Algebră (pregătire) de duminică

Inversa unei matrici.

$\mathbb{Z}_5$ $\quad \bar{2} \cdot \bar{x} = \bar{1} \pmod 5$ $\quad$ Deci inversul e $\bar{3}$

$\bar{2} \cdot \bar{3} = \bar{1} \pmod 5$

$\bar{6} = \bar{1} \pmod 5$

$\bar{1} = \bar{1} \pmod 5$

$$A^{-1} = \frac{1}{\det A} \cdot B^T$$

$$\begin{pmatrix} \bar{1} & \bar{2} & \bar{0} \\ \bar{1} & \bar{1} & \bar{2} \\ \bar{0} & \bar{1} & \bar{1} \end{pmatrix} = A \in \mathcal{M}_3(\mathbb{Z}_5) \qquad A^{-1} = \frac{1}{\det \bar{A}} \cdot B^T$$

$$b_{ij} = (-1)^{i+j} \cdot \det A_{ij}$$

$B = (b_{ij})$

A_{ij} - tai linia i și coloana j

$$b_{11} = (-1)^{1+1} \begin{vmatrix} \bar{1} & \bar{2} \\ \bar{1} & \bar{1} \end{vmatrix} = -\bar{1} = \bar{4}$$

$$b_{12} = (-1)^{1+2} \begin{vmatrix} \bar{1} & \bar{2} \\ \bar{0} & \bar{1} \end{vmatrix} = \bar{4}$$

$$b_{13} = (-1)^{1+3} \begin{vmatrix} \bar{1} & \bar{1} \\ \bar{0} & \bar{1} \end{vmatrix} = \bar{1}$$

$$b_{21} = (-1)^{2+1} \begin{vmatrix} \bar{2} & \bar{0} \\ \bar{1} & \bar{1} \end{vmatrix} = (-1) \cdot \bar{2} = -\bar{2} = \bar{3}$$

$$b_{22} = (-1)^{2+2} \begin{vmatrix} \bar{1} & \bar{0} \\ \bar{0} & \bar{1} \end{vmatrix} = \bar{1}$$

$$b_{23} = (-1)^{2+3} \begin{vmatrix} \bar{1} & \bar{2} \\ \bar{0} & \bar{1} \end{vmatrix} = -\bar{1} = \bar{4}$$

$$b_{31} = (-1)^{3+1} \begin{vmatrix} \bar{2} & \bar{0} \\ \bar{1} & \bar{2} \end{vmatrix} = \bar{4}$$

$$A^{-1} = \bar{3}\begin{pmatrix} \bar{4} & \bar{4} & \bar{1} \\ \bar{3} & \bar{1} & \bar{4} \\ \bar{4} & \bar{3} & \bar{4} \end{pmatrix}^T$$

$$= \bar{3}\begin{pmatrix} 4 & 3 & 4 \\ 4 & 1 & 3 \\ 1 & 4 & 4 \end{pmatrix} = \begin{pmatrix} \overline{12} & \bar{9} & \overline{12} \\ \overline{12} & \bar{3} & \bar{9} \\ \bar{3} & \overline{12} & \overline{12} \end{pmatrix} = \begin{pmatrix} \bar{2} & \bar{4} & \bar{2} \\ \bar{2} & \bar{3} & \bar{4} \\ \bar{3} & \bar{2} & \bar{2} \end{pmatrix}$$

$$A \cdot A^{-1} = \begin{pmatrix} \bar{1} & \bar{2} & \bar{0} \\ \bar{1} & \bar{1} & \bar{2} \\ \bar{0} & \bar{1} & \bar{1} \end{pmatrix}\begin{pmatrix} \bar{2} & \bar{4} & \bar{2} \\ \bar{2} & \bar{3} & \bar{4} \\ \bar{3} & \bar{2} & \bar{2} \end{pmatrix} = \begin{pmatrix} \bar{1} & \bar{0} & \bar{0} \\ --- & \bar{1} & --- \\ ---- & & \bar{1} \end{pmatrix}$$

$\bar{1} \cdot \bar{2} + \bar{2} \cdot \bar{2} + \bar{0} \cdot \bar{3} = \bar{6} = \bar{1}$

$\bar{1} \cdot \bar{4} + \bar{2} \cdot \bar{3} + \bar{0} \cdot \bar{2} = \overline{10} = \bar{0}$

$\bar{1} \cdot \bar{2} + \bar{2} \cdot \bar{4} + \bar{0} \cdot \bar{2} = \overline{10} = \bar{0}$

Exerciții:

Determinantul matricii $\begin{pmatrix} \bar{0} & \bar{1} & \bar{1} \\ \bar{2} & \bar{3} & \bar{3} \\ \bar{1} & \bar{1} & \bar{2} \end{pmatrix} \in \mathcal{M}_3(\mathbb{Z}_4)$

este:

