

# Magnetic Materials

## Basics of Magnetism:

Magnetic flux density (Magnetic induction)  $\rightarrow \vec{B} = \mu \vec{H}$  magnetic field intensity or Magnetic field strength.

Unit of  $\vec{B} \rightarrow \text{weber/m}^2$ .

$$\mu = \mu_0 \mu_r$$

$\hookrightarrow$  absolute permeability of free space or vacuum.  
 $\hookrightarrow$  permeability of the medium.

→ If the substance is placed in a magnetic field, the substance will get magnetised.

$$\begin{aligned} \vec{B} &= \mu \vec{H} = \mu_0 \mu_r \vec{H} + \mu_0 \vec{H} - \mu_0 \vec{H} \\ &= \mu_0 \vec{H} + \mu_0 (\mu_r - 1) \vec{H} \end{aligned}$$

Magnetization  $M$ .

where  $\vec{M} = (\mu_r - 1) \vec{H}$

Magnetization (Magnetic dipole moment per unit volume)

Unit of  $\vec{M} = \text{A/m}$ .

$\boxed{\vec{B} = \mu_0 (\vec{H} + \vec{M})} \rightarrow \text{when placed in magnetic field.}$

where  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ .

$\boxed{\vec{M} = \chi_m \vec{H}}$

magnetic susceptibility.

$\boxed{\mu_r = 1 + \chi_m}$

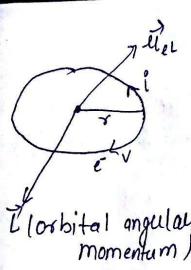
$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi_m \vec{H})$$

$$= \mu_0 \vec{H} (1 + \chi_m) = \mu \vec{H}$$

## Origin of Magnetism.

Atomic magnetic dipoles originates from

- ① Orbital motion of electrons
- ② Spin motion of electrons
- ③ Spin motion of nucleus

① 

$$\vec{m}_{el} = -g_e \frac{\vec{l}_{orbital}}{\hbar}$$
  
 (orbital angular momentum)      (orbital magnetic moment)

$g_e = 1$  (constant)  
 (orbital 'g' factor or orbital gyromagnetic ratio)

$\mu_b = \text{Bohr magneton} = \frac{e\hbar}{2m}$   
 $= \frac{e\hbar}{4\pi m}$   
 $\mu_b = \frac{1.6 \times 10^{-19} \times 6.626 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} =$   
 $\boxed{\mu_b = 9.27 \times 10^{-24} \text{ Amp-m}^2}$

$\vec{m}_{el} = \frac{\mu_b}{\hbar} \sqrt{l(l+1)} \hat{t}$        $\Rightarrow \vec{m}_{el} = \mu_b \sqrt{l(l+1)}$   
 ↑ magnitude of magnitude of moment      l = orbital quantum number

②  $\vec{m}_{es} = -g_s \frac{\vec{\mu}_b}{\hbar} \hat{s}$   
 magnetic moment associated with spin motion.  
 $\omega$  = spin quantum number       $\boxed{m_{es} = g_s \mu_b \sqrt{s(s+1)} \hat{s}}$

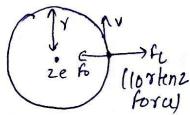
③ Nuclear spin  
 Nuclear magneton  $\mu_n = \frac{e\hbar}{2m_p}$   
 $m_p$  = mass of proton,  
 $\mu_n = \frac{e\hbar}{4\pi m_p} = \frac{1.6 \times 10^{-19} \times 6.626 \times 10^{-34}}{4 \times 3.14 \times 1.67 \times 10^{-27}} = 5.05 \times 10^{-27} \text{ Amp}^{-1}$   
 Since  $m_p \gg m_e$  so nuclear spin is generally negligible.

Classification of Magnetic material.  
 (i) Diamagnetic material  
 (ii) Paramagnetic material  
 (iii) Ferromagnetic material  
 (iv) Ferrimagnetic material  
 (v) Antiferromagnetic material

Diamagnetic material  
 Diamagnetic material are those materials which when placed in external magnetic field acquire small magnetization in direction opposite to the applied field.  
 Susceptibility of diamagnetic substances is -ve.  
 Diamagnetic materials have a tendency to repel the magnetic lines of forces due to external field.  
 Superconductor also repel magnetic lines of forces hence are the example of perfect diamagnetism.  
 e.g. Cu, Ag, Au, Zn, Hg, He, Ar.

Langvin Theory of dia magnetism.  
 (i) External magnetic field  
  
 (ii) direction of revolving of electron  
 (iii) centripetal force ( $f_0$ ).  
 Without the external magnetic field  
 $f_0 = \frac{mv_0^2}{r}$ ;  $v_0$  = velocity of electron in absence of external magnetic field.  
 $f_0 = \frac{mv_0^2}{r} = \perp \frac{(ze)e}{4\pi\epsilon_0 r^2}$ ;  $\omega_0$  = angular velocity of electron in absence of magnetic field.  
 $\perp$  (coulomb force)  
 $\Rightarrow m \frac{(rv_0)^2}{r} = \perp \frac{2e}{4\pi\epsilon_0 r^3} \Rightarrow \omega_0^2 = \frac{2e^2}{4\pi\epsilon_0 m r^3}$  - (1)

→ In presence of external magnetic field let the velocity of electron become  $v$  and velocity  $w$ , in presence of external magnetic field



$$f_L = \text{Lorentz force} = -e(\vec{v} \times \vec{B}) = evB$$

Now eq. of motion force can be written as

$$f_m = f_0 - |f_L| \\ \frac{mv^2}{r} = \frac{2e^2}{4\pi G_0 r^2} - evB$$

$$\rightarrow \frac{m(rw)^2}{r} = \frac{2e^2}{4\pi G_0 r^2} - eB(rw)$$

$$\Rightarrow mrw^2 = \frac{2e^2}{4\pi G_0 r^2} - erwB$$

$$w^2 = \frac{2e^2}{4\pi G_0 m r^3} - \frac{erwB}{m}$$

from eq ①

$$w^2 + \frac{erwB}{m} = w_0^2 - ②$$

$$w = \frac{-(eB)}{(m)} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + w_0^2} \Rightarrow w = -\frac{(eB)}{(2m)} \pm \sqrt{\left(\frac{eB}{2m}\right)^2 + w_0^2}$$

for small values of  $B$ ,  $\frac{eB}{2m} \ll w_0$ .

$$w = \pm w_0 - \frac{(eB)}{(2m)} - ③$$

$\Delta w = -\frac{(eB)}{(2m)}$  → change in freq. when magnetic field applied.

$$\omega = 2\pi\nu \Rightarrow \Delta\omega = 2\pi\Delta\nu$$

$$\rightarrow 2\pi\Delta\nu = -\frac{(eB)}{(2m)} \Rightarrow \boxed{\Delta\nu = -\frac{eB}{4\pi m}}$$

Change in linear freq in presence of magnetic field

Magnetic moment of electron.

$$M = iA = \left(\frac{e}{T}\right) \pi r^2 = e \left(\frac{1}{T}\right) \pi r^2 \\ = e\tau \pi r^2$$

$T$  = time period.

Change in magnetic moment.

$$\Delta M = e(\Delta t) \pi r^2 = e \left(\frac{-eB}{4\pi m}\right) \pi r^2$$

$$\boxed{\Delta M = -\frac{e^2 B}{4m} \pi r^2} - ④$$

↑  
this is change in magnetic moment due to one electron.

Summing over all electrons in the atom.  
the change in magnetic moment per atom.

$$\boxed{\Delta M = -\frac{e^2 B}{4m} \sum r^2} - ⑤$$

Since, core electrons have different radii,

$$\text{so, } \boxed{\sum r^2 = Z \langle r^2 \rangle}$$

where  $\langle r^2 \rangle^{1/2}$  is the average distance from the field of electron from the field axis. i.e. 2-axis (axis along which we can apply Ampere's law).

$$\Delta M = -\frac{e^2 B Z \langle r^2 \rangle}{4m} - ⑤$$

if the orbit lies in x-y plane then

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle - ⑥$$

If  $\langle r_0^2 \rangle^{1/2}$  is the avg. distance of electron from the nucleus then

$$\langle r_0^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle - ⑦$$

if the atom has spherical symmetry then

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$$

$$\text{from ⑥ } \langle r^2 \rangle = 2\langle x^2 \rangle$$

$$\text{from ⑦ } \langle r^2 \rangle = 3\langle x^2 \rangle$$

$$\langle r^2 \rangle = \frac{r_0^2}{3}$$

$$\boxed{\langle r^2 \rangle = \frac{2}{3} \langle r_0^2 \rangle}$$

from eq ⑤.

$$\Delta U = -\frac{e^2 B z}{4m} \langle r^2 \rangle$$

$$\Delta U = -\frac{e^2 B z}{4m} \frac{2}{3} \langle r_0^2 \rangle$$

$$\boxed{\Delta U = -\frac{e^2 B z}{6m} \langle r_0^2 \rangle}$$

If there are  $N$  atoms per unit volume, then total magnetization  $M$  will be given as  $M$  (total magnetic moment per unit volume)

$$\boxed{M = N \Delta U = -\frac{e^2 B z N}{6m} \langle r_0^2 \rangle}$$

$$\text{Susceptibility } \gamma_s = \frac{M}{H} = -\frac{e^2 B z N}{6m} \langle r_0^2 \rangle$$

$$\boxed{\gamma_{dia} = -\frac{e^2 B z N}{6m} \langle r_0^2 \rangle} \Rightarrow -ve \text{ of temp independent}$$

### Paramagnetic materials

Paramagnetic materials are those materials, which when placed in external strong magnetic field can get weakly magnetized in the field direction.

