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(54) **STATEFUL HASH-BASED SIGNATURES**

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H04L 9/32 (2006.01)
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CPC ***H04L 9/3247*** (2013.01); ***H04L 9/30*** (2013.01); ***H04L 9/3239*** (2013.01)

(58) **Field of Classification Search**
None
See application file for complete search history.

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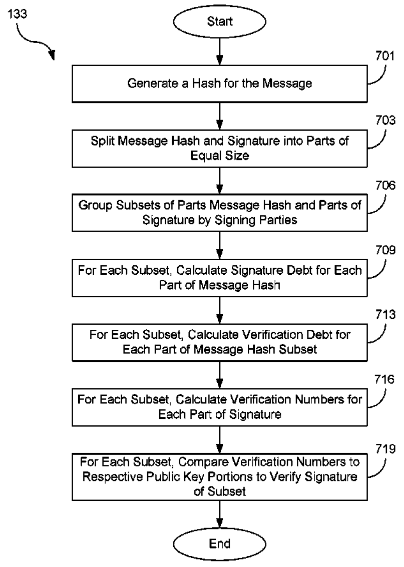
(57) **ABSTRACT**

Disclosed are various embodiments for generating and verifying stateful hash-based signatures. A message hash of a message is generated using a cryptographic hash function. Then, the message hash is split into a predefined number of parts of equal size. Next, a value of a signature debt is calculated for each of the predefined number of parts of the message hash. Subsequently, a respective signature part is calculated for each of the predefined number of parts of the message hash by applying the cryptographic hash function to a respective portion of a private key for a number of times equal to the value of the signature debt. Next, the respective signature parts are concatenated to generate a signature for the message.

20 Claims, 8 Drawing Sheets


133
Start
701
Generate a Hash for the Message
703
Split Message Hash and Signature into Parts of Equal Size
706
Group Subsets of Parts Message Hash and Parts of Signature by Signing Parties
709
For Each Subset, Calculate Signature Debt for Each Part of Message Hash
713
For Each Subset, Calculate Verification Debt for Each Part of Message Hash Subset
716
For Each Subset, Calculate Verification Numbers for Each Part of Signature
719
For Each Subset, Compare Verification Numbers to Respective Public Key Portions to Verify Signature of Subset
End

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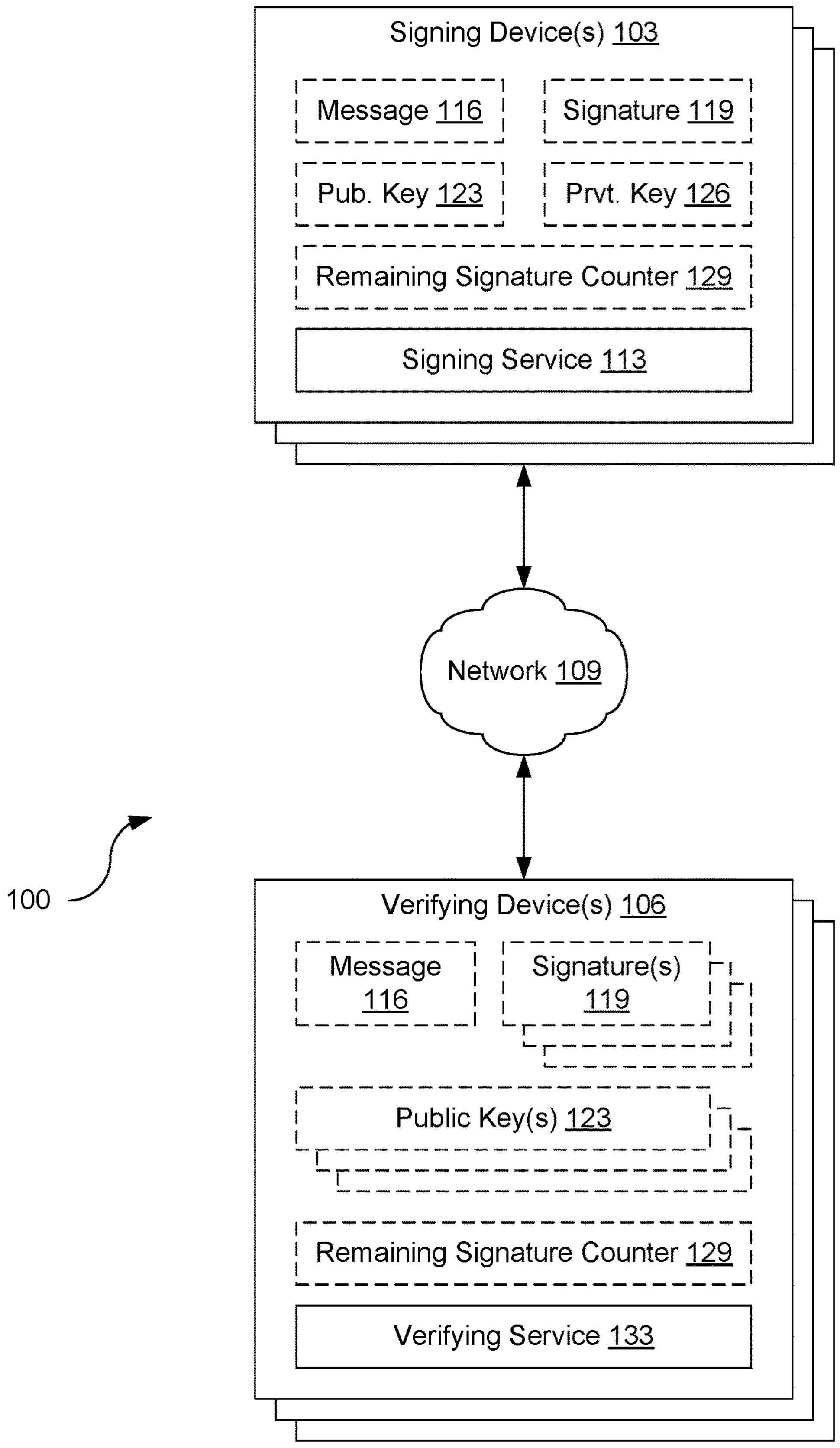
Signing Device(s) 103
Message 116
Signature 119
Pub. Key 123
Prvt. Key 126
Remaining Signature Counter 129
Signing Service 113
Network 109
100
Verifying Device(s) 106
Message 116
Signature(s) 119
Public Key(s) 123
Remaining Signature Counter 129
Verifying Service 133

FIG. 1

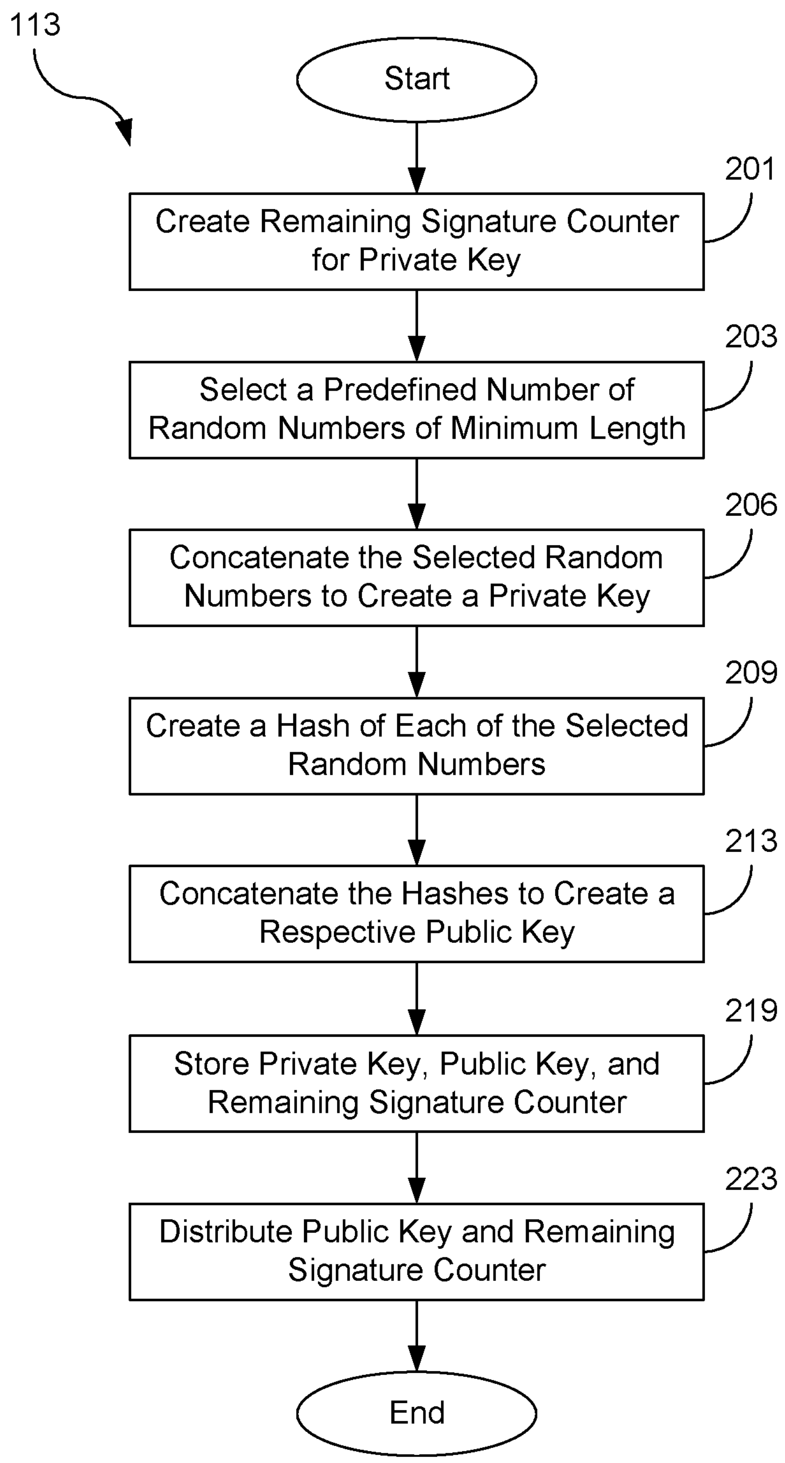
113
Start
201
Create Remaining Signature Counter for Private Key
203
Select a Predefined Number of Random Numbers of Minimum Length
206
Concatenate the Selected Random Numbers to Create a Private Key
209
Create a Hash of Each of the Selected Random Numbers
213
Concatenate the Hashes to Create a Respective Public Key
219
Store Private Key, Public Key, and Remaining Signature Counter
223
Distribute Public Key and Remaining Signature Counter
End

