POLITECNICO DI MILANO

Multibody System Dynamics: MBDyn Hydraulics Modeling

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Outline

MBDyn

- Introduction
- Modeling Scales
- Multibody/Multiphysics Dynamics:
    - MBDyn Software
    - Modeling Approach
- Hydraulic Library Overview
- Examples

Introduction

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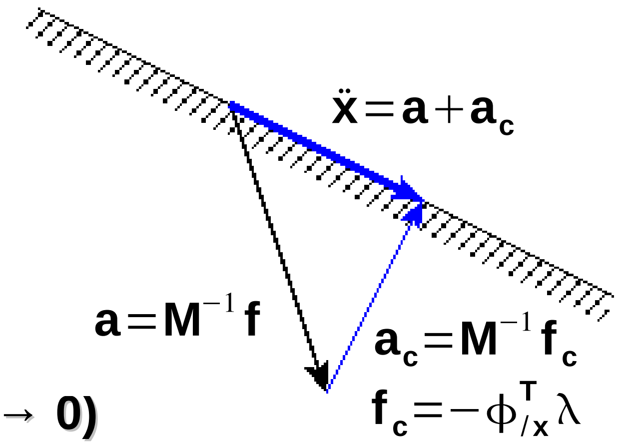
- **Multibody Dynamics: unconstrained mechanical systems...**

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{t})$$

- **… plus kinematic constraints: constrained mechanical systems**

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} + \phi_{/\mathbf{x}}^{\mathbf{T}}\lambda = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{t})$$
$$\phi(\mathbf{x}, \mathbf{t}) = 0$$



- **System of differential-algebraic equations (DAE)**

- **Infinitely “fast” dynamics (time scale → 0)**

- **Requires unconditionally stable integration, algorithmic dissipation → implicit A/L-stable schemes**

Introduction

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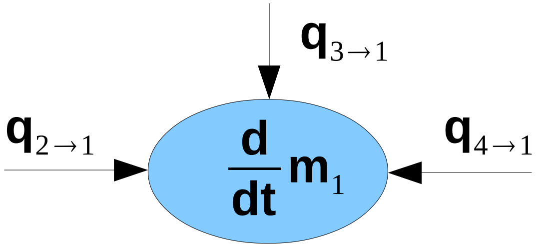
- **Hydraulic system dynamics:**
  - **approximate or neglect local dynamics**
  - **spatial resolution: 0D & 1D**
  - **circuit theory: node pressures, branch flows**

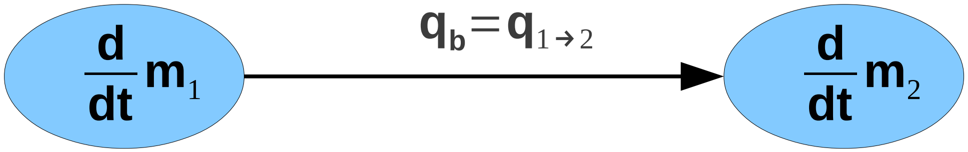
- **Flow balance at nodes**

$$\sum q_b = \frac{d}{dt} m_n$$



- **Constitutive properties of branches (e.g. pressure loss)**

$$\psi\left(q_b, \dot{q}_b, p_{n1}, \dot{p}_{n1}, p_{n2}, \dot{p}_{n2}, t\right) = 0$$



Introduction

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- **Constitutive properties of fluid: linearization about reference condition**

$$\frac{d}{dt} m = \frac{d}{dt} \left(\rho V \right) = \dot{\rho} V + \rho \dot{V}$$

$$\dot{\rho} = \frac{\partial \rho}{\partial p} \dot{p} + \frac{\partial \rho}{\partial T} \dot{T} = \rho_0 \left(\frac{1}{\beta} \dot{p} - \alpha \dot{T} \right)$$

volume change (e.g. actuator)

compressibility can be neglected if:
- **m small**
- **bulk modulus large**
- **pressure rate small**

thermal expansion can be neglected if:
- **m small**
- **coefficient small**
- **temperature rate small**

$$\frac{d}{dt} m = \frac{m}{\beta} \dot{p} - \cancel{m \alpha \dot{T}} + \rho \dot{V}$$

assume temperature rate small compared to time scale of hydro-mechanical processes

Modeling Scales

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- Multiphysics problem: interaction between different domains (e.g. mechanical and hydraulic)

- Interaction described in terms of:
  - Frequency, bandwidth (how rapid phenomena are)
  - Power (how much work is transferred between domains)

- Determine how interaction can be simplified:
  - Truncation
  - Steady approximation
  - Quasi-steady approximation
  - Complete dynamics coupling

Modeling Scales

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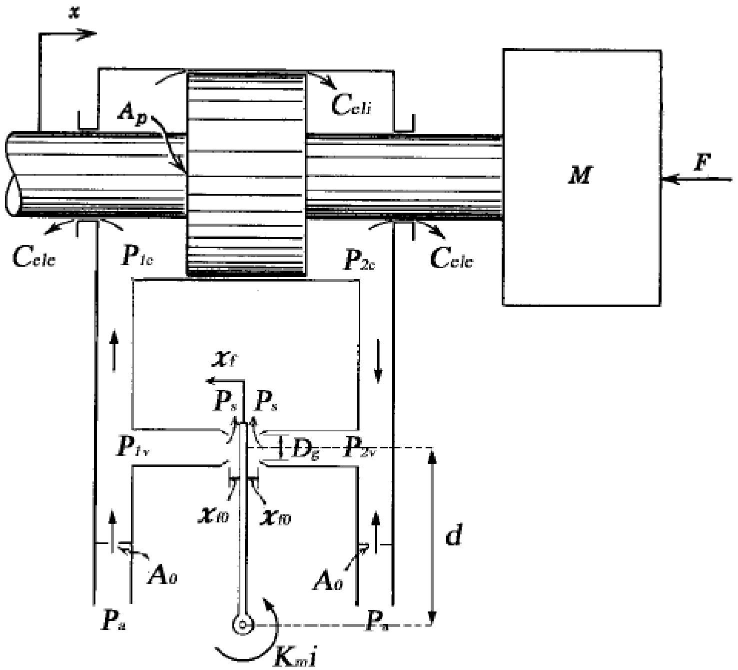
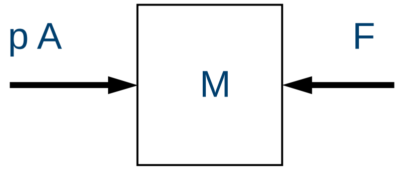
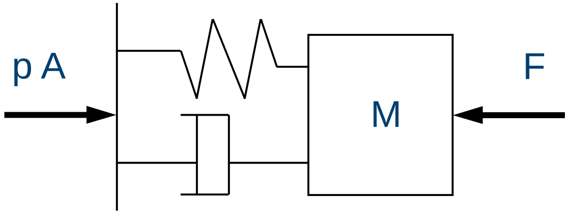
- **Example: actuator**





truncated model: commanded force independed from dynamics



(quasi-)static model: commanded force depends on dynamics approx. as mass-spring-damper

Approximations

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- **Coupled dynamic problem: state-space representation (e.g. linear)**

