Control with Matlab

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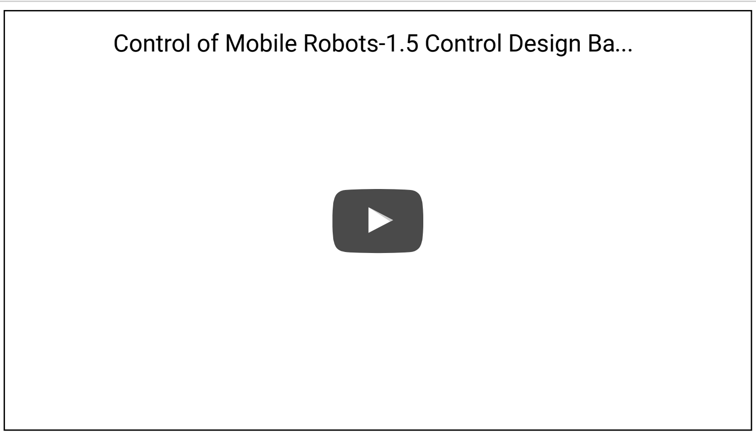
1. PID Control

- 1.5 Control Design Basics | Control of Mobile Robots

In [1]:

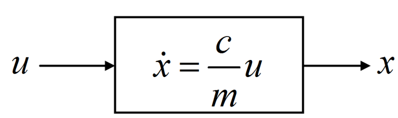
```
%%html
<iframe src="https://www.youtube.com/embed/DJuo9kLdr4M?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```



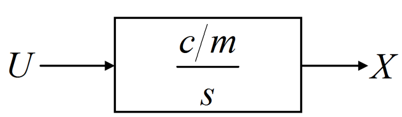
For the given car model

$$\dot{x} = \frac{c}{m}u \quad \text{for the velocity of a car, } x$$

In a bloack diagram



in a Laplace transform



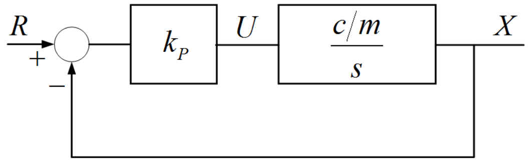
We want to achieve

$$x \to r \quad \text{as} \quad t \to \infty \; (e = r - x \to 0)$$

Attempt 2: P Regulator

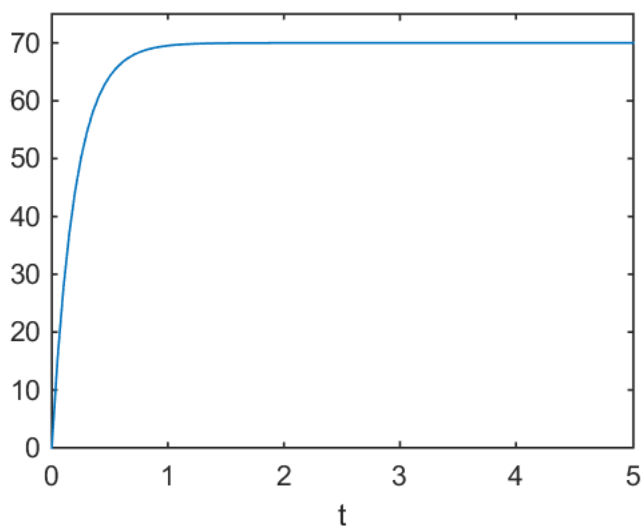
$$u = ke$$

- small error yeilds small control signals
- nice and smooth
- so-called proportional regulation (P regulator)



In [2]:

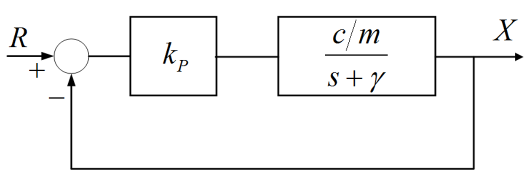
```
c = 1;
m = 1;

G = tf(c/m,[1 0]);

k = 5;
C = k;

Gcl = feedback(C*G,1,-1);

t = linspace(0,5,100);
r = 70*ones(size(t));   % reference

x0 = 0;
[y,tout] = lsim(Gcl,r,t,x0);
plot(tout,y), xlabel('t'), ylim([0,75])
```



Out[2]:

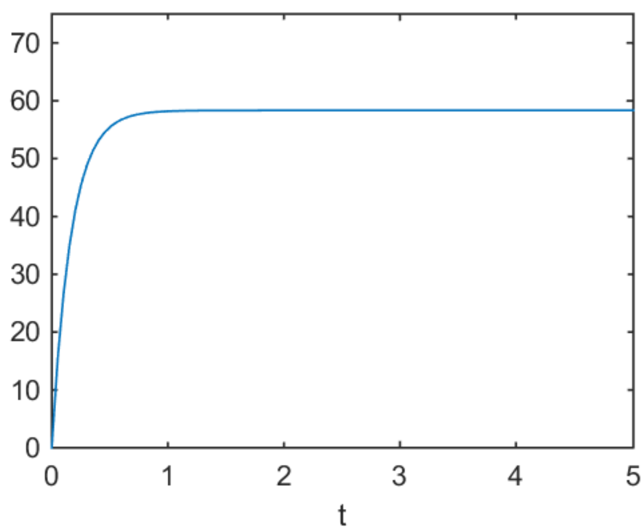
What if the true system is:

$$\dot{x} = \frac{c}{m}u - \gamma x$$



In [3]:

```
gamma = 1;
Gtr = tf(c/m,[1 gamma]);
C = k;

Gcl = feedback(C*Gtr,1,-1);

x0 = 0;
t = linspace(0,5,100);
r = 70*ones(size(t));
[y,tout] = lsim(Gcl,r,t,x0);
plot(tout,y), xlabel('t'), ylim([0,75])
```



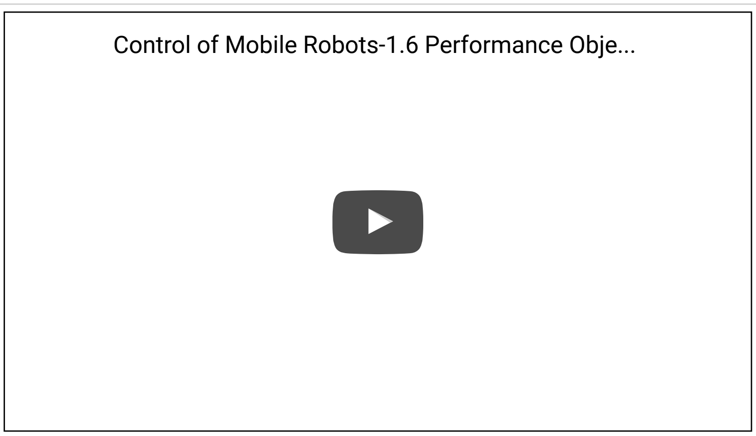
Out[3]:

Attempt 3

- 1.6 Performance Objectives | Control of Mobile Robots
- 1.7 PID Control | Control of Mobile Robots

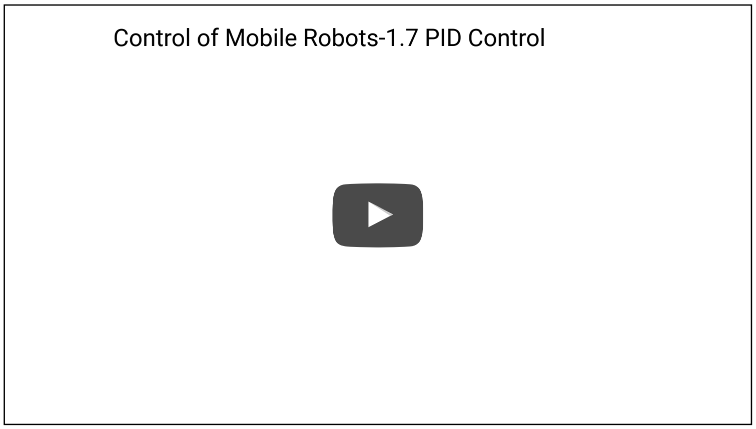
In [4]:

```
%%html
<iframe src="https://www.youtube.com/embed/cQhqx65kLfM?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```



In [5]:

```
%%html
<iframe src="https://www.youtube.com/embed/Mk1ygHj4zxw?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```



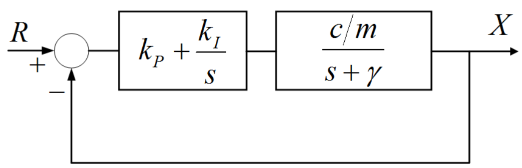
$$u = ke + \gamma\frac{m}{c}x$$

However, all of sudden we have to know all these physical parameters that we typically do not know - not robust !!!

Attempt 4: PI Regulators

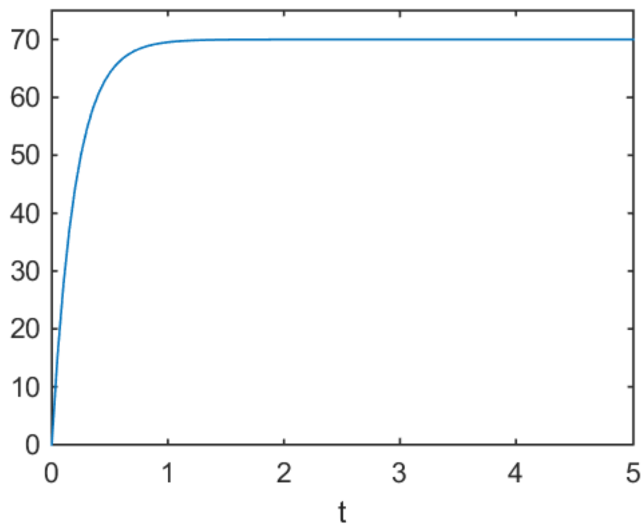
- Stability (BIBO)
- Tracking
- Robustness

$$u(t) = k_P\, e(t) + k_I \int_0^t e(\tau)d\tau$$



In [6]:

```
Gtr = tf(c/m,[1 gamma]);
kP = 5;
kI = 5;
C = tf([kP kI],[1 0]);