In the absence of external magnetic field, paramagnetic materials do not exhibit resultant magnetic moment due to randomization of magnetic field. However, if the external field is applied, the magnetic dipoles tend to align in the field direction and now, the para magnetic materials have some magnetization.

→ Susceptibility of paramagnetic material is greater than zero but small.

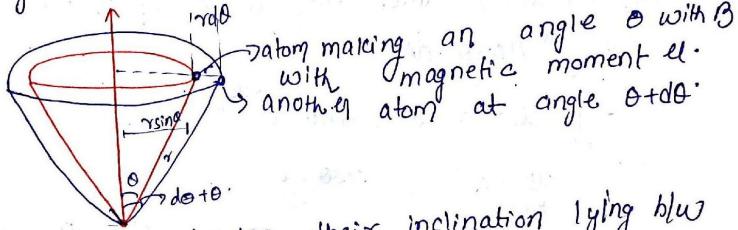
→ Susceptibility is temp dependent.

Ex:- Platinum, Al, Mn, Cr, O.

### Langevin theory of Paramagnetism

Let us consider a paramagnetic substance having  $N$  atoms/m<sup>3</sup> (molecules/m<sup>3</sup>, magnetic dipole moment).

And let every atom is having the permanent magnetic moment  $\vec{\mu}$ .



No. of dipoles having their inclination lying b/w  $\theta$  and  $\theta + d\theta$

$$dN \propto \exp\left(-\frac{eB}{kT}\right) d\Omega$$

Also, the potential energy of a dipole making an angle  $\theta$  with  $B$

$$U = -\vec{\mu} \cdot \vec{B} = -eB \cos \theta$$

$$d\Omega = \frac{\pi r^2 \sin \theta \cos \theta}{r^2} = 2\pi \sin \theta d\theta$$

$$dN = C \exp\left(\frac{eB \cos \theta}{kT}\right) \cdot (2\pi \sin \theta d\theta)$$

$$dN = 2\pi C \sin \theta e^{\frac{eB \cos \theta}{kT}} d\theta \quad \dots \text{---(1)}$$

$\theta \rightarrow 0$  to  $\pi$   
Total no. of dipoles lying b/w  $0$  &  $\pi$

$$N = 2\pi C \int_0^\pi \sin \theta e^{\frac{eB \cos \theta}{kT}} d\theta$$

$$\text{let } a = \frac{eB}{kT} \quad \cos\theta = x \\ -\sin\theta d\theta = dx$$

$$N = \alpha\pi C \int_1^{\infty} e^{ax} dx \Rightarrow N = \alpha\pi C \left[ \frac{e^{ax}}{a} \right]_1^{\infty}$$

$$N = \alpha\pi C \left[ \frac{e^a - e^{-a}}{a} \right] \Rightarrow C = \frac{Na}{2\pi(e^a - e^{-a})} \quad \text{---(2)}$$

The contribution to magnetic field by a dipole would be  $\mu \cos\theta$ .

Total magnetic moment per unit volume i.e. Magnetization due to all  $N$  dipoles.

$$M = \int_0^{\infty} \mu \cos\theta dN$$

$$= \int_0^{\infty} \mu \cos\theta (\alpha\pi C \sin\theta e^{\frac{eB \cos\theta}{kT}} d\theta)$$

$$\frac{eB}{kT} = a, \quad \cos\theta = x$$

$$M = \alpha\pi C \mu \int_{-1}^1 x(-dx) e^{ax} = \alpha\pi C \mu \int_{-1}^1 x e^{ax} dx$$

$$= \alpha\pi C \mu \left[ x \frac{e^{ax}}{a} - \frac{e^{ax}}{a^2} \right]_1^{\infty}$$

$$= \alpha\pi C \mu \left[ \left( \frac{e^a}{a} - \frac{e^a}{a^2} \right) - \left( -\frac{e^{-a}}{a} - \frac{e^{-a}}{a^2} \right) \right]$$

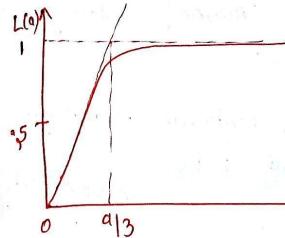
$$= \alpha\pi C \mu \left[ \frac{e^a}{a} - \frac{e^a}{a^2} + \frac{e^{-a}}{a} + \frac{e^{-a}}{a^2} \right]$$

$$= \alpha\pi \mu \frac{1}{\alpha\pi} \left[ \frac{Na}{e^a - e^{-a}} \right] \left[ \frac{e^a + e^{-a}}{a} - \frac{e^a - e^{-a}}{a^2} \right]$$

$$= \mu N \left[ \coth a - \frac{1}{a} \right]$$

$\hookrightarrow L(a) = \text{Langevin function}$

$$\boxed{M = \mu N L(a)}$$



For  $eB \ll kT$  i.e.  $a \gg 1$   
 $\Rightarrow L(a) = a/3$

$$M = N \mu \frac{a}{3} = N \mu \frac{eB}{3kT}$$

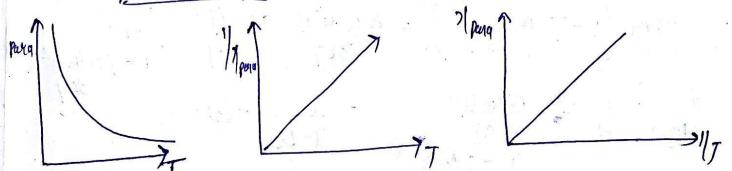
$$M = \frac{N \mu^2 B}{3kT}$$

$$\gamma_{\text{para}} = \frac{N}{H} = \frac{N \mu^2 \mu_0 H}{3kT H}$$

$$\boxed{\gamma_{\text{para}} = \frac{N \mu^2 \mu_0}{3kT}} \quad \text{Curie Law}$$

$$\boxed{\gamma_{\text{para}} = \frac{C}{T}}$$

\* where  $C = \frac{N \mu^2 \mu_0}{3k}$  = Curie constant



for  $a \gg 1$  i.e.  $eB \gg kT$ ,  $L(a) = 1$ .

$M_s = M = \mu N$  = saturation magnetization.

$\hookrightarrow$  complete alignment of magnetic dipole in direction of magnetic field.

### Weiss Theory of paramagnetism.

$\hookrightarrow$  Langevin Theory could not explain the complicated behaviour / dependence of susceptibility on temperature exhibited by the paramagnetic substances.

$\hookrightarrow$  In addition Langevin theory did not throw the light b/w the relationship of para and ferromagnetic material.

→ Weiss theory is based on internal molecular field.

Internal molecular field  $H_i \propto N$ .

$$\Rightarrow H_i = \gamma M.$$

$\gamma$  = Molecular field coefficient (Internal field coefficient)

Therefore, the net effective field,  $H_e = H + H_i$  (Applied external field)

$$H_e = H + \gamma M \quad \text{---(2)}$$

also,  $M = NL u(L)$ ;  $u$  = permanent magnetic dipole of one atom.

$$\therefore M = Nu \left(\frac{4}{3}\right) \quad \left(\because L(L) = \frac{q}{3} \text{ for smaller value of } q\right)$$

$$= Nu \left(\frac{4B_0}{3kT}\right) = \frac{Nu^2}{3kT} B_0 H_e$$

$$\text{from eq.(2)} \quad M = \frac{Nu^2}{3kT} u_0 (H + \gamma M)$$

$$\Rightarrow M \left(1 - \gamma \frac{Nu_0 u^2}{3kT}\right) = \frac{Nu_0 u^2 H}{3kT} \Rightarrow M = \frac{Nu_0 u^2}{H} \frac{3kT}{1 - \gamma \frac{Nu_0 u^2}{3kT}}$$

$$\left\{ \begin{array}{l} \theta_C = \frac{M}{H} = \frac{Nu_0 u^2}{3k} \\ \theta_{para} = \frac{C}{T - \gamma \frac{Nu_0 u^2}{3kT}} \end{array} \right. \quad = \frac{C}{T - \theta_C} \quad \text{---(3)}$$

$$\frac{\theta_{para}}{\theta_C} = \frac{C}{T - \theta_C} \quad \text{curve Weiss Law.}$$

$$\left[ C = \frac{N u_0 u^2}{3k}, \theta_C = \gamma \frac{N u_0 u^2}{3k} = \gamma C \right]$$

$$\theta_C = \gamma C \quad \theta_C = \text{paramagnetic Curie-temp.}$$

$T \gamma \theta_C \Rightarrow$  paramagnetic  
 $T < \theta_C \Rightarrow$  ferromagnetic.

### Quantum Theory of Paramagnetism

$$\chi_{para} = \frac{N u_0 u^2}{3kT}$$

$j$  = total angular momentum quantum no.

