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Waez et al.

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(54) DYNAMIC BOUNDING BOX

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B60W 60/00	(2020.01)
G06T 7/20	(2017.01)

(52) **U.S. Cl.**

CPC ***G06T 7/20*** (2013.01); ***B60W 60/0027*** (2020.02); *G06T 2207/20081* (2013.01); *G06T 2207/20084* (2013.01); *G06T 2207/30241* (2013.01)

(58) **Field of Classification Search**

CPC G06T 7/20; G06T 2207/20081; G06T 2207/20084; G06T 2207/30241; G06T 2207/30252; B60W 60/0027

See application file for complete search history.

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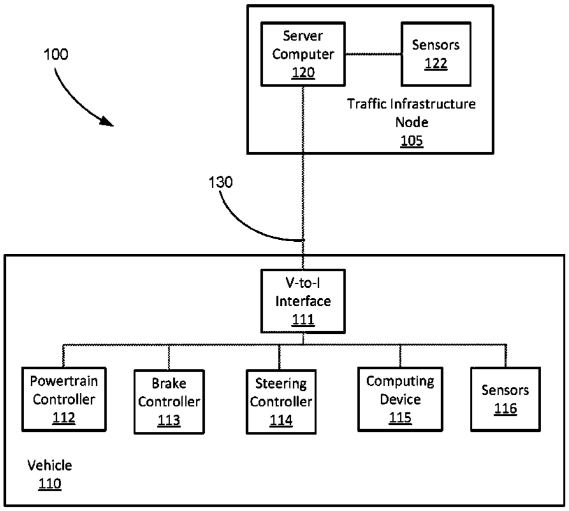
(57) ABSTRACT

A computer that includes a processor and a memory can determine a final trajectory for a vehicle by determining a candidate trajectory of a first object based on a detected second object. The candidate trajectory can be input to a reachable polyhedral marching processor to determine dynamic occupancy polyhedrals based on a shape of the candidate trajectory. A reachable tube can be determined based on combining the dynamic occupancy polyhedrals and the final trajectory can be determined based on the reachable tube avoiding the detected second object.

20 Claims, 8 Drawing Sheets



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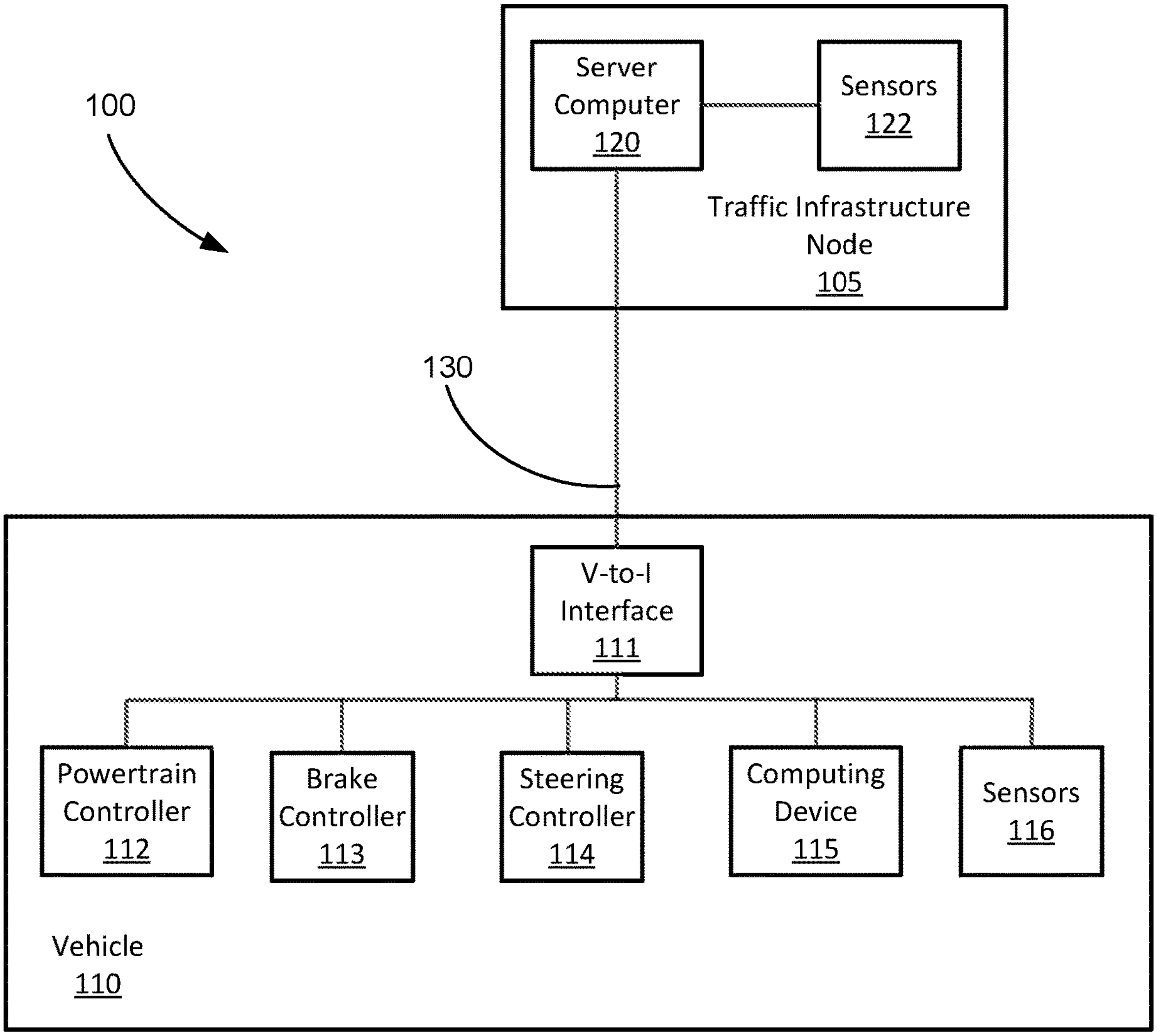
100
Server Computer 120
Sensors 122
Traffic Infrastructure Node 105
130
V-to-I Interface 111
Powertrain Controller 112
Brake Controller 113
Steering Controller 114
Computing Device 115
Sensors 116
Vehicle 110