Gcl = feedback(C*Gtr,1,-1);

x0 = 0;
t = linspace(0,5,100);
r = 70*ones(size(t));
[y,tout] = lsim(Gcl,r,t,x0);
plot(tout,y), xlabel('t'), ylim([0,75])
```



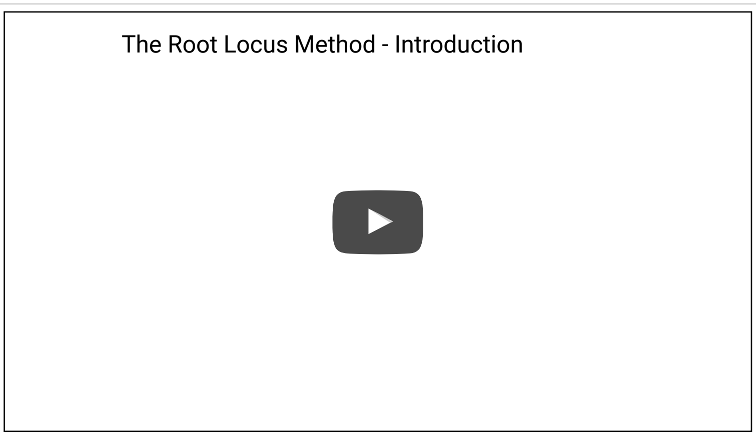
Out[6]:

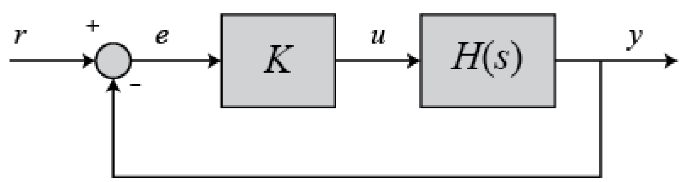
2. Root Locus

- The Root Locus Method by Brian Douglas
- from umich control (http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlRootLocus)

In [7]:

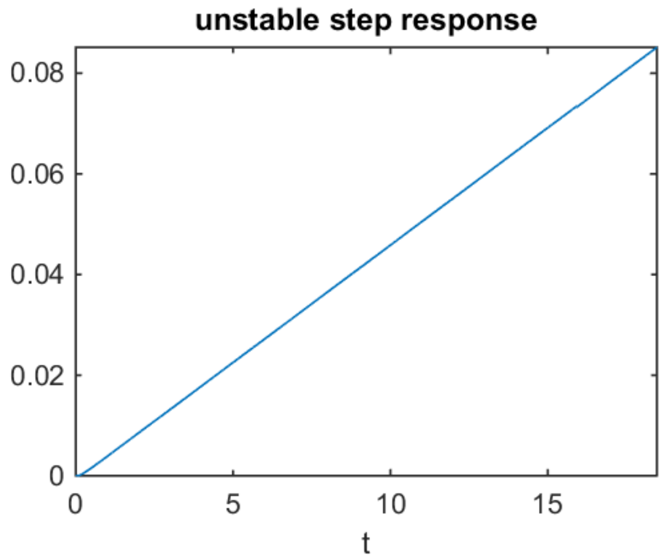
```
%%html
<iframe src="https://www.youtube.com/embed/CRvVDoQJjYI?list=PLUMWjy5jgHK1NC52DXXrriwihVrYZKqjk"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```





In [8]:

```
s = tf('s');
sys = (s + 7)/(s*(s + 5)*(s + 15)*(s + 20));
[y,tout] = step(sys); % unstable
plot(tout,y), axis tight, xlabel('t'), title('unstable step response')
```



Out[8]:

In [9]:

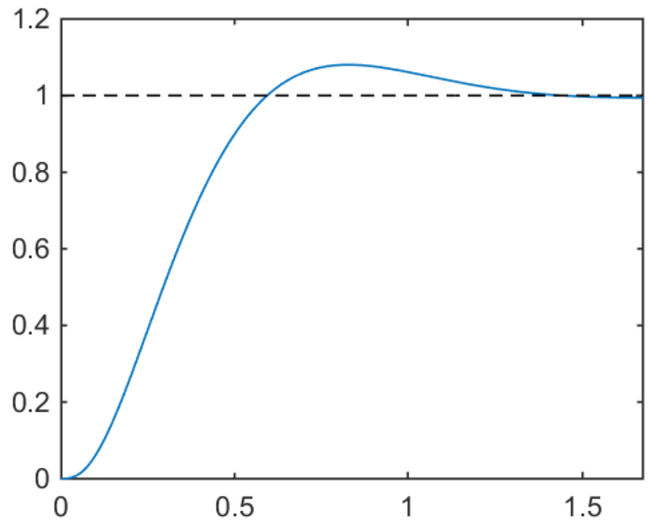
```
pole(sys)
G = sys;
```

Out[9]:

```
ans =

         0
  -20.0000
  -15.0000
   -5.0000
```

In [10]:

```
%k = 10;
k = 800

Gcl = feedback(k*G,1,-1)
pole(Gcl)
[y,tout] = step(Gcl);
plot(tout,y,tout,ones(size(tout)),'k--'), axis tight, ylim([0,1.2])
```

