LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

1. Definitii
   - gramatica
   - gramatica independenta de context
   - limbaj independent de context
2. Forme normale
   - definitie
   - exemple
   - aducerea la forma normala Chomsky
3. Lema de pompare
4. Operatii de inchidere
   - $\mathcal{L}_2$ este inchisa la reuniune, concatenare, operatia *, omomorfism
   - $\mathcal{L}_2$ nu este inchisa la intersectie si complementara
5. Automatul pushdown

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

- $\{0^n 1^n \mid n \geq 0\}$
- memoria finita a unui AFD nu poate memora numere **n** foarte mari.
- gramaticile independente de context (GIC)
- structura recursiva
- Domenii de utilizare a GIC:
  - studiul limbilor naturale,
- Domenii de aplicabilitate
  - specificarea si compilarea limbajelor de programare
  - sintaxa unui limbaj de programnare
  - parsere

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Reamintim ca:

1. Limbajele generate / descrise de G.I.C. se numesc L.I.C.;
2. Clasa $\mathcal{L}_2 \supset \mathcal{L}_3$;
3. Un mecanism, care s-a dovedit a fi echivalent cu cel al G.I.C., este constituit de APD.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Definitia 1

Gramatica = (V, Σ, P, S) unde:

V = multime finita, nevida, ale carei elemente se numesc variabile sau neterminale;

Σ = multime finita, nevida, numita alfabet de intrare, ale carei elemente se numesc terminale; $V \cap \Sigma = \emptyset$;

P = multime finita, nevida, ale carei elemente se numesc productii sau reguli de substitutie;

$S \in V$ se numeste variabila (simbolul) de start.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

<u>Exemplu</u>

G_1=({A,B}, {0,1,#}, {A → 0A1 | B, B → #}, A)

Reprezentarea derivarilor:

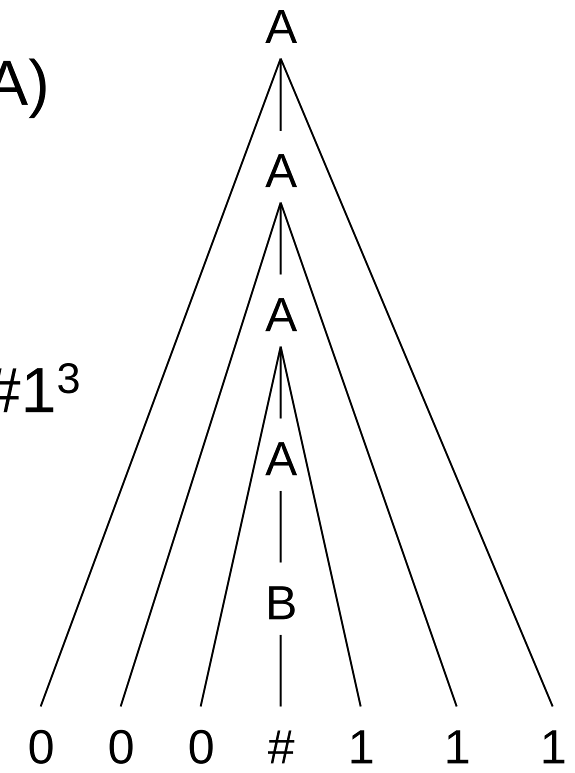
- linear:

$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 0^3B1^3 \Rightarrow 0^3\#1^3$

- sintetic:

$A \ {}_{G}\!\Rightarrow^* 0^3\#1^3$

- arbore de derivare:



LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Definitia 2

Gramatica independenta de context = (V,Σ, P, S) a.i.
$\forall\ \alpha \to \beta \in P: |\alpha|=1$ si $\alpha \in V$.

Definitia 3

Limbaj independent de context = L.I.C. =
$L(G)=\{ w \in \Sigma^* \mid \exists\ S \ {}_G\!\Rightarrow^* w$ si $G=G.I.C. \}$

Exemplu

$G_2= (\{S\}, \{a,b\}, \{S \to aSb \mid \lambda\}, S) \Rightarrow L_2=\{a^nb^n \mid n \geq 0\}$.