FIG. 2

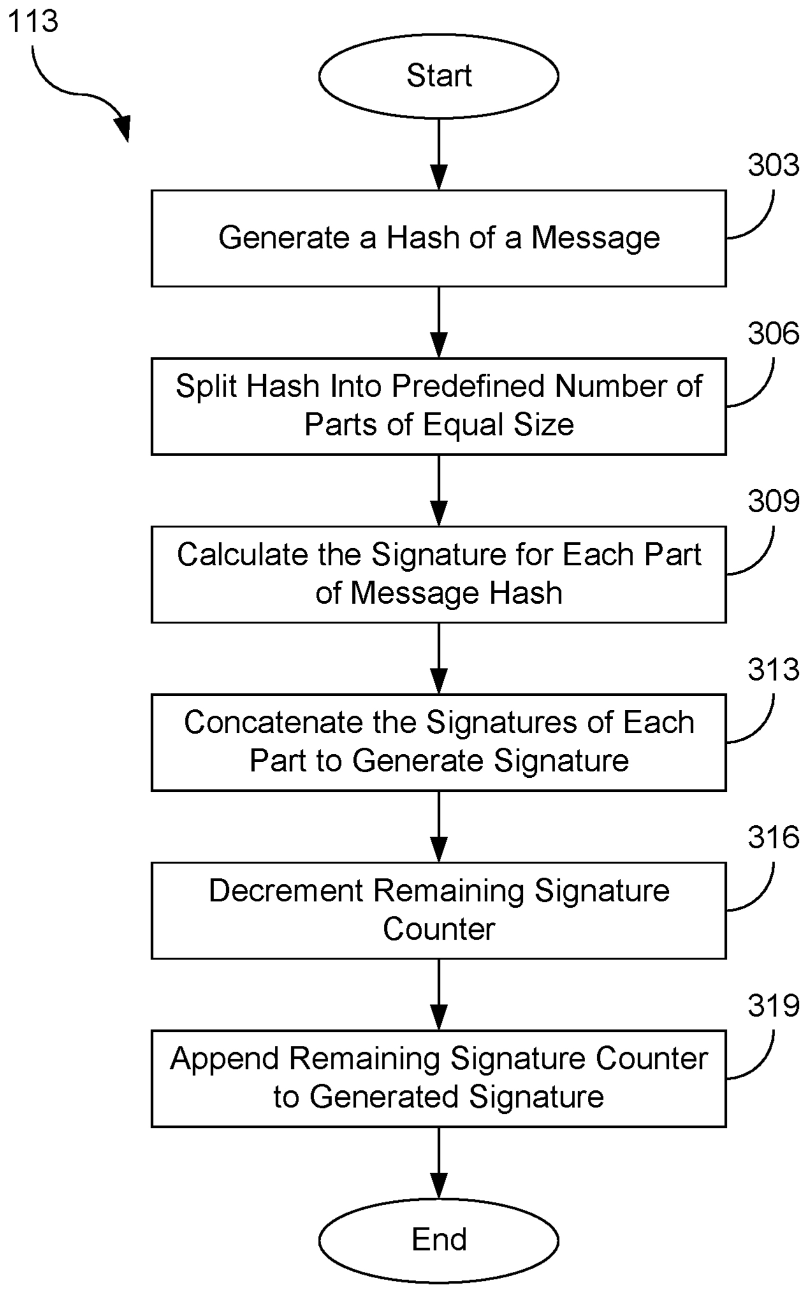
113
Start
Generate a Hash of a Message
303
Split Hash Into Predefined Number of Parts of Equal Size
306
Calculate the Signature for Each Part of Message Hash
309
Concatenate the Signatures of Each Part to Generate Signature
313
Decrement Remaining Signature Counter
316
Append Remaining Signature Counter to Generated Signature
319
End

FIG. 3

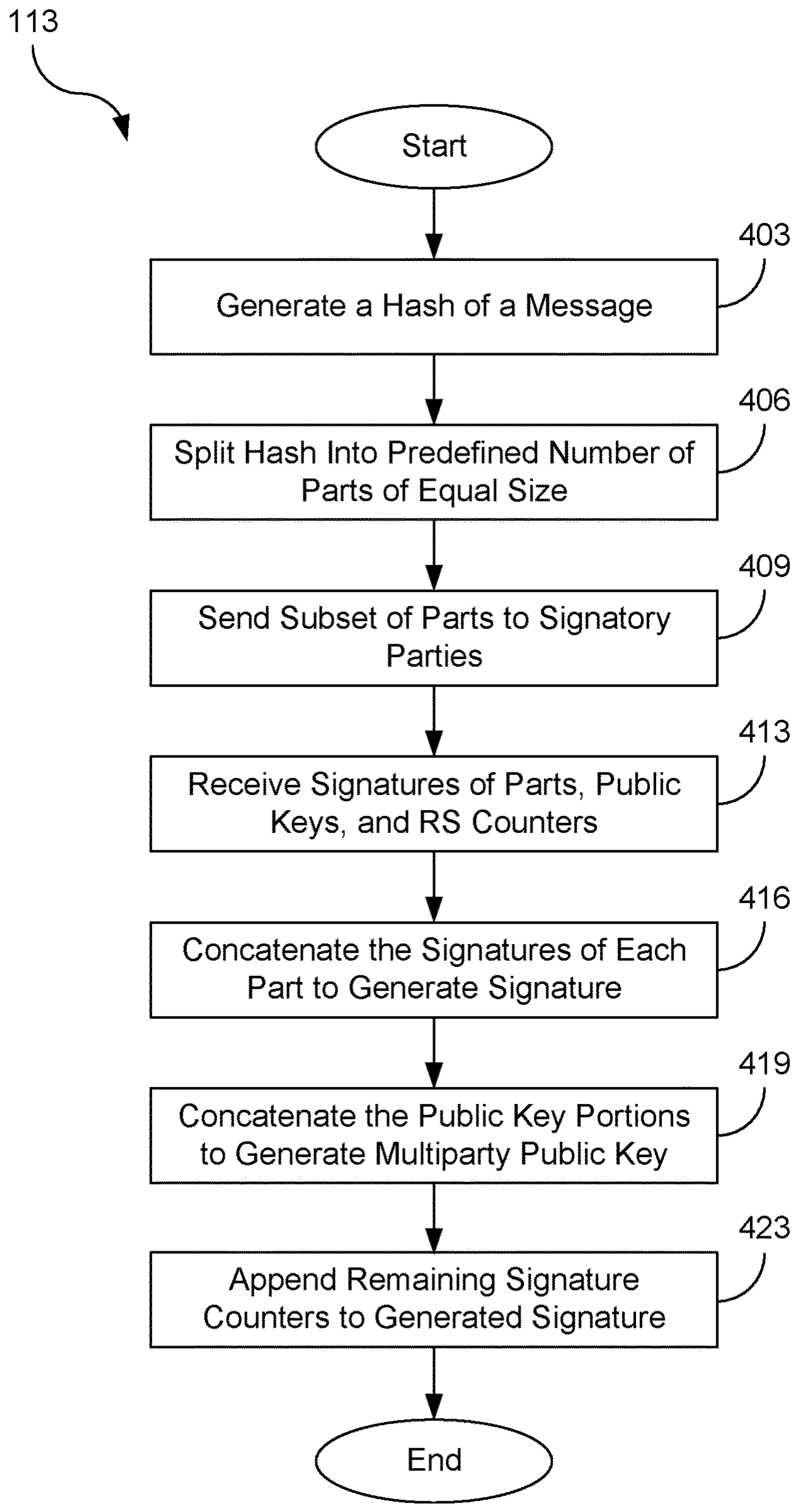
113
Start
Generate a Hash of a Message
403
Split Hash Into Predefined Number of Parts of Equal Size
406
Send Subset of Parts to Signatory Parties
409
Receive Signatures of Parts, Public Keys, and RS Counters
413
Concatenate the Signatures of Each Part to Generate Signature
416
Concatenate the Public Key Portions to Generate Multiparty Public Key
419
Append Remaining Signature Counters to Generated Signature
423
End