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}\\ \mathbf{y} &= \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u}\end{aligned}$$

- **States partitioned based on frequency separation ("slow" vs. "fast"):**

slow

fast

$$\begin{Bmatrix}\dot{\mathbf{x}}_s\\ \dot{\mathbf{x}}_f\end{Bmatrix} = \begin{bmatrix}\mathbf{A}_{ss} & \mathbf{A}_{sf}\\ \mathbf{A}_{fs} & \mathbf{A}_{ff}\end{bmatrix}\begin{Bmatrix}\mathbf{x}_s\\ \mathbf{x}_f\end{Bmatrix} + \begin{bmatrix}\mathbf{B}_s\\ \mathbf{B}_f\end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix}\mathbf{C}_s & \mathbf{C}_f\end{bmatrix}\begin{Bmatrix}\mathbf{x}_s\\ \mathbf{x}_f\end{Bmatrix} + \mathbf{D}\,\mathbf{u}$$

- **Approximations consist in reducing the system to the "slow" dynamics while preserving information about the "fast" dynamics**

Approximations

- **Truncation: only consider "slow" states**

slow

$$\begin{Bmatrix} \dot{\mathbf{x}}_s \\ 0 \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{A}_{sf} \\ \mathbf{A}_{fs} & \mathbf{A}_{ff} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_s \\ 0 \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_s \\ \mathbf{B}_f \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C}_s & \mathbf{C}_f \end{bmatrix} \begin{Bmatrix} \mathbf{x}_s \\ 0 \end{Bmatrix} + \mathbf{D}\mathbf{u}$$

- **Reduced system:**

$$\dot{\mathbf{x}}_s = \mathbf{A}_{ss}\mathbf{x}_s + \mathbf{B}_s\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}_s\mathbf{x}_s + \mathbf{D}\mathbf{u}$$

Approximations

- **Steady approximation: statically approximate “fast” states**

$$\begin{Bmatrix} \dot{\mathbf{x}}_{\mathbf{s}} \\ \cancel{0} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_{\mathbf{ss}} & \mathbf{A}_{\mathbf{sf}} \\ \mathbf{A}_{\mathbf{fs}} & \mathbf{A}_{\mathbf{ff}} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{\mathbf{s}} \\ \mathbf{x}_{\mathbf{f}} \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_{\mathbf{s}} \\ \mathbf{B}_{\mathbf{f}} \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C}_{\mathbf{s}} & \mathbf{C}_{\mathbf{f}} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_{\mathbf{s}} \\ \mathbf{x}_{\mathbf{f}} \end{Bmatrix} + \mathbf{D}\mathbf{u}$$

- **Reduced system:**

$$\dot{\mathbf{x}}_{\mathbf{s}} = \left(\mathbf{A}_{\mathbf{ss}} - \mathbf{A}_{\mathbf{sf}} \mathbf{A}_{\mathbf{ff}}^{-1} \mathbf{A}_{\mathbf{fs}} \right) \mathbf{x}_{\mathbf{s}} + \left(\mathbf{B}_{\mathbf{s}} - \mathbf{A}_{\mathbf{sf}} \mathbf{A}_{\mathbf{ff}}^{-1} \mathbf{B}_{\mathbf{f}} \right) \mathbf{u}$$

$$\mathbf{x}_{\mathbf{f}} = -\mathbf{A}_{\mathbf{ff}}^{-1} \mathbf{A}_{\mathbf{fs}} \mathbf{x}_{\mathbf{s}} - \mathbf{A}_{\mathbf{ff}}^{-1} \mathbf{B}_{\mathbf{f}} \mathbf{u}$$

$$\mathbf{y} = \left(\mathbf{C}_{\mathbf{s}} - \mathbf{C}_{\mathbf{f}} \mathbf{A}_{\mathbf{ff}}^{-1} \mathbf{A}_{\mathbf{fs}} \right) \mathbf{x}_{\mathbf{s}} + \left(\mathbf{D} - \mathbf{C}_{\mathbf{f}} \mathbf{A}_{\mathbf{ff}}^{-1} \mathbf{B}_{\mathbf{f}} \right) \mathbf{u}$$

- **Note: the original system becomes differential-algebraic (DAE) of index 1, thus reducible to ODE by direct substitution**

Approximations

MBDyn

- **Quasi-steady approximation: use low-order dynamics**
- **In Laplace's domain:**

$$\mathbf{y}(\mathbf{s})=\left(\mathbf{C}(\mathbf{s}\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}\right)\mathbf{u}(\mathbf{s})=\mathbf{H}(\mathbf{s})\mathbf{u}(\mathbf{s})$$

Taylor

$$\mathbf{y}(\mathbf{s})\simeq\left(\mathbf{H}(0)+\mathbf{s}\left(\frac{\mathbf{dH}}{\mathbf{ds}}\right)_{\mathbf{s}=0}+\frac{\mathbf{s}^2}{2}\left(\frac{\mathbf{d}^2\mathbf{H}}{\mathbf{ds}^2}\right)_{\mathbf{s}=0}+\ldots+\frac{\mathbf{s}^{\mathbf{n}}}{\mathbf{n}!}\left(\frac{\mathbf{d}^{\mathbf{n}}\mathbf{H}}{\mathbf{ds}^{\mathbf{n}}}\right)_{\mathbf{s}=0}\right)\mathbf{u}(\mathbf{s})$$

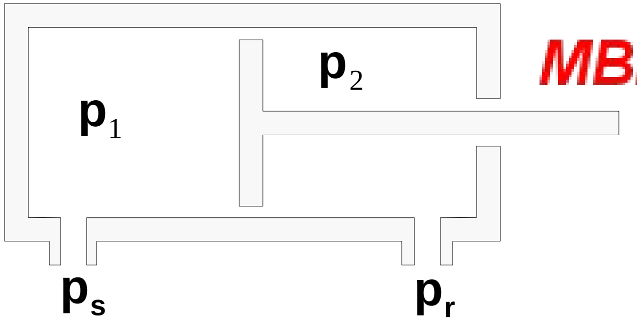
$$\mathbf{H}(0)=-\mathbf{C}\mathbf{A}^{-1}\mathbf{B}+\mathbf{D}$$
$$\mathbf{H}'(0)=-\mathbf{C}\mathbf{A}^{-2}\mathbf{B}$$
$$\mathbf{H}''(0)=-2\mathbf{C}\mathbf{A}^{-3}\mathbf{B}$$
$$\ldots$$
$$\mathbf{H}^{(\mathbf{n})}(0)=-(\mathbf{n}!)\mathbf{C}\mathbf{A}^{-(\mathbf{n}+1)}\mathbf{B}$$

- **Back to time domain:**

$$\mathbf{y}(\mathbf{t})=\mathbf{H}(0)\mathbf{u}(\mathbf{t})+\mathbf{H}'(0)\dot{\mathbf{u}}(\mathbf{t})+\frac{1}{2}\mathbf{H}''(0)\ddot{\mathbf{u}}(\mathbf{t})+\ldots$$

- **Note: time derivative of input!**
  (Only makes sense when u is state of interacting system)