$G_3 = (\{S\}, \{a,b\}, \{S \to aSb \mid SS \mid \lambda\}, S) \Rightarrow$
$L_3=\{\lambda, ab, a^nb^n, a^nbab^n, a^n(ba)^kb^n, \ldots \}$.

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LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

<u>Definitia 4</u>

$G \in$ G.I.C. se afla in forma normala GREIBACH (FNG) $\Leftrightarrow$

$\forall\ p \in P,\ \ p : A \rightarrow aB,$ unde:

$A \in V,$

$a \in \Sigma$

$B \in (V \cup \Sigma)^*.$

<u>Teorema 1</u>

$\forall\ L=L(G) \in$ L.I.C., $\lambda \notin L$: $\exists\ G'$ in FNG a.i. $L=L(G')$.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Definitia 5

$G \in$ G.I.C. se afla in forma normala CHOMSKY (FNC) $\Leftrightarrow$

$\forall\, p \in P$, $p : A \rightarrow BC$ sau $A \rightarrow a$, unde:

$A, B, C \in V$, $B \neq S \neq C$,

$a \in \Sigma$,

In plus, $S \rightarrow \lambda \in P$.

Teorema 2

$L=L(G) \in$ L.I.C.: $\exists\, G'$ in FNC a.i. $L=L(G')$.

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LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Lema de pompare pentru L.I.C.

Fie $A \subseteq \Sigma^*$, $A \in$ L.I.C. =>

$\exists\ p \in N$ (constanta=lungimea de pompare) a.i.

$\forall s \in A$, $|s| \geq p \rightarrow \exists\ u,v,x,y,z \in \Sigma^*$ cu proprietatile:

(i) $s = uvxyz$,

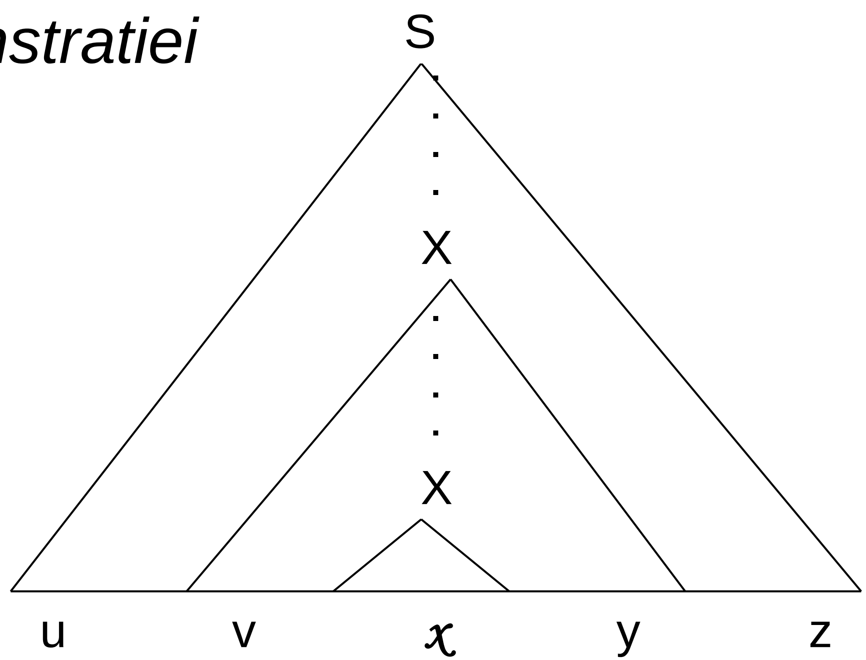
(ii) $\forall i \geq 0$: $uv^ixy^iz \in A$,