FIG. 4

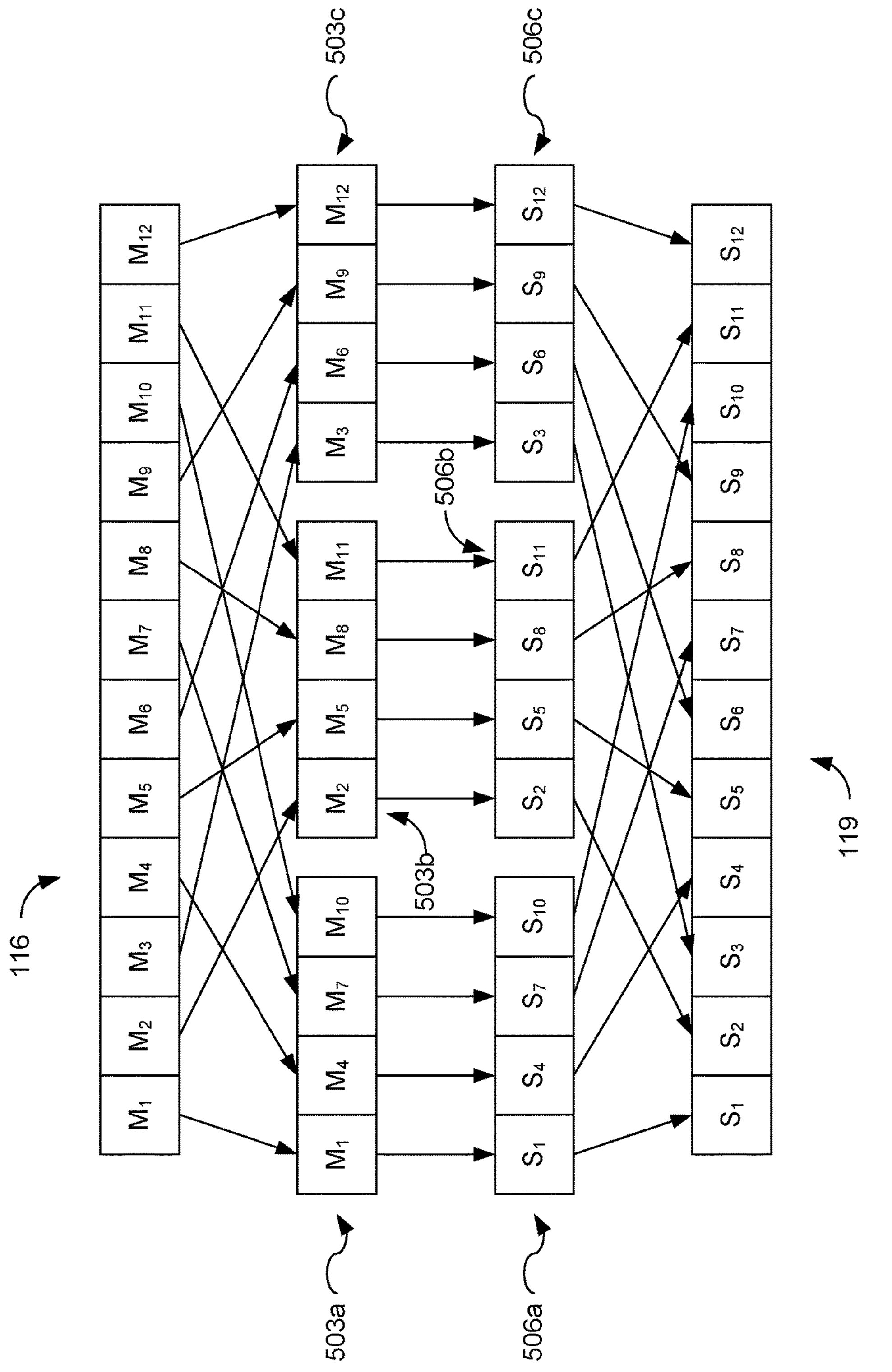
116
M_1 M_2 M_3 M_4 M_5 M_6 M_7 M_8 M_9 M_{10} M_{11} M_{12}
503a
503b
503c
506a
506b
506c
S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 S_{10} S_{11} S_{12}
119

FIG. 5

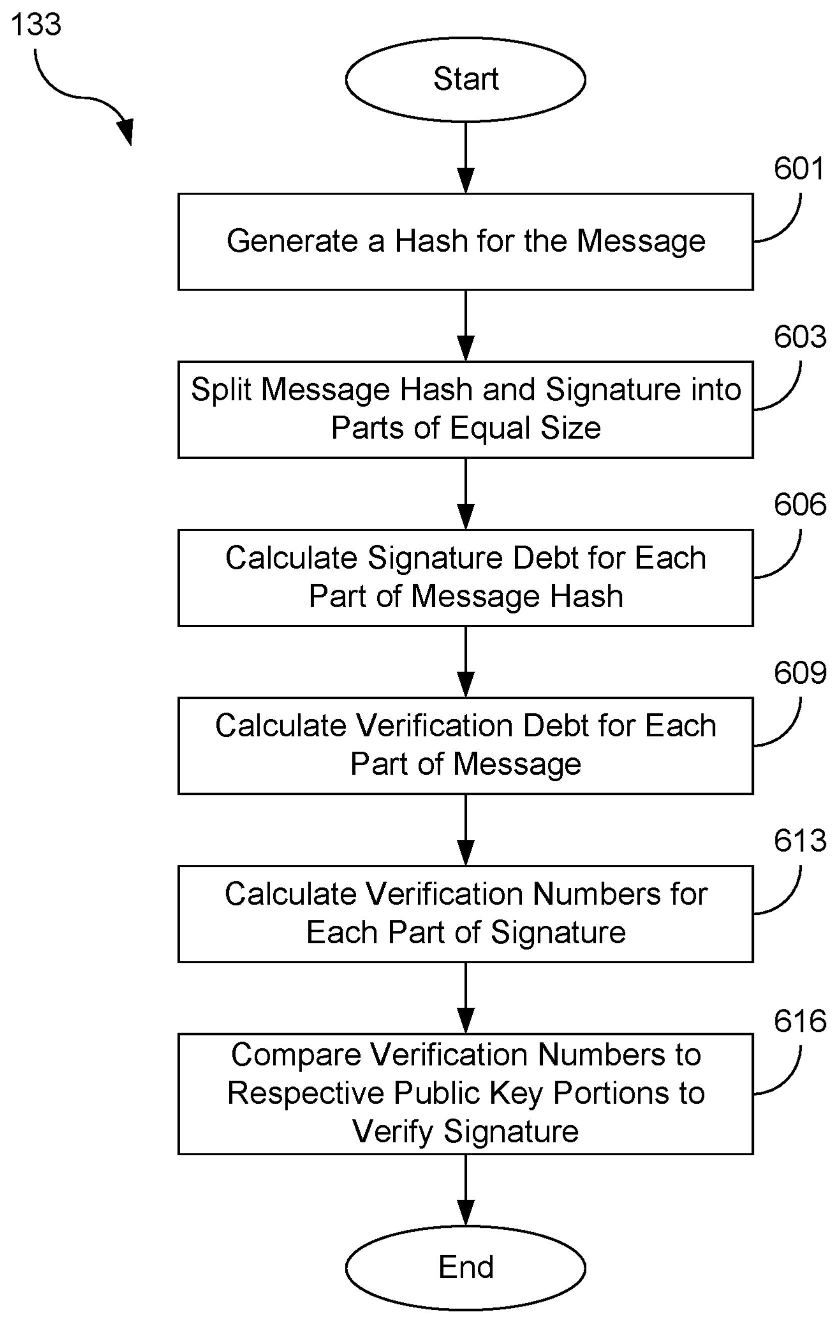
133
Start
601
Generate a Hash for the Message
603
Split Message Hash and Signature into Parts of Equal Size
606
Calculate Signature Debt for Each Part of Message Hash
609
Calculate Verification Debt for Each Part of Message
613
Calculate Verification Numbers for Each Part of Signature
616
Compare Verification Numbers to Respective Public Key Portions to Verify Signature
End

FIG. 6

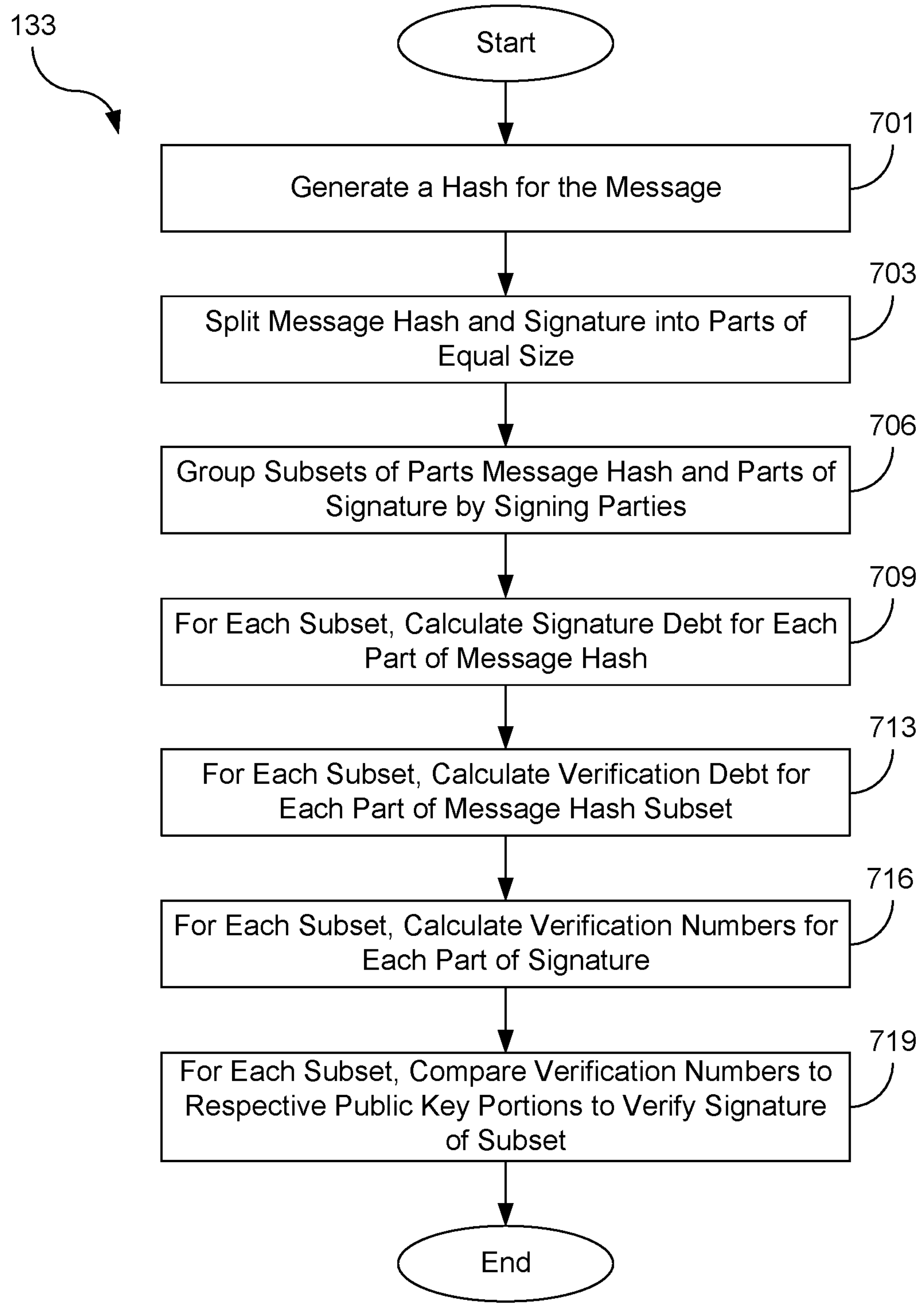
133
Start
Generate a Hash for the Message
701
Split Message Hash and Signature into Parts of Equal Size
703
Group Subsets of Parts Message Hash and Parts of Signature by Signing Parties
706
For Each Subset, Calculate Signature Debt for Each Part of Message Hash
709
For Each Subset, Calculate Verification Debt for Each Part of Message Hash Subset
713
For Each Subset, Calculate Verification Numbers for Each Part of Signature
716
For Each Subset, Compare Verification Numbers to Respective Public Key Portions to Verify Signature of Subset
719
End