Approximations



Example: actuator; equations:

- **Flow balance in chamber 1**

$$V_1/\beta\,\dot{p}_1 = -A_p\,\dot{x} - C_{eli}\,A_{eli}\left(p_1 - p_2\right) - C_{ele}\,A_{ele}\,p_1 - C_{els}\,A_{els}\left(p_s - p_1\right)$$

- **Flow balance in chamber 2**

$$V_2/\beta\,\dot{p}_2 = +A_p\,\dot{x} - C_{eli}\,A_{eli}\left(p_2 - p_1\right) - C_{ele}\,A_{ele}\,p_2 - C_{elr}\,A_{elr}\left(p_2 - p_r\right)$$

- **Equilibrium of piston**

$$m\,\ddot{x} = A_p\left(p_1 - p_2\right) - r\,\dot{x} + F$$

- **State-space realization**

$$\begin{Bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{x} \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} -C_{e11}\,\beta/V_1 & C_{e12}\,\beta/V_1 & 0 & -A_p\,\beta/V_1 \\ C_{e12}\,\beta/V_2 & -C_{e22}\,\beta/V_2 & 0 & A_p\,\beta/V_2 \\ 0 & 0 & 0 & 1 \\ A_p/m & -A_p/m & 0 & -r/m \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \\ x \\ v \end{Bmatrix} + \begin{Bmatrix} C_{e10}\,\beta/V_1\,p_s \\ C_{e20}\,\beta/V_2\,p_r \\ 0 \\ F/m \end{Bmatrix}$$

Approximations

MBDyn

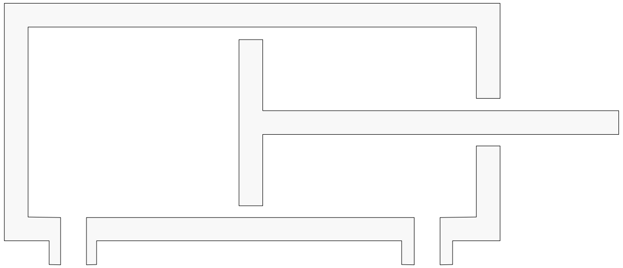
Truncation approximation:

- **Neglect hydraulics (velocities small, infinite power available)**

$$m\ddot{x} = A_p(p_s - p_r) - r\dot{x} + F$$

- **Directly control the force exerted by the fluid**

$$\begin{Bmatrix} \dot{x} \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -r/m \end{bmatrix} \begin{Bmatrix} x \\ v \end{Bmatrix} + \begin{Bmatrix} 0 \\ A_p/m(p_s - p_r) + F/m \end{Bmatrix}$$



Static approximation:

- **Neglect compressibility (volume is small, bulk modulus is high)**

$$\begin{aligned} 0 &\simeq -A_p\dot{x} - C_{eli}A_{eli}(p_1 - p_2) - C_{ele}A_{ele}p_1 - C_{els}A_{els}(p_s - p_1) \\ 0 &\simeq +A_p\dot{x} - C_{eli}A_{eli}(p_2 - p_1) - C_{ele}A_{ele}p_2 - C_{elr}A_{elr}(p_2 - p_r) \\ m\ddot{x} &= A_p(p_1 - p_2) - r\dot{x} + F \end{aligned} \qquad \Rightarrow p_1,\ \ p_2 \div \dot{x}$$

- **Pressure depends on velocity: → equivalent damping**

$$\begin{Bmatrix} \dot{x} \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -r/m - c_{eq}/m \end{bmatrix} \begin{Bmatrix} x \\ v \end{Bmatrix} + \begin{Bmatrix} 0 \\ A_p/m(p_s - p_r) + F/m \end{Bmatrix}$$

Software

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- **AMESim:**
  - **Monolithic software (can be interfaced as dynamic module to other solvers)**
  - **Models hydraulic networks in detail**
- **ADAMS:**
  - **Module for basic solver**
  - **Introduces modeling capabilities of hydraulic components**
- **Modelica (Dymola, MathModelica, OpenModelica?)**
  - **Modeling language, based on open library of elements**
  - **Broad library for general-purpose components**
  - **Many fields, including hydraulics and multibody**
  - **Needs specific (closed) solver**
- **MBDyn**

Software

MBDyn

- **MBDyn:**
- **Developed at Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano**
- **Monolithic multibody/multiphysics general-purpose software**
- **It's free: released under GPL (GNU General Public License)**
- **Includes integrated hydraulic library**

Software

MBDyn

- **Web site: http://www.mbdyn.org/**
- **Distributed as source code**
- **Developed for Linux**
- **Needs Un*x-like build environment**
  - **Linux**
  - **Linux in virtual machine**
  - **MSYS/MinGW in windows**
  - **Cygwin in windows**

Software

MBDyn

- Command-line software
- Prepare an input file using your favourite text editor
- Execute:

```
# mbdyn -f input_file -o output_file_prefix
```

- Output in files with specific extensions
- Load output files in math environment (octave, scilab, matlab, …) for plotting and further manipulation

Software

MBDyn

- Output can be reformatted for some post-processing tools
  - built-in: EasyAnim
  - ...

- Ongoing third-party project (“Blender & MBDyn”) about using Blender http://www.blender.org/ for pre/post-processing:

  http://www.baldwintechnology.com/

Software

MBDyn

- The model and the analysis are defined in a textual input file
- Use your preferred editor to prepare the input file
- The structure and the syntax of the statements are described here
  http://www.mbdyn.org/ → Download
  (pick the manual for the version in use, or follow instructions)
- A set of tutorials and other documentation is presented here
  http://www.mbdyn.org/ → Documentation
  including example input files

Modeling Approach

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- Nodes instantiate degrees of freedom and the corresponding balance equations

- Static structural nodes only instantiate equilibrium equations

$$0 = \sum \mathbf{f}$$
$$0 = \sum \mathbf{m}$$

- Dynamic structural nodes also instantiate momentum and momenta moment definitions

$$\mathbf{M}\dot{\mathbf{x}} = \beta$$
$$\mathbf{J}\omega = \gamma$$
$$\dot{\beta} = \sum \mathbf{f}$$
$$\dot{\gamma} = \sum \mathbf{m}$$