$u_b$  = Bohr magneton

$\mu_j$  = Effective no. of Bohr magnetons.

$$\mu_j = g \sqrt{j(j+1)}$$

$$g = 1 + \frac{j(j+1) + S(S+1) - l(l+1)}{2J(J+1)}$$

$g$  = Landé  $g$ -factor.

$l$  = orbital quantum number.

Q) calculate the effective Bohr magneton number for rare earth ions  $Ce^{3+}(4f^1)$  and  $Dy^{3+}(4f^9)$  in their paramagnetic states.

$Ce^{3+}(4f^1)$

↓  
1 valence electron

$l = 3$ .

$m_l = -3 \text{ to } 3$   
 $= 3, 2, 1, 0, -1, -2, -3$

$L = |\sum m_l| = 3$

$S = |\sum m_s| = 1/2$ .

$m_l$	3	2	1	0	-1	-2	-3
$m_s$	↑						

Only one electron so consider only this  $m_l$   
(just one shell has one electron)  
 $m_l$  ko add karke)

this subshell is less than half filled.

so,  $J = L - S = 3 - 1/2 = 5/2$ .

$$g = 1 + \left[ \frac{(5/2)(7/2) + (1/2)(3/2) - 3(4)}{2 \times (5/2)(7/2)} \right] = 1 + \left[ \frac{\frac{19}{2} - 12}{\frac{35}{2}} \right]$$

$$= 1 - \frac{5}{35} = 6/7.$$

$$\mu_j = \frac{6}{7} \sqrt{\frac{5}{2} \left(\frac{7}{2}\right)} = \frac{3\sqrt{35}}{7}$$

$Dy^{3+}(4f^9)$

$l = 3$

$m_l = -3 \text{ to } 3$

$m_l$	3	2	1	0	-1	-2	-3
$m_s$	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓

↓  
Paired so these do not contribute

$$L = |\Sigma_{m_L}| = |1+0+(-1)+(-1)| = 5$$

$$S = |\Sigma_{m_S}| = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2^S$$

For more than half

$$J = L+S \\ = 5 + \frac{5}{2} = 15/2 = 7.5$$

$$J = 1 + \left[ \frac{(7.5)(\frac{15}{2}) + (\frac{5}{2})(\frac{7}{2}) - 30}{2 + 15 \times \frac{17}{2}} \right]$$

$$= 1 + \left[ \frac{7.5 \times 17 + 2.5 \times 7 - 60}{15 \times 17} \right]$$

$$= 1 + \left[ \frac{85}{15 \times 17} \right] = 1 + \frac{1}{3} = 4/3 \\ = 1.33$$

$$B_{eff} = g \sqrt{J(J+1)} \\ = 1.33 \sqrt{\frac{15 \times 17}{2}} \\ = 1.33 \times \frac{15.9687}{2} = 10.61$$

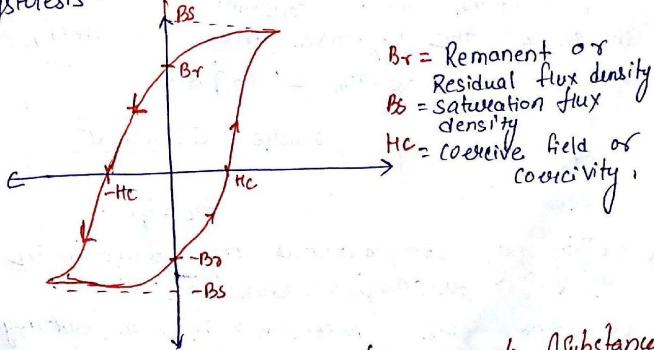
### Ferromagnetic Substances.

Ferromagnetic substances are those substances which when placed in weak external magnetic field get strongly magnetized in the field direction. These materials possess permanent magnetic moment even in absence of external magnetic field. The susceptibility for these substances is very large.  $\chi_m \approx 10^5$ .

These materials strongly attract the magnetic lines of forces due to external field. Ferromagnetism is observed upto a certain temperature known as critical temperature and beyond which material is paramagnetic.

e.g.: Ni, Co, Fe, Cd.

These materials exhibits hysteresis loop. The phenomenon of  $B$  lagging  $H$  is called hysteresis.



$B_r$  = Remanent or Residual flux density  
 $B_s$  = Saturation flux density  
 $H_c$  = Coercive field or coercivity

### Spontaneous Magnetization in ferromagnetic Substances

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\text{If } H=0 \quad \vec{B}_r = \mu_0 \vec{M}_r \rightarrow \text{Spontaneous Magnetization}$$

$$\text{Typically } B_r = 1 \text{ T} \quad M_r = \frac{1}{\mu_0} = 10^6 \text{ A/m}$$

$M_T = 10^6 \text{ A/m}$   
 In absence of external field there is large magnetization in these materials.

If each atom of the ferromagnetic substance has a magnetic moment of  $1 \mu_B$ .

$$M_T = N \times \mu_B$$

$$N = \frac{M_T}{\mu_B} = \frac{10^6}{927 \times 10^{-23}}$$

i.e if external magnetic field is zero, then also there are  $10^{23}$  atoms/m<sup>3</sup> which are in the field direction, so there is spontaneous magnetization.

### Weiss theory of ferromagnetism.

internal molecular field  $H_m \propto M$ .

$$H_m = \gamma M$$

$\gamma$  = internal field constant.  
 Therefore, the effective magnetic field,

$$H_{\text{eff}} = H + H_m = H + \gamma M$$

$$\left| \frac{M}{M_s} = \frac{C}{T - \Theta_c} \right|$$

$$\text{where } C = \frac{N \mu_0 \mu^2}{K}$$

$$\Theta_c = \gamma C$$

### Possibility of spontaneous Magnetization in ferromagnetic substances

Let there are  $N$  spins/m<sup>3</sup> in a ferromagnetic substance and each spin contributes to magnetic moment of  $1 \mu_B$  then

Magnetization is

$$M = N \mu_0 \tanh \left[ \frac{\mu_0 \mu_B H_e}{K T} \right]$$

$\frac{\mu_0 \mu_B}{K T}$  is very small so,  $\tanh \theta \rightarrow 0$

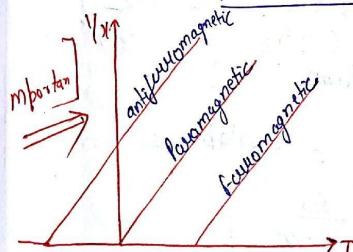
$$\text{for high value of temp.} \\ \tanh \left[ \frac{\mu_0 \mu_B}{K T} (H + \gamma M) \right] = \frac{\mu_0 \mu_B}{K T} \left[ \frac{H + \gamma M}{K T} \right]$$

$$\therefore N = N \mu_0 \left[ \frac{\mu_0 \mu_B}{K T} \left[ \frac{H + \gamma M}{K T} \right] \right]$$

$$\Rightarrow M \left[ 1 - \frac{N \mu_0 \mu_B^2 \gamma}{K T} \right] = \frac{N \mu_0 \mu_B^2 H}{K T}$$

$$\left| \frac{M}{M_s} = \frac{N \mu_0 \mu_B^2}{K T} \frac{H}{1 - \frac{N \mu_0 \mu_B^2 \gamma}{K T}} \right| \Rightarrow \left| \frac{M}{M_s} = \frac{C}{T - \Theta_c} \right|$$

$$\text{where } \left| C = \frac{N \mu_0 \mu_B^2}{K} \right|, \theta = \gamma C$$



If  $H=0$ ,

$$M = N \mu_0 \tanh \left[ \frac{\mu_0 \mu_B \gamma M}{K T} \right]$$

$$M_s = M_{\max} = N \mu_0 \mu_B$$

Saturation magnetization

$$\left| \frac{M}{M_s} = \tanh \left[ \alpha \right] \right| \text{ where } -1$$

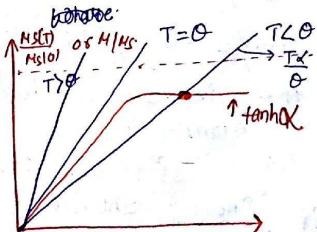
$$\alpha = \frac{\mu_0 \mu_B \gamma M}{K T} \Rightarrow M = \frac{\alpha K T}{\mu_0 \mu_B \gamma}$$

$$\frac{M}{M_s} = \frac{M}{N \mu_0} = \frac{(K T \alpha)}{\frac{\mu_0 \mu_B \gamma}{N \mu_0}}$$

$$\Rightarrow \left| \frac{M(T)}{M_s(0)} = \frac{T \alpha}{\Theta} \right| - (i) \quad \Theta = \frac{N \mu_0 \mu_B^2}{K}$$

$$\left| \frac{M}{M_s} = \frac{M(T)}{M_s(0)} = T \alpha \right| \quad \Theta = \frac{N \mu_0 \mu_B^2}{K}$$

$\Rightarrow$  Spontaneous polarization is possible when  $T < \Theta$



### Exchange interaction in magnetic materials

The large value of Weiss field in ferro magnetic materials is explained by Heisenberg using exchange interaction b/w the spins.

$$\text{imb } E_{\text{exch}} = -2J \vec{s}_1 \cdot \vec{s}_2 \leftarrow \begin{matrix} \text{spin of atom 2} \\ \uparrow \\ \text{exchange energy} \end{matrix} \quad \begin{matrix} \text{spin of atom 1} \\ \downarrow \\ \text{exchange integral} \end{matrix}$$

→ If spins of  $s_1$  and  $s_2$  are parallel, then  $J > 0$  (ferromagnetic substance) and if spins are antiparallel, then  $J < 0$  (paramagnetic substance)

→  $J$  depends on the separation b/w the atoms as well as the overlap of electron charge cloud b/w the atoms.