FIG. 7

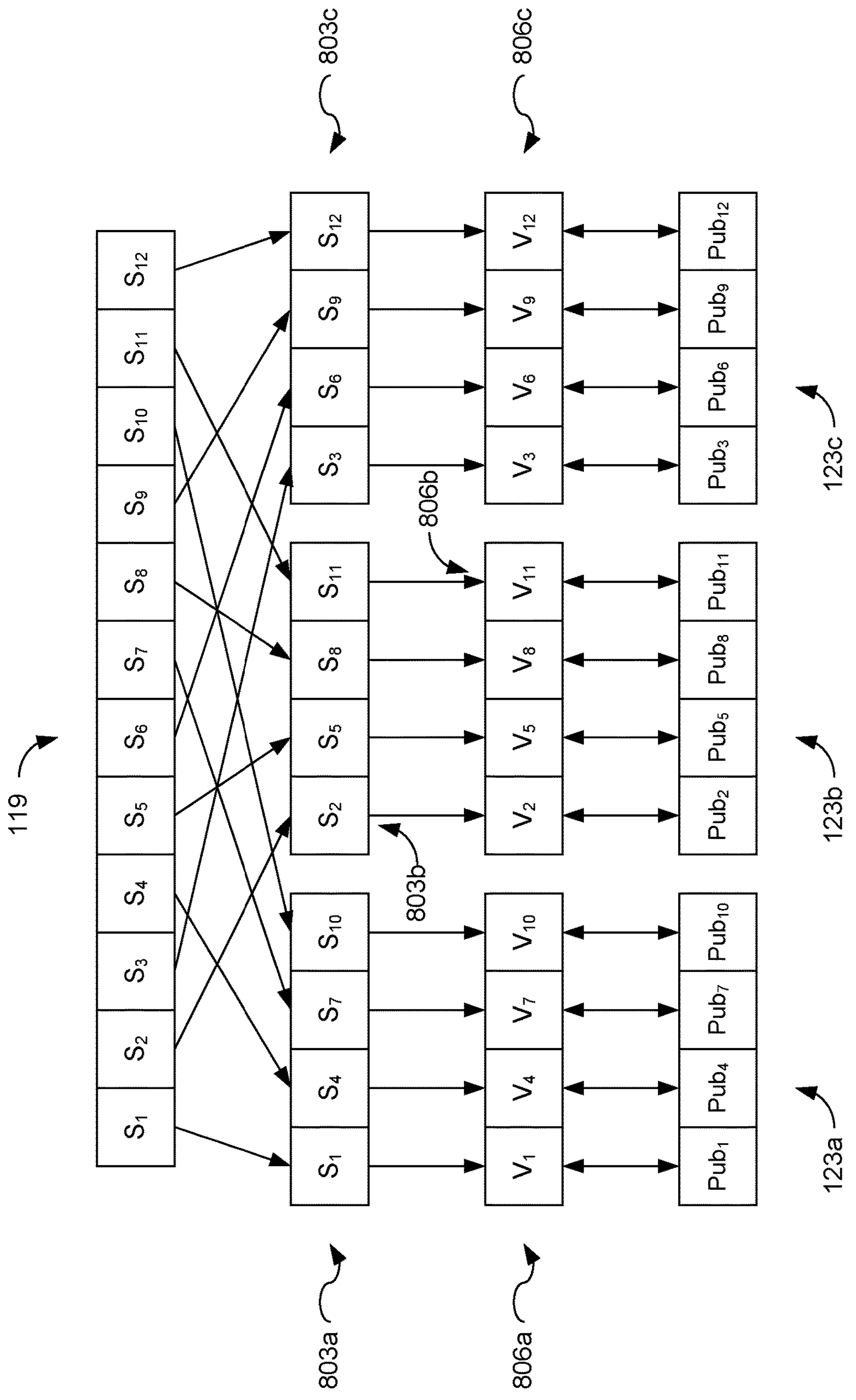
119
S_1
S_2
S_3
S_4
S_5
S_6
S_7
S_8
S_9
S_{10}
S_{11}
S_{12}
803a
803b
803c
S_1
S_4
S_7
S_{10}
S_2
S_5
S_8
S_{11}
S_3
S_6
S_9
S_{12}
806a
806b
806c
V_1
V_4
V_7
V_{10}
V_2
V_5
V_8
V_{11}
V_3
V_6
V_9
V_{12}
Pub_1
Pub_4
Pub_7
Pub_{10}
Pub_2
Pub_5
Pub_8
Pub_{11}
Pub_3
Pub_6
Pub_9
Pub_{12}
123a
123b
123c

FIG. 8

STATEFUL HASH-BASED SIGNATURES

BACKGROUND

Cryptographic signatures are used to verify the integrity, validity, and veracity of messages or data transmitted between one party to another. Multiparty signatures allow for multiple parties to attest to the integrity, validity, and veracity of messages or data. Generally, multiparty signature schemes are sequential, with a first party signing the message or data, then a second party signing the combination of both the message or data and the first signature, a third party signing the combination of both the message or data, the first signature, and the second signature, etc. Accordingly, multiparty signatures may be slow to perform or verify because the signatures of the signing parties are generated sequentially and must also be verified sequentially. Moreover, existing signature schemes often rely on algorithms or protocols that are decades old, and may be susceptible to cryptanalysis by advancing computing technology.

BRIEF DESCRIPTION OF THE DRAWINGS

Many aspects of the present disclosure can be better understood with reference to the following drawings. The components in the drawings are not necessarily to scale, with emphasis instead being placed upon clearly illustrating the principles of the disclosure. Moreover, in the drawings, like reference numerals designate corresponding parts throughout the several views.

FIG. **1** is a drawing of a network environment according to various embodiments of the present disclosure.

FIG. **2** is a flowchart depicting an example of the creation of an asymmetric key pair in the network environment of FIG. **1** according to various embodiments of the present disclosure.

FIG. **3** is a flowchart depicting an example of the generating a signature for a message in the network environment of FIG. **1** according to various embodiments of the present disclosure.

FIG. **4** is a flowchart depicting an example of the generating a signature for a message in the network environment of FIG. **1** according to various embodiments of the present disclosure.

FIG. **5** is a graphical representation of the creation of a signature for a message in the network environment of FIG. **1** according to various embodiments of the present disclosure.

FIG. **6** is a flowchart depicting an example of verifying a signature for a message in the network environment of FIG. **1** according to various embodiments of the present disclosure.

FIG. **7** is a flowchart depicting an example of verifying a signature for a message in the network environment of FIG. **1** according to various embodiments of the present disclosure.

FIG. **8** is a graphical representation of the verification of a signature for a message in the network environment of FIG. **1** according to various embodiments of the present disclosure.

DETAILED DESCRIPTION

Disclosed are various approaches for using stateful hashed-based signatures for signing and verifying the integrity of messages or data. The various embodiments of the present disclosure allow for multiple parties to sign a single

message or other item of data in parallel by the parties. Similarly, the various embodiments of the present disclosure allow for signatures to be verified in parallel. As a result, signature creation and verification can be performed more quickly and efficiently compared to other approaches where multiple parties sequentially sign a message or item of data. Moreover, the various embodiments of the present disclosure can leverage algorithms that are resistant against cryptanalytic attacks by quantum computers, making them more secure compared to digital signature schemes that are susceptible to cryptanalysis by quantum computers.

In the following discussion, a general description of the system and its components is provided, followed by a discussion of the operation of the same. Although the following discussion provides illustrative examples of the operation of various components of the present disclosure, the use of the following illustrative examples does not exclude other implementations that are consistent with the principals disclosed by the following illustrative examples.

With reference to FIG. **1**, shown is a network environment **100** according to various embodiments. The network environment **100** can include one or more signing devices **103** and one or more verifying devices **106**. The signing devices **103** and the verifying devices **106** can be in data communication with each other via a network **109**.

The network **109** can include wide area networks (WANs), local area networks (LANs), personal area networks (PANs), or a combination thereof. These networks can include wired or wireless components or a combination thereof. Wired networks can include Ethernet networks, cable networks, fiber optic networks, and telephone networks such as dial-up, digital subscriber line (DSL), and integrated services digital network (ISDN) networks. Wireless networks can include cellular networks, satellite networks, Institute of Electrical and Electronic Engineers (IEEE) 802.11 wireless networks (i.e., WI-FI®), BLUETOOTH® networks, microwave transmission networks, as well as other networks relying on radio broadcasts. The network **109** can also include a combination of two or more networks **109**. Examples of networks **109** can include the Internet, intranets, extranets, virtual private networks (VPNs), and similar networks.