Fig. 1

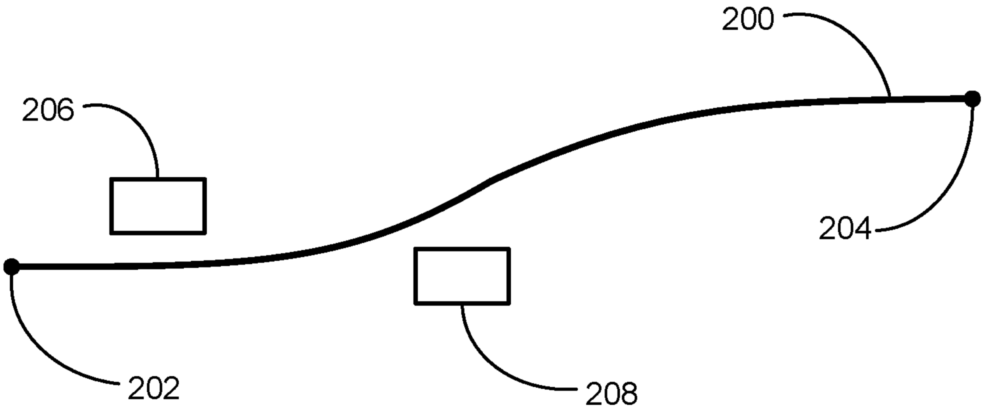

Fig. 2

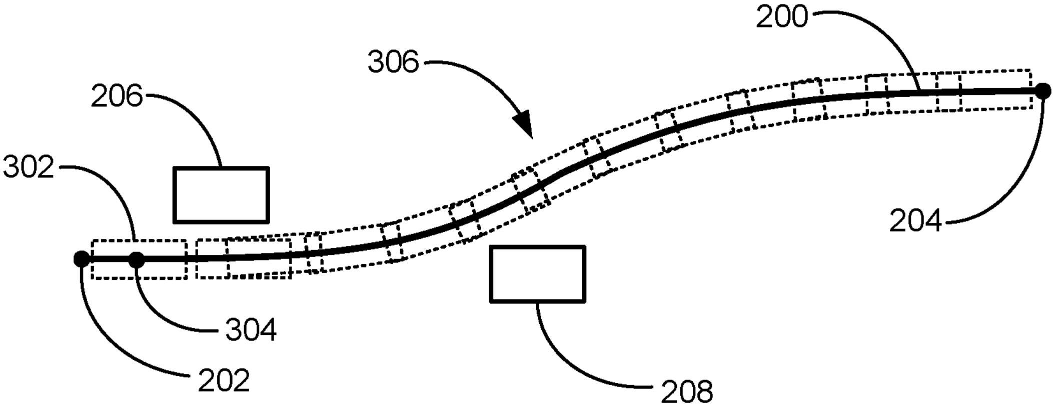

Fig. 3

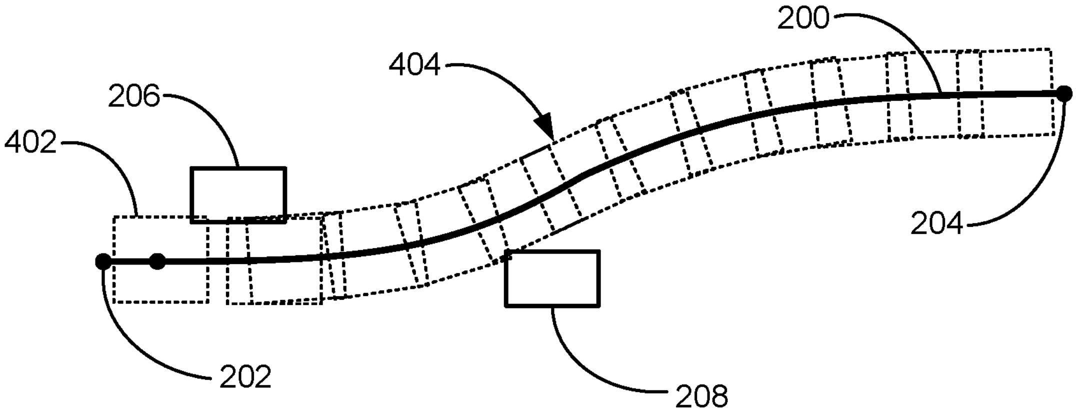

Fig. 4

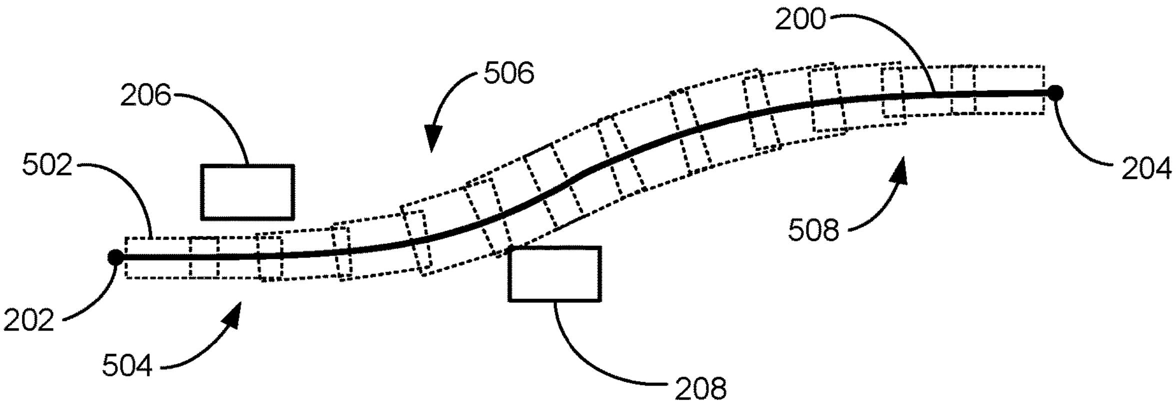

Fig. 5

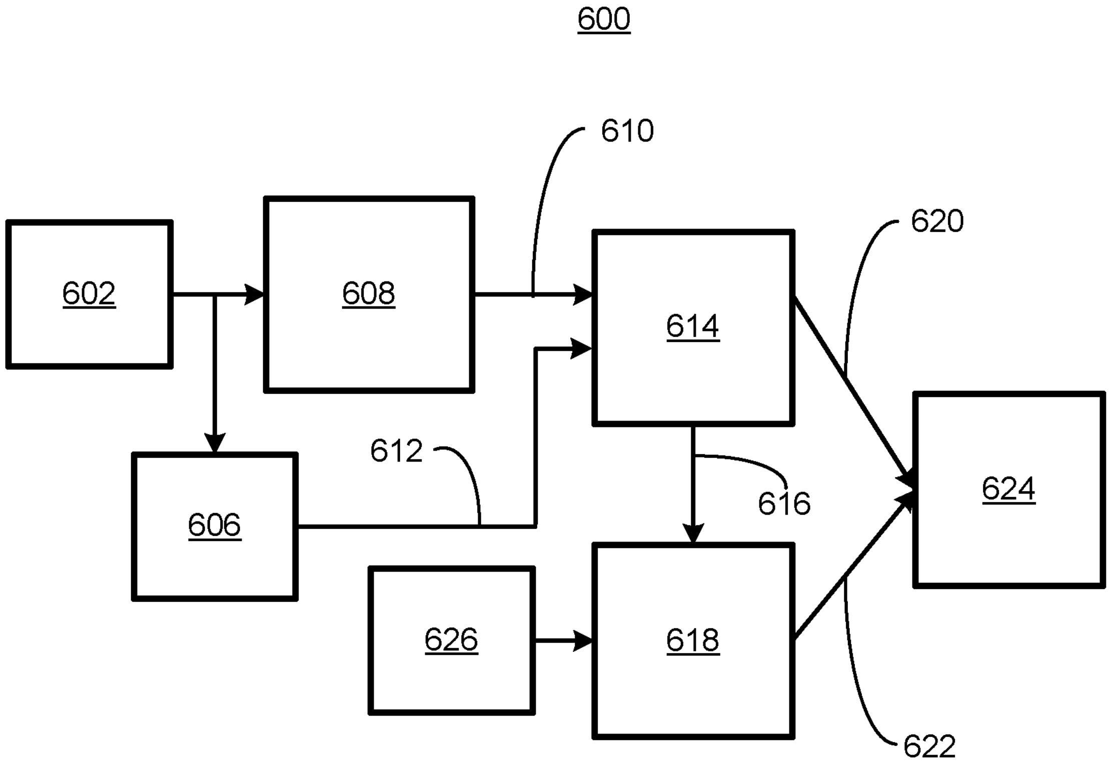

Fig. 6

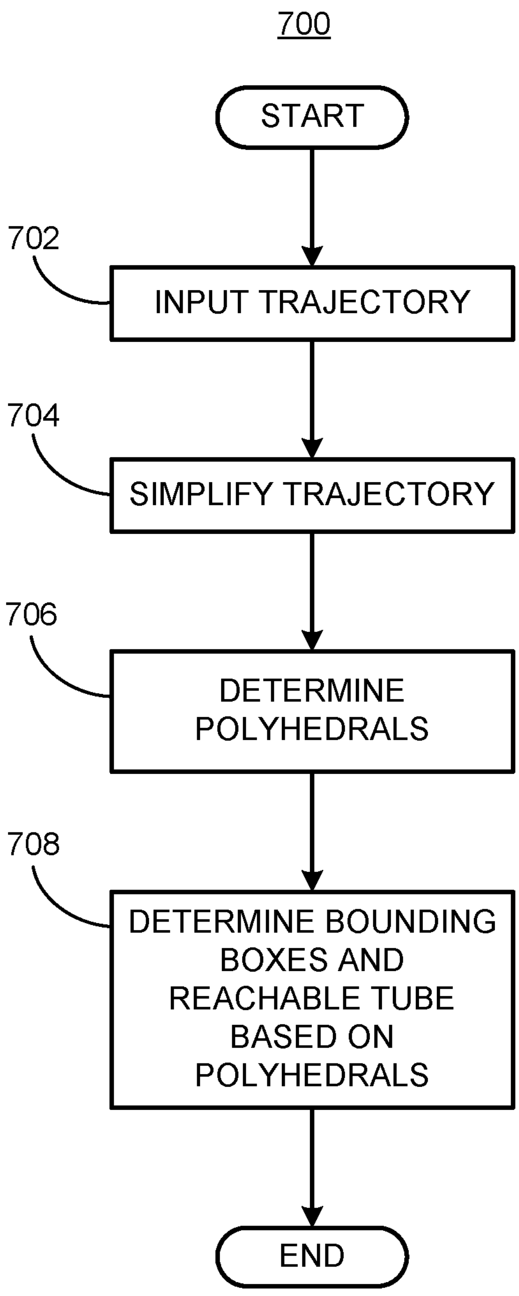
700
START
702
INPUT TRAJECTORY
704
SIMPLIFY TRAJECTORY
706
DETERMINE POLYHEDRALS
708
DETERMINE BOUNDING BOXES AND REACHABLE TUBE BASED ON POLYHEDRALS
END