Overlap.  
(more is the overlap more is the value of  $J$ )

The estimate of overlap is

$$\text{overlap of electron charge cloud b/w the atoms.} \quad S = \frac{2r}{R-2r}$$

$R$  = interatomic distance  
(distance b/w two nuclei)

$r$  = radius of magnetic orbital

for a ferromagnetic material,

$$E_{\text{exch}} = -2 \sum_{j=i+1}^N J_{ij} \vec{s}_i \cdot \vec{s}_j \quad i^{\text{th}} \text{ atom}, \quad j^{\text{th}} \text{ atom.}$$

if the nearest neighbours are  $z$  of  $i^{\text{th}}$  atom then

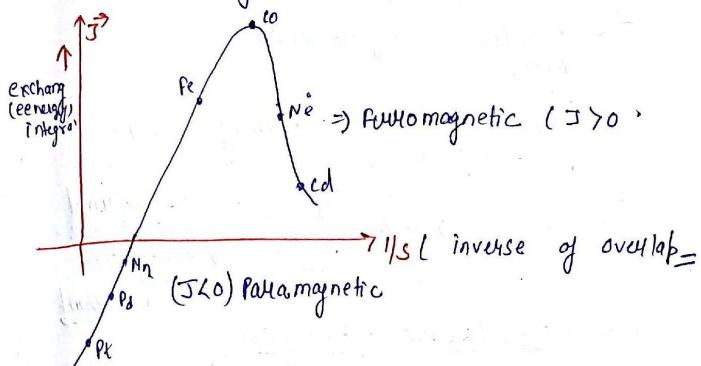
$$E_{\text{exch}} = -z(2Jz^2) = -2Jz^2. \quad \textcircled{1}$$

(this energy should be equal to  $kT_c$  ( $k$  = boltzmann constant) because at  $T_c$ , ferromagnetism is destroyed)

$$E_{\text{exch}} = -kT_c \quad \textcircled{2}$$

$$\Rightarrow 2Jz^2 = kT_c \Rightarrow J = \frac{kT_c}{2z^2}$$

$S$  = spin of individual atom.



### Stoner expression

$$\text{Exchange energy } U_i = -2 \sum_{j=1}^{z-1} J_{ij} \vec{s}_i \cdot \vec{s}_j = -2J \sum_{j=1}^{z-1} \vec{s}_i \cdot \vec{s}_j = -2Jz (\vec{s}_i \cdot \vec{s}_j) \quad (z \text{ no. of nearest neighbours})$$

(Assuming that  $J$  is constant for all the atoms and  $z$  is the nearest neighbours).

$$\textcircled{2} \quad U_i = -2Jz (s_{xi} s_{xj} + s_{yi} s_{yj} + s_{zi} s_{zj})$$

Suppose magnetization is along  $z$  direction.

$$U_i = -2Jz (s_{zi} s_{zj}) \quad \textcircled{2}$$

$$(s_{xi} = s_{yj} = 0)$$

$$M = N g \mu_B s_z$$

$N$  = no. of spins.

$$s_{zj} = \frac{M}{N g \mu_B}$$

$$U_i = -2Jz \left[ s_{zi} \frac{M}{N g \mu_B} \right] = -\frac{2Jz s_{zi} M}{N g \mu_B} \quad \textcircled{3}$$

$$\text{also, } \nabla U = -\mu_0 B_E$$

↳ Magnetic moment along z-dir

$$= -g \mu_B S_{z1} B_E \quad \text{(i)}$$

from eq (3) & (i)

$$-\frac{2JZ}{N} S_{z1} M = -g \mu_B S_{z1} B_E$$

$N \mu_B$

$$\Rightarrow B_E = \frac{2JZM}{Ng^2 \mu_B^2} \Rightarrow \text{exchange field.}$$

$$B_E = \gamma M$$

where

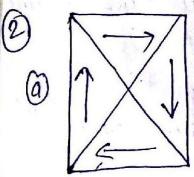
$$\gamma = \frac{2JZ}{Ng^2 \mu_B^2} = \text{internal field constant or Weiss field constant}$$

### DOMAIN MODEL (Weiss hypothesis)

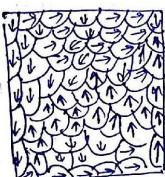
(i) All ferromagnetic solids consist large number of small regions, these small regions are known as domain.

The domain size may vary from  $10^{-6}$  to entire volume of the crystal.

Each domain contains  $10^9$  to  $10^{15}$  atoms.



Single crystal

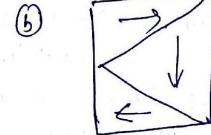
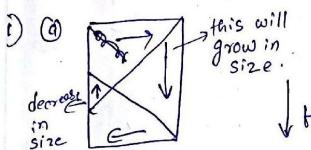


Polycrystal  
Net magnetization = 0

The spin magnetic moments in each domain are aligned in one particular direction. However, the magnetization of domains are so arranged, that the net magnetization is zero.

### In presence of external magnetic field H.

Two independent process occur:



The parallel or nearly parallel domains can grow in size at the expense of antiparallel domains.

(ii) The magnetic moments of the domains rotate along the direction of magnetic field to give the non-zero magnetization.

If the external magnetic field is increased continuously then the magnetic moments in the domain will be completely aligned along the direction of the external magnetic field to give the saturation magnetic field  $H_s$ .

### Hysteresis

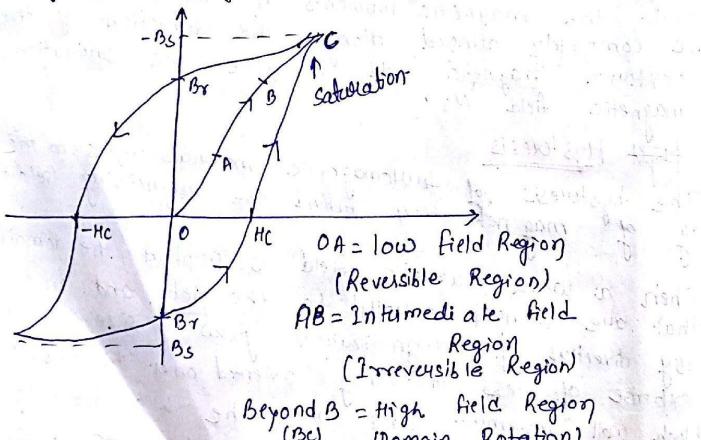
The hysteresis of ferromagnetic materials refers to the lag of magnetization behind the magnetizing field.

When a weak magnetic field is applied, the domains that are aligned parallel to the field and in the easy direction of magnetization grow in size at the expense of less favourably oriented ones. This results in Bloch wall movement and when the weak field is removed the domains reverse back to their original state. This reversible wall displacement is indicated by OA of the magnetization curve.

When the field becomes stronger the Bloch wall movements continues and it is mostly irreversible movement. This is indicated by AB.

At the point B all domains have got magnetized along their easy directions. Application of still higher

fields rotates the domain ~~hor~~ into the field direction which may be away from the easy direction thereby storing anisotropy energy. Once the domain rotation is complete the specimen is saturated (i) ON removal of the field the specimen tends to attain the original configuration by the movement of Bloch walls. But this movement is hampered by impurities, lattice, imperfections etc, and so more energy must be supplied to overcome the opposing forces. This means that a coercive field is required to reduce the magnetization of the specimen to zero.



Q) Why the entire crystal of iron does not organize itself to form single domain?

$\Rightarrow$  Gibbs free energy  $G = E - TS - \text{Entropy}$

magnetic materials are highly ordered.

that's why entropy ( $S$ ) is very less  
 $\Rightarrow S \approx 0$ ,  $T_S$  is negligible

A/C thermodynamics, for a system to be stable, energy / i.e. should be min:

$$G = E \text{ (and } E \text{ is not min)}$$

so, if the crystal is single domain energy is not minimum and so, does not exist.

Inherent energy consists of four terms

### i) Anisotropy Energy:

There are two types of direction.

(i) Easy direction

(ii) Hard direction:

Suppose dipoles are  $\Rightarrow$  in this direction and if we want to magnetize along this direction only then it is easy direction and if we want to magnetize along opposite direction then hard direction.

The excess energy required to magnetize the sample along the easy direction is called anisotropy energy.

ii) Magnetostatic Energy or Magnetic field energy  
Energy required to assemble the magnetic moment in particular domain is called magnetostatic energy.