The signing device(s) **103** and the verifying device(s) **106** are representative of computing devices that include a processor, a memory, and/or a network interface. For example, the computing devices can be configured to perform computations on behalf of other computing devices or applications. As another example, such computing devices can host and/or provide content to other computing devices in response to requests for content.

Various applications or other functionality can be executed by a signing device **103**. The components executed by a signing device **103** can include a signing service **113**. Other applications or services can also be executed by a signing device **103** according to various embodiments of the present disclosure.

Various data can also be stored on a signing device **103**. This data can include a message **116**, a signature **119**, a public key **123** and a respective private key **126**, and a remaining signature counter **129**.

The signing service **113** can be executed by the signing device **103** to generate a signature **119** for a message **116** using the private key **126**. In situations where multiple signing devices **103** are cooperating to generate a signature **119** for the message **116**, the signing service **113** can generate a signature **119** for a portion of the message assigned to the signing device **103**.

The message **116** can represent any item of data to be signed by the signing service **113**. Examples of messages **116** can include payment transaction requests, payment transaction authorizations, web service requests or responses, files, etc. Messages **116** can come from any source and, in some implementations, can be assembled by multiple parties that utilize a predefined message schema. For example, if the message **116** represented a credit or charge card transaction, the message could have a schema that specified the location within the message that the account number, transaction amount, credit card verification code (CVC), account holder name, merchant identifier, and/or additional information should be located.

The signature **119** can represent a cryptographic signature of the message **116**. The signature **119** can be generated by the signing service **113** using the private key **126**, thereby allowing the signature **119** to be verified by the verifying service **133** using the respective public key **123** for the private key **126**.

The public key **123** and the private key **126** can represent respective members of an asymmetric cryptographic key pair. The public key **123** and the private key **126** can be generated by the signing service **113** using the approaches discussed later in the present disclosure.

The remaining signature counter **129** is the state counter representing the number of remaining possible signatures that can be generated using the private key **126**. In some implementations, the remaining signature counter **129** can be implemented as a decrementing counter that tracks the number of times the private key **126** has been used by the signing service **113** to generate a signature **119**. The initial value of the remaining signature counter **129** can be set to any value greater than or equal to one that is appropriate for the security model or threat environment envisioned for a particular deployment. Where it is desired to replace the public key **123** and private key **126** frequently, the initial value of the remaining signature counter **129** can be set to a low initial value. Where it is desired to replace the public key **123** and the private key **126** infrequently, the initial value of the remaining signature counter **129** can be set to a higher initial value. Once the remaining signature counter **129** reaches a value of zero, thereby indicating that the private key **126** can no longer be used for generating new signatures **119**, the signing service **113** can generate a new asymmetric key pair that includes a new public key **123** and a new private key **126**. The signing service **113** can also reset the remaining signature counter **129** to the initial value once the new asymmetric key pair is generated.

Various applications or other functionality can be executed by a verifying device **106**. The components executed by a verifying device **106** can include a verifying service **133**. Other applications or services can also be executed by a verifying device **106** according to various embodiments of the present disclosure.

Various data can also be stored on a verifying device **106**. This data can include a message **116**, one or more signatures **119** of the message **116**, one or more public keys **123** for verifying the signature **119**, and a remaining signature counter **129**.

The verifying service **133** can be executed by the verifying device(s) **106** to verify the integrity and validity of the signature **119** of the message **116**. In situations where multiple signing devices **103** signed portions of the message **116** to create separate signatures **119**, the verifying service **133** could verify the integrity and/or validity of the signatures **119** of the message **116** in parallel.

Next, a general description of the operation of the various components of the network environment **100** is provided. More detailed descriptions of the various operations and interactions are provided in the discussion accompanying FIGS. **2-8**.

First, a cryptographic hash function f is selected for use by the signing service **113** and the verifying service **133**. For reference, the notation $f_n(x)$ indicates that the cryptographic hash function f is applied n times to the input x. For example, $f_3(x)=f(f(f(x)))$ would indicate that the cryptographic hash function f has been applied three times.

Examples of cryptographic hash functions include the Secure Hash Algorithm 2 (SHA-2) family of cryptographic hash functions, including SHA-256 and SHA-512, the Secure Hash Algorithm 3 (SHA-3) cryptographic hash function, the BLAKE2 cryptographic hash function, the BLAKE3 cryptographic hash function, etc. The cryptographic has function f has a length Len(f), which is the length of the output of f in bits. For example, if f is the SHA-256 cryptographic hash function, then Len(f) would be 256. The Len(f) might be selected as a multiple of eight, in order to simplify calculations, but this is not a requirement. The Len(f) should also be a composite number.

The number of divisions to be used by the signing service **113** and the size of each division are also selected. These could be selected or negotiated by the signing service **113** and the verifying service **133** each time a signature **119** is to be generated, or they could be preselected in advance for all signatures to be generated. The product of the number of divisions, denoted as d, and the size of the divisions, denoted as m, must equal the length Len(f) of the cryptographic hash function f. For example, if length Len(f)=256 bits, then acceptable combinations of d and m would be d=32 and m=8, d=8 and m=32, d=16 and m=16, d=4 and m=64, d=2 and m=128, etc.

The signing service **113** can then generate a public key **123** and a private key **126**. At the time the public key **123** and private key **126** are generated, the signing service **113** can also set the remaining signature counter **129** to an initial value c, where $c \geq 1$. The signing service **113** can also select a value for the schema debt, denoted n, where $n > c*(2^m+1)$.

To sign a message **116**, the signing service can use the hash function f to calculate a hash of the message **116**. The hash of the message **116** can then be split into d parts of m length. A signature **119** for each part can be generated using the private key **126**. In some implementations, one or more subsets containing a portion of the parts of the message **116** can be signed using different private keys **126**. This could be done in parallel by multiple instances of the signing service **113** or by multiple signing devices **103**. This might occur, for example, when multiple parties wish to sign the same message **116** or a portion of the message **116**. The signature **119** can then be assembled by concatenating the signatures **119** in the same order as the d parts of the hash of the message **116**.

Once the signature **119** is generated, the signing service **113** can decrement the remaining signature counter **129**. In some implementations, the signing service **113** can append the decremented remaining signature counter **129** to the signature **119** in order to synchronize the remaining signature counter **129** with the verifying service **133**. However, other approaches for synchronizing the remaining signature counter **129** can also be used according to various embodiments of the present disclosure. The signature **119** can then be sent by the signing service **113** to the verifying service **133**. In other implementations, the signature **119** could be relayed or provided to the verifying service **133** through intermediary or third-party applications. In some implementations, the signing service **113** could also provide the message **116** to the verifying service **133**. In other implementations, the message **116** could be provided to the verifying service **133** by other applications or services.

The verifying service **133** could then verify the signature **119** of the message **116**. First, the verifying service **133** could split the message **116** and the signature **119** into d parts of m length. Then, verifying service **133** could calculate a key portion for each of the into d parts of the signature **119**. If each key portion matches a respective portion of the public key **123** used to create that portion of the signature **119**, then the verifying service **133** could determine that the signature **119** is valid for the message **116**. If multiple private keys **126** were used to generate portions o the signature **119**, then those portions of the signature **119** could be verified in parallel by the verifying service **133** or by multiple instances of the verifying service **133** executing on multiple verifying devices **106**.

Referring next to FIG. **2**, shown is a flowchart that provides one example of the operation of a portion of the signing service **113** to generate an asymmetric key pair that includes a public key **123** and a private key **126** for signing messages **116** in various implementations of the present disclosure. The flowchart of FIG. **2** provides merely an example of the many different types of functional arrangements that can be employed to implement the operation of the depicted portion of the signing service **113**. As an alternative, the flowchart of FIG. **2** can be viewed as depicting an example of elements of a method implemented within the network environment **100**.

Beginning with block **201**, the signing service **113** can create or initialize a remaining signature counter **129** for the private key **126**. The value of the remaining signature counter **129** could be set to a predefined value according to the security requirements of the private key **126**, with a lower value for the remaining signature counter **129** causing the public key **123** and the private key **126** to be replaces more frequently and a larger value for the remaining signature counter **129** causing the public key **123** and the private key **126** to be replaced less frequently. A value of one for the remaining signature counter **129** would cause the public key **123** and private key **126** to effectively act as a single use key pair. For simplicity in the following formulas, the initial value of the remaining signature counter **129** is referred to by the variable c when the initial value of the remaining signature counter **129** is used to calculate another value.

Proceeding to block **203**, the signing service **113** can select d number of random numbers, denoted as P_1 to P_d. Each of the random numbers P that are selected should have predefined minimum length Ly that provides sufficient entropy to avoid brute force attacks. In general, $L_p \geq \mathrm{Len}(f)$.

Then, at block **206**, the signing service **113** can generate the private key **126** by concatenating the random numbers $P_1| \ldots |P_d$ that were selected at block **203**. Although the strength of the private key **126** is related to L_p, the actual length of the private key is $d{*}L_p$.

Next, at blocks **209** and **213**, the signing service **113** can create the respective public key **123** for the private key **126** generated at block **206**. First, at block **209**, the signing service **113** apply the cryptographic hash function f a total of n times to each random number P used for the private key **126**, where $n=c{*}(2^m+1)$.

Moving on to block **213**, the signing service **113** can then concatenate the resulting hashes in the same order as the respective numbers used for the private key **126**. Accordingly, the public key **123** could be depicted as the concatenation of $f_n(P_1)| \ldots |f_n(P_d)$.

Referring to block **219**, the signing service **113** could save the public key **123**, private key **126**, the remaining signature counter **129** on the signing device **103** for later use. Then, at block **223**, the signing service **113** could, in some implementations, distributed the public key **123** and/or the remaining signature counter **129** to one or more verifying devices **106** using various communications channels and/or approaches.

Referring next to FIG. **3**, shown is a flowchart that provides one example of the operation of a portion of a single instance of the signing service **113** for signing a message **116**. The flowchart of FIG. **3** provides merely an example of the many different types of functional arrangements that can be employed to implement the operation of the depicted portion of the signing service **113**. As an alternative, the flowchart of FIG. **3** can be viewed as depicting an example of elements of a method implemented within the network environment **100**.