Fig. 7

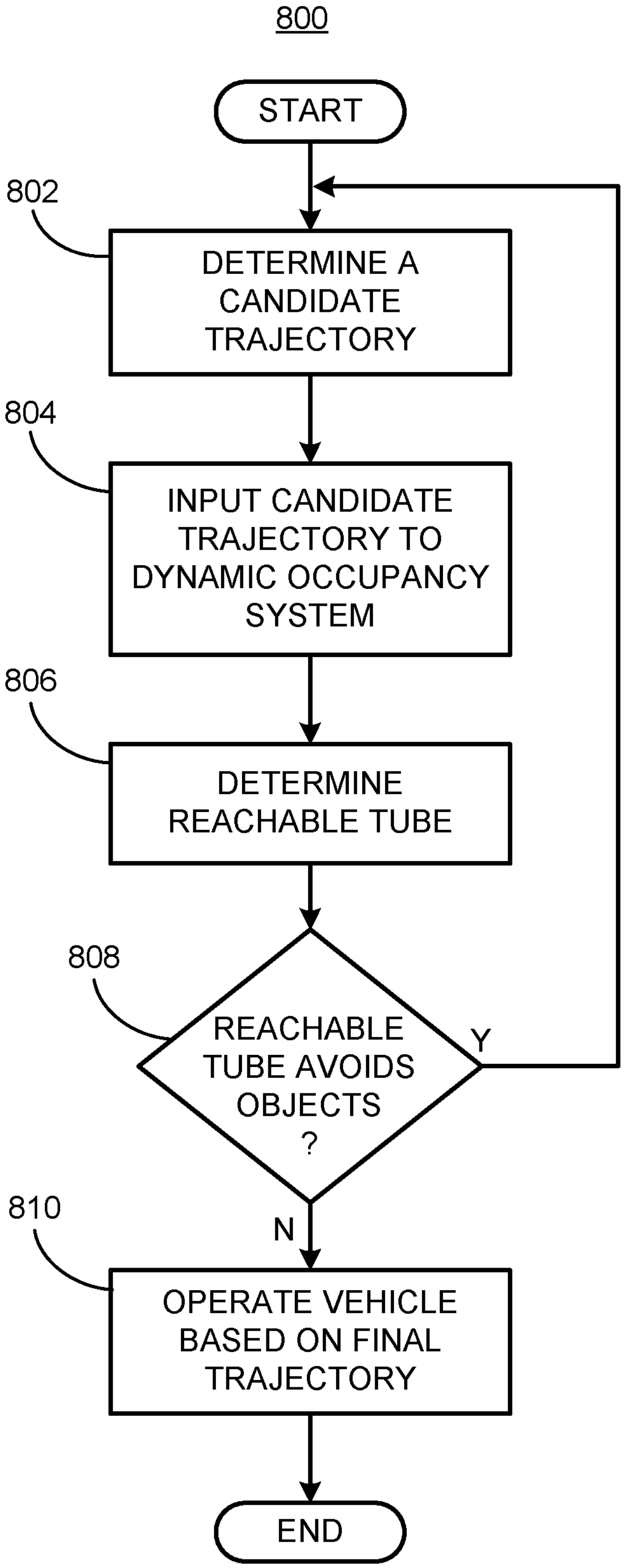
800
START
802
DETERMINE A CANDIDATE TRAJECTORY
804
INPUT CANDIDATE TRAJECTORY TO DYNAMIC OCCUPANCY SYSTEM
806
DETERMINE REACHABLE TUBE
808
REACHABLE TUBE AVOIDS OBJECTS ?
Y
N
810
OPERATE VEHICLE BASED ON FINAL TRAJECTORY
END

Fig. 8

DYNAMIC BOUNDING BOX

BACKGROUND

Images can be acquired by sensors and processed using a computer to determine data regarding objects in an environment around a system. Operation of a sensing system can include acquiring accurate and timely data regarding objects in the system's environment. A computer can acquire images from one or more sensors that can be processed to determine data regarding objects in the system's environment. Data regarding objects in the environment can be combined with data regarding location of a system by a computer and used to operate systems including vehicles, robots, security systems, and/or object tracking systems.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a block diagram of an example traffic infrastructure system.

FIG. **2** is a diagram of an example vehicle trajectory.

FIG. **3** is a diagram of an example vehicle trajectory including narrow static bounding boxes.

FIG. **4** is a diagram of an example vehicle trajectory including broad static bounding boxes.

FIG. **5** is a diagram of an example vehicle trajectory including dynamic bounding boxes.

FIG. **6** is a diagram of an example dynamic occupancy system.

FIG. **7** is a flowchart diagram of an example process to generate dynamic bounding boxes.

FIG. **8** is a flowchart diagram of an example process to operate a vehicle based on dynamic bounding boxes.

DETAILED DESCRIPTION

As described herein, one or more software programs executing on a computer in a system can be used to determine data regarding objects in the system's environment and operate the system in the environment. Vehicle operation is used herein as an example of a system that acquires sensor data regarding objects in the vehicle's environment, processes the sensor data and operates the vehicle based on the processed sensor data. For example, a vehicle can acquire data regarding roadway lane markers, other vehicles, or pedestrians from sensors included in the vehicle. A computing device can determine a trajectory upon which to operate the vehicle based on predicted locations of the vehicle and determined locations of the objects in the environment. Lane keeping, where a vehicle stays within a lane, roadway exiting and entering, lane change maneuvers and object avoidance are examples of vehicle operations that can be performed based on predicted vehicle trajectories and object locations. Techniques described herein can be applied to other systems such as robots, security systems or object tracking systems.

A computer can acquire sensor data regarding objects in an environment around a vehicle and software programs included in the computer can identify and locate portions of the environment including roadway lanes and objects such as other vehicles and pedestrians. Based on the identified and located portions of the environment, a vehicle trajectory can be determined. A vehicle trajectory is a set of location values that includes velocities, e.g., speeds and headings, and lateral and longitudinal accelerations. Lateral and longitudinal accelerations can be determined based on the shape of the trajectory which indicate rate of change in lateral and longitudinal locations. The vehicle trajectory can be determined based on polynomial functions, for example. Advantageously, techniques described herein enhance vehicle operation by predicting locations of a vehicle based on dynamic occupancy. Techniques discussed herein can efficiently determine dynamic occupancy states for a vehicle based on acquired data regarding actual locations of vehicles with respect to predicted trajectories and determine dynamic bounding boxes for the vehicle based on the dynamic occupancy states. Dynamic bounding boxes can enhance the accuracy of predicting future locations of a vehicle as the vehicle travels on a predicted trajectory.

In examples of predicting locations of a system based on dynamic occupancy, the system can be a robot. In examples where the system is a robot, a candidate trajectory can be determined for a gripper attached to a robotic arm. A computing device in the robot can determine a candidate trajectory based on controlling actuators that can control movement of a gripper and a robotic arm to a position to permit the gripper to pick up a workpiece, for example. The candidate trajectory can be determined to avoid contact with a second object detected by sensors included in the robot. For example, a second object can be a second robot or portions of a conveyor belt that supports the workpiece. Dynamic occupancy techniques can enhance the operation of the robot by predicting locations of the gripper and robotic arm as it moves past the detected second object based on the candidate trajectory. Dynamic occupancy accurately predicts locations of the gripper and robotic arm based on the speed, lateral accelerations and longitudinal accelerations included in the candidate trajectory, to neither underestimate nor overestimate possible deviations from the candidate trajectory.