(iii) $|vy| > 0$,

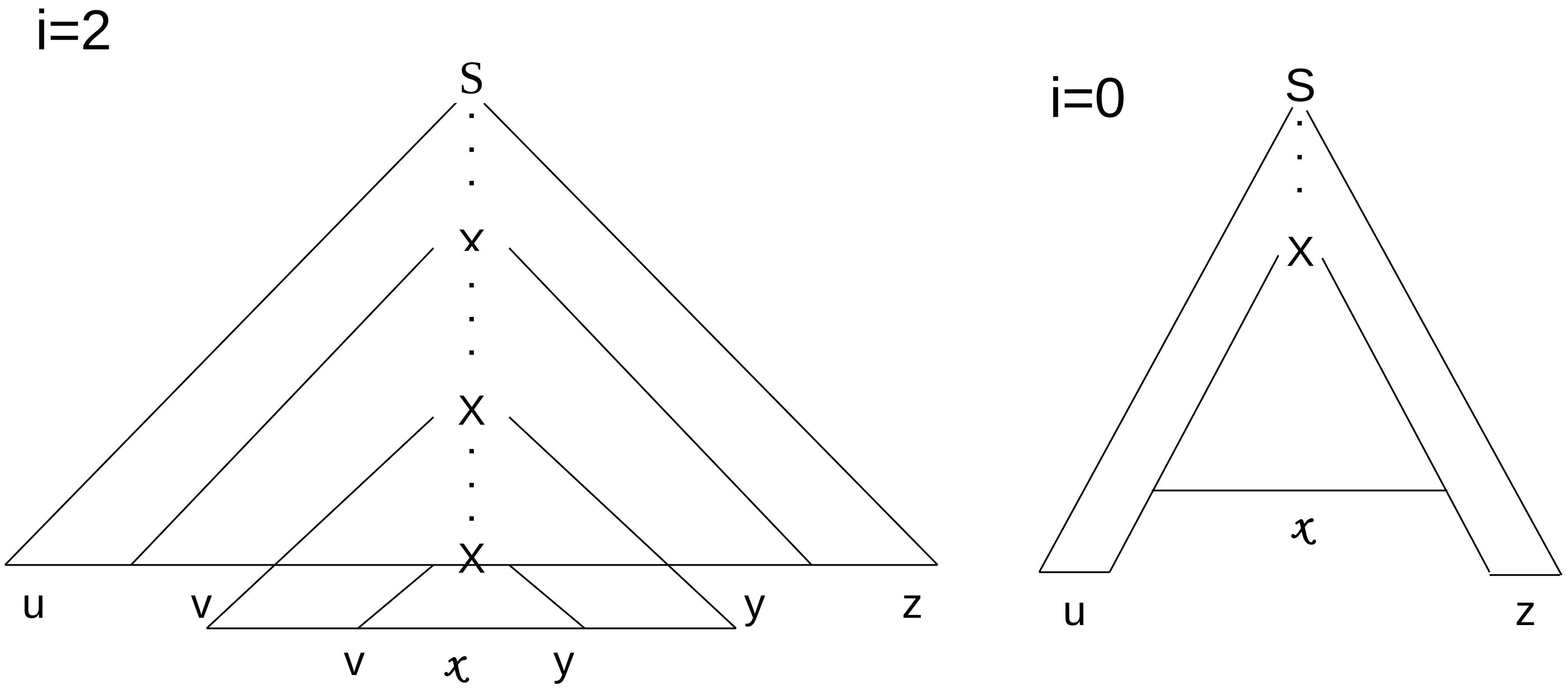
(iv) $|vxy| \leq p$.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Ideea demonstratiei



LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)



LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Definitii 6

$\forall L_1, L_2 \subseteq \Sigma^*$:

$L_1 \cup L_2 = \{w \in \Sigma^* \mid w \in L_1 \text{ sau } w \in L_2\}$,

$L_1 \cap L_2 = \{w \in \Sigma^* \mid w \in L_1 \text{ si } w \in L_2\}$,

$L_1 \setminus L_2 = \{w \in \Sigma^* \mid w \in L_1 \text{ si } w \notin L_2\}$,

$L_1 \circ L_2 = \{w_1 w_2 \in \Sigma^* \mid w \in L_1 \text{ si } w \in L_2\}$,

$mi(L) = \{mi(w) \mid w \in L\}$.

$$\overline{L} = \{w \in \Sigma^* \mid w \notin L\} = \Sigma^* \setminus L,$$

$$L^n : \begin{cases} L^0 = \lambda \;\; si \\ L^{n+1} = L \cdot L^n = L \cdot L^n, \forall n \in N, \end{cases}$$

$$\mathrm{L} \cdot \varnothing = \varnothing \cdot L = \varnothing,$$

$$\mathrm{L} \cdot \{\lambda\} = \{\lambda\} \cdot L = L,$$

$$L^* = \bigcup_{i=0}^{\infty} L^i, \quad L^+ = \bigcup_{i=1}^{\infty} L^i.$$

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Lema 3

L.I.C. este inchisa la reuniune, concatenare si operatia *.