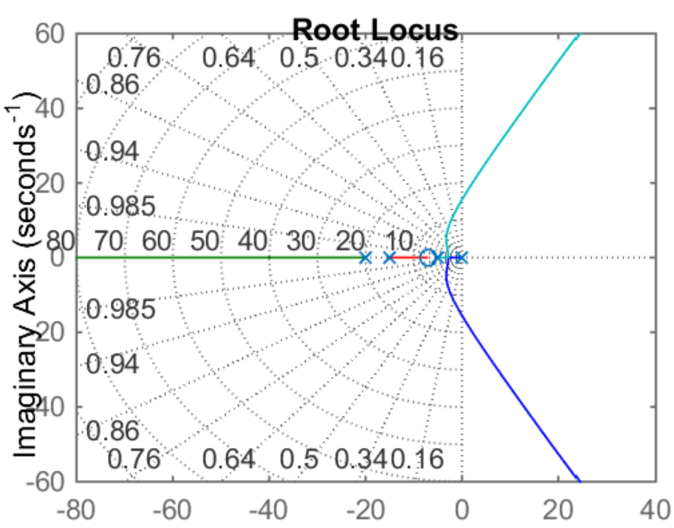


Out[10]:

```
k =

   800


Gcl =

                800 s + 5600
  -------------------------------------
  s^4 + 40 s^3 + 475 s^2 + 2300 s + 5600

Continuous-time transfer function.


ans =

 -23.5466 + 0.0000i
 -10.1226 + 0.0000i
  -3.1654 + 3.6708i
  -3.1654 - 3.6708i
```

In [11]:

```
rlocus(G), grid
```

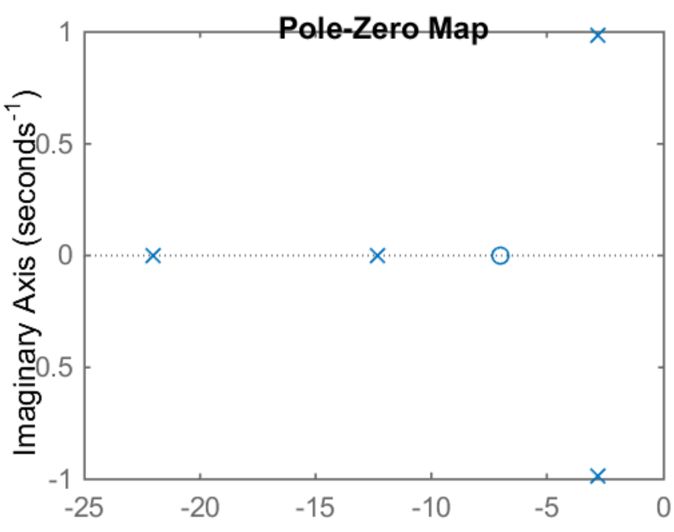


Out[11]:

In [12]:
```
help rlocus
```

Out[12]:
```
RLOCUS  Evans root locus.

    RLOCUS(SYS) computes and plots the root locus of the single-input,
    single-output LTI model SYS. The root locus plot is used to analyze
    the negative feedback loop

                    +-----+
        ---->O----->| SYS |----+---->
            -|      +-----+    |
             |                 |
             |       +---+     |
             +-------| K |<----+
                     +---+

    and shows the trajectories of the closed-loop poles when the feedback
    gain K varies from 0 to Inf.  RLOCUS automatically generates a set of
    positive gain values that produce a smooth plot.

    RLOCUS(SYS,K) uses a user-specified vector K of gain values.

    RLOCUS(SYS1,SYS2,...) draws the root loci of several models SYS1,SYS2,...
    on a single plot. You can specify a color, line style, and marker for
    each model, for example:
       rlocus(sys1,'r',sys2,'y:',sys3,'gx').

    [R,K] = RLOCUS(SYS) or R = RLOCUS(SYS,K) returns the matrix R of
    complex root locations for the gains K.  R has LENGTH(K) columns
    and its j-th column lists the closed-loop roots for the gain K(j).

    See RLOCUSPLOT for additional graphical options for root locus plots.

    See also RLOCUSPLOT, SISOTOOL, POLE, ISSISO, LTI.

    Overloaded methods:
       DynamicSystem/rlocus
       resppack.ltisource/rlocus

    Reference page in Help browser
       doc rlocus
```

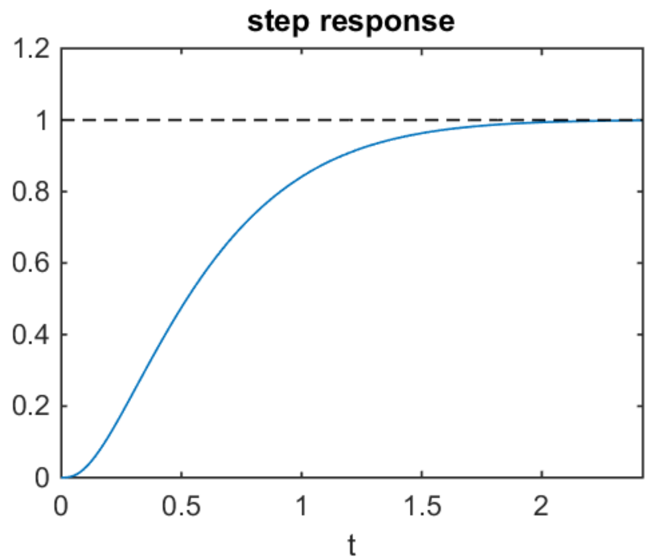
In [13]:
```
K = 350;
sys_cl = feedback(K*sys,1,-1) % negative feedback

pzmap(sys_cl)
```



