Calculation of Satellite Position from Ephemeris Data

Table A3-1. Representation of GPS Broadcast Ephemeris

	Time Parameters
t_{0e}	Reference time, ephemeris parameters (s)
t_{0c}	Reference time, clock parameters (s)
a_0, a_1, a_2	Polynomial coefficients for clock correction (bias (s), drift (s/s), drift rate (aging) (s/s^2))
	Keplerian Parameters
$\sqrt{A}$	Square root of the semi-major axis (m$^{1/2}$)
e	Eccentricity (dimensionless)
i_0	Inclination angle at reference time (semicircles)
Ω_0	Longitude of ascending node at reference time (semicircles)
ω	Argument of perigee (semicircles)
$\overline{M}_0$	Mean anomaly at reference time (semicircles)
	Pertubation Parameters
Δn	Mean motion difference from computed value (semicircles/s)
$\dot{\Omega}$	Rate of change of right ascension (semicircles/s)
$\dot{i}$	Rate of change of inclination (semicircles/s)
C_{us}	Amplitude of the sine harmonic correction term to the argument of latitude (rad)
C_{uc}	Amplitude of the cosine harmonic correction term to the argument of latitude (rad)
C_{is}	Amplitude of the sine harmonic correction term to the angle of inclination (rad)
C_{ic}	Amplitude of the cosine harmonic correction term to the angle of inclination (rad)
C_{rs}	Amplitude of the sine harmonic correction term to the orbit radius (m)
C_{rc}	Amplitude of the cosine harmonic correction term to the orbit radius (m)

Source: Seeber (2003).

The individual satellite time, t_{SV}, is corrected to GPS system time, t, using:

$$t = t_{SV} - \Delta t_{SV}$$

in which

$$\Delta t_{SV} = a_0 + a_1(t - t_{0c}) + a_2(t - t_{0c})^2 \qquad \text{(A3-1)}$$

Differentiating Eq. A3-1 with respect to time yields satellite clock drift.

The satellite coordinates in the WGS-84 Cartesian system are computed for a given epoch, t. The time, t_k, elapsed since the reference epoch, t_{0e}, is $t_k = t - \Delta t_{0e}$.

Table A3-2. Calculating Satellite Coordinates from GPS Broadcast Ephemeris

	Constants
$GM = 3.986005 \cdot 10^{14}\ \text{m}^3/\text{s}^2$	WGS-84 value for the product of gravitational constant G and the mass of the Earth M
$\omega_e = 7.292115 \cdot 10^{-5}$ rad/s	WGS-84 value of the Earth's rotation rate
$\pi = 3.1415926535898$ (exactly)	
	Keplerian Parameters to ECEF Coordinates
$T = 2\pi/\sqrt{GM/A^3}$	Satellite orbital period
$n_0 = \sqrt{\dfrac{GM}{A^3}}$	Computed mean motion
$n = n_0 + \Delta n$	Corrected mean motion
$\overline{M}_k = \overline{M}_0 + nt_k$	Mean anomaly
$E_k = \overline{M}_k + e\sin E_k$	Kepler's equation of eccentric anomaly is solved by iteration. Because of the small eccentricity of GPS orbits ($e < 0.001$), two steps are usually sufficient: $E_0 = \overline{M}$, $E_i = \overline{M} + e\sin E_{i-1}$, $i = 1, 2, 3, \ldots$
$\cos \upsilon_k = \dfrac{\cos E_k - e}{1 - e\cos E_k}$	True anomaly
$\sin \upsilon_k = \dfrac{\sqrt{1-e^2}\sin E_k}{1 - e\cos E_k}$	True anomaly
$\Phi_k = \upsilon_k + \omega$	Argument of latitude
$\delta u_k = C_{uc}\cos 2\Phi_k + C_{us}\sin 2\Phi_k$	Argument of latitude correction
$\delta r_k = C_{rc}\cos 2\Phi_k + C_{rs}\sin 2\Phi_k$	Radius correction
$\delta i_k = C_{ic}\cos 2\Phi_k + C_{is}\sin 2\Phi_k$	Inclination correction
$u_k = \Phi_k + \delta u_k$	Corrected argument of latitude
$r_k = A(1 - e\cos E_k) + \delta r_k$	Corrected radius
$i_k + i_0 + \dot{i}t_k + \delta i_k$	Corrected inclination
$X'_k = r_k \cos u_k$	Position in the orbital plane
$Y'_k = r_k \sin u_k$	Position in the orbital plane
$\Omega_k = \Omega_0 + (\dot{\Omega} - \omega_e)t_k - \omega_e t_{0e}$	Corrected longitude of ascending node
$X_k = X'_k \cos \Omega_k - Y'_k \sin \Omega_k \cos i_k$	Earth-fixed geocentric satellite coordinate
$Y_k = X'_k \sin \Omega_k + Y'_k \cos \Omega_k \cos i_k$	Earth-fixed geocentric satellite coordinate
$Z_k = Y'_k \sin i_k$	Earth-fixed geocentric satellite coordinate

ECEF, Earth-centered, Earth-fixed.
Source: Seeber (2003).