- Hydraulic nodes (pressure) instantiate flow balance equations

Modeling Approach

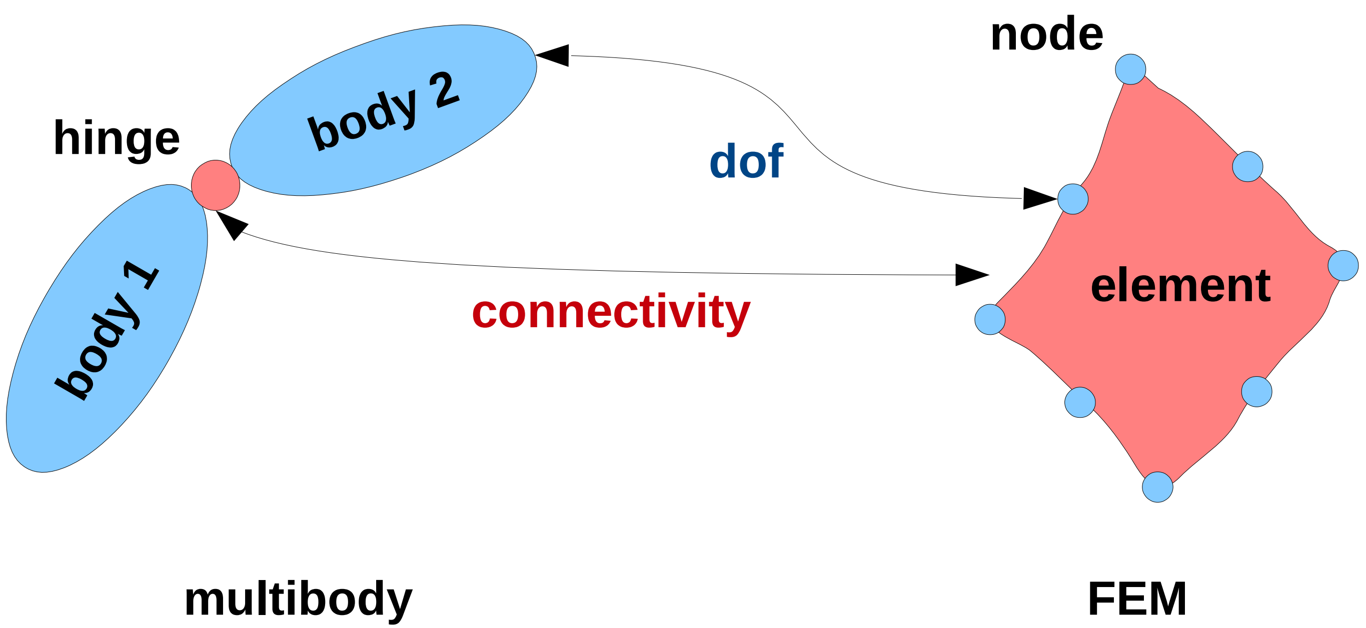
MBDyn

- Elements write contributions to equations instantiated by nodes
- Elements represent “connectivity” and “constitutive properties”
- Elements can add further, “private” equations and variables (e.g. algebraic constraints and Lagrange multipliers)
- Mechanical elements typically add forces and moments to equilibrium equations
- Mechanical constraints (“joints”) may add algebraic relationships between kinematic degrees of freedom
- Hydraulic elements typically add flow contributions to flow balance equations in nodes

Modeling Approach

MBDyn

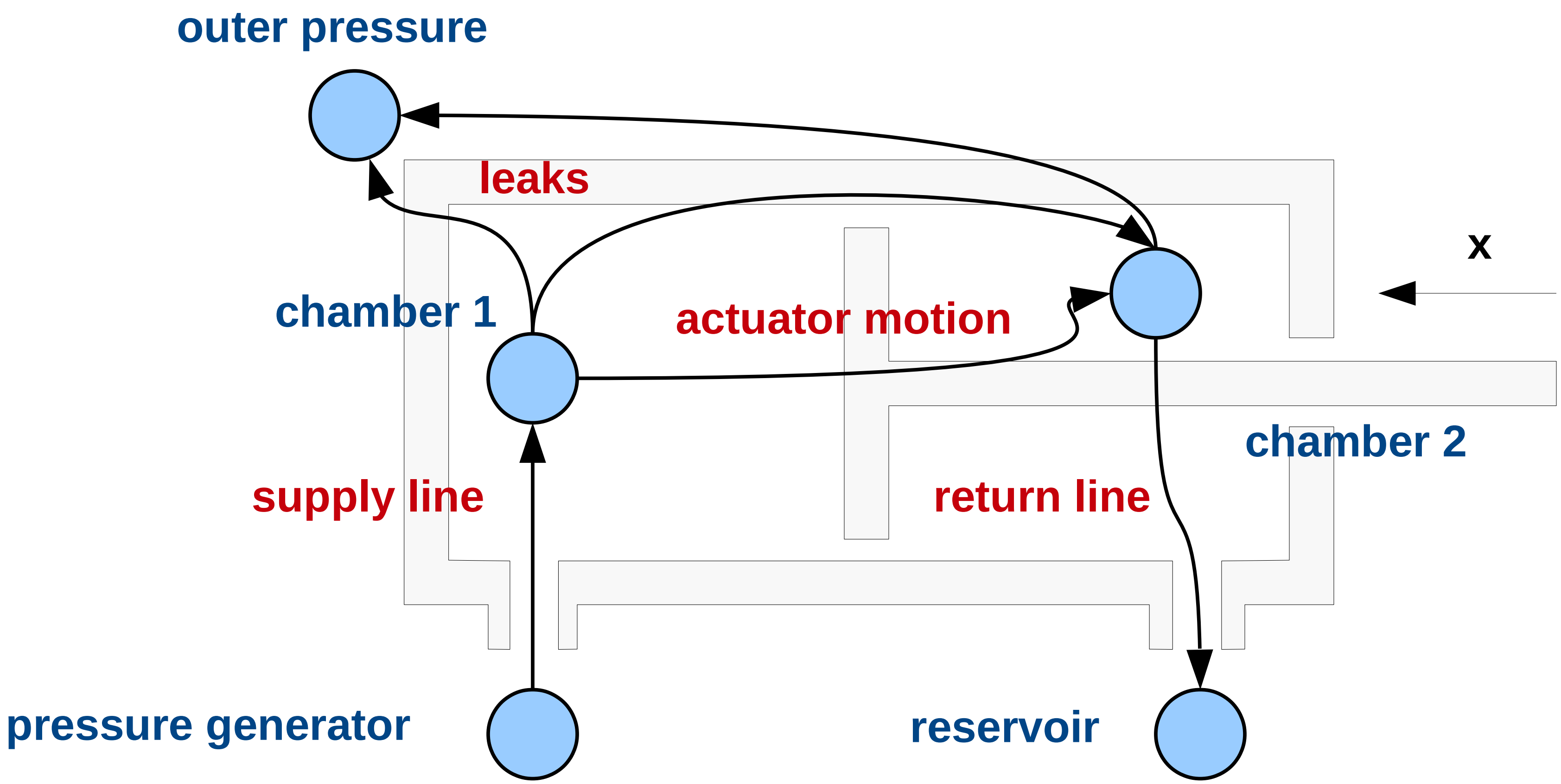
- Multibody vs. FEM: nodes at bodies vs. nodes at frontier
  (node ↔ body; element ↔ hinge)



Modeling Approach

MBDyn

- **Hydraulic modeling: network**



Hydraulic Element Library: Pressure Generator

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- **Not specifically implemented; use a “clamp” genel instead**
- **This element adds a scalar algebraic equation that enforces a specific value (possibly time-dependent) on a scalar node**

$$\mathbf{p}=\mathbf{p}_0(\mathbf{t})$$

- **This algebraic equation implies a Lagrange multiplier on the flow balance equation related to the pressure node**

$$\sum \mathbf{q}+\lambda=0$$

- **The Lagrange multiplier represents the flow required to grant the imposed pressure value**