A method is disclosed herein, including determining a final trajectory by: determining a candidate trajectory of a first object based on a detected second object, inputting the candidate trajectory to a reachable polyhedral marching processor to determine dynamic occupancy polyhedrals based on a shape of the candidate trajectory, determining a reachable tube based on combining the dynamic occupancy polyhedrals, and determining the final trajectory based on the reachable tube avoiding the detected second object. The first object can be operated based on the final trajectory by controlling one or more actuators to control movement of the first object. The first object can be a vehicle and controlling the actuators to control the movement of the vehicle can include controlling one or more of vehicle powertrain, vehicle steering and vehicle brakes. The first object can be a robot and controlling the actuators to control the movement of the robot can include controlling motion of one or more of a gripper and a robotic arm. Inputting the candidate trajectory to a first neural network can include simplifying the candidate trajectory. The candidate trajectory can be a T*n matrix that includes x and y locations, velocities, and heading angles at a plurality of time steps t and simplifying the trajectory can reduce the T*n matrix to a vector with m dimensions, where m<T*n. The reachable polyhedral marching processor can be programmed based on weights output from a second neural network.

The second neural network can determine density distributions based on solving Liouville partial differential equations. The second neural network can be trained based on a plurality of trajectories acquired from real world vehicles. The second neural network can include fully connected neurons with ReLU activation and is trained based on stochastic gradient descent with L2-norm reconstruction loss. The dynamic occupancy polyhedrals can include two-dimensional regions wherein a probability that the first object will occupy locations within the reachable polyhedrals is higher than an empirically determined threshold. A size of the dynamic occupancy polyhedrals can be based on curvature and a distance between samples of segments of the candidate trajectory. The dynamic occupancy polyhedrals can have between six and eight sides. The second object can be one or more of a vehicle, a pedestrian, or a lane marker.

Further disclosed is a computer readable medium, storing program instructions for executing some or all of the above method steps. Further disclosed is a computer programmed for executing some or all of the above method steps, including a computer apparatus, programmed to determine a final trajectory by: determining a candidate trajectory of a first object based on a detected second object, inputting the candidate trajectory to a reachable polyhedral marching processor to determine dynamic occupancy polyhedrals based on a shape of the candidate trajectory, determining a reachable tube based on combining the dynamic occupancy polyhedrals, and determining the final trajectory based on the reachable tube avoiding the detected second object. The first object can be operated based on the final trajectory by controlling one or more actuators to control movement of the first object. The first object can be a vehicle and controlling the actuators to control the movement of the vehicle can include controlling one or more of vehicle powertrain, vehicle steering and vehicle brakes. The first object can be a robot and controlling the actuators to control the movement of the robot can include controlling motion of one or more of a gripper and a robotic arm. Inputting the candidate trajectory to a first neural network can include simplifying the candidate trajectory. The candidate trajectory can be a T*n matrix that includes x and y locations, velocities, and heading angles at a plurality of time steps t and simplifying the trajectory can reduce the T*n matrix to a vector with m dimensions, where m<T*n. The reachable polyhedral marching processor can be programmed based on weights output from a second neural network.

The instructions can include further instructions to determine density distributions based on solving Liouville partial differential equations with a second neural network. The second neural network can be trained based on a plurality of trajectories acquired from real world vehicles. The second neural network can include fully connected neurons with ReLU activation and is trained based on stochastic gradient descent with L2-norm reconstruction loss. The dynamic occupancy polyhedrals can include two-dimensional regions wherein a probability that the first object will occupy locations within the reachable polyhedrals is higher than an empirically determined threshold. A size of the dynamic occupancy polyhedrals can be based on curvature and a distance between samples of segments of the candidate trajectory. The dynamic occupancy polyhedrals can have between six and eight sides. The second object can be one or more of a vehicle, a pedestrian, or a lane marker.

FIG. **1** is a diagram of a sensing system **100** that can include a traffic infrastructure node **105** that includes a server computer **120** and stationary sensors **122**. Sensing system **100** includes a vehicle **110**, operable in autonomous ("autonomous" by itself in this disclosure means "fully autonomous"), semi-autonomous, and occupant piloted (also referred to as non-autonomous) mode. One or more vehicle **110** computing devices **115** can receive data regarding the operation of the vehicle **110** from sensors **116**. The computing device **115** may operate the vehicle **110** in an autonomous mode, a semi-autonomous mode, or a non-autonomous mode.

The computing device **115** includes a processor and a memory such as are known. Further, the memory includes one or more forms of computer-readable media, and stores instructions executable by the processor for performing various operations, including as disclosed herein. For example, the computing device **115** may include programming to operate one or more of vehicle brakes, propulsion (i.e., control of acceleration in the vehicle **110** by controlling one or more of an internal combustion engine, electric motor, hybrid engine, etc.), steering, climate control, interior and/or exterior lights, etc., as well as to determine whether and when the computing device **115**, as opposed to a human operator, is to control such operations.

The computing device **115** may include or be communicatively coupled to, i.e., via a vehicle communications bus as described further below, more than one computing devices, i.e., controllers or the like included in the vehicle **110** for monitoring and/or controlling various vehicle components, i.e., a powertrain controller **112**, a brake controller **113**, a steering controller **114**, etc. The computing device **115** is generally arranged for communications on a vehicle communication network, i.e., including a bus in the vehicle **110** such as a controller area network (CAN) or the like; the vehicle **110** network can additionally or alternatively include wired or wireless communication mechanisms such as are known, i.e., Ethernet or other communication protocols.

Via the vehicle network, the computing device **115** may transmit messages to various devices in the vehicle and/or receive messages from the various devices, i.e., controllers, actuators, sensors, etc., including sensors **116**. Alternatively, or additionally, in cases where the computing device **115** actually comprises multiple devices, the vehicle communication network may be used for communications between devices represented as the computing device **115** in this disclosure. Further, as mentioned below, various controllers or sensing elements such as sensors **116** may provide data to the computing device **115** via the vehicle communication network.

In addition, the computing device **115** may be configured for communicating through a vehicle-to-infrastructure (V2X) interface **111** with a remote server computer **120**, i.e., a cloud server, via a network **130**, which, as described below, includes hardware, firmware, and software that permits computing device **115** to communicate with a remote server computer **120** via a network **130** such as wireless Internet (WI-FI®) or cellular networks. V2X interface **111** may accordingly include processors, memory, transceivers, etc., configured to utilize various wired and/or wireless networking technologies, i.e., cellular, BLUETOOTH®, Bluetooth Low Energy (BLE), Ultra-Wideband (UWB), Peer-to-Peer communication, UWB based Radar, IEEE 802.11, and/or other wired and/or wireless packet networks or technologies. Computing device **115** may be configured for communicating with other vehicles **110** through V2X (vehicle-to-everything) interface **111** using vehicle-to-vehicle (V-to-V) networks, i.e., according to including cellular communications (C-V2X) wireless communications cellular, Dedicated Short Range Communications (DSRC) and/or the like, i.e., formed on an ad hoc basis among nearby vehicles **110** or formed through infrastructure-based networks. The computing device **115** also includes nonvolatile memory such as is known. Computing device **115** can log data by storing the data in nonvolatile memory for later retrieval and transmittal via the vehicle communication network and a vehicle to infrastructure (V2X) interface **111** to a server computer **120** or user mobile device **160**.

As already mentioned, generally included in instructions stored in the memory and executable by the processor of the computing device **115** is programming for operating one or more vehicle **110** components, i.e., braking, steering, propulsion, etc., without intervention of a human operator. Using data received in the computing device **115**, i.e., the sensor data from the sensors **116**, the server computer **120**, etc., the computing device **115** may make various determinations and/or control various vehicle **110** components and/or operations without a driver to operate the vehicle **110**. For example, the computing device **115** may include programming to regulate vehicle **110** operational behaviors (i.e., physical manifestations of vehicle **110** operation) such as speed, acceleration, deceleration, steering, etc., as well as tactical behaviors (i.e., control of operational behaviors typically in a manner intended to achieve efficient traversal of a route) such as a distance between vehicles and/or amount of time between vehicles, lane-change, minimum gap between vehicles, left-turn-across-path minimum, time-to-arrival at a particular location and intersection (without signal) minimum time-to-arrival to cross the intersection.