Out[13]:
```
sys_cl =

                350 s + 2450
  ------------------------------------
  s^4 + 40 s^3 + 475 s^2 + 1850 s + 2450

Continuous-time transfer function.
```

In [14]:

```
[y,tout] = step(sys_cl);
plot(tout,y,tout,ones(size(tout)),'k--'),
axis tight, ylim([0,1.2]), xlabel('t'), title('step response')
```



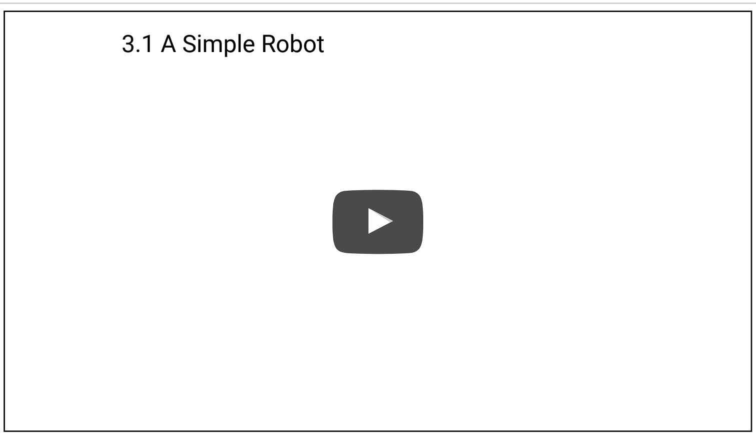
Out[14]:

3. State Space Representation

- from 3.1 A Simple Robot | Control of Mobile Robots
- from 3.2 State Space Models | Control of Mobile Robots

In [15]:

```
%%html
<iframe src="https://www.youtube.com/embed/kQNUpNh6nBc?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```



In [16]:

```
%%html
<iframe src="https://www.youtube.com/embed/W6AUOyj5bFA?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```

Controlling a point mass

Given a point mass on a line whose acceleration is directly controlled:

$$\ddot{p} = u$$

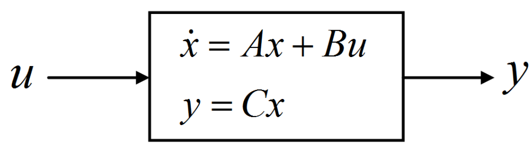
want to write this on a compact/general form

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

on a state space form

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = p = x_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



3.1. The car model

If we care about/can measure the velocity:

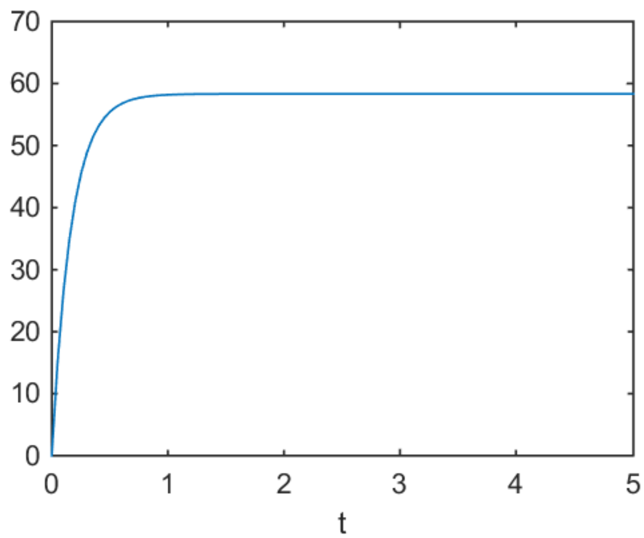
$$A = -\gamma, \qquad B = \frac{c}{m}, \qquad C = 1$$

If we care about/can measure the position we have the same general equation with different matrices:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\gamma \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \frac{c}{m} \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

In [17]:

```
% system in ss
c = 1;
m = 1;
gamma = 1;

A = -gamma;
B = c/m;
C = 1;
D = 0;

Gss = ss(A,B,C,D);

% P controller
k = 5;
C = k;

% close loop
Gcl = feedback(C*Gss,1,-1);

x0 = 0;
t = linspace(0,5,100);
r = 70*ones(size(t));
[y,tout] = lsim(Gcl,r,t,x0);
plot(tout,y), xlabel('t'), ylim([0,70])
```