Beginning with block **303**, the signing service **113** can generate a hash of the message **116**. For example, the entire message **116**, denoted as M, could be supplied as an input to the cryptographic hash function f, such that the message hash $HM=f(M)$.

Then, at block **306**, the signing service **113** can split the message hash HM into d values HM_1 to HM_d, such that $HM=HM_1| \ldots |HM_d$. As previously mentioned in the discussion of FIG. **1**, the length $\mathrm{Len}(HM_i)=m$, and the product of $d{*}m=\mathrm{Len}(HM)$.

Next, at block **309**, the signing service **113** can calculate the signature for each part HM_1 to HM_d. First, the signing service **113** can calculate the value of the signature debt for each portion of the message hash HM to be signed, denoted as s_debt_i. The value of $s\_debt_i=HM_i+(rs-1)(2^m+1)$, where "rs" is the current value of the remaining signature counter **129**. Then the individual parts of d parts of the signature **119** can be calculated using the respective random numbers P_1 to P_d of the private key **126** using the formula $S_i=f_{s\_debt\,i}(P_i)$ for each i from 1 to d.

Moving on to block **313**, the signing service **113** can concatenate the signature portions S_i for each i from 1 to d, such that the signature **119**, denoted as S, is represented by the formula $S=S_1|S_2| \ldots |S_d$.

Proceeding to block **316**, the signing service **113** can decrement the remaining signature counter **129** after signature **119** is completed at block **313**.

Subsequently, at block **319**, the signing service **113** can append the remaining signature counter **129** to the signature **119** generated at block **313**. This could be done, for example, to communicate the remaining signature counter **129** to the verifying service **133** in order to synchronize the verifying service **133** with the signing service **113**. However, other approaches could also be used to communicate the updated remaining signature counter **129** to the verifying service **133**.

Referring next to FIG. **4**, shown is a flowchart that provides one example of the operation of a portion of the signing service **113** to generate signatures **119** for a message **116** in a multiparty signing scheme. The flowchart of FIG. **4** provides merely an example of the many different types of functional arrangements that can be employed to implement the operation of the depicted portion of the signing service **113**. As an alternative, the flowchart of FIG. **4** can be viewed as depicting an example of elements of a method implemented within the network environment **100**.

Beginning with block **403**, the signing service **113** can generate a hash of the message **116**. For example, the entire message **116**, denoted as M, could be supplied as an input to the cryptographic hash function f, such that the hashed message HM=f(M).

Then, at block **406**, the signing service **113** can split the message hash HM into d values HM_1 to HM_d, such that $HM=HM_1| \dots |HM_d$. As previously mentioned in the discussion of FIG. **1**, the length $Len(HM_i)=m$, and the product of d*m=Len(HM).

Next, at block **409**, the signing service **113** could select send a subset of the parts of the hashed message HM to additional signatory parties (e.g., another signing service **113** executing on another signing device **103**). In the example, depicted in the flowchart of FIG. **4**, the signing service **113** could send a first third of the portions of the message hash HM to a first signing party, a second third of the portions of the message hash HM to a second signing party, and a remaining third of the portions of the message hash HM to a third signing party. Each of the signing parties could, in parallel, sign their respective portions of the hashed message using the approach previously described at block **309** of FIG. **3**.

Moving on to block **413**, the signing service **113** can receive the signatures of the respective portions of the message hash HM, the respective portions of the public keys **123** to be used to verify the signatures of the respective portions of the message hash HM, and the respective remaining signature counters **129** of the signing parties.

Although the discussion of blocks **409** and **413** discusses the signing service **113** providing portions of the message hash HM to various parties to sign, it should be noted that the signing service **113** could act as a signatory for one portion of the message hash HM while other parties are provided with other portions of the message hash HM to sign. For example, at blocks **409** and **413**, the signing service **113** could sign the remaining third of the portions of the message hash HM, while providing the first and second thirds of the message hash HM to other signatories.

Subsequently, at block **416**, the signing service **113** can concatenate the signature portions S_i received at block **413** for each i from 1 to d, such that the signature **119**, denoted as S, is could be split such that $S=S_1|S_2| \dots |S_d$ and the message **116**, denoted as M, could be split such that $M=M_1|M_2| \dots |Md$.

Then, at block **419**, the signing service **113** can create a new multiparty public key **123** that can be used to verify the multiparty signature created at block **419**. For example, if the first party created S_1 and S_4 using $f_n(P_1)$ and $f_n(P_4)$ for the first party, the second party created S_2 and S_5 using $f_n(P_2)$ and $f_n(P_5)$ for the second party, and third party created S_3 and S_6 using $f_n(P_3)$ and $f_n(P_6)$ for the third party, then the new multiparty public key **123** would equal the concatenation of $f_n(P_1)|f_n(P_2)|f_n(P_3)|f_n(P_4)|f_n(P_5)|f_n(P_6)$ used by the first, second, and third parties. The new multiparty public key **123** could then be provided to the verifying service **133** through various communications channels to allow the verifying service **133** to verify the signature **119** of the message **116**. These communications channels can include or provide various levels of security, authentication, and access controls as appropriate for particular implementations.

Proceeding to block **423**, the signing service **113** can append the remaining signature counters **129** of the signing parties to the signature **119** generated at block **416**. This could be done, for example, to communicate the remaining signature counters **129** of the signing parties to the verifying service **133** in order to synchronize the verifying service **133** with the signing service **113**. However, other approaches could also be used to communicate the updated remaining signature counters **129** to the verifying service **133**.

FIG. **5** provides a graphical representation of the multiparty signing process described by the flowchart of FIG. **4**. As shown, a message **116** is divided in to d parts of m size, indicated as M_1-M_{12} in the illustrated example. The message **116** is split into multiple message subsets **503***a*, **503***b*, and **503***c* (collectively "message subsets **503**" or generically a "message subset **503**"). Message subset **503***a* contains the concatenation of $M_1|M_4|M_7|M_{10}$. Message subset **503***b* contains the concatenation of $M_2|M_5|M_8|M_{11}$. Message subset **503***c* contains the concatenation of $M_3|M_6|M_9|M_{12}$.

Using the process described in FIG. **4**, each message subset **503** could be signed using a separate private key **126** of a separate signing party to generate respective signature subsets **506***a*, **506***b*, and **506***c* (collectively "signature subsets **506**" or generically a "signature subset **506**"). As shown, signature subset **506***a* contains the concatenation of $S_1|S_4S_7|S_{10}$, as derived from $M_1|M_4|M_7|M_{10}$. Signature subset **506***b* contains the concatenation of $S_2|S_5|S_8|S_{11}$, as derived from $M_2|M_5|M_8|M_{11}$. Signature subset **506***c* contains the concatenation of $S_3|S_6|S_9|S_{12}$, as derived from $M_3|M_6|M_9|M_{12}$.

The signature subsets **506** could then be recombined to create a signature **119**. To allow for the multiparty signature **119** to be verified, the signature portions S_1-S_{12} could be reordered as illustrated to match the order of the respective message parts M_1-M_{12} in the illustrated message **116**.

Referring next to FIG. **6**, shown is a flowchart that provides one example of the operation of a portion of the verifying service **133** to verify a signature **119** of a message **116**. The flowchart of FIG. **6** provides merely an example of the many different types of functional arrangements that can be employed to implement the operation of the depicted portion of the verifying service **133**. As an alternative, the flowchart of FIG. **6** can be viewed as depicting an example of elements of a method implemented within the network environment **100**.

Beginning with block **601**, the verifying service **113** can generate a hash of the message **116**. For example, the entire message **116**, denoted as M, could be supplied as an input to the cryptographic hash function f, such that the hashed message HM=f(M).

Next, at block **603**, the verifying service **133** can split the message hash HM and the signature **119** into d parts of equal size m. Accordingly, the signature **119** could be split such that $S=S_1|S_2| \dots |S_d$ and the message hash HM could be split such that $HM=HM_1| \dots |HM_d$.

Then, at block **606**, the verifying service **133** can calculate the signature debt for each part HM_1 to HM_d of the message hash HM of the message **116**. First, the verifying service **133** can calculate the value of the signature debt for each portion of the message hash HM of the message that was signed, denoted as s_debt_i. The value of $s\_debt_i=HM_i+(rs-1)(2^m+1)$, where "rs" is the current value of the remaining signature counter **129**. As previously discussed the value "rs" may be have been appended to the signature **119** being verified in order to synchronize the remaining signature counter **129** between the signing service **113** and the verifying service **133**.

Moving to block **609**, the verifying service **133** can calculate the verification debt for each part of the message **116**. The verification debt can represent the number of iterations of the cryptographic hash function f to be performed on each portion of the signature **119** to reach the public key **123** for said portion. Mathematically, the verification debt is equal to the difference between the schema debt n and the signature debt, as represented by the equation $v\_debt_i = n - s\_debt_i$.

Next, at block **613**, the verifying service **133** can calculate a verification number V_i for each signature portion S_i. For any given signature portion S_i, $V_i = f_{vdebti}(S_i)$.

Proceeding to block **616**, the verifying service **133** can compare the verification numbers V_i to respective portions of the public key **123** Pub_i. If each $V_i = Pub_i$, then the verifying service **133** can determine that the signature **119** is valid. However, if one or more of the verification numbers fails to match respective components of the public key **123**, then the verifying service **133** can determine that the signature **119** is invalid.

Referring next to FIG. **7**, shown is a flowchart that provides one example of the operation of a portion of the verifying service **133** to verify a signature **119** of a message **116**, wherein the signature **119** was generated by multiple signing parties. The flowchart of FIG. **7** provides merely an example of the many different types of functional arrangements that can be employed to implement the operation of the depicted portion of the verifying service **133**. As an alternative, the flowchart of FIG. **7** can be viewed as depicting an example of elements of a method implemented within the network environment **100**.