Controllers, as that term is used herein, include computing devices that typically are programmed to monitor and/or control a specific vehicle subsystem. Examples include a powertrain controller **112**, a brake controller **113**, and a steering controller **114**. A controller may be an electronic control unit (ECU) such as is known, possibly including additional programming as described herein. The controllers may communicatively be connected to and receive instructions from the computing device **115** to actuate the subsystem according to the instructions. For example, the brake controller **113** may receive instructions from the computing device **115** to operate the brakes of the vehicle **110**.

The one or more controllers **112**, **113**, **114** for the vehicle **110** may include known electronic control units (ECUs) or the like including, as non-limiting examples, one or more powertrain controllers **112**, one or more brake controllers **113**, and one or more steering controllers **114**. Each of the controllers **112**, **113**, **114** may include respective processors and memories and one or more actuators. The controllers **112**, **113**, **114** may be programmed and connected to a vehicle **110** communications bus, such as a controller area network (CAN) bus or local interconnect network (LIN) bus, to receive instructions from the computing device **115** and control actuators based on the instructions.

Sensors **116** may include a variety of devices known to provide data via the vehicle communications bus. For example, a radar fixed to a front bumper (not shown) of the vehicle **110** may provide a distance from the vehicle **110** to a next vehicle in front of the vehicle **110**, or a global positioning system (GPS) sensor disposed in the vehicle **110** may provide geographical coordinates of the vehicle **110**. The distance(s) provided by the radar and/or other sensors **116** and/or the geographical coordinates provided by the GPS sensor may be used by the computing device **115** to operate the vehicle **110** autonomously or semi-autonomously, for example.

The vehicle **110** is generally a land-based vehicle **110** capable of autonomous and/or semi-autonomous operation and having three or more wheels, i.e., a passenger car, light truck, etc. The vehicle **110** includes one or more sensors **116**, the V2X interface **111**, the computing device **115** and one or more controllers **112**, **113**, **114**. The sensors **116** may collect data related to the vehicle **110** and the environment in which the vehicle **110** is operating. By way of example, and not limitation, sensors **116** may include, i.e., altimeters, cameras, LIDAR, radar, ultrasonic sensors, infrared sensors, pressure sensors, accelerometers, gyroscopes, temperature sensors, pressure sensors, hall sensors, optical sensors, voltage sensors, current sensors, mechanical sensors such as switches, etc. The sensors **116** may be used to sense the environment in which the vehicle **110** is operating, i.e., sensors **116** can detect phenomena such as weather conditions (precipitation, external ambient temperature, etc.), the grade of a road, the location of a road (i.e., using road edges, lane markings, etc.), or locations of target objects such as neighboring vehicles **110**. The sensors **116** may further be used to collect data including dynamic vehicle **110** data related to operations of the vehicle **110** such as velocity, yaw rate, steering angle, engine speed, brake pressure, oil pressure, the power level applied to controllers **112**, **113**, **114** in the vehicle **110**, connectivity between components, and accurate and timely performance of components of the vehicle **110**.

Vehicles can be equipped to operate in autonomous, semi-autonomous, or manual modes. By a semi- or fully-autonomous mode, we mean a mode of operation wherein a vehicle can be piloted partly or entirely by a computing device as part of a system having sensors and controllers. For purposes of this disclosure, an autonomous mode is defined as one in which each of vehicle propulsion (i.e., via a powertrain including an internal combustion engine and/or electric motor), braking, and steering are controlled by one or more vehicle computers; in a semi-autonomous mode the vehicle computer(s) control(s) one or more of vehicle propulsion, braking, and steering. In a non-autonomous mode, none of these are controlled by a computer. In a semi-autonomous mode, some but not all of them are controlled by a computer.

A traffic infrastructure node **105** can include a physical structure such as a tower or other support structure (i.e., a pole, a box mountable to a bridge support, cell phone tower, road sign support, etc.) on which infrastructure sensors **122**, as well as server computer **120**, can be mounted, stored, and/or contained, and powered, etc. One traffic infrastructure node **105** is shown in FIG. **1** for ease of illustration, but the system **100** could and likely would include tens, hundreds, or thousands of traffic infrastructure nodes **105**. The traffic infrastructure node **105** is typically stationary, i.e., fixed to and not able to move from a specific geographic location. The infrastructure sensors **122** may include one or more sensors such as described above for the vehicle **110** sensors **116**, i.e., lidar, radar, cameras, ultrasonic sensors, etc. The infrastructure sensors **122** are fixed or stationary. That is, each sensor **122** is mounted to the infrastructure node so as to have a substantially unmoving and unchanging field of view.

Server computer **120** typically has features in common with the vehicle **110** V2X interface **111** and computing device **115**, and therefore will not be described further to avoid redundancy. Although not shown for ease of illustration, the traffic infrastructure node **105** also includes a power source such as a battery, solar power cells, and/or a connection to a power grid. A traffic infrastructure node **105** server computer **120** and/or vehicle **110** computing device **115** can receive sensor **116**, **122** data to monitor one or more objects. An "object," in the context of this disclosure, is a physical, i.e., material, structure or thing that can be detected by a vehicle sensor **116** and/or infrastructure sensor **122**.

FIG. **2** is a diagram of a vehicle trajectory **200** that can be determined by a computing device **115** in a vehicle **110**. Vehicle trajectory **200** can include an initial location **202** for vehicle **110** and an end point **204**. Trajectory **200** can be determined based on polynomial equations that connect initial location **202** and end point **204**. The polynomial equations are typically defined in a plane parallel to a roadway and can be of the form

$$t(x,y)=ax^2+bxy+cy^2+d \tag{1}$$

Where t(x,y) are the coordinates of the vehicle with respect to the roadway and the polynomial equation is a second degree polynomial in x and y. A vehicle trajectory can be determined based on upper and lower limits for lateral and longitudinal accelerations predicted for vehicle **110** as it travels along vehicle trajectory **200**. Computing device **115** in vehicle **110** can also receive as input data from sensors **116** included in vehicle **110** to determine locations of objects **206**, **208** near trajectory **200**. For example, computing device can acquire video images from video sensors and input them to a neural network trained to locate objects **206**, **208** in video images. Objects **206**, **208** can be other vehicles, pedestrians, or traffic barriers, for example. Vehicle trajectory **200** can be determined by computing device **115** to avoid contact between vehicle **110** and objects **206**, **208**.

FIG. **3** is a diagram of an example traffic scene including detected objects and can be used to illustrate a technique for determining contact between a vehicle **110** traveling on a vehicle trajectory **200** and objects **206**, **208**. In FIG. **3**, a bounding box **302** (dotted lines) that minimally encloses (i.e., is the smallest possible rectangle that can be drawn and still encompass all points on the vehicle) vehicle **110** at a time step to **304** can be centered on the vehicle trajectory **200**. Centered on the vehicle trajectory **200** means that the center of the bounding box **302**, which can also be the center of the vehicle **110** which the bounding box **302** encloses, is placed on the vehicle trajectory **200**. A plurality of bounding boxes can be located at a plurality of locations centered on trajectory **200** at additional time steps $t_1 \ldots t_n$ to form a tube **306**. Tube **306** predicts the locations that will be occupied by vehicle **110** as it travels along trajectory **200**. That is, the tube **306** defines a location of the vehicle **110** on the trajectory **200** according to a center point of a line segment drawn perpendicularly to the trajectory **200** (or a tangent thereof where the trajectory **200** is curved) and extends from a first to a second side of the tube **306**.

FIG. **3** indicates that vehicle **110** will be able to travel on trajectory **200** without contacting objects **206**, **208**. An issue with determining contact between a vehicle **110** and objects **206**, **208** in this fashion is that the initial locations **202** of the vehicle **110** and the locations of vehicle **110** as it travels along trajectory **200** will include errors. Sensors **116** included in the vehicle **110** can vary in the accuracy with which the initial location **202** of vehicle **110** is determined. Operation of vehicle powertrain, steering and brakes can also vary in response to commands sent to powertrain, steering, and braking controllers **112**, **113**, **114** which can cause variance in how the vehicle **110** travels on the trajectory **200**. Variance in the initial position of vehicle **110** and variance in how vehicle **110** travels on the trajectory **200** can cause the vehicle **110** to occupy locations outside of tube **306**. This can cause a probability of contact between vehicle **110** and objects **206**, **208** to be greater than zero.