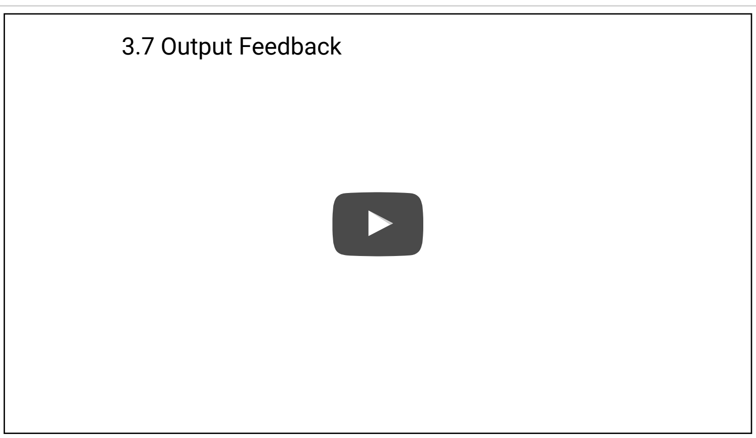
Out[17]:

3.2. Back to the World's Simplest Robot (Output Feedback)

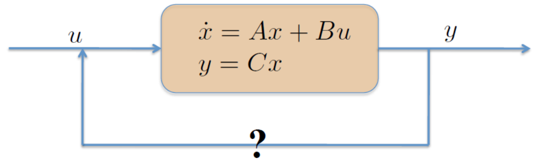
- from 3.7 Output Feedback | Control of Mobile Robots

In [18]:

```
%%html
<iframe src="https://www.youtube.com/embed/HmqOnsRH73w?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```

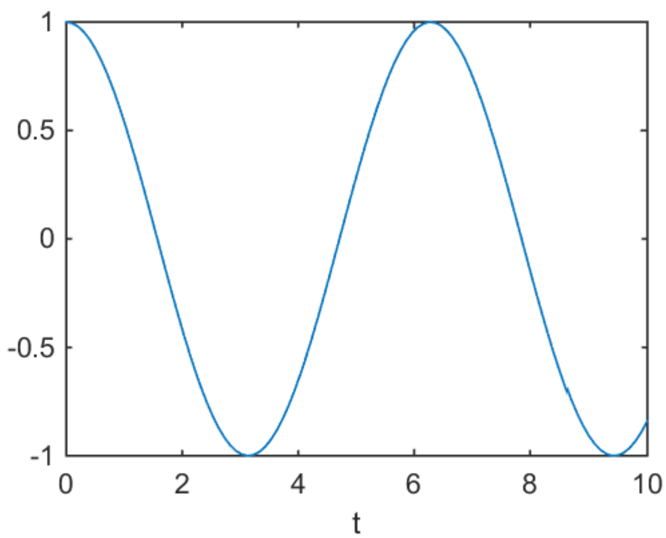


$$u = -y$$



In [19]:

```
% to move towards the origin
% u = -y

A = [0 1;0 0];
B = [0 1]';
C = [1 0];
D = 0;
G = ss(A,B,C,D);

K = 1;
Gcl = feedback(G,K,-1);
x0 = [1 0]';
t = linspace(0,10,100);
r = zeros(size(t));
[y,tout] = lsim(Gcl,r,t,x0);
plot(tout,y), xlabel('t')
```



Out[19]:

In [20]:

```
eig(Gcl)
```

Out[20]:

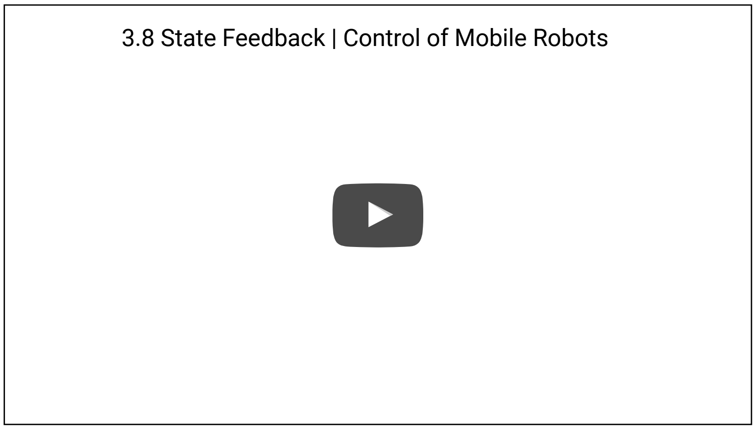
```
ans =

   0.0000 + 1.0000i
   0.0000 - 1.0000i
```

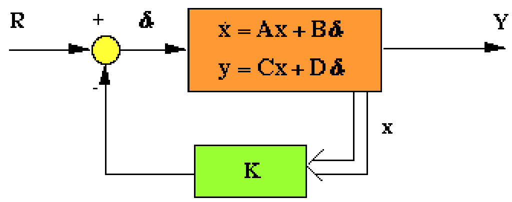
4. State Feedback

- from 3.8 State Feedback | Control of Mobile Robots

In [21]:

```
%%html
<iframe src="https://www.youtube.com/embed/yl5IiJOYQps?list=PLciAw3uhNCiD3dkLTPJgHoMnsu8XgCt1m"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```