Lema 4

L.I.C. este inchisa la operatia mirror si la omomorfism.

Lema 5

L.I.C. NU este inchisa la intersectie si complementara.

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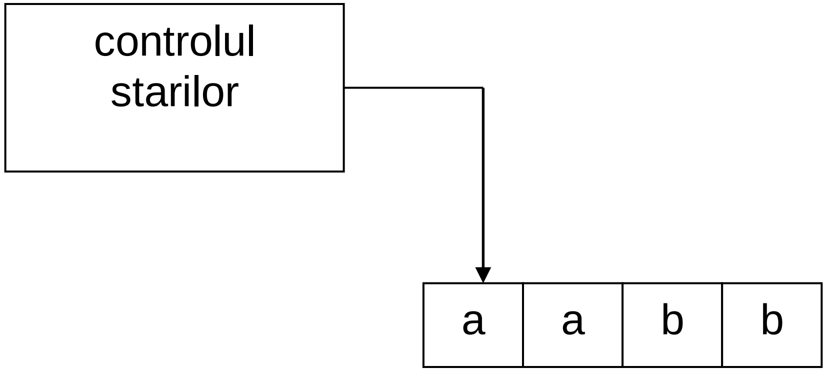
LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

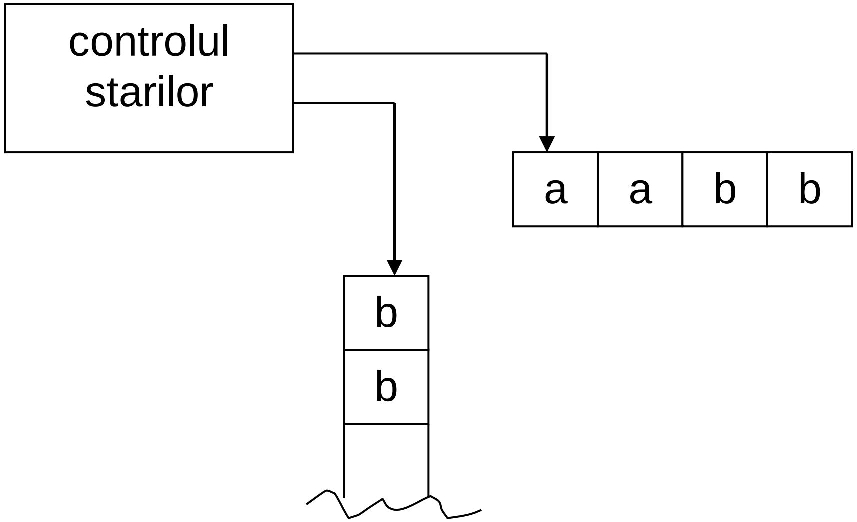
APD reprezinta un nou model de calculabilitate

Stiva

AFN



APD



$\{0^n 1^n \mid n \in N\}$

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Observatii

AFD $\Leftrightarrow$ AFN

APDN $\geq$ APD

$\{0^n 1^n \mid n \geq 0\}$: APDN, APD;

$\{ww^r \mid w \in \{0,1\}^*\}$: APDN.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Deosebiri intre APD si AFD:

(i) Doua alfabete, Σ si Γ

(ii) $dom(\delta) = Q \times (\Sigma \cup \{\lambda\}) \times (\Gamma \cup \{\lambda\})$

(iii) $codom(\delta) = \mathcal{P}(Q \times (\Gamma \cup \{\lambda\}))$.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Definitia 7

Automat pushdown=APD=(Q, Σ, Γ, δ, q_0, F), unde:

Q = multime finita, nevida, ale carei elemente se numesc stari;

Σ = multime finita, nevida, numita alfabet de intrare, ale carei elemente se numesc simboluri, ($\Sigma_\lambda = \Sigma \cup \{\lambda\}$);

Γ= multime finita, nevida, numita alfabetul stivei, ($\Gamma_\lambda = \Gamma \cup \{\lambda\}$);