FIG. **4** is a diagram of an example traffic scene including detected objects and can be used to illustrate a technique for determining contact between a vehicle **110** traveling on a trajectory **200** and objects **206**, **208** that assumes an error in determining vehicle **110** locations. The error can be determined empirically by operating a vehicle **110** along a trajectory **200** while measuring the location of the vehicle **110** using external means. For example, an externally mounted stationary lidar sensor can acquire range data of the vehicle **110** as it travels on trajectory **200** a plurality of times and the externally acquired location data can be compared to the location data acquired by sensors **116** internal to the vehicle. Statistics, including means and standard deviations can be calculated based on the comparisons to determine probabilities of errors in location. For example, an error term can be determined based on an average of the error measurements as the vehicle travels on a trajectory **200**.

Sample trajectories **200** can be determined that include respective ranges of vehicle velocities and lateral and longitudinal accelerations over specified segments (or lengths) of the trajectory that make up a total length or distance of the trajectory. A vehicle **110** can be operated on the sample trajectories **200**, i.e., within the specified ranges over the specified segments, a plurality of times, for example greater than 10 times, and errors, indicated by the vehicle **110** occupying locations outside of the minimal tube **306**, can be recorded. An average error term can be determined based on the sample trajectories **200** and added to a minimal bounding box **302** as described in relation to FIG. **3**, above to determine bounding box **402**, centered on vehicle trajectory **200** in FIG. **4**. Bounding box **402**, adjusted by adding an error term, is typically larger than the rectangular box **302** described above. Bounding box **402** can be determined to include the outline of the vehicle **110** plus an allowance for a statistical measure of the error in location based on the empirically acquired data. Bounding box **402** can provide greater confidence in avoiding objects by incorporating an error term in the predicted locations of a vehicle **110** as it travels on a vehicle trajectory **200**.

The adjusted rectangular box **402** can be propagated along the trajectory **200** to determine a tube **404** that indicates the locations the vehicle **110** can assume as it travels along trajectory **200** including errors. For example, tube **404** can indicate potential locations of vehicle **110** including errors. Including three standard deviations of error statistics would yield a 99% probability that the vehicle **110** would operate within tube **404**, assuming that the error terms follow Gaussian statistics. That is, in distributions that follow Gaussian statistics, 99% of the observations, in this example errors in vehicle locations, fall within three standard deviations of the average. Assuming that error terms are distributed equally in both directions from the vehicle trajectory **200**, the average error would be 0. Tube **404** indicates that vehicle **110** would not have a 99% certainty of avoiding both objects **206**, **208**, and computing device **115** would not operate vehicle **110** on trajectory **200** from initial location **202**, to end point **204**.

An issue with error analysis as applied to vehicle **110** trajectories **200** as illustrated in FIG. **3** is that it can overstate probabilities of contact between a vehicle **110** and objects **206**, **208** because the error in vehicle **110** location with respect to vehicle trajectory **200** is not constant as the vehicle **110** travels along a trajectory **200**. Errors in the location of a vehicle **110** with respect to a vehicle trajectory **200** are a function of speed and lateral acceleration, etc. For example, the faster a vehicle **110** is changing direction, the more lateral acceleration the vehicle **110** is experiencing. High lateral acceleration in short time periods can generate greater errors in vehicle **110** control systems, for example. In general, changing directions can generate greater errors than following a straight line when traveling on a vehicle trajectory **200**.

Techniques described herein can enhance the determination of probabilities of contact between a first object such as a vehicle **110** and other (or second) objects **206**, **208** by dynamically determining first object, e.g., vehicle **110**, occupancy as a function of trajectory. Vehicle occupancy, which is the location of an object such as vehicle **110** according to a user defined probability, can be determined as a function of velocity and lateral and longitudinal accelerations along a trajectory by operating a vehicle **110** along sample trajectories **200** a plurality of times and measuring errors in vehicle locations. Density distributions of error including mean and standard deviations are determined as a function of velocity and lateral and longitudinal accelerations. The resulting density distributions then be applied to a trajectory to be operated on by a vehicle **110** to determine probabilities of contact with objects **206**, **208** at points or segments on the trajectory.

FIG. **5** illustrates a tube **506** formed by dynamic bounding boxes **502** determined according to techniques discussed herein. Dynamic bounding boxes **502** are determined dynamically, meaning that the size of the dynamic bounding box **502** is determined based on an estimate of the error included in the location of the vehicle determined at the location on the vehicle trajectory **200** upon which the dynamic bounding box **502** is located. The estimated error is the two-dimensional shape that indicates that probability that the vehicle **110** will occupy the location and orientation at a given location on a vehicle trajectory **200**. The two-dimensional shape that indicates the probability that the vehicle **110** will occupy a location with an indicated orientation on a vehicle trajectory **200** can be determined based the estimated error being greater than a user selected value. For example, the two-dimensional shape can indicate a region where the probability that the vehicle **110** will be located is greater than 99%.

The two-dimensional shape that indicates the probability that a vehicle **110** will be located can be determined based on the curvature and distance between sample points of the trajectory **200**. The curvature and distance between sample points of the trajectory **200** indicate the velocity and lateral and longitudinal accelerations of the vehicle **110** at that point. The estimated error in determining the location and orientation of the vehicle **110** is a function of the velocity and lateral and longitudinal accelerations of the vehicle **110**. For example, rectangular boxes **504**, **508** near the initial location **202** and the end point **204** are small relative to the vehicle trajectory **200** because at these locations vehicle **110** is moving in an approximately straight line, and errors in determining the location of the vehicle with respect to the trajectory **200** are low. In portions of the vehicle trajectory **200** where the vehicle **110** is changing direction, lateral accelerations are higher and the dynamic bounding boxes **502** are larger because the uncertainty in the location of the vehicle **110** is higher.

Determining vehicle trajectories **200** based on dynamic occupancy with dynamic bounding boxes **502** as illustrated in FIG. **5** permits greater range of operation for vehicles **110** while maintaining a desired level of confidence for avoiding contact, for example 99% confidence. In examples such as FIG. **5** dynamic bounding boxes **502** indicate that object **206** can likely be avoided while object **208** is likely not avoided. Determining potential contact in this fashion permits a computing device **115** in a vehicle **110** to make adjustments to avoid object **208**. For example, trajectory **200** can be adjusted to avoid object **208**. For example, the speed and/or the lateral acceleration of vehicle **110** can be adjusted to dynamic bounding boxes **502** near object **208** smaller. Lowering the speed and/or lateral acceleration can thereby reduce the size of the dynamic bounding boxes **502** and hence the dynamic occupancy of vehicle **110** in the vicinity of object **208**. Speed and lateral acceleration can be adjusted by changing the vehicle trajectory to lower speeds and make turns begin sooner and change vehicle direction more slowly, for example.

FIG. **6** is a diagram of a dynamic occupancy system **600** that calculates reachability tubes for vehicle trajectories **200** configured for training. A dynamic occupancy system **600** determines the locations that are likely to be occupied by a vehicle **110** dynamically as a function of vehicle **110** speed and lateral and longitudinal accelerations. Vehicle occupancy is indicated by a reachability tube determined by combining dynamic bounding boxes **502** that indicates the outer limit of locations that can be likely occupied by vehicle **110** (i.e., reachability refers to a location that a vehicle **110** can reach). FIG. **6** illustrates a dynamic occupancy system **600** configured for training and testing. A dynamic occupancy system **600** can be trained and tested on a server computer **120** and transmitted to a computing device **115** in a vehicle **110** for operation. A candidate trajectory **602** can be determined by motion planning software included in the vehicle **110** as discussed above in relation to FIG. **2**. This trajectory is referred to a candidate trajectory **602** because it has not been determined to be appropriate for operation of the vehicle **110** until it is processed by dynamic occupancy system **600**. The candidate trajectory **602** can be output to a vehicle or simulation software **606**. In examples where a vehicle **110** is employed, the vehicle **110** is operated and the motion of the vehicle **110** in response to the candidate trajectory **602** is recorded. In examples where simulation software is employed, the simulation software determines simulated motion of a vehicle in response to the candidate trajectory **602**.

