MAT4220   Problem Set 1 (due September 12, 2019)

1. Solve the equation $xu_x - yu_y = u$ with initial condition $u(x, 1) = f(x)$.

2. Prove that the equation $xu_x + yu_y = -u$ in the domain $x^2 + y^2 \leq a^2$ has only one solution $u \equiv 0$.

3. Prove that the equation $a(x, y)u_x + b(x, y)u_y = -u$ in the domain $x^2 + y^2 \leq a^2$ has only one solution $u \equiv 0$ if $xa(x, y) + yb(x, y) > 0$ on the boundary $x^2 + y^2 = a^2$.

(Hint: Show that, in $x^2 + y^2 \leq a^2$, we have $max\, u \leq 0$ and $min\, u \geq 0$.)