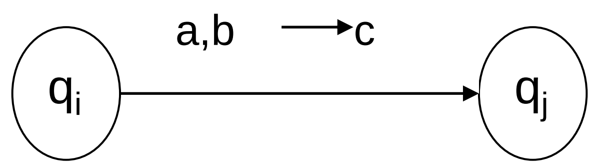
$\delta : Q \times \Sigma_\lambda \times \Gamma_\lambda \rightarrow \mathcal{P}(Q \times \Gamma_\lambda)$, numita functia de tranzitie;

$q_0 \in Q$, numita starea initiala;

$F \subseteq Q$ numita multimea starilor finale.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Notatie



Observatie

(i) Metoda standard de testare a vidarii stivei: $\$ \in \Gamma$;

(ii) Metoda standard de testare a terminarii secventei de intrare: trecerea intr-o stare finala.

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

Teorema 3

Fie L $\subseteq \Sigma^*$;$\exists$ G $\in$ G.I.C.: L=L(G) $\Leftrightarrow$ $\exists$ A $\in$ APD: L=L(A).

Demonstratie "$\Rightarrow$"

Fie L $\in$ LIC $\Rightarrow$ $\exists$ G=(V,Σ,P,S) $\in$ GIC ai. L=L(G);

putem defini un APD R=(Q, Σ_λ,Γ_λ, δ, q_0, F) cu ajutorul lui G astfel:

Q = $\{q_{start}, q_{loop}, q_{accept}\} \cup$ E:

E = multimea starilor auxiliare necesare implementarii depunerii in stiva a secventelor intermediare din derivarea S $\Rightarrow^*$ w, w $\in$ L;

Σ_λ,Γ_λ depind de limbajul L considerat;

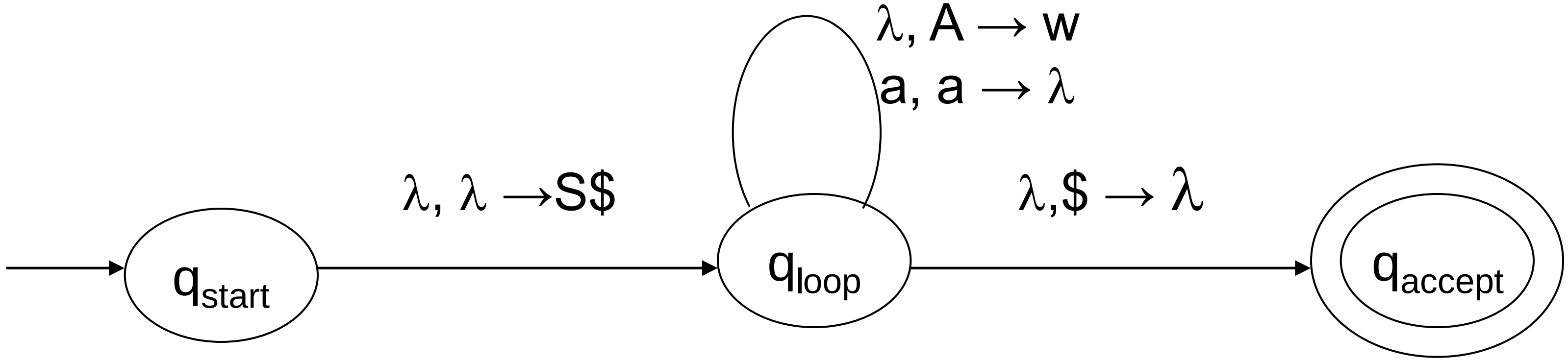
$q_0= q_{start}$;

F = $\{q_{accept}\}$;

LIMBAJE INDEPENDENTE DE CONTEXT (RECAPITULARE)

$\delta : Q \times \Sigma_\lambda \times \Gamma_\lambda \rightarrow P(Q \times \Gamma_\lambda)$ definita prin:

$\delta(q_{start},\lambda,\lambda)=\{(q_{loop},S\$)\}$
$\delta(q_{loop},\lambda,A)=\{(q_{loop},w) \mid \exists\ A \rightarrow w \in P\}$
$\delta(q_{loop},a,a)=\{(q_{loop}, \lambda)\}$
$\delta(q_{loop},\lambda,\$)=\{(q_{accept}, \lambda)\}$



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