A. $\bar{0}$

B. $\bar{1}$

C. $\bar{2}$

D. $\bar{3}$

$$\begin{vmatrix} \bar{0} & \bar{1} & \bar{1} \\ \bar{2} & \bar{3} & \bar{3} \\ \bar{1} & \bar{1} & \bar{2} \end{vmatrix} = \bar{0} + \bar{3} + \bar{2} - \bar{3} - \bar{0} - \bar{4}$$

$$= \bar{5} - \bar{7} = \bar{2}$$

V_2: $C_2 - C_3 = \begin{vmatrix} \cancel{\bar{0}} & \cancel{\bar{0}} & \cancel{\bar{1}} \\ \bar{2} & \bar{0} & \cancel{\bar{3}} \\ \bar{1} & -\bar{1} & \cancel{\bar{2}} \end{vmatrix} = \begin{vmatrix} \bar{2} & \bar{0} \\ 1 & -1 \end{vmatrix} = -\bar{2} = \bar{2}$

$$\det A = \sum (-1)^{i+j} \cdot a_{ij} \cdot \det A_{ij}$$

Din curs
Exemplul ii:

$$\begin{vmatrix} 1 & -6 & 0 & 0 \\ 1 & 1 & -6 & 0 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -6 & 0 & 0 \\ 0 & 7 & -6 & 0 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 7 & -6 & 0 \\ 1 & 1 & -6 \\ 0 & 1 & 1 \end{vmatrix}$$

prima linie rămâne pe loc.

$= 7 + 42 + 6 = 55$

$\mathbb{Z}_{67}$ $\quad \bar{5}^{-1} = ?$

$-\bar{2} = \overline{65}$

$\overline{67} = \bar{0}$

$\bar{5} \cdot \bar{x} = \bar{1} \quad | \cdot \overline{13}$

$\overline{65} \cdot \bar{x} = \overline{13}$

$\overline{27} \cdot \bar{5} = 135 \overset{67}{\equiv} 1$

$-2 \cdot \bar{x} = +\overline{13} = \overline{54}$

$134 = 67 \cdot 2$

$\bar{x} = \overline{27}$

$67 - 13 = 54$

Ex: Inversul lui $\bar{5}$ în corpul $\mathbb{Z}_{67}$ este

A. $\overline{25}$ $\quad$ Poate fi și așa (să nu ne imuncăm!)

B. $\overline{26}$

C. $\overline{94} = \overline{27}$

D. $\overline{28}$

$x^4+x^3+2x^2+7x+5=0$

x_1, x_2, x_3, x_4 rădăcinile polinomului.

$x_1+x_2+x_3+x_4=-\frac{1}{1}=-1$

$x_1^2+x_2^2+x_3^2+x_4^2=(x_1+x_2+x_3+x_4)^2-2(x_1x_2+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_3x_4)=(-1)^2-2\cdot 2$
$=-3$

$x_1x_2+\dots x_3x_4=\frac{2}{1}=2$

Câte rădăcini reale are?

0, 1, 2, 3, 4

sigur 4 nu e corect

conjugatul $\overline{a+bi}=a-bi$ $\qquad \bar{2}=\bar{2}$

x_1 rădăcină pt ec $\Rightarrow \bar{x}_1$ rădăcină ec

Anulăm și pe ~~3~~ și pe ~~1~~

Poate fi doar 0 sau 2

! −1 este rădăcină. Aducem 1 la loc → 0, 1, 2

$x^3(x+1)+(x+1)(2x+5)=0$

$\underbrace{(x+1)}_{=0}\underbrace{(x^3+2x+5)}_{=0}=0$

$(-1)^3+2\cdot(-1)+5=2$ deci −1 se numără o singură dată
deci răspunsul va fi $\underline{\underline{2}}$

$(-2)^3 + 2(-2) + 5 = -7$

$(\underbrace{-2, -1}_{\text{Aici se mai găsește o rădăcină}})$

Polinoame —> Viète ! Alt exercițiu de tip grilă:

$$\begin{cases} x + 2y + 3z = 6 \\ x - 3y + 5z = 3 \\ 7x - y - z = 5 \end{cases}$$

$x + y + z = ?$

A.
B.
C.
D.

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & -3 & 5 \\ 7 & -1 & -1 \end{vmatrix} = 3 + 70 + (-3)$$

$63 + 5 + 2$

$= 140$

$$x = \frac{\begin{vmatrix} 6 & 2 & 3 \\ 3 & -3 & 5 \\ 5 & -1 & -1 \end{vmatrix}}{140}$$

$x = 1$

$y = 1$

$z = 1$

$x + y + z = 3$

Dacă det $\neq 0$ atunci are sol unică (x, y, z)

$$y = \frac{\begin{vmatrix} 1 & 6 & 3 \\ 1 & 3 & 5 \\ 7 & 5 & -1 \end{vmatrix}}{140} = ---$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 6 \\ 1 & -3 & 3 \\ 7 & -1 & 5 \end{vmatrix}}{140} = ---$$