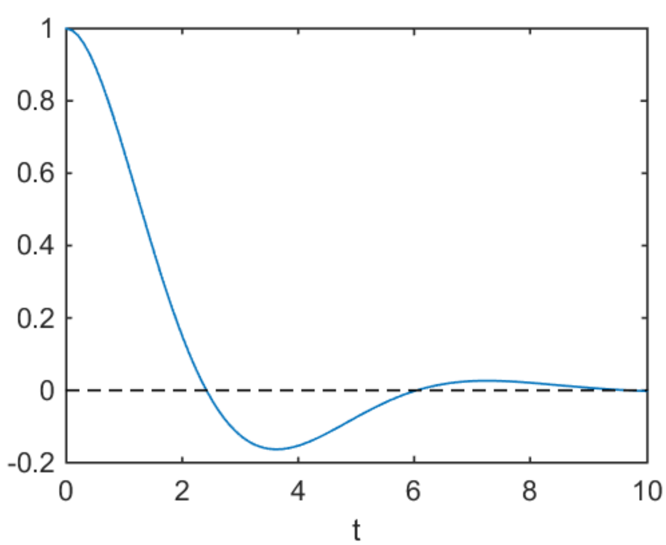




To move forwards origin, $R = 0$

In [22]:

```
A = [0 1;0 0];
B = [0 1]';
C = [1 0];
D = 0;
G = ss(A,B,C,D);

k1 = 1;
k2 = 1;
K = [k1 k2];
Gcl = ss(A-B*K,B,C,D);

x0 = [1 0]';
t = linspace(0,10,100);
r = zeros(size(t));
[y,tout] = lsim(Gcl,r,t,x0);
plot(tout,y,tout,zeros(size(tout)),'k--'), xlabel('t')
```



Out[22]:

In [23]:
```
eig(Gcl)
```
Out[23]:
```
ans =

  -0.5000 + 0.8660i
  -0.5000 - 0.8660i
```

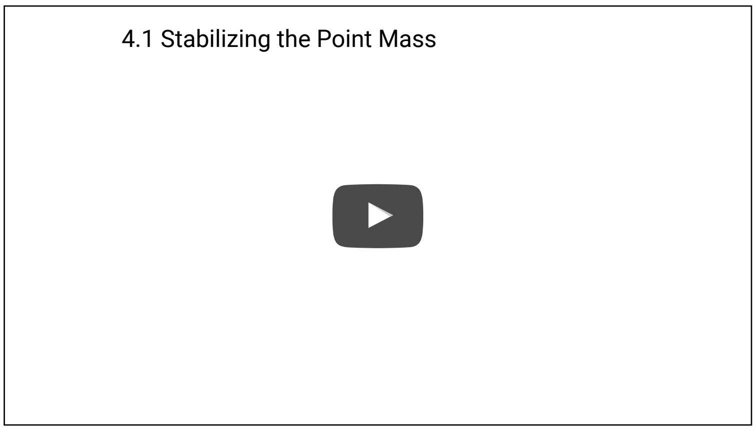
Eigenvalues Matter

- It is clear that some eigenvalues are better than others. Some cause oscillations, some make the system respond too slowly, and so forth ...
- In the next module we will see how to select eigenvalues and how to pick control laws based on the output rather than the state.

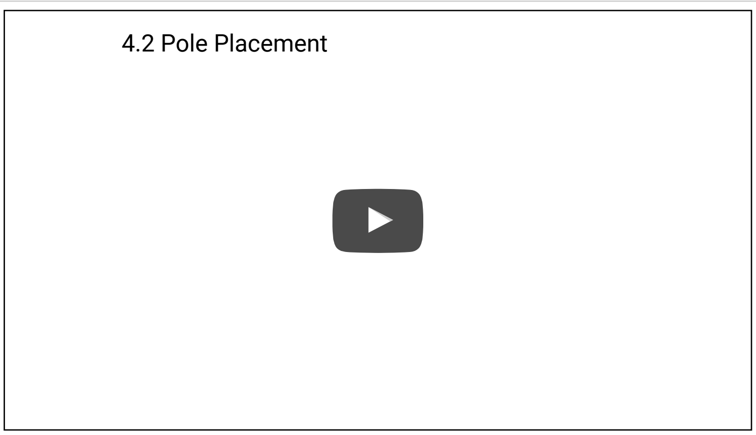
4.1. Pole Placement

- from 4.1 Stabilizing the Point Mass | Control of Mobile Robots
- from 4.2 Pole Placement | Control of Mobile Robots