### iii) Domain wall energy or the Bloch wall energy:

A thin region that separates adjacent domains magnetized in different directions is called domain walls or Bloch wall. In going from one domain to another oriented differently, the electron spin changes gradually. This is because the exchange energy is lower when the change is gradual than when it occurs abruptly. But the anisotropy energy is ~~lower~~ only when spin change abruptly. Hence

Change in the exchange energy when the angle b/w the spins of domain change from 0 to  $\phi_0$ .

$$\begin{aligned}\Delta E_{\text{exch}} &= E_{\text{exch}}(\phi_0) - E_{\text{exch}}(0) \\ &= -2JS^2(1-\frac{\phi_0^2}{2}) + 2JS^2 \\ &= JS^2\phi_0^2.\end{aligned}$$

$\hookrightarrow$  exchange integral

The angle b/w spins for  $N$  equals steps is  $\phi_0$ , therefore angle for two neighbouring spins is  $(\phi/N)$ . So, the change in exchange energy for two neighbouring atoms/spins is

$$(\Delta E_{\text{exch}})_{\text{1 pair}} = JS^2 \frac{\phi_0^2}{N^2}.$$

Therefore, the total change in exchange energy for  $N$  steps.

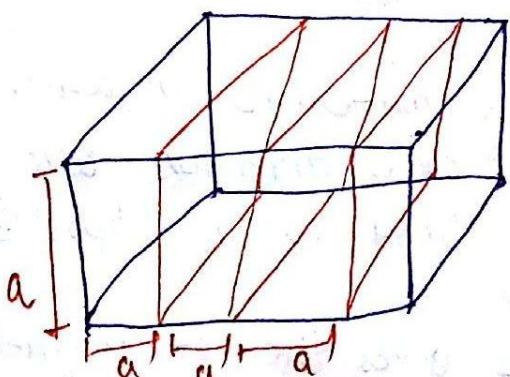
$$(\Delta E_{\text{exch}})_{\text{total}} = JS^2 \frac{\phi_0^2}{N^2} \times N = JS^2 \frac{\phi_0^2}{N}$$

$\therefore$  The total change in exchange energy per unit area of the wall (Bloch wall)

$$(\Delta E_{\text{exch}})_{\text{total per unit area}} = JS^2 \frac{\phi_0^2}{Na^2}.$$

$a$  = lattice constant

$\hookrightarrow$  distance b/w two spins/atoms



$N$  is very large, so,  $\Delta E_{\text{exch}}$  is min, and if  $N$  is large anisotropy will increase.

Anisotropy energy per unit area of the wall - (2)

$$\mathcal{E}_{\text{anis}} = \frac{KNA}{N^2} \quad \begin{matrix} K \\ \rightarrow \text{anisotropy constant} \end{matrix}$$

Bloch wall energy is equilibrium energy of  $\Delta E_{\text{exg}}$  and  $\mathcal{E}_{\text{aniso}}$

Therefore, total energy per unit area

$$E = \frac{JS^2 \phi_0^2}{N^2} + KNA \quad \text{--- (3)}$$

for eq. energy.  $\frac{dE}{dN} = 0$

$$\Rightarrow KA - \frac{JS^2 \phi_0^2}{N^2} = 0 \Rightarrow N^2 = \frac{JS^2 \phi_0^2}{KA}$$

$$\Rightarrow N = \frac{S\phi_0}{a} \sqrt{\frac{J}{KA}}$$

∴ At equilibrium, the wall energy is

$$E_w = \frac{JS^2 \phi_0^2}{a^2} \frac{a}{S\phi_0} \sqrt{\frac{KA}{J}} + KA \frac{S\phi_0}{a} \sqrt{\frac{J}{KA}}$$

$$E_w = S\phi_0 \sqrt{\frac{KJ}{a}} + S\phi_0 \sqrt{\frac{JK}{a}}$$

$$E_{\text{wall}} = 2S\phi_0 \sqrt{\frac{KJ}{a}} \quad \begin{matrix} \rightarrow \text{important} \end{matrix}$$

$\phi_0$  = angle b/w the two domains.

For iron,  $E_w = 10^{-3} \text{ J/m}^2$

$$N \approx 300$$

### Antiferromagnetic Material

→ Antiferromagnetic materials are those materials in which atomic or ionic dipoles in one direction are of unequal magnitude from those lined up in opposite direction

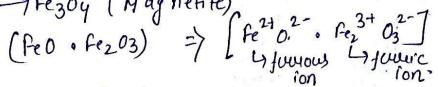
This type of arrangement of dipoles gives the net magnetic moment

→ General formula:  $[AB_2O_4]$

$$A = Fe, Ti, Ni, Cd, Mn, Zn, Cr$$

$B = Fe$  These materials are also known as ferrites.

$\rightarrow Fe_3O_4$  (Magnetite)



$Fe_3O_4 \rightarrow$  one formula unit

→ One unit cell contains 8 formula units i.e.  $8Fe_3O_4$ .  $(8Fe_3O_4) \Rightarrow (8Fe^{2+} 16Fe^{3+} 32O^{2-})$

→ There are two directions along which ionic dipoles are oriented

+ In a unit cell i.e.  $8Fe_3O_4$

(i)  $8Fe^{3+}$  ions occupy A sites (i.e. tetrahedral sites) (i.e. tetrahedrally coordinated by 4 O atoms)

(ii) Remaining  $8Fe^{3+}$  and  $8Fe^{2+}$  ions occupy B sites (octahedral sites (i.e. octahedrally coordinated by 6 O atoms))

Inb Distribution of spin, magnetic moments in unit cell of  $Fe_3O_4$  ( $8Fe_3O_4$ )

Cation	Octahedral lattice sites (B sites)	Tetrahedral lattice sites (A-sites)	Net magnetic moment
$Fe^{3+}$ (ferric ions)	$\uparrow \uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow \uparrow$	$\downarrow \downarrow \downarrow \downarrow$ $\downarrow \downarrow \downarrow \downarrow$	Exact / complete cancellation
$Fe^{2+}$ (ferrrous ions)	$\uparrow \uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow \uparrow$	-	$\uparrow \uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow \uparrow$

Therefore the contribution to magnetic moment in a formula unit i.e.  $Fe_3O_4$  is due to  $Fe^{2+}$  ion i.e.  $4/6$  b.

(2/3)

\* The contribution to magnetic moment in a unitcell of  $\text{Fe}_3\text{O}_4$  (i.e.  $\text{Fe}_3\text{O}_4$ ) is due to  $4\text{Fe}^{2+}$  ions i.e.  $8 \times 4 \mu_B = 32 \mu_B$ .

Cation	spin magnetic moment ( $\mu_{\text{B}}$ )
$\text{Fe}^{3+}$	$5\mu_B$
$\text{Fe}^{2+}$	$4\mu_B$
$\text{Mn}^{2+}$	$2\mu_B$
$\text{Co}^{2+}$	$3\mu_B$
$\text{Cu}^{2+}$	$4\mu_B$

### Garnet

General formula:  $\text{M}_3\text{Fe}_5\text{O}_4$ .

Where  $\text{M} = \text{Y} \text{ (Yttrium)}, \text{Rare earth ions} \text{ (Sm, Eu, Gd)}$

$\text{Y}_3\text{Fe}_5\text{O}_4$  (YIG, yttrium iron garnet)

### Spinel

cubic ferrites have spinel structure

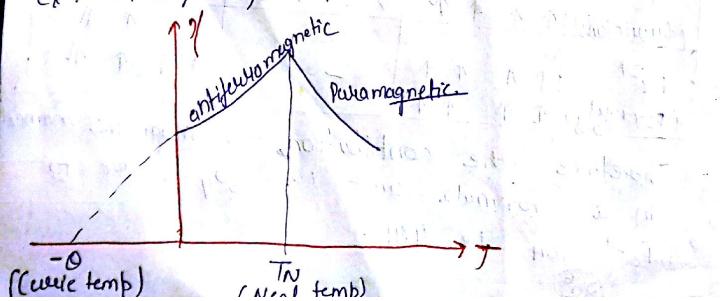
### Antiferro Magnetic Materials



→ Antiferro magnetic substances are those substances in which the spin magnetic moment of neighboring atoms are in antiparallel order.

These substances are feebly (weakly) magnetized when placed in external strong magnetic field.