Beginning with block **701**, the verifying service **113** can generate a hash of the message **116**. For example, the entire message **116**, denoted as M, could be supplied as an input to the cryptographic hash function f, such that the hashed message $HM = f(M)$.

Then, at block **703**, the verifying service **133** can split the message hash HM and the signature **119** into d parts of equal size m. Accordingly, the signature **119** could be split such that $S = S_1|S_2| \ldots |S_d$ and the message hash HM could be split such that $HM = HM_1| \ldots |HM_d$.

Proceeding to block **706**, the verifying service **133** can group individual portions HM_i of the message hash HM into subsets of the message hash HM. The verifying service **133** can also group individual portions S_i of the signature **119** into subsets of the signature **119**. In some implementations, specific portions of the message hash HM and the signature **119** can be selected based at least in part on the identity of a signing party. For example, if the message **116** were a transaction request associated with a credit card payment, then information provided by a merchant (e.g., transaction amount, merchant identifier, etc.) might need to be verified with a merchant's public key **123**, while information provided by a credit card issuer (e.g., account number, preauthorization code or token, etc.) might need to be verified with the issuer's public key **123**.

Moving on to block **709**, the verifying service **133** can calculate the signature debt for each portion HM_i of the message hash HM included in the subset. First, the verifying service **133** can calculate the value of the signature debt for each portion of the message hash HM of the message that was signed, denoted as s_debt_i. The value of $s\_debt_i = HM_i + (rs-1)(2^m+1)$, where "rs" is the current value of the remaining signature counter **129** associated with the public key **123** used to sign the portions HM_i of the message hash HM included in the subset. As previously discussed, the value "rs" may be have been appended to the signature **119** being verified in order to synchronize the remaining signature counter **129** between the signing service **113** and the verifying service **133**.

Next, at block **713**, the verifying service **133** can calculate the verification debt for each respective part M_i of the message **116** associated with the portions of the signature **119** included in each subset. The verification debt can represent the number of iterations of the cryptographic hash function f to be performed on each portion of the signature **119** to reach the public key **123** for said portion. Mathematically, the verification debt is equal to the difference between the schema debt n and the signature debt, as represented by the equation $v\_debt_i = n - s\_debt_i$.

Then, at block **716**, the verifying service **133** can calculate a verification number V_i for each signature portion S_i included in each subset. For any given signature portion S_i, $V_i = f_{vdebti}(S_i)$.

Proceeding to block **719**, the verifying service **133** can compare the verification numbers V_i to respective portions of the public key **123** Pub_i associated with each signature portion S_i included in each subset. If each $V_i = Pub_i$, then the verifying service **133** can determine that the signature **119** is valid. However, if one or more of the verification numbers fails to match respective components of the public key **123**, then the verifying service **133** can determine that the signature **119** is invalid.

It should be noted that where multiple subsets of the message hash HM and signature **119** have been identified, the subsets can be processed in parallel. For example, the operations on each subset described in blocks **709** to **719** can be performed in parallel to improve performance, either by the same instance of the verifying service **133** on the verifying device **106** (e.g., by using separate CPU cores, threads, or processes), or by multiple instances of the verifying service **133** operating on multiple, respective verifying devices **106**.

FIG. **8** provides a graphical representation of the process for verifying a multiparty signature **119** as described by the flowchart of FIG. **7**. To simplify the example, verification of the multiparty signature **119** illustrated in FIG. **5** is depicted in FIG. **8**.

For example, a signature **119** could be divided in to d parts of m size, indicated as S_1-S_{12} in the illustrated example. The signature **119** could be split into multiple signature subsets **803***a*, **803***b*, and **803***c* (collectively "signature subsets **803**" or generically a "signature subset **803**"). Signature subset **803***a* could contain the concatenation of $S_1|S_4S_7|S_{10}$, for the respective message subset **503***a* containing $M_1|M_4|M_7|M_{10}$. Signature subset **803***b* could contain the concatenation of $S_2|S_5S_8|S_{11}$ for the respective message subset **503***b* containing $M_2|M_5|M_8|M_{11}$. Signature subset **803***c* could contain the concatenation of $S_3|S_6|S_9|S_{12}$ for the respective message subset **503***c* containing $M_3|M_6|M_9|M_{12}$.

Verification subsets **806***a*, **806***b*, and **806***c* (collectively "verification subsets **803**" or generically a "verification subset **803**") could then be generated from the respective signatures subsets **803**. Each verification subset **806** could contain a respective verification number V_i for each respective signature portion S_i. For example, as illustrated, verification subset **806***a* could contain verification values $V_1|V_4|V_7|V_{10}$ for respective signature values $S_1|S_4|S_7|S_{10}$. Verification subset **806***b* could contain verification values $V_2|V_5|V_8|V_{11}$ for respective signature values $S_2|S_5|S_8|S_{11}$. Verification subset **806***c* could contain verification values $V_3|V_6|V_9|V_{12}$ for respective signature values $S_3|S_6|S_9|S_{12}$.

The verification values V_i could then be compared to the portions Pub_i of the public keys **123***a*, **123***b*, and **123***c* (collectively "public keys **123**" or generically a "public key **123**") of respective signing private keys **126**. For example, if each verification value V_i in verification subset **806***a* matched each portion Pub_i of public key **123***a*, then the first portion of the signature **119** would be valid. Likewise, if each verification value V_i in verification subset **806***b* matched each portion Pub_i of public key **123***b*, then the second portion of the signature **119** would be valid. Similarly, if each verification value V_i in verification subset **806***c* matched each portion Pub_i of public key **123***c*, then the third portion of the signature **119** would be valid. Moreover, if every verification value V_i matches its respective portion Pub_i of a public key **123**, then the entire signature **119** could be determined to be valid.

The various embodiments of the present disclosure have a number of use cases for improving the security of existing computing systems and/or computerized transactions. In the following paragraphs, a number of example use cases are described. However, these are not the only use cases which could take advantage of the various embodiments of the present disclosure.

As previously discussed, the various embodiments of the present disclosure allow for multiparty signing of messages or data without having to share private keys or to generate multiple signatures. For example, in a situation where multiple parties need to sign the same message, transaction, etc., all of the signing parties can independently sign the message, transaction, etc., without synchronization between the signing parties. Moreover, the signing parties can generate their signatures in parallel. Likewise, the signature of messages or data could be verified by multiple parties without synchronization between the verifying parties, and the verification of the respective signatures of the signing parties could also be performed in parallel.

In a first use case, multiple merchants could sign the same transaction. Although in this use case, all of the participating merchants would need to sign the same transaction, the individual merchants would not need to synchronize their signatures or signing keys with each other. The signed transaction could then be sent to a verifying party (e.g., a payment processor, credit card issuer, etc.), which could verify the separate signatures of the merchants.

In a second use case, a customer could participate in a signature process along with one or more merchants. For example, if a customer were paying by credit card, the point of sale (POS) device (e.g., credit card terminal, mobile phone, tablet, personal computer, etc.). In this use case, the POS device could sign portions of a transaction authorization request or similar message that were customer specific (e.g., account identifier, card verification value (CVV) or card security code (CSC), card expiration date, customer billing address, transaction amount, etc.), the merchant could sign portions that were merchant specific (e.g., the merchant identifier, transaction date and/or time, transaction amount, etc.), and the merchant could concatenate the portions of the signature generated by the customer with the portions of the signature generated by the merchant to create a signature for the transaction authorization request. The signature could be appended to the transaction authorization request by the merchant to create a signed transaction authorization request, which could be submitted to a verifying party (e.g., a payment processor, credit card issuer, etc.) through preexisting authorization channels or workflows. The verifying party could then determine that both the merchant and the customer signed the transaction authorization request using the previously described implementations of the present disclosure. The verifying party could then return a transaction authorization to the merchant in response. In some implementations, the verifying party could return a transaction approval to the customer using a separate communications channel.

In a third use case, which could be viewed as an extension of the second use case, a credit card issuer could participate in the signature generation process. In these use cases, a credit card issuer could provide a pre-authorization code or token to a merchant for a transaction. This could occur, for example, when a merchant has requested a hold for a requested amount prior to authorization or completion of the transaction. The credit card issuer could participate in a multiparty signature by providing the preauthorization code and/or preauthorization amount to the merchant and signing the preauthorization code or token provided to the merchant and/or the preauthorization amount. The merchant could include the preauthorization code or token and/or the preauthorization amount in the transaction authorization request. The portions of the transaction authorization request signed by the merchant and the user could be concatenated with the signature generated by the credit card issuer for the preauthorization code or token and/or preauthorization amount to create a multiparty signature for the entire transaction authorization request. The credit card issuer could then verify the signature of the transaction authorization request by verifying the separate portions of the signature created by the credit card issuer, merchant, and user. The verifying party could then return a transaction authorization to the merchant in response. In some implementations, the verifying party could return a transaction approval to the customer using a separate communications channel.

A number of software components previously discussed are stored in the memory of the respective computing devices and are executable by the processor of the respective computing devices. In this respect, the term "executable" means a program file that is in a form that can ultimately be run by the processor. Examples of executable programs can be a compiled program that can be translated into machine code in a format that can be loaded into a random access portion of the memory and run by the processor, source code that can be expressed in proper format such as object code that is capable of being loaded into a random access portion of the memory and executed by the processor, or source code that can be interpreted by another executable program to generate instructions in a random access portion of the memory to be executed by the processor. An executable program can be stored in any portion or component of the memory, including random access memory (RAM), read-only memory (ROM), hard drive, solid-state drive, Universal Serial Bus (USB) flash drive, memory card, optical disc such as compact disc (CD) or digital versatile disc (DVD), floppy disk, magnetic tape, or other memory components.