In [24]:
```
%%html
<iframe src="https://www.youtube.com/embed/S4WZTmEnbrY?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```



In [25]:
```
%%html
<iframe src="https://www.youtube.com/embed/5tWhOK8Klo0?list=PLp8ijpvp8iCvFDYdcXqqYU5Ibl_aOqwjr"
width="560" height="315" frameborder="0" allowfullscreen></iframe>
```



back to the point-mass, again

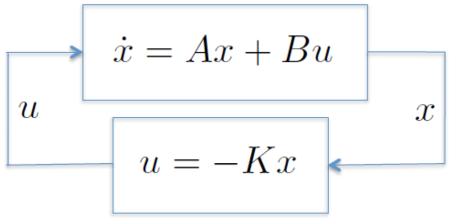
$$u = -Kx \rightarrow \dot{x} = (A - BK)x$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 1 \\ -k_1 & -k_2 \end{vmatrix} = \lambda^2 + \lambda k_2 + k_1$$

Desired Eigenvalues: let's pick both eigenvalues at -1

$$(\lambda + 1)(\lambda + 1) = \lambda^2 + 2\lambda + 1$$
$$k_1 = 2, k_2 = 1$$



Pick the control gains such that the eigenvalues (poles) of the closed loop system match the desired eigenvalues

Questions

- Is this always possible? (No)
- How should we pick the eigenvalues? (Mix of art and science)

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$A - BK = \begin{bmatrix} 2 - k_1 & -k_2 \\ 1 - k_1 & 1 - k_2 \end{bmatrix}$$
$$\varphi = \lambda^2 + \lambda(-3 + k_1 + k_2) + 2 - k_1 - k_2$$

Suppose

$$\varphi = (\lambda + 1)^2 = \lambda^2 + \lambda(-3 + k_1 + k_2) + 2 - k_1 - k_2$$

Let's pick both eigenvalues at -1

$$-3 + k_1 + k_2 = 2 \quad \text{and} \quad 2 - k_1 - k_2 = 1$$

$\rightarrow$ no k_1 and k_2 exist

What's at play here is a lack of `controllability`, i.e., the effect of the input is not sufficiently rich to influence the system enough

In [26]:

```
A = [2 0;
     1 -1];

B = [1 1]';
C = [1 0];

P = [-0.5 + 1j, -0.5 - 1j];
%P = [-0.1 + 1j, -0.1 - 1j];
%P = [-0.5, -1];
%P = [-5, -4];

K = place(A,B,P)
```

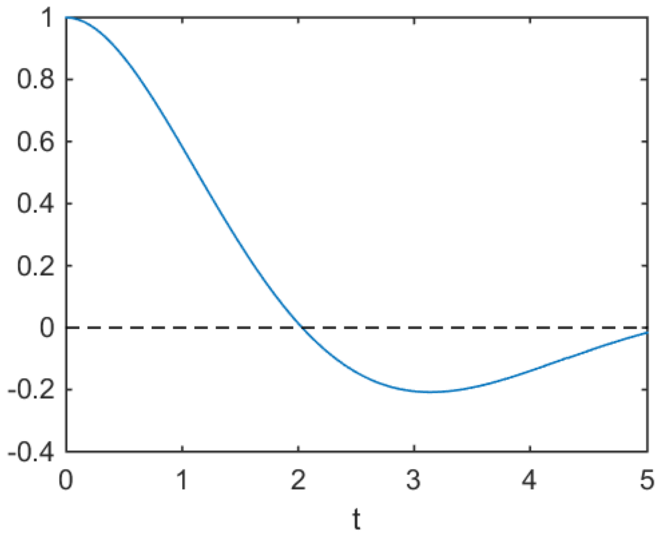
Out[26]:

```
K =

    2.6250   -0.6250
```

$$\dot{x} = Ax + Bu = Ax - BKx = (A - BK)x$$

In [27]:

```
x0 = [1 1]';
Gcl = ss(A-B*K,B,C, 0);

t = linspace(0,5,100);
u = zeros(size(t));

[y, tout] = lsim(Gcl,u,t,x0);
plot(tout,y,tout,zeros(size(tout)),'k--'), xlabel('t')
```



Out[27]:

4.2. Controllability

- When can we place the eigenvalues using state feedback?
- When is B matrix (the actuator configuration) rich enough so that we can make the system do whatever we want it to do?
- The answer revolves around the concept of `controllability`

Given a discrete-time system

$$x_{k+1} = Ax_k + Bu_k$$

We would like to drive this system in n steps to a particular target state x^*

$$\begin{aligned}
x_1 &= Ax_0 + Bu_0 = Bu_0 \\
x_2 &= Ax_1 + Bu_1 = ABu_0 + Bu_1 \\
x_3 &= Ax_2 + Bu_2 = A^2Bu_0 + ABu_1 + Bu_2 \\
&\vdots \\
x_n &= A^{n-1}Bu_0 + \cdots + Bu_{n-1}
\end{aligned}$$

We want to solve

$$x^* = \left[B \quad AB \quad \cdots \quad A^{n-1}B \right] \begin{bmatrix} u_{n-1} \\ \vdots \\ u_1 \\ u_0 \end{bmatrix}$$

This is possible for any target state if and only if

$$\text{rank}\left(\left[B \quad AB \quad \cdots \quad A^{n-1}B \right]\right) = n$$

Example 1

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

In [28]:

```
A = [2 0;
     1 1];
B = [1 1]';

G = ctrb(A,B)
rank(G)
```

Out[28]:

```
G =

     1     2
     1     2


ans =

     1
```

Example 2

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

In [29]:

```
A = [0 1;
     0 0];
B = [0 1]';

G = ctrb(A,B)
rank(G)
```

Out[29]:

```
G =

     0     1
     1     0


ans =

     2
```

In [30]:

```
%%javascript
$.getScript('https://kmahelona.github.io/ipython_notebook_goodies/ipython_notebook_toc.js')
```