Ex: -  $\text{NiO}, \text{MnO}, \text{FeO}, \text{MnS}, \text{CoO}$



$$H \left[ 1 + \frac{N_0 \mu_B^2}{3kT} (\alpha + \beta) \right] = 2M \frac{N_0 \mu_B^2}{3kT}$$

$$\chi_{\text{antiferro.}} = \frac{M}{H} = \frac{2N_0 \mu_B^2}{3k} \frac{1}{T + N_0 \mu_B^2 (\alpha + \beta)}$$

$$\Rightarrow \chi_{\text{antiferro.}} = \frac{C}{T + \theta}$$

$\theta = \text{Curie temp}$

$C = \text{Curie constant}$

$$C = \frac{2N_0 \mu_B^2}{3k}, \theta = \frac{(\alpha + \beta) C}{2}$$

When  $T = T_N$

and  $M_B$  have finite value even in absence of H

$$M_A = \frac{N_0 \mu_B^2}{3kT} [H - \alpha M_A - \beta M_B]$$

$$M_B = \frac{N_0 \mu_B^2}{3kT} [H - \alpha M_B - \beta M_A]$$

getting  $T = T_N$  at  $H = 0$ .

$$M_A = \frac{N_0 \mu_B^2}{3kT_N} [-(\alpha M_A + \beta M_B)]$$

$$\Rightarrow \left( 1 + \frac{N_0 \mu_B^2}{3kT_N} \alpha \right) M_A + \frac{N_0 \mu_B^2}{3kT_N} \beta M_B = 0 \quad \text{--- (i)}$$

$$\text{and } M_B = \frac{N_0 \mu_B^2}{3kT_N} [0 - (\beta M_A + \alpha M_B)]$$

$$\Rightarrow \frac{N_0 \mu_B^2}{3kT_N} \beta M_A + \left( 1 + \frac{N_0 \mu_B^2}{3kT_N} \alpha \right) M_B = 0 \quad \text{--- (ii)}$$

$$\begin{cases} 1 + \frac{N_0 \mu_B^2}{3kT_N} \alpha & \frac{N_0 \mu_B^2 \beta}{3kT_N} \\ \frac{N_0 \mu_B^2}{3kT_N} \beta & 1 + \frac{N_0 \mu_B^2}{3kT_N} \alpha \end{cases} = 0 \quad \begin{cases} \alpha_1 \alpha_2 = 0 \\ b_1 b_2 = 0 \\ |a_1 a_2| = 0 \\ b_1 b_2 = 0 \end{cases}$$

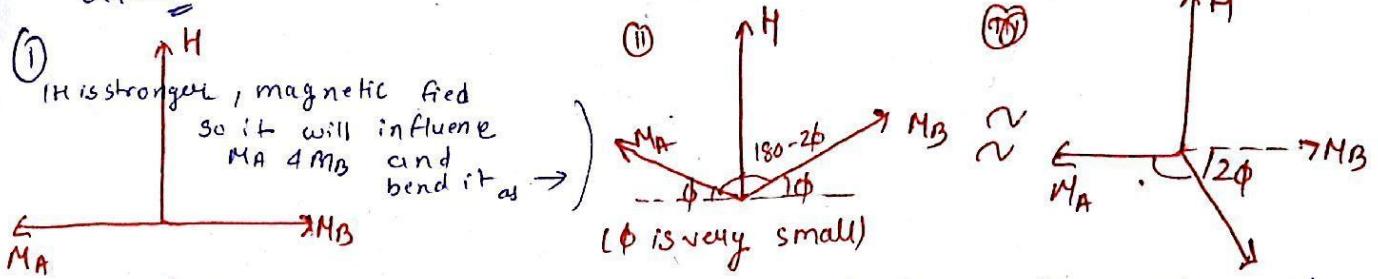
$$\left[ 1 + \frac{N_0 \mu_B^2}{3kT_N} \alpha \right]^2 - \left[ \frac{N_0 \mu_B^2}{3kT_N} \beta \right]^2 = 0 \Rightarrow 1 + \frac{N_0 \mu_B^2}{3kT_N} \alpha = \frac{N_0 \mu_B^2 \beta}{3kT_N}$$

$$\Rightarrow 1 = \frac{N_0 \mu_B^2}{3kT_N} [\beta - \alpha] \Rightarrow T_N = \frac{N_0 \mu_B^2 (\beta - \alpha)}{3k}$$

$$\boxed{\frac{T_N}{2} = \frac{\beta - \alpha}{2}}$$

$$\boxed{T_N = \frac{(\beta - \alpha)}{2} k}$$

When  $T < T_N$   
 external magnetic field is  $\perp$  to the spin axis  
 The external magnetic field is parallel to the spin axis.



The component of  $M_B$  along the direction of  $H = -M_B \sin 2\phi$   
 $= -M_B (2\phi)$   
 The field on sublattice A due to this magnetization (assuming only A-B type interaction i.e.  $\alpha = 0$ )  
 $= -M_B (2\phi) \beta$ .

$$\text{In equilibrium } H = (2\phi) M_B \beta \quad \text{--- (1)}$$

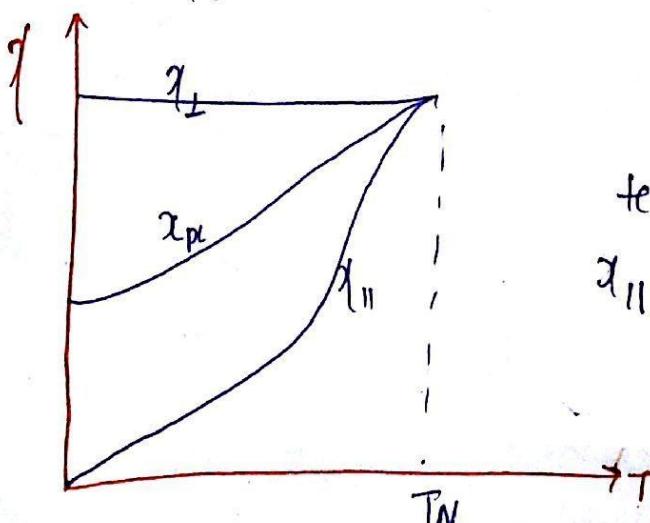
$$\begin{aligned} \text{from fig (i)} \quad M &= M_A \sin \phi + M_B \sin \phi \\ &= 2 M_B \sin \phi \quad (\text{as } M_A \approx M_B) \\ &= 2 M_B \phi \quad \text{--- (2)} \end{aligned}$$

from eq (1) & (2)

$$\chi_{\perp} = \frac{H}{M} = \frac{2 M_B \phi}{2 M_B \phi \beta} = 1/\beta$$

$$\chi_{\perp} = 1/\beta \quad \text{--- (3) (not dependent on temp)}$$

i) if external magnetic field is parallel to spin axis i.e., the net magnetization will be equal to zero.  $\chi_{\parallel} = 0$  at  $T = 0K$ .



$\chi_{\perp}$  is constant for all temp for  $T < T_N$  However,  $\chi_{\parallel}$  increases from 0 to  $\chi_{\parallel}(T_N)$

Q Prove  $\frac{[\bar{\gamma}_{pc}]}{[\bar{\gamma}_{pc}]_{T=0K}} = \frac{2}{3}$

$$[\bar{\gamma}_{pc}]_{T=T_N}$$

Polycrystal.

$$[\bar{\gamma}_{pc}] = \frac{1}{3}x_{II}(0) + \frac{2}{3}x_{\perp}(0)$$

$$[\bar{\gamma}_{pc}]_{T=0K} = 0 + \frac{2}{3}x_{\perp}(T_N) \quad \text{--- (1)}$$

$$\text{as } [x_{\perp}(T_N)] = x_{\perp}(0)$$

$$[\bar{\gamma}_{pc}]_{T=T_N} = \frac{1}{3}x_{II}(T_N) + \frac{2}{3}x_{\perp}(T_N)$$

$$= \frac{1}{3}x_{\perp}(T_N) + \frac{2}{3}x_{\perp}(T_N)$$

$$(at T_N, x_{\perp} = x_{II})$$

$$= x_{\perp}(T_N) \quad \text{--- (ii)}$$

divide (i) & (ii)

$$\frac{[\bar{\gamma}_{pc}]_{T=0K}}{[\bar{\gamma}_{pc}]_{T=T_N}} = \frac{\frac{2}{3}x_{\perp}(T_N)}{x_{\perp}(T_N)} = \frac{2}{3}$$

The next question is  
whether it is true that  
the density of states  
is proportional to

1-29

# Superconductivity

Date \_\_\_\_\_

For metal

$$\frac{R}{\downarrow} = \frac{\rho l}{A} \rightarrow \text{specific resistance or resistivity}$$

resistance.

(Residual Resistivity)

For pure metals.  $\Rightarrow T \downarrow \Rightarrow \rho \downarrow$ at  $T=0K \Rightarrow \rho=0$ For impure metals.  $T \downarrow \Rightarrow \rho \downarrow$ at  $T=0K \Rightarrow \rho = \frac{\rho_0}{\uparrow \text{Residual Resistivity}}$ 

Q Why there is resistance in metals?

→ Flow of free electrons leads to conductivity, if the temp. of the metal is raised, impurity ions or impurity atoms start vibrating and there is obstruction in the free flow of electron.

vijeta

Date \_\_\_\_\_

Scattering of free electrons with impurity ions or atoms leads to resistance. More would be the temp of the metal, larger would be the amplitude of vibration of ions, so larger will be the scattering and therefore, resistance of is increased.

Date \_\_\_\_\_

therefore,  $T_c = 4.18\text{K}$  for Hg.

$T_c = 7\text{K}$  for Pb

$T_c = 9\text{K}$  for Nb

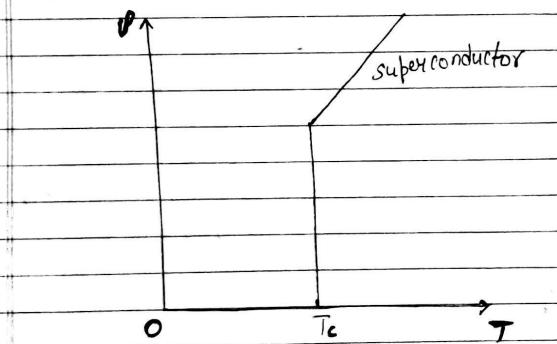
$T_c = 23\text{K}$  for  $\text{Nb}_3\text{Ge}$

for ceramic metals like

$\text{YBa}_2\text{Cu}_3\text{O}_7$

$T_c = 123\text{K}$ .