The memory includes both volatile and nonvolatile memory and data storage components. Volatile components are those that do not retain data values upon loss of power. Nonvolatile components are those that retain data upon a loss of power. Thus, the memory can include random access memory (RAM), read-only memory (ROM), hard disk drives, solid-state drives, USB flash drives, memory cards accessed via a memory card reader, floppy disks accessed via an associated floppy disk drive, optical discs accessed via an optical disc drive, magnetic tapes accessed via an appropriate tape drive, or other memory components, or a combination of any two or more of these memory components. In addition, the RAM can include static random access memory (SRAM), dynamic random access memory (DRAM), or magnetic random access memory (MRAM) and other such devices. The ROM can include a programmable read-only memory (PROM), an erasable programmable read-only memory (EPROM), an electrically erasable programmable read-only memory (EEPROM), or other like memory device.

Although the applications and systems described herein can be embodied in software or code executed by general purpose hardware as discussed above, as an alternative the same can also be embodied in dedicated hardware or a combination of software/general purpose hardware and dedicated hardware. If embodied in dedicated hardware, each can be implemented as a circuit or state machine that employs any one of or a combination of a number of technologies. These technologies can include, but are not limited to, discrete logic circuits having logic gates for implementing various logic functions upon an application of one or more data signals, application specific integrated circuits (ASICs) having appropriate logic gates, field-programmable gate arrays (FPGAs), or other components, etc. Such technologies are generally well known by those skilled in the art and, consequently, are not described in detail herein.

The flowcharts show the functionality and operation of an implementation of portions of the various embodiments of the present disclosure. If embodied in software, each block can represent a module, segment, or portion of code that includes program instructions to implement the specified logical function(s). The program instructions can be embodied in the form of source code that includes human-readable statements written in a programming language or machine code that includes numerical instructions recognizable by a suitable execution system such as a processor in a computer system. The machine code can be converted from the source code through various processes. For example, the machine code can be generated from the source code with a compiler prior to execution of the corresponding application. As another example, the machine code can be generated from the source code concurrently with execution with an interpreter. Other approaches can also be used. If embodied in hardware, each block can represent a circuit or a number of interconnected circuits to implement the specified logical function or functions.

Although the flowcharts show a specific order of execution, it is understood that the order of execution can differ from that which is depicted. For example, the order of execution of two or more blocks can be scrambled relative to the order shown. Also, two or more blocks shown in succession can be executed concurrently or with partial concurrence. Further, in some embodiments, one or more of the blocks shown in the flowcharts can be skipped or omitted. In addition, any number of counters, state variables, warning semaphores, or messages might be added to the logical flow described herein, for purposes of enhanced utility, accounting, performance measurement, or providing troubleshooting aids, etc. It is understood that all such variations are within the scope of the present disclosure.

Also, any logic or application described herein that includes software or code can be embodied in any non-transitory computer-readable medium for use by or in connection with an instruction execution system such as a processor in a computer system or other system. In this sense, the logic can include statements including instructions and declarations that can be fetched from the computer-readable medium and executed by the instruction execution system. In the context of the present disclosure, a "computer-readable medium" can be any medium that can contain, store, or maintain the logic or application described herein for use by or in connection with the instruction execution system. Moreover, a collection of distributed

computer-readable media located across a plurality of computing devices (e.g, storage area networks or distributed or clustered filesystems or databases) may also be collectively considered as a single non-transitory computer-readable medium.

The computer-readable medium can include any one of many physical media such as magnetic, optical, or semiconductor media. More specific examples of a suitable computer-readable medium would include, but are not limited to, magnetic tapes, magnetic floppy diskettes, magnetic hard drives, memory cards, solid-state drives, USB flash drives, or optical discs. Also, the computer-readable medium can be a random access memory (RAM) including static random access memory (SRAM) and dynamic random access memory (DRAM), or magnetic random access memory (MRAM). In addition, the computer-readable medium can be a read-only memory (ROM), a programmable read-only memory (PROM), an erasable programmable read-only memory (EPROM), an electrically erasable programmable read-only memory (EEPROM), or other type of memory device.

Further, any logic or application described herein can be implemented and structured in a variety of ways. For example, one or more applications described can be implemented as modules or components of a single application. Further, one or more applications described herein can be executed in shared or separate computing devices or a combination thereof. For example, a plurality of the applications described herein can execute in the same computing device, or in multiple computing devices in the same network environment **100**.

Disjunctive language such as the phrase "at least one of X, Y, or Z," unless specifically stated otherwise, is otherwise understood with the context as used in general to present that an item, term, etc., can be either X, Y, or Z, or any combination thereof (e.g., X; Y; Z; X or Y; X or Z; Y or Z; X, Y, or Z; etc.). Thus, such disjunctive language is not generally intended to, and should not, imply that certain embodiments require at least one of X, at least one of Y, or at least one of Z to each be present.

It should be emphasized that the above-described embodiments of the present disclosure are merely possible examples of implementations set forth for a clear understanding of the principles of the disclosure. Many variations and modifications can be made to the above-described embodiments without departing substantially from the spirit and principles of the disclosure. All such modifications and variations are intended to be included herein within the scope of this disclosure and protected by the following claims.

Therefore, the following is claimed:

1. A method, comprising:

hashing a message to generate a message hash;

splitting the message hash into a predefined number of parts, each of the predefined number of parts being of equal size;

splitting a signature of the message into the predefined number of parts, each of the predefined number of parts being of equal size;

calculating a value for a respective signature debt for each of the predefined number of parts of the message hash, wherein the value for the respective signature debt is based at least in part on a current value of a remaining signature counter;

calculating a value for a verification debt for each of the predefined number of parts of the message hash, the value for the verification debt for each of the predefined number of parts being based at least in part on the value of the respective signature debt for each of the predefined number of parts of the message hash;

calculating a verification value for each of the predefined number of parts of the signature, the verification value being the result of hashing a respective part of the signature by a number of times equal to the value of the verification debt for the respective part of the signature;

splitting a public key into the predefined number of parts, each of the predefined number of parts being of equal size; and

comparing the verification value for each of the predefined number of parts of the signature to a respective one of the predefined number of parts of the public key, wherein a mismatch between the verification value for one of the parts of the signature and a respective one of the parts of the public key indicates that the signature is invalid.

2. The method of claim **1**, further comprising synchronizing, with a signing computing device, a current value of the remaining signature counter.

3. The method of claim **1**, wherein the remaining signature counter is appended to the signature of the message.

4. The method of claim **2**, wherein calculating the signature debt for each of the predefined number of parts of the message hash is further based at least in part on the current value of the remaining signature counter.

5. The method of claim **1**, further comprising receiving the message or the signature from a signing computing device.

6. The method of claim **1**, wherein a length of the message hash is equal to a size of the predefined number of parts multiplied by the predefined number of parts.

7. The method of claim **1**, wherein the verification debt represents a difference between a schema debt and the respective signature debt for each of the predefined number of parts of the message hash.

8. A system, comprising:

a computing device comprising a processor and a memory; and

machine-readable instructions stored in the memory that, when executed by the processor, cause the computing device to at least:

hash a message to generate a message hash;

split the message hash into a predefined number of parts, each of the predefined number of parts being of equal size;

split a signature of the message into the predefined number of parts, each of the predefined number of parts being of equal size;

calculate a value for a respective signature debt for each of the predefined number of parts of the message hash, wherein the value for the respective signature debt is based at least in part on a current value of a remaining signature counter;

calculate a verification debt for each of the predefined number of parts of the message hash, the verification debt for each of the predefined number of parts being based at least in part on the value of the respective signature debt for each of the predefined number of parts of the message hash;

calculate a verification value for each of the predefined number of parts of the signature, the verification value being the result of hashing the respective part of the signature by a number of times equal to a value of the verification debt for the respective part of the signature;

split a public key into the predefined number of parts, each of the predefined number of parts being of equal size; and

compare the verification value for each of the predefined number of parts of the signature to a respective one of the predefined number of parts of the public key, wherein a mismatch between the verification value for one of the parts of the signature and a respective one of the parts of the public key indicates that the signature is invalid.

9. The system of claim **8**, wherein computing device is a first computing device and the machine-readable instructions further cause the first computing device to at least synchronize with a second computing device a current value of a remaining signature counter.

10. The system of claim **8**, wherein a remaining signature counter is appended to the signature of the message.

11. The system of claim **9**, wherein the machine-readable instructions that cause the first computing device to at least calculate the signature debt for each of the predefined number of parts of the message hash further cause the computing device to calculate the signature debt based at least in part on the current value of the remaining signature counter.

12. The system of claim **8**, wherein the computing device is a first computing device and the machine-readable instructions further cause the first computing device to at least receive, from a second computing device, the message or the signature of the message.

13. The system of claim **8**, wherein a length of the message hash is equal to a size of the predefined number of parts multiplied by the predefined number of parts.

14. The system of claim **8**, wherein the verification debt represents a difference between a schema debt and the respective signature debt for each of the predefined number of parts of the message hash.

15. A method, comprising:

generating a message hash of a message using a cryptographic hash function;

splitting the message hash into a predefined number of parts of equal size;

calculating a value of a signature debt for each of the predefined number of parts of the message hash;

calculating a respective signature part for each of at least a subset of the predefined number of parts of the message hash by applying the cryptographic hash function to a respective portion of a private key for a number of times equal to the value of the signature debt; and

concatenating the respective signature part for each of at least the subset of the predefined number of parts of the message hash to generate a signature for the message.

16. The method of claim **15**, further comprising:

decrementing a remaining signature counter associated with the private key; and

appending the remaining signature counter to the signature.

17. The method of claim **15**, wherein a length of the message hash, in bits, is a composite number.

18. The method of claim **15**, further comprising generating the private key by:

selecting a predefined number of random numbers, each of the predefined number of random numbers having a length that is greater than or equal to the length of the output of the cryptographic hash function; and

concatenating the predefined random numbers to create the private key.

19. The method of claim **18**, further comprising generating a public key for the private key by:

hashing each of the predefined number of random numbers for a number of times equal to a debt of a signature schema to generate a respective predefined number of hashes; and

concatenating the respective predefined number of hashes to create the public key.

20. The method of claim **19**, wherein each of the predefined number of random numbers having a length that is greater than or equal to the length of the output of the cryptographic hash function.

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