The candidate trajectory **602** is input to a first neural network **608** which simplifies the candidate trajectory **602** to determine a simplified trajectory **610**. A candidate trajectory **602** can be a T*n matrix that can include x and y locations, velocities, and heading angles at a plurality of time steps t. First neural network **608** reduces the candidate trajectory **602** to a simplified trajectory **610** that is a learned latent vector indicated by m dimensions, where m<n. The simplified trajectory **610** is input to a second neural network **614**. Second neural network **614** has a plurality of layers including fully connected neurons with ReLU activation between layers. ReLU activation is a function between layers of the second neural network **614** that is a piecewise linear function that outputs the input directly if it is positive and outputs zero if it is negative.

The second neural network **614** determines density distributions for the input simplified trajectory **610** assuming that the density distributions are described by Liouville partial differential equations. The Liouville equation describes the time evolution of a distribution function along a trajectory and is derived from statistical mechanics. The Liouville equation:

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \sum\nolimits_{i=1}^{n}\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i + \frac{\partial\rho}{\partial p_i}\dot{p}_i\right) = 0 \tag{2}$$

Describes a system with canonical coordinates q_i and conjugate momenta p_i, where the phase distribution ρ(p,q) determines the probability ρ(p,q)d^nqd^np that the system will be found in the system will be found in the phase space volume d^nqd^np.

The second neural network **614** is trained by comparing the output vehicle trajectories **620** to ground truth measured vehicle trajectories **612** from real world vehicles **110** or simulation software **606**. The second neural network **614** is further trained based on stochastic gradient descent with L2-norm reconstruction loss because Liouville partial differential equations are continuously differentiable. An L2-norm calculates the Euclidian distance between the density distribution output from the second neural network and the ground truth. The weights from the trained second neural network **614** are output to a polyhedral marching system **618** which inputs initial density distribution data **626** for the initial location **202** of the candidate trajectory **602**. The initial density distribution data **626** can be determined empirically based on the speed and direction of the vehicle **110** at the initial location **202**.

Training the second neural network **614** based on recorded behavior of vehicles **110** determines weights that control the behavior of the layers of the second neural network **614**. The weights control the sizes of density distributions that indicate the distances from a point on the trajectory that a vehicle will occupy based on the curvature of the trajectory for a user determined probability. For example, based on real world vehicle **110** data, the second neural network **614** can determine density distributions that indicate that a vehicle **110** will occupy a distance from the trajectory with 99% probability. Weights indicating these density distributions determined as a function of trajectory curvature can be output to polyhedral marching system **618**. Polyhedral marching system **618** can then determine vehicle occupancy based on trajectory curvature without having to calculate density distributions to permit the polyhedral marching system **618** to determine dynamic bounding boxes **502** in real time in a computing device **115** in a vehicle **110**.

Polyhedral marching system **618** generates polyhedrals **622**. A polyhedral **622** is an n-sided geometric shape determined at each time step t of the candidate trajectory **602**. The sides of the polyhedral **622** are based on the weights determined by training the second neural network **614** and are the distances from the candidate trajectory **602** where the probability of encountering any portion of the vehicle **110** is lower than an empirically determined threshold without having to calculate density distributions. The empirically determined threshold can be determined to avoid a user determined percentage of contacts between a vehicle and an object, for example 99%. Polyhedral marching system **618** begins by selecting a first point on a candidate trajectory **602**. The beginning point can be and initial location **202** for vehicle **110**, for example. Polyhedral marching system **618** then determines a polyhedral shape centered at the first point. A polyhedral is a two-dimensional shape having n sides, where n is a user-selected value. Setting n=4 produces rectangles, setting n=5 produces pentagrams, etc. Higher values of n can produce more accurate predictions of vehicle reachability but require greater time to compute. Values of n=6-8 are good compromises between accuracy and compute time.

Polyhedral marching system **618** proceeds by calculating polyhedrals indicating that the probability that a vehicle will occupy a location along a line from the first point perpendicular to a first side of the polyhedral is greater than the predetermined probability. The first side of the polygon is placed at a distance from the first point on the candidate trajectory **602** based on the weights input from the trained second neural network. The distance from the point on the candidate trajectory can be determined without having to calculate the density distribution. The data from the calculation of the density distribution as a function of trajectory curvature and sample distance is included in the weights input from the second neural network. Sample distance is the distance between adjacent points on a trajectory where the location of the vehicle **110** was predicted. Polyhedral marching then selects the next side of the polyhedral and calculates the location of the second side of the polyhedral in similar fashion and so on until distances to all the sides of the polyhedral are selected.

Polyhedral marching system **618** then selects a next point on the candidate trajectory **602** at the next time step and determines a next polyhedral based on the speed and lateral and longitudinal accelerations can be based on the curvature and distance between samples of a segment of the candidate trajectory **602** at the next point on the candidate trajectory **602**. The next polyhedral is combined with the first polyhedral by logically ORing the two polyhedrals together. The polyhedral marching system **618** then proceeds along the candidate trajectory **602**, incrementing the time step and selecting the point on the candidate trajectory **602** at the incremented time step and combining the resulting polyhedral with the previously determined polyhedrals to produce polyhedrals **622**.

Polyhedrals **622** output by the polyhedral marching system **618** can be combined to form dynamic bounding boxes **502** that can be further combined to form a reachable tube **624** which describes two-dimensional regions around a candidate trajectory **602** that has a user-selected probability of including the vehicle **110**. For example, a reachable tube **624** encloses the candidate trajectory **602** and indicates that a vehicle **110** traveling along the candidate trajectory **602** has a less than a selected probability, 1% for example, of contacting an object that lies outside of the reachable tube **624**. The reachable tube **624** enhances the ability to operate a vehicle **110** by including dynamical bounding boxes **502** that are sized based on vehicle speed and rate of change of direction to accurately determine probabilities of contact with external objects **206**, **208**. In examples where the objects are roadway lane markers the reachable tube **624** can indicate the probability that a candidate trajectory **602** will permit the vehicle **110** to remain within a roadway lane, e.g., lane keeping. In examples where the objects include both lane markers and other vehicles, the reachable tube **624** can indicate the probability that a candidate trajectory **602** will permit a vehicle **110** to change lanes in traffic.

Determining vehicle trajectories **200** based on dynamic occupancy system **600** that calculates reachability tubes enhances determination of vehicle trajectories **200** by determining a reachable tube **624** based on polyhedrals **622** that indicate probabilities of contact with objects **206**, **208** at a plurality of locations along the trajectory **200** in real time in a computing device **115** included in a vehicle **110**. Each polyhedral **622** is of a size and shape that matches the density distributions of output vehicle trajectories **620** determined by second neural network **614** which was trained based on ground truth measured vehicle trajectories **612** from a plurality of vehicles or simulation software, without having to determine the density distributions in real time. Dynamic occupancy system **600** can determine reachable tubes **624** in real time as a vehicle **110** is being operated on a roadway at highway speeds using a computing device **115** included in a vehicle **110**.

FIG. **7** is a flowchart, described in relation to FIGS. **1-6** of a process **700** for determining a trajectory for a vehicle **110**. Process **700** can be implemented by a processor of a computing device **115**, taking as input a candidate trajectory **602**, executing commands, and outputting a reachable tube

624. Process **700** includes multiple blocks that can be executed in the illustrated order. Process **700** could alternatively or additionally include fewer blocks or can include the blocks executed in different orders.

Process **700** begins at block **702**, where a candidate trajectory **602** is input from motion control software included in a computing device **115** in a vehicle **110** along with locations of objects in an environment around vehicle **110**. A candidate trajectory **602** is discussed in relation to FIG. **2**, above.

At block **704** a first neural network **608** inputs the candidate trajectory **602** and outputs a simplified trajectory **610**.

At block **706** a polyhedral marching system **618** determines polyhedrals based on an the simplified trajectory **610**, initial density distribution data **626** and weights **616** output by the second neural network **614** at training time.

At block **708** polyhedrals **622** are combined to form dynamic bounding boxes **502** and further combined to form a reachable tube **624**. The reachable tube **624** along with can be output to computing device **115**. Following block **708** process **700** ends.

FIG. **8** is a flowchart, described in relation to FIGS. **1-7** of a process **800** for operating a vehicle **110** based on a reachable tube **624** determined by a dynamic occupancy system **600**. Process **800** can be implemented by a processor of a computing device **115**, taking as input a vehicle trajectory, executing commands, and operating a vehicle. Process **800** includes multiple blocks that can be executed in the illustrated order. Process **800** could alternatively or additionally include fewer blocks or can include the blocks executed in different orders.