Superconductors.



$R = 0$  for  $T = 0$  to  $T = T_c$ .  
Where  $T_c$  is the critical temp.

"1911 by K. Onnes (first invention for superconductor)  
for Mercury (Hg)

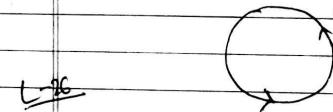
$R = 0.08\ \Omega$  at  $T = 4.20\text{K}$

$R = 3 \times 10^{-6}\ \Omega$  at  $T = 4.18\text{K}$

Properties of superconducting materials.

(i) Persistent current

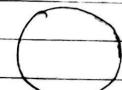
Take a superconductor material, form a ring and connect it to current source. Current continues to flow even after the removal of source.



NEXT  
Effect of magnetic field. (MEISSNER EFFECT) [1933]



$T > T_c$   
 $H = \text{any value}$



$T < T_c$   
 $H < H_c$

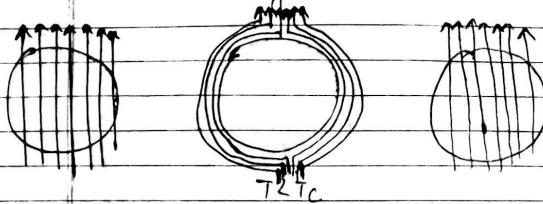


$T < T_c$   
 $H > H_c$

$T_c$  = Critical temperature  
 $H_c$  = Critical magnetic field

Date \_\_\_\_\_

Let us consider a spherical sample of metal



at  $T > T_c$   
any value  
of  $H$

(a)

$H < H_c$

(b)

$H = \text{external magnetic field}$

$T < T_c$

(c)

$H > H_c$

(d)  $T > T_c$  and at any value of  $H$ .

Magnetic flux enters through the sample.

(e)  $T < T_c$ ,  $H < H_c$

Magnetic flux does not enter the sample because it is thrown out of the material  
 $\therefore B=0$  and  $P=0$

(Meissner effect)

$P=0$  for  $T \leq T_c$

in superconductor

vijeta

$$B = \mu_0 (H + M)$$

$$\Rightarrow \mu_0 (H + M) = 0$$

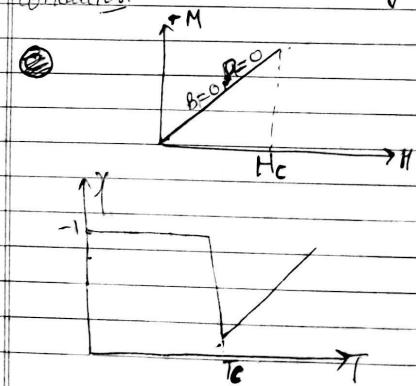
$$M = -H$$

$$\gamma = -1$$

Perfect diamagnetic

$B = 0 \Rightarrow \text{Perfect diamagnetic}$   
 $P = 0 \Rightarrow \text{perfect conductor}$

Superconductor is simultaneously an ideal diamagnet as well as ideal conductor. On the other hand a perfect conductor is only an ideal conductor.



(f)  $T < T_c$ ,  $H > H_c$

Magnetic flux enters through the sample i.e.  $B \neq 0$  and  $P \neq 0$ .

vijeta

In this case ~~superconductivity~~  
is destroyed.

Relation between critical magnetic field and critical temp:

When a superconductor is subjected to a strong magnetic field, its superconductivity is destroyed, this value is known as critical magnetic field.

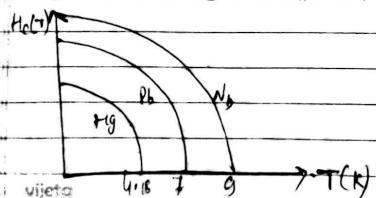
$$H_c(T) = H_{c(0)} \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

where  $H_{c(0)}$  = critical magnetic field at a certain temp  $T$ .

$H_{c(0)}$  = critical magnetic field at a temp  $T=0$ .

If  $T=T_c$ ,  $H_c=0$ :  
at  $T=0$ ,  $H_c=H_{c(0)}$

$H_{c(0)}$  is max. value of  $H_c(T)$   
i.e.  $H_c(T)$  is max at  $T=0\text{K}$ .



How to apply the Magnetic field?



$$H = \frac{I}{2\pi r}$$

$H \propto I$

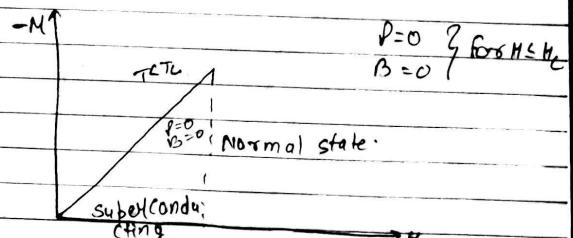
$$\text{If } I = I_c \text{ then } H = H_c \\ \text{i.e. } I_c = 2\pi r H_c$$

Critical current

if  $H > H_c \Rightarrow$  superconductivity is destroyed.

Type I and Type II ~~superconductors~~  
superconductors.

Type I superconductors.

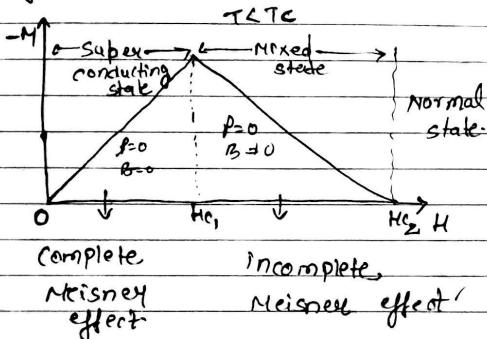


Those ~~superconductors~~ which shows complete Meissner effect for the field  $H \leq H_c$  is known as ~~super~~.

### Type I Superconductor

for eg. Pb,  $H_c = 48 \times 10^{-4}$  A/m.

### Type II Superconductor



Those superconductors which show complete Meissner effect for the field  $H < H_c$ , and incomplete Meissner effect b/w  $H_{c1}$  &  $H_{c2}$ . This is known as Type II superconductor.

Type II superconductors are practically useful as they have largest value of critical magnetic field.

→ Type I superconductors is also known as soft superconductors because they have low value of critical temperature.

→ Type II superconductors are made up of more than one superconductor (alloys).

for eg. Pb + (20% wt Nb)

$$H_{c1} = 0.6 \times 10^{-4} \text{ A/m}$$

$$H_{c2} = 30 \times 10^{-4} \text{ A/m}$$

### Isotope effect in superconductors

Isotope :- same atomic number but different atomic mass.

$$T_c \propto N^{-x}$$

Transition temp of a superconductor decreases with increase of isotopic mass.

$N$  = isotopic mass of the element.

$\alpha$  = isotope effect coefficient.

Date \_\_\_\_\_

for Most superconductors  
 $\alpha \rightarrow 0.45 \text{ to } 0.5$ .

i.e.  $T_c \propto M^{1/2}$   
 $T_c M^{1/2} = \text{constant}$ .  
 $T_c^{(1)} M_1^{1/2} = T_c^{(2)} M_2^{1/2}$

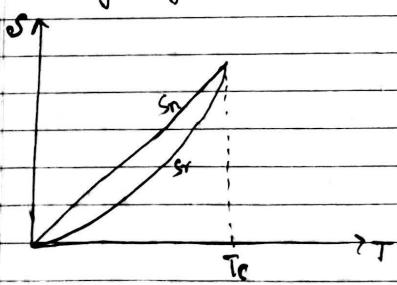
$M_1$  and  $M_2$  are the isotopic mass.

$$\Rightarrow \frac{T_c^{(1)}}{T_c^{(2)}} = \left(\frac{M_2}{M_1}\right)^{1/2}$$

For e.g. for Hg  
 $T_c$  changes from  
 4.85 K to 4.146 K.

as isotopic mass changes from  
 199 amu to 203 amu.

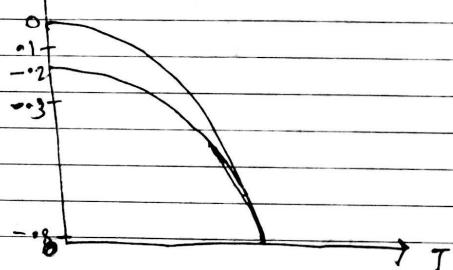
Entropy of a superconductor.



Elections are more ordered in superconductor than in metals so metals have more entropy than superconductors.

free energy of a superconductor.

$$F(\text{mJ/mole})$$



L-28  
Imp London's Theory.

