg \psi \rightarrow \neg \varphi) \rightarrow (\varphi \rightarrow \psi)$$

$$(1) \quad \vdash \varphi \rightarrow \varphi$$

$$(2) \quad \vdash (\varphi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi))$$

$$(3) \quad \vdash (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\psi \rightarrow (\varphi \rightarrow \chi))$$

$$(4) \quad \vdash \varphi \rightarrow (\varphi \rightarrow \psi)$$

$$(5) \quad \vdash \neg \varphi \rightarrow (\varphi \rightarrow \psi)$$

$$(6) \quad \vdash (\neg \neg \varphi \rightarrow \varphi)$$

$$(7) \quad \vdash (\varphi \rightarrow \neg \neg \varphi)$$

$$(8) \quad \vdash (\varphi \rightarrow \psi) \rightarrow (\neg \psi \rightarrow \neg \varphi)$$

$$(9) \quad \vdash (\varphi \rightarrow \neg \varphi) \rightarrow \neg \varphi$$

$$(10) \quad \vdash (\neg \varphi \rightarrow \varphi) \rightarrow \varphi$$

$$(11) \quad \vdash \varphi \rightarrow (\neg \psi \rightarrow \neg (\varphi \rightarrow \psi))$$

$$(12) \quad \vdash \varphi \rightarrow (\varphi \lor \psi)$$

$$(13) \quad \vdash \psi \rightarrow (\varphi \lor \psi)$$

$$(14) \quad \vdash \varphi \lor \psi \rightarrow \psi \lor \varphi$$

$$(15) \quad \vdash (\varphi \rightarrow \chi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\varphi \lor \psi \rightarrow \chi))$$

$$(16) \quad \vdash \varphi \land \psi \rightarrow \varphi$$

$$(17) \quad \vdash \varphi \land \psi \rightarrow \psi$$

$$(18) \quad \vdash (\chi \rightarrow \varphi) \rightarrow ((\chi \rightarrow \psi) \rightarrow (\chi \rightarrow \varphi \land \psi))$$

$$(19) \quad \vdash \varphi \land \psi \rightarrow \psi \land \varphi$$

$$(20) \quad \vdash \varphi \rightarrow (\psi \rightarrow \varphi \land \psi)$$

$$(21) \quad \vdash (\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\varphi \land \psi \rightarrow \chi)$$

$$(22) \quad \vdash (\varphi \land \psi \rightarrow \chi) \rightarrow (\varphi \rightarrow (\psi \rightarrow \chi))$$

$$(23) \quad \vdash (\varphi \rightarrow \psi) \rightarrow (\varphi \land \chi \rightarrow \psi \land \chi)$$

$$(24) \quad \vdash (\varphi \rightarrow \psi) \rightarrow (\varphi \land \chi \rightarrow \psi \land \chi)$$

$$(25) \quad \vdash (\varphi \land \psi \rightarrow \chi) \rightarrow (\varphi \land \neg \chi \rightarrow \neg \psi)$$

$$(26) \quad \vdash \varphi \lor \neg \varphi$$

 $\begin{array}{ll} (27) & \vdash ((\varphi \to \chi) \land (\psi \to \delta)) \to (\varphi \land \psi \to \chi \land \delta) \\ (28) & \vdash ((\varphi \to \chi) \land (\psi \to \delta)) \to (\varphi \lor \psi \to \chi \lor \delta) \end{array}$

 $\begin{array}{ll} (29) & \vdash (\varphi \lor (\psi \lor \chi)) \leftrightarrow ((\varphi \lor \psi) \lor \chi) \\ (30) & \vdash (\varphi \land (\psi \land \chi)) \leftrightarrow ((\varphi \land \psi) \land \chi) \\ (31) & \vdash (\varphi \land (\psi \lor \chi)) \leftrightarrow ((\varphi \land \psi) \lor (\varphi \land \chi)) \\ (32) & \vdash (\varphi \lor (\psi \land \chi)) \leftrightarrow ((\varphi \lor \psi) \land (\varphi \lor \chi)) \end{array}$