Hydraulic Element Library: Imposed Flow

MBDyn

- **Not specifically implemented, use an “abstract” force instead**
- **This element adds a contribution, possibly time-dependent, to an arbitrary scalar equation**

$$\mathbf{q}=\mathbf{q}(\mathbf{t})$$

- **Note: in MBDyn, a negative flow enters the circuit at the given node, a positive flow leaves the circuit at the given node**

Hydraulic Element Library: Dynamic Pipeline

MBDyn

- Dynamic pipes are formulated using a finite-volume approach
- Degenerate into static when pressure time derivatives neglected
- Differential mass balance:

$$\frac{D}{Dt} dm = 0 \rightarrow q_{/x} + A \rho_{/t} = 0$$

- Differential momentum balance:

$$\frac{D}{Dt} dq = df \rightarrow q_{/t} + \left(\frac{q^2}{\rho A} + A p \right)_{/x} = f_v$$

- Discretization:

$$q(x,t) = \left[\frac{1-\xi}{2} \quad \frac{1+\xi}{2} \right] \left\{ \begin{matrix} q_1(t) \\ q_2(t) \end{matrix} \right\}$$

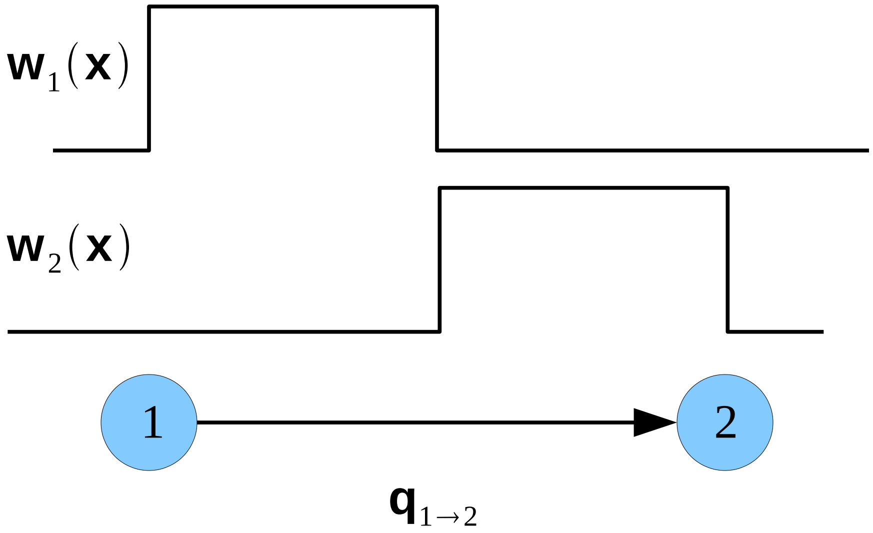
$$p(x,t) = \left[\frac{1-\xi}{2} \quad \frac{1+\xi}{2} \right] \left\{ \begin{matrix} p_1(t) \\ p_2(t) \end{matrix} \right\}$$

$$x = x(\xi)$$

Hydraulic Element Library: Dynamic Pipeline

MBDyn

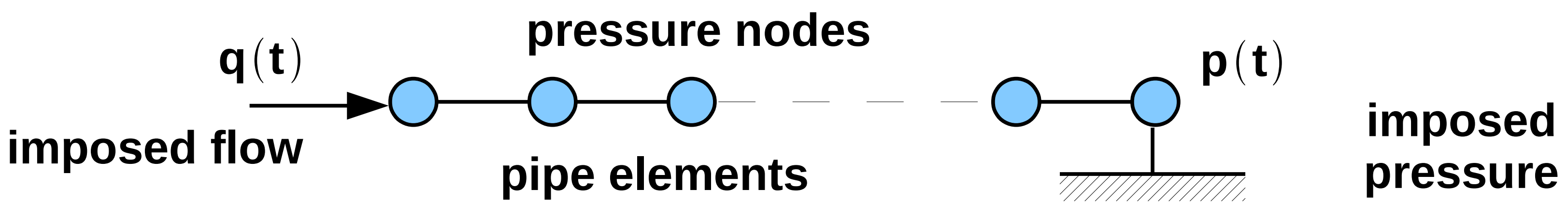
- **Piecewise-constant weight functions:**
- **Weight the mass and momentum balance equations**
- **Four constitutive equations in pressure and flow at nodes**



