Formulele lui Viete

Rădăcini Polinomiale și Coeficienți

de ordin 2

Quadratic: $ax^2 + bx + c = 0$

$ax^2 + bx + c \equiv a(x-\alpha)\cdot(x-\beta)$

$\alpha + \beta = -\frac{b}{a}$ & $\alpha \cdot \beta = \frac{c}{a}$

de ordin 3

Cubic: $ax^3 + bx^2 + cx + d = 0$ → Ar trebui să aibă 3 rădăcini

$$ax^3 + bx^2 + cx + d \equiv a(x-\alpha)(x-\beta)(x-\gamma) \quad | a$$

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} \equiv (x-\alpha)(x-\beta)(x-\gamma)$$

$$\cancel{x^3} + \underbrace{\frac{b}{a}}x^2 + \underbrace{\frac{c}{a}}x + \frac{d}{a} \equiv \cancel{x^3} \underbrace{-\alpha x^2 - \beta x^2 - \gamma x^2} + \alpha\beta x + \underbrace{\beta\gamma x + \alpha\gamma x} - \alpha\beta\gamma$$

$$\equiv -(\alpha+\beta+\gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma$$

$\therefore \alpha + \beta + \gamma = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

de ordin 4

Quartic: $ax^4+bx^3+cx^2+dx+e=0$

4 Rădăcini

$$a(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=0$$

$$\alpha+\beta+\gamma+\delta=-\frac{b}{a}$$

$$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=\frac{c}{a}$$

$$\alpha\beta\gamma+\alpha\beta\delta+\beta\gamma\delta+\alpha\gamma\delta=-\frac{d}{a}$$

$$\alpha\beta\gamma\delta=\frac{e}{a}$$

$$x^2+5x-5=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Nu există formulă pt gradul 5 și ↗

Pt gradul 3 și gradul 4 folosim aproximări numerice (analiză numerică)

Considerăm:

$$x^2 + x + 1 = 0$$

$\Delta = -3 < 0 \Rightarrow$ nu avem soluții reale

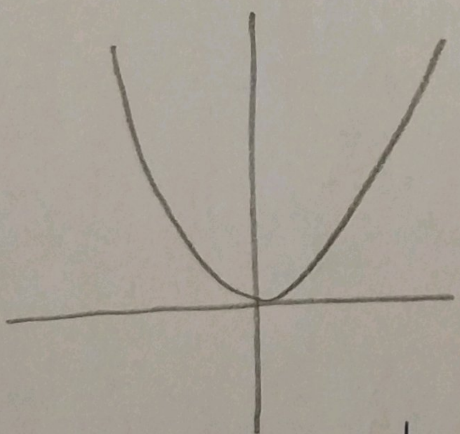
$\alpha + \beta = -1$

$\alpha\beta = 1$

$$x = \frac{-1 \pm \sqrt{-3}}{2} \Rightarrow \underset{(\alpha)}{x_1} = \frac{-1 - \sqrt{-3}}{2} \text{ și } \underset{(\beta)}{x_2} = \frac{-1 + \sqrt{-3}}{2}$$

$$\alpha + \beta = -\frac{1}{2} + \frac{-1}{2} = -1$$

$$\alpha\beta = \frac{(-1-\sqrt{-3})(-1+\sqrt{-3})}{2 \cdot 2} = \frac{(-1)^2 - (\sqrt{-3})^2}{4} = \frac{1-(-3)}{4} = 1$$



Deși sunt în afara graficului, aceste numere există (α, β)

$\frac{6}{3} = 2$ A pt că $6 = 2 \cdot 3$

$\frac{1}{0} = x$

$1 = 0 \cdot x$ F

Exerc:

1.

$x^2+2x+5=0$

Găsește: (i) $\alpha+\beta$ (iii) $(\alpha+2)(\beta+2)$

(ii) $\alpha\cdot\beta$

(i) $-\frac{b}{a}=-2$

(ii) $\frac{c}{a}=5$

(iii) $(\alpha+2)(\beta+2)=\alpha\beta+2\alpha+2\beta+4$

$=\alpha\beta+2(\alpha+\beta)+4$

$=5+2(-2)+4=5$

2.

$2x^3-x^2+4x+3=0$

Găsește: (i) $\alpha+\beta+\gamma$ (iii) $\alpha\beta\gamma$ (v) $\alpha^2+\beta^2+\gamma^2$

(ii) $\alpha\beta+\beta\gamma+\alpha\gamma$ (iv) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

(i) $-\frac{b}{a}=\frac{1}{2}$ (ii) $\frac{c}{a}=2$ (iii) $-\frac{d}{a}=-\frac{3}{2}$

(iv) $\frac{\beta\gamma}{\alpha\beta\gamma}+\frac{\alpha\gamma}{\alpha\beta\gamma}+\frac{\alpha\beta}{\alpha\beta\alpha}=\frac{2}{\left(-\frac{3}{2}\right)}=-\frac{4}{2}$

(v) Încercăm: $(\alpha+\beta+\gamma)^2=(\alpha^2+\underline{\alpha\beta}+\underline{\alpha\gamma})(\underline{\alpha\beta}+\beta^2+\underline{\beta\gamma})(\underline{\alpha\gamma}+\underline{\beta\gamma}+\gamma^2)$

$=(\alpha^2+\beta^2+\gamma^2)+2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$\alpha^2+\beta^2+\gamma^2=\frac{1}{4}-4=-\frac{15}{4}$