At block **802** sensors **116** included in a vehicle **110** can input data regarding an environment around the vehicle **110**. For example, sensor **116** can input image data regarding the lane markers and objects **206**, **208** on the roadway. A computing device **115** in the vehicle **110** can input the sensor data to software programs included in the computing device **115** to determine a candidate trajectory **602** upon which to operate the vehicle **110**.

At block **804** the computing device can input the candidate trajectory **602** and data regarding objects **206**, **208** around a vehicle **110** to a dynamic occupancy system **600** as described in relation to FIG. **7**, above.

At block **806** the dynamic occupancy system **600** outputs a reachable tube **624** including dynamic bounding boxes **502** based on the candidate trajectory **602** to computing device **115**.

At block **808** process **800** determines whether the reachable tube **624** avoids objects **206**, **208**. Objects **206**, **208** can include lane markers, other vehicles, and pedestrians, for example. Avoiding objects **206**, **208** can include lane keeping, avoiding contact with other vehicles and pedestrians, and successfully performing a lane change maneuver, for example. In examples where reachable tube **624** does not successfully avoid objects **206**, **208**, computing device **115** can return to block **802** to determine a new candidate trajectory **602** that reduces the probability of contact with objects **206**, **208** by moving the candidate trajectory **602** away from the objects **206**, **208**, slowing the speed of the vehicle **110** or reducing the lateral and longitudinal accelerations in the vicinity of the object **206**, **208**, for example. When reachable tube **624** does not intersect an object **206**, **208**, candidate trajectory **602** becomes the final trajectory and process **800** passes to block **810**.

At block **810** the computing device **115** in vehicle **110** determines commands to output to controllers **112**, **113**, **114** to control vehicle powertrain, vehicle steering and vehicle brakes to operate vehicle **110** based on the final trajectory. Following block **810** process **800** ends.

Computing devices such as those discussed herein generally each includes commands executable by one or more computing devices such as those identified above, and for carrying out blocks or steps of processes described above. For example, process blocks discussed above may be embodied as computer-executable commands.

Computer-executable commands may be compiled or interpreted from computer programs created using a variety of programming languages and/or technologies, including, without limitation, and either alone or in combination, Java™, C, C++, Python, Julia, SCALA, Visual Basic, Java Script, Perl, HTML, etc. In general, a processor (i.e., a microprocessor) receives commands, i.e., from a memory, a computer-readable medium, etc., and executes these commands, thereby performing one or more processes, including one or more of the processes described herein. Such commands and other data may be stored in files and transmitted using a variety of computer-readable media. A file in a computing device is generally a collection of data stored on a computer readable medium, such as a storage medium, a random access memory, etc.

A computer-readable medium (also referred to as a processor-readable medium) includes any non-transitory (i.e., tangible) medium that participates in providing data (i.e., instructions) that may be read by a computer (i.e., by a processor of a computer). Such a medium may take many forms, including, but not limited to, non-volatile media and volatile media. Instructions may be transmitted by one or more transmission media, including fiber optics, wires, wireless communication, including the internals that comprise a system bus coupled to a processor of a computer. Common forms of computer-readable media include, for example, RAM, a PROM, an EPROM, a FLASH-EEPROM, any other memory chip or cartridge, or any other medium from which a computer can read.

All terms used in the claims are intended to be given their plain and ordinary meanings as understood by those skilled in the art unless an explicit indication to the contrary in made herein. In particular, use of the singular articles such as "a," "the," "said," etc. should be read to recite one or more of the indicated elements unless a claim recites an explicit limitation to the contrary.

The term "exemplary" is used herein in the sense of signifying an example, i.e., a candidate to an "exemplary widget" should be read as simply referring to an example of a widget.

The adverb "approximately" modifying a value or result means that a shape, structure, measurement, value, determination, calculation, etc. may deviate from an exactly described geometry, distance, measurement, value, determination, calculation, etc., because of imperfections in materials, machining, manufacturing, sensor measurements, computations, processing time, communications time, etc.

In the drawings, the same candidate numbers indicate the same elements. Further, some or all of these elements could be changed. With regard to the media, processes, systems, methods, etc. described herein, it should be understood that, although the steps or blocks of such processes, etc. have been described as occurring according to a certain ordered sequence, such processes could be practiced with the described steps performed in an order other than the order described herein. It further should be understood that certain steps could be performed simultaneously, that other steps could be added, or that certain steps described herein could be omitted. In other words, the descriptions of processes herein are provided for the purpose of illustrating certain embodiments, and should in no way be construed so as to limit the claimed invention.

The invention claimed is:

1. A system, comprising:

a computer that includes a processor and a memory, the memory including instructions executable by the processor to determine a final trajectory by:

determining a candidate trajectory of a first object based on a detected second object;

inputting the candidate trajectory to a reachable polyhedral marching processor to determine dynamic occupancy polyhedrals based on a shape of the candidate trajectory wherein the sizes of the dynamic occupancy polyhedrals are based on estimates of errors;

determining a reachable tube based on combining the dynamic occupancy polyhedrals; and

determining the final trajectory based on the reachable tube avoiding the detected second object.

2. The system of claim **1**, wherein the instructions include further instructions to operate the first object based on the final trajectory by controlling actuators to control movement of the first object.

3. The system of claim **2**, wherein the first object is a vehicle and controlling the actuators to control the movement of the vehicle include controlling one or more of vehicle powertrain, vehicle steering and vehicle brakes.

4. The system of claim **2**, wherein the first object is a robot and controlling the actuators to control the movement of the robot include controlling movement of one or more of a gripper and a robotic arm.

5. The system of claim **1**, the instructions including further instructions to input the candidate trajectory to a first neural network to simplify the candidate trajectory.

6. The system of claim **5**, wherein the candidate trajectory is a T*n matrix that includes x and y locations, velocities, and heading angles at a plurality of time steps t and simplifying the trajectory reduces the T*n matrix to a vector with m dimensions, where m<T*n.

7. The system of claim **1**, wherein the reachable polyhedral marching processor is programmed based on weights output from a second neural network.

8. The system of claim **7**, wherein the second neural network determines density distributions based on solving Liouville partial differential equations.

9. The system of claim **8**, wherein the second neural network is trained based on a plurality of trajectories acquired from real world vehicles.

10. The system of claim **9**, wherein the second neural network includes fully connected neurons with ReLU activation and is trained based on stochastic gradient descent with L2-norm reconstruction loss.

11. The system of claim **1**, wherein the dynamic occupancy polyhedrals include two-dimensional regions wherein a probability that the first object will occupy locations within the reachable polyhedrals is higher than an empirically determined threshold.

12. The system of claim **1**, wherein the sizes of the dynamic occupancy polyhedrals are based on curvature and a distance between samples of segments of the candidate trajectory.

13. A method, comprising:

determining a final trajectory by:

determining a candidate trajectory of a first object based on a detected second object;

inputting the candidate trajectory to a reachable polyhedral marching processor to determine dynamic occupancy polyhedrals based on a shape of the candidate trajectory wherein the sizes of the dynamic occupancy polyhedrals are based on estimates of errors;

determining a reachable tube based on combining the dynamic occupancy polyhedrals; and

determining the final trajectory based on the reachable tube avoiding the detected second object.

14. The method of claim **13**, wherein the first object is operated based on the final trajectory by controlling one or more actuators to control movement of the first object.

15. The method of claim **14**, wherein the first object is a vehicle and controlling the actuators to control the movement of the vehicle includes controlling one or more of vehicle powertrain, vehicle steering and vehicle brakes.

16. The method of claim **14**, wherein the first object is a robot and controlling the actuators to control the movement of the robot includes controlling motion of one or more of a gripper and a robotic arm.

17. The method of claim **13**, further comprising inputting the candidate trajectory to a first neural network to simplify the candidate trajectory.

18. The method of claim **17**, wherein the candidate trajectory is a T*n matrix that includes x and y locations, velocities, and heading angles at a plurality of time steps t and simplifying the trajectory reduces the T*n matrix to a vector with m dimensions, where m<T*n.

19. The method of claim **13**, wherein the reachable polyhedral marching processor is programmed based on weights output from a second neural network.

20. The method of claim **19**, wherein the second neural network determines density distributions based on solving Liouville partial differential equations.

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