Example: Pressure Wave in Pipeline

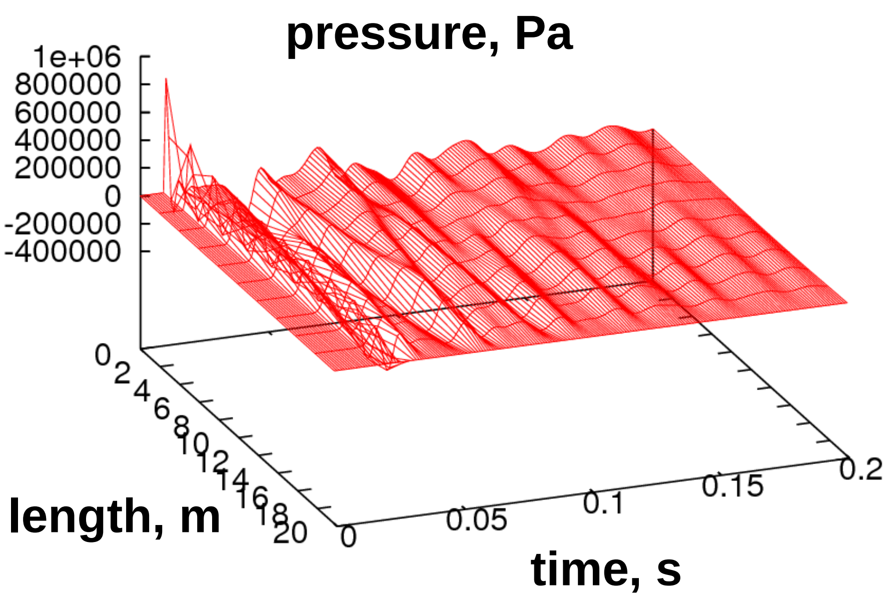
MBDyn

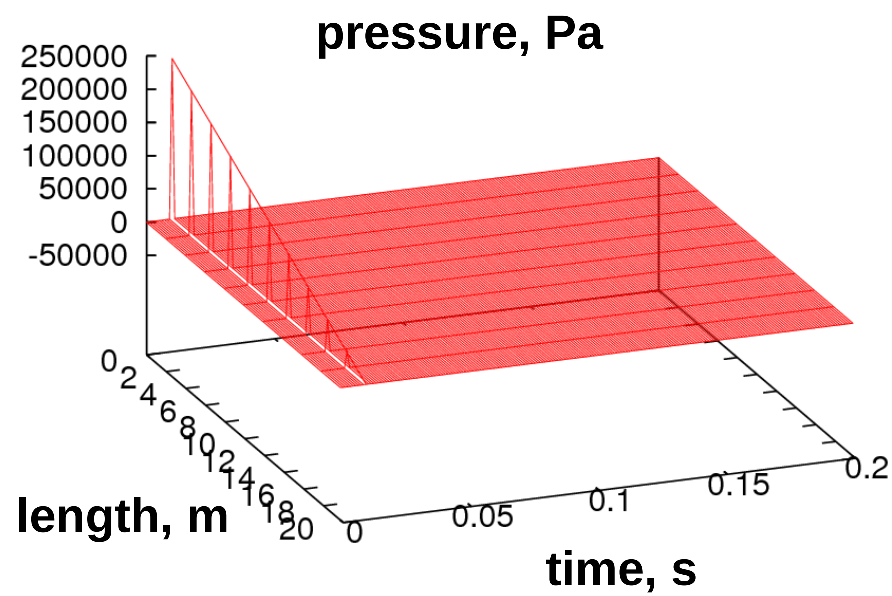
- From: R. Piché, A. Ellman, "A Fluid Transmission Line Model for Use with ODE Simulators", 8th Bath International Fluid Power Workshop, Sep. 20-22 1995, University of Bath, UK
  - Length: 19.74 m
  - Radius: 6.17e-3 m
  - Density: 870 kg/m^3
  - Viscosity: 8.e-5 m^2/s
  - Sound celerity: 1.4e3 m/s
  - Impulsive flow: 0.001 m^3/s
  - Duration: 0.1e-3 s
- Impulsive dynamics



Example: Pressure Wave in Pipeline

- **Dynamic pipe (10 "dynamic pipe" elements) compared to static pipe, "impulsive" flow input:**





Example: Pressure Wave in Pipeline

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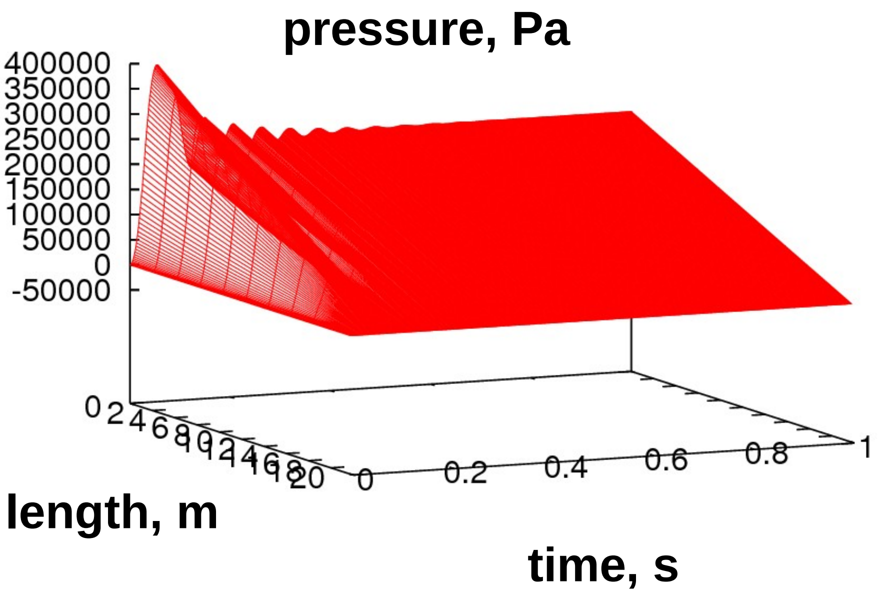
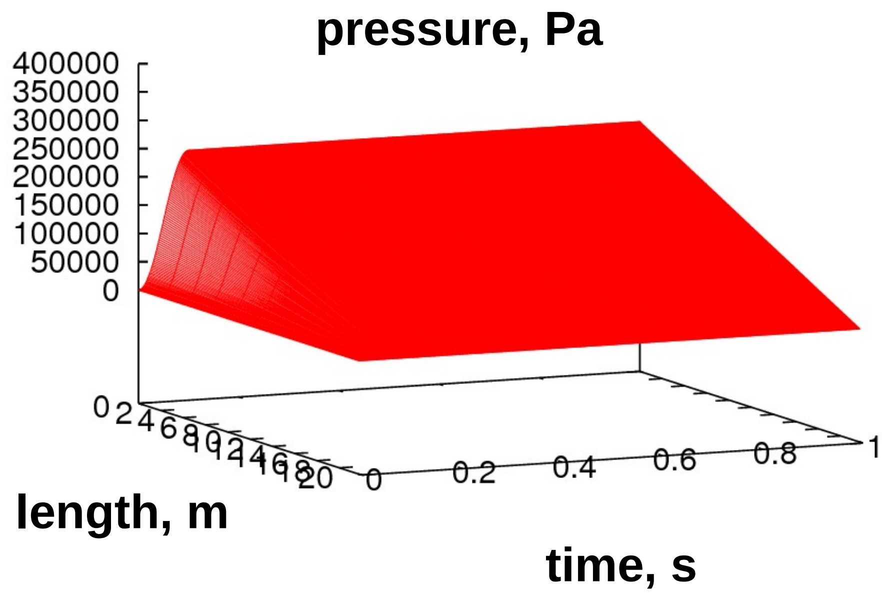
- Dynamic pipe (10 “dynamic pipe” elements)
  compared to static pipe, “smooth” flow input:

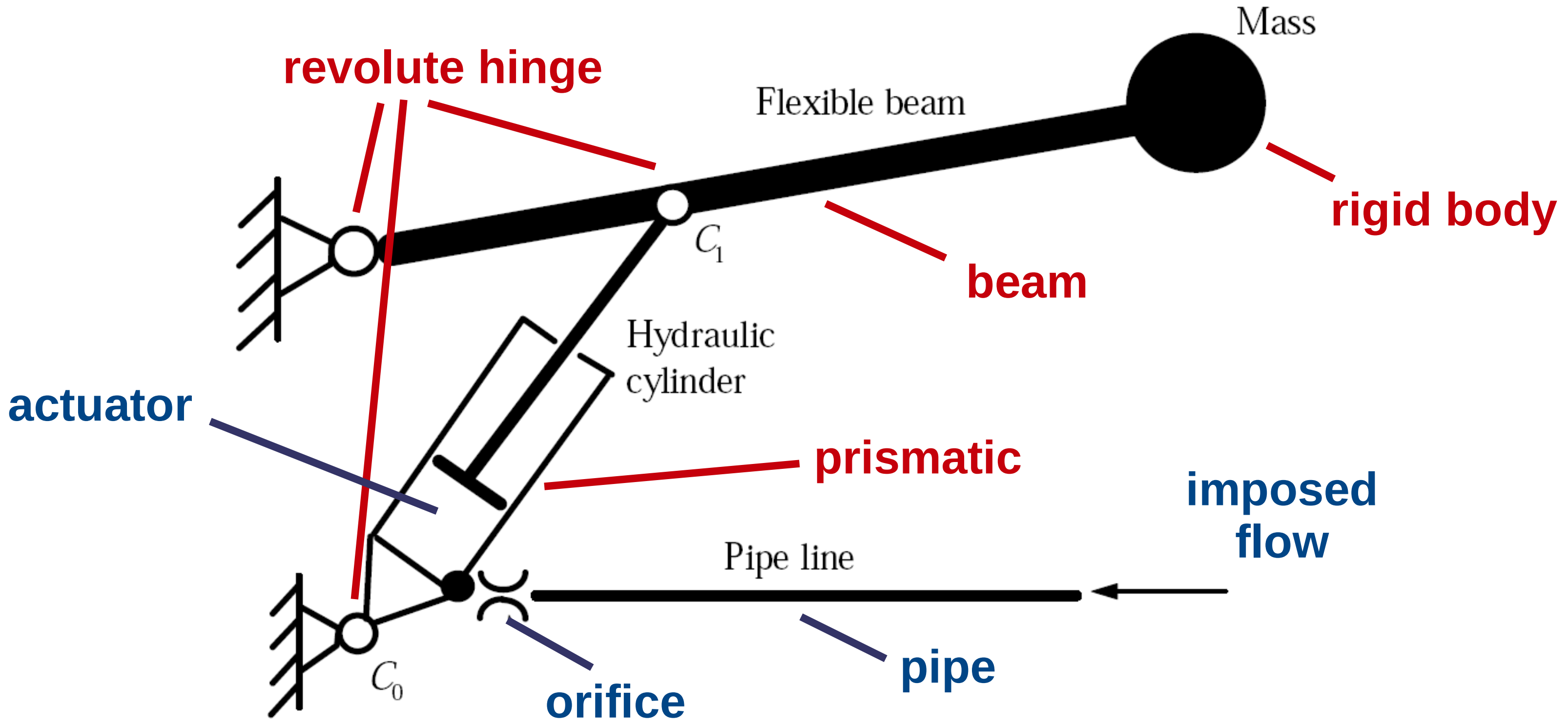




Example: Hydraulically Actuated Beam

MBDyn

- From: J. Mäkinen, A. Ellman, R. Piché, “Dynamic Simulations of Flexible Hydraulic-Driven Multibody Systems using Finite Strain Beam Theory”, 5th Scandinavian International Conference on Fluid Power, Linköping, 1997, Sweden



- Input file: https://www.mbdyn.org/userfiles/documents/examples/actuator

Example: Hydraulically Actuated Beam

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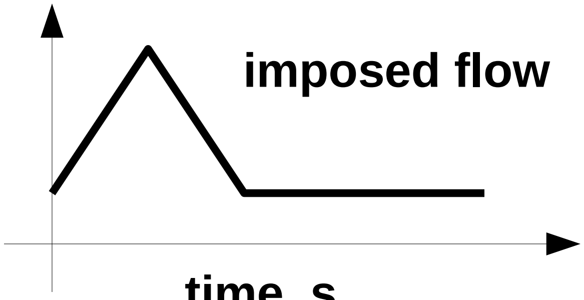
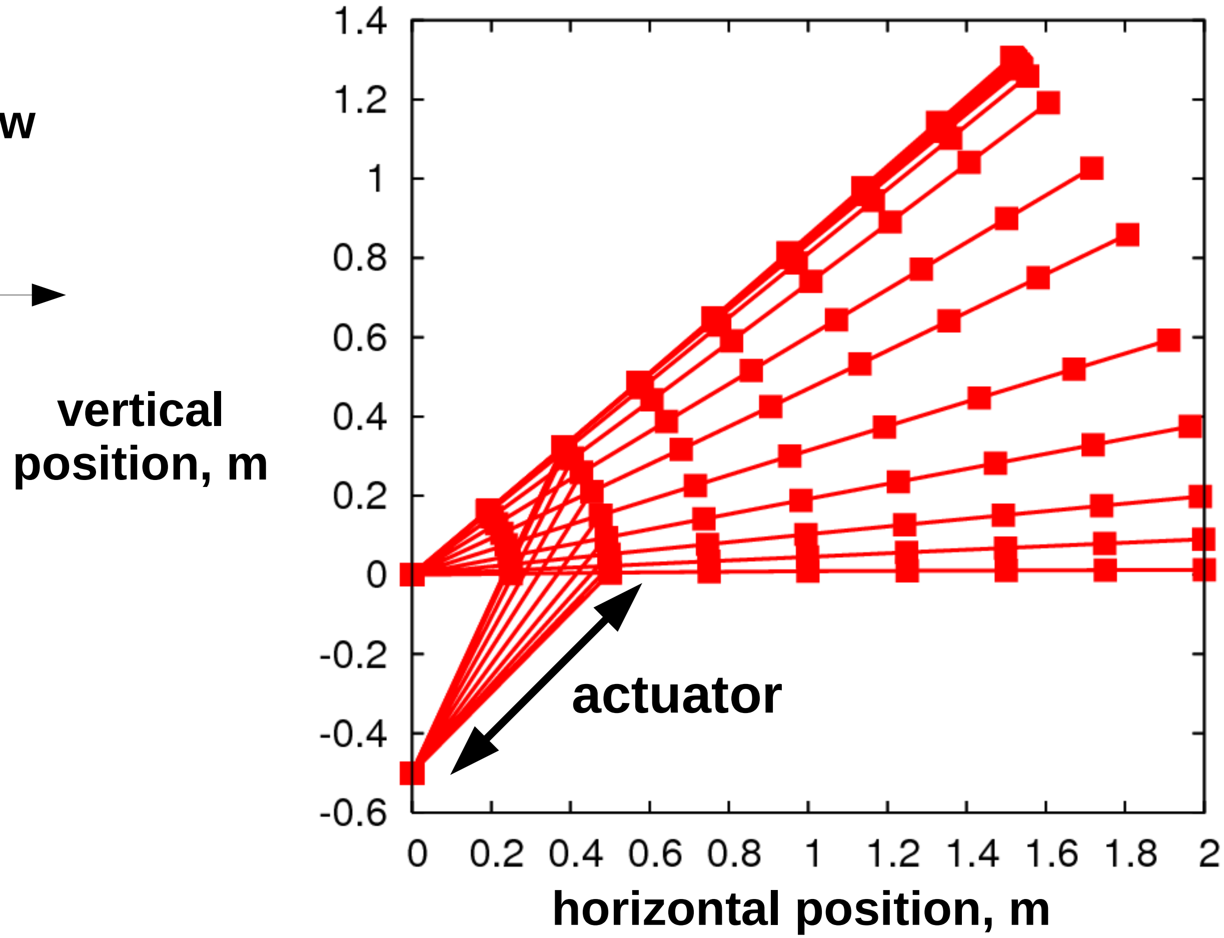
- 4 3-node beam elements