Background:-

$$V = iR$$

$$= i \left( \frac{P}{A} \right) \quad \cancel{\text{or}}$$

$$E \times \frac{1}{l} = i \frac{P}{A} = jP$$

But, for super conductor,  $P=0, B=0$ :  
 So,  $\Rightarrow E=0$ .  
 from Maxwell eqn.

vijeta

10.1

Date \_\_\_\_\_

$$\textcircled{1} \quad \vec{J} + \vec{E} = -\frac{8B}{st}$$

$$\text{if } \vec{E} = 0 \Rightarrow 0 = -\frac{8B}{st}$$

i.e.  $B$  is a constant.  
(not changing with time)

However, As per Meissner effect  
 $\vec{B} = 0$ .

Therefore Maxwell eq. and  
Meissner effect are inconsistent.  
Or Maxwell eq. fails to explain  
the superconductivity. Therefore,  
there was a need of  
new Theory.

The new Theory was given  
London so, it called London's  
theory.

Assumptions: The conduction electrons  
are classified in two

Categories

(i) Super electrons

(ii) Normal electrons.

$\rightarrow$  Super electrons are not scattered  
by the thermal vibration or  
impurity ions (and when there  
is no scattering conductivity is max).

$\rightarrow$  Normal electrons are scattered  
by thermal vibration  
(so, have some resistivity).

- $\textcircled{i}$  At  $T=0K$ , all the electrons  
are superelectrons (max conduct)
- $\textcircled{ii}$  As  $T$  is increased from  $T=0K$   
assumption to  $T=T_c$ , the proportion of Normal  
electrons increases.
- $\textcircled{iii}$  At  $T=T_c$ , all the electrons are  
normal electrons

Let there are ' $n$ ' number of  
conduction electrons per unit  
volume in the superconductor,  
 $\Rightarrow N = n_s + n_n$

$\downarrow$  normal electron density  
 $\uparrow$  super electron density

Let  $\vec{v}_s$  = velocity of super electrons.  
 $\vec{v}_n$  = velocity of normal electrons

vijeta

$$G_n = \frac{\eta e^2 \tau}{m}$$

current density for superelectrons  
 $\vec{J}_s = e n_s \vec{v}_s$

current density for normal electrons  
 $\vec{J}_n = e n_n \vec{v}_n$

$$\vec{J} = \vec{J}_s + \vec{J}_n$$

$\uparrow$   
 Total current density.

Under the action of external electric field, the eq. of motion for superelectrons,

$$\frac{d\vec{p}}{dt} = \vec{F} \quad (\text{Newton's Second Law})$$

(change in momentum = applied force)

$$\frac{d(m\vec{v}_s)}{dt} = e\vec{E}$$

$$\Rightarrow \frac{d\vec{v}_s}{dt} = \frac{e\vec{E}}{m}$$

$$\Rightarrow \frac{d(e n_s \vec{v}_s)}{dt} = \frac{e n_s e \vec{E}}{m}$$

$$\Rightarrow \frac{d\vec{J}_s}{dt} = \frac{e^2 n_s \vec{E}}{m} \quad (1)$$

(London's first equation)

Similarly for normal electrons  
 $\vec{J}_n = G_n \vec{E}$

$$= \frac{n_n e^2 \tau}{m} \vec{E}$$

$\tau$  = relaxation time - (11).

taking curl of eq (1)

$$\nabla \times \frac{d\vec{J}_s}{dt} = \frac{e^2 n_s}{m} \nabla \times \vec{E}$$

$$\Rightarrow \frac{d}{dt} (\nabla \times \vec{J}_s) = \frac{e^2 n_s}{m} \left( -\frac{\delta \vec{B}}{\delta t} \right) \quad (3)$$

Also, from Maxwell

$$\text{eq. } \vec{\nabla} \times \vec{B} = 4\pi \vec{J}_s$$

( $\because D = 0$ )

- (1)

$$\vec{\nabla} \times \vec{B}$$

$$= 4\pi \vec{J}_s$$

$$+ 4\pi \frac{\delta \vec{B}}{\delta t}$$

$$\frac{\delta}{\delta t}$$

$$D = 0 \text{ for}$$

$$\text{superconductor}$$

taking curl of eq (6)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = 4\pi (\vec{\nabla} \times \vec{J}_s)$$

$$\frac{\delta}{\delta t} (\vec{\nabla} \times \vec{\nabla} \times \vec{B}) = 4\pi \frac{\delta}{\delta t} (\vec{\nabla} \times \vec{J}_s)$$

$$\Rightarrow \frac{\delta}{\delta t} (\vec{\nabla} (\vec{\nabla} \cdot \vec{B}) - \vec{\nabla}^2 \vec{B}) = 4\pi \frac{e^2 n_s}{m} \left( -\frac{\delta \vec{B}}{\delta t} \right)$$

$$\vec{0} \quad (\vec{\nabla} \cdot \vec{B} = 0)$$

$$\Rightarrow -\frac{\delta}{\delta t} (\vec{\nabla}^2 \vec{B}) = -4\pi \frac{e^2 n_s}{m} \frac{\delta \vec{B}}{\delta t} \quad (\text{from (3)})$$

vijeta

vijeta

Date \_\_\_\_\_

new

$$\nabla^2 \vec{B} = \frac{4\pi e^2 n_s}{m} \vec{B}$$

↑  
derivative  
 $B \propto t^{\frac{1}{2}}$

$$\nabla^2 \vec{B} = \frac{\vec{B}}{\lambda_L^2} \quad - (5)$$

London's 2nd equation.

$$\lambda_L^2 = \frac{m}{4\pi e^2 n_s}$$

$\lambda_L$  = London's penetration depth.

If  $\vec{B}$  is satisfying eq. 5 then  $\vec{B}$  should also satisfy eq. 5.

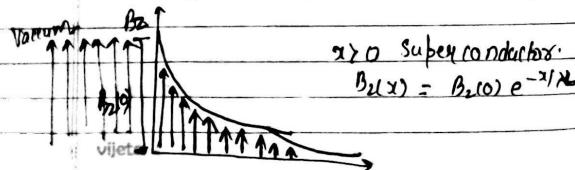
$$\text{So, } \boxed{\nabla^2 \vec{B} = \vec{B}} \quad - (6)$$

Solution for eq. 6 is

$$\vec{B} = 0$$

General Solution is

$$B_2(x) = B_{2(0)} e^{-x/\lambda_L} \quad - (7)$$



$x > 0$  Superconductor

$$B_2(x) = B_{2(0)} e^{-x/\lambda_L}$$

London's eq. nearly satisfies Meissner effect. ( $B$  decays to zero exponentially not exactly  $B$ ).

$$\text{If } x = \lambda_L, B_2(\lambda_L) = \frac{B_{2(0)}}{e}$$

$$\text{If } x = 2\lambda_L, B_2(2\lambda_L) = \frac{B_{2(0)}}{e^2}$$

$$\text{Typically } n_s = 10^{28}/\text{m}^3 \\ \lambda_L = 532 \text{ nm}$$

so, London's eq. satisfies Meissner effect nearly not accurately =

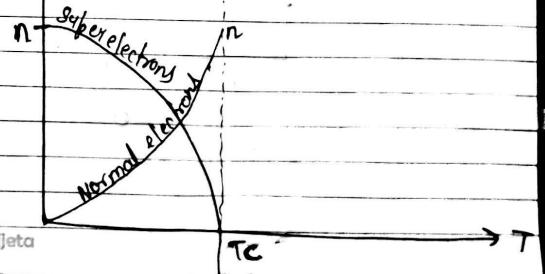
At  $T=0$  (all the electrons are super electrons).

$$n_s = n_s(0) = n$$

$$n_n = 0$$

$$\text{At } T=T_c, n_s = 0, n_n = n$$

(Not imp.)



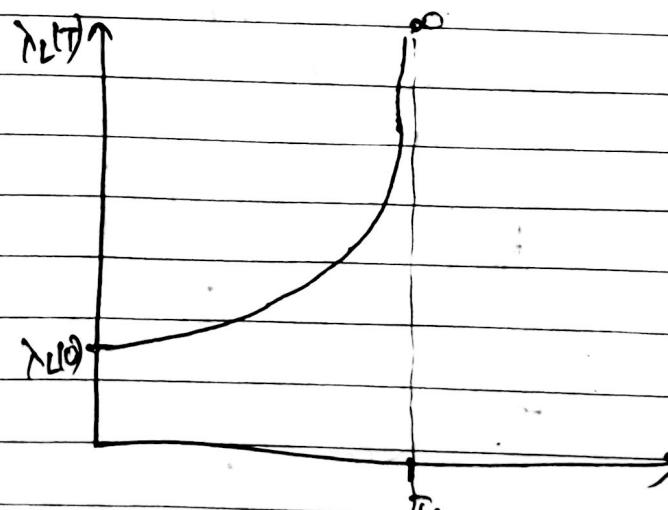
$$n_b(T) = n_{b(0)} \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]$$

So,  $\lambda_L^2(0) = m$   
 $n_b(0)e^2 k_{\text{B}}$

$$\lambda_L^2(T) = \frac{m}{n_b(T) e^2 k_{\text{B}}}$$

$$= \frac{m}{n_{b(0)} e^2 k_{\text{B}} \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]}$$

$$\lambda_L^2(T) = \frac{\lambda_L^2(0)}{1 - \left( \frac{T}{T_c} \right)^4}$$



$$T = T_c, \lambda_L^2(T) = \infty$$

$$T = 0 = \lambda_L^2(0)$$