Example: Hydraulically Actuated Beam

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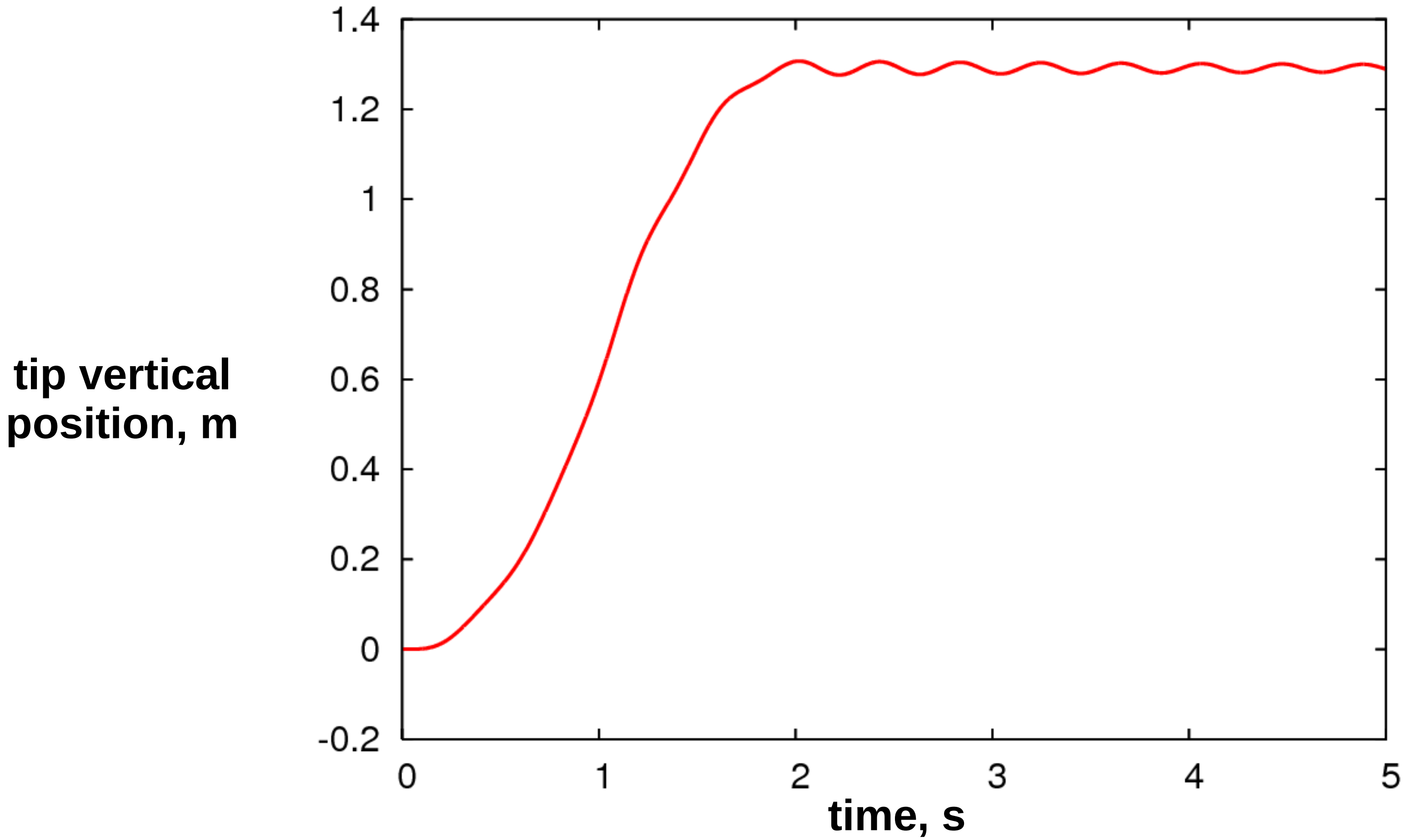
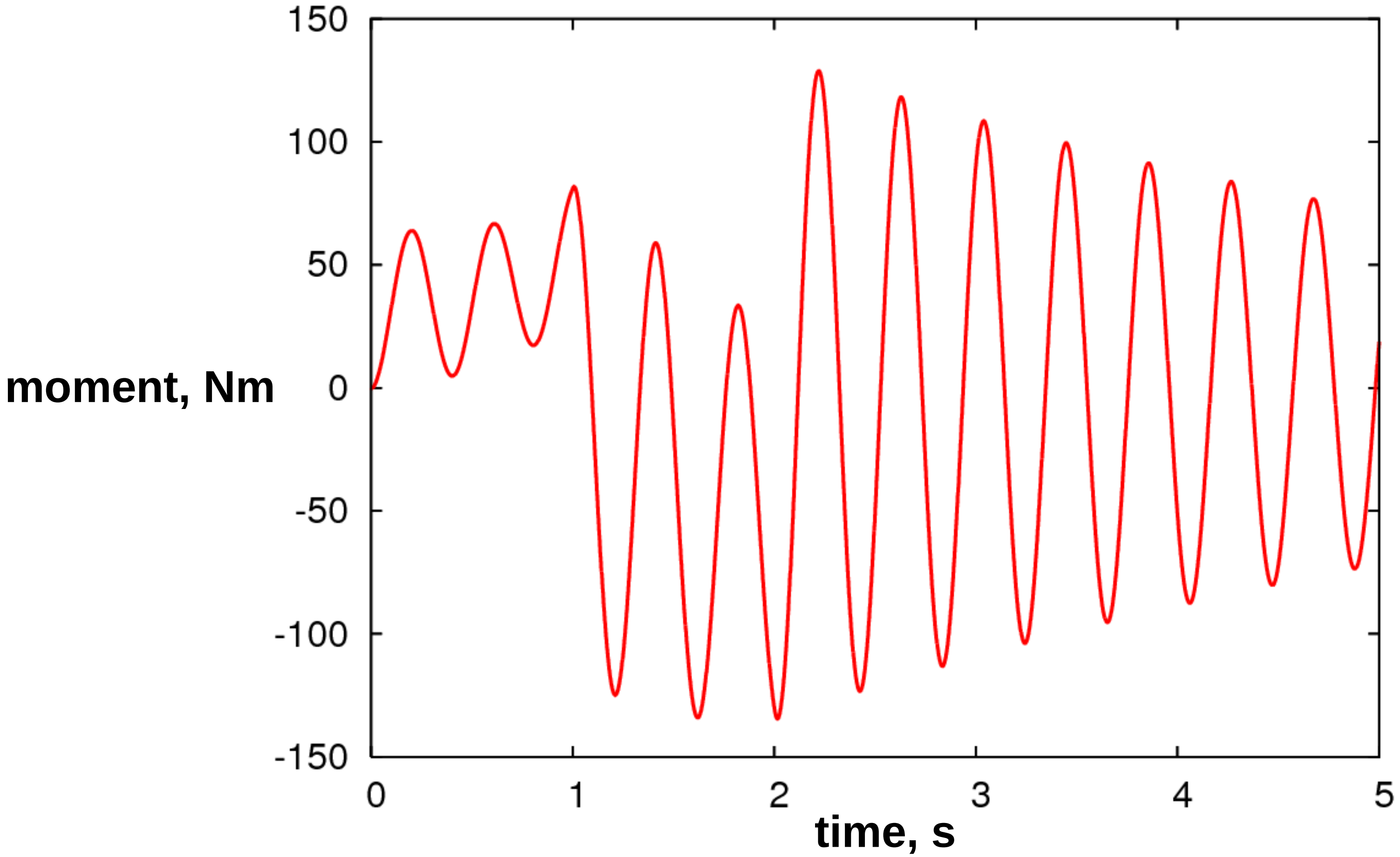
- **Tip vertical displacement**



Example: Hydraulically Actuated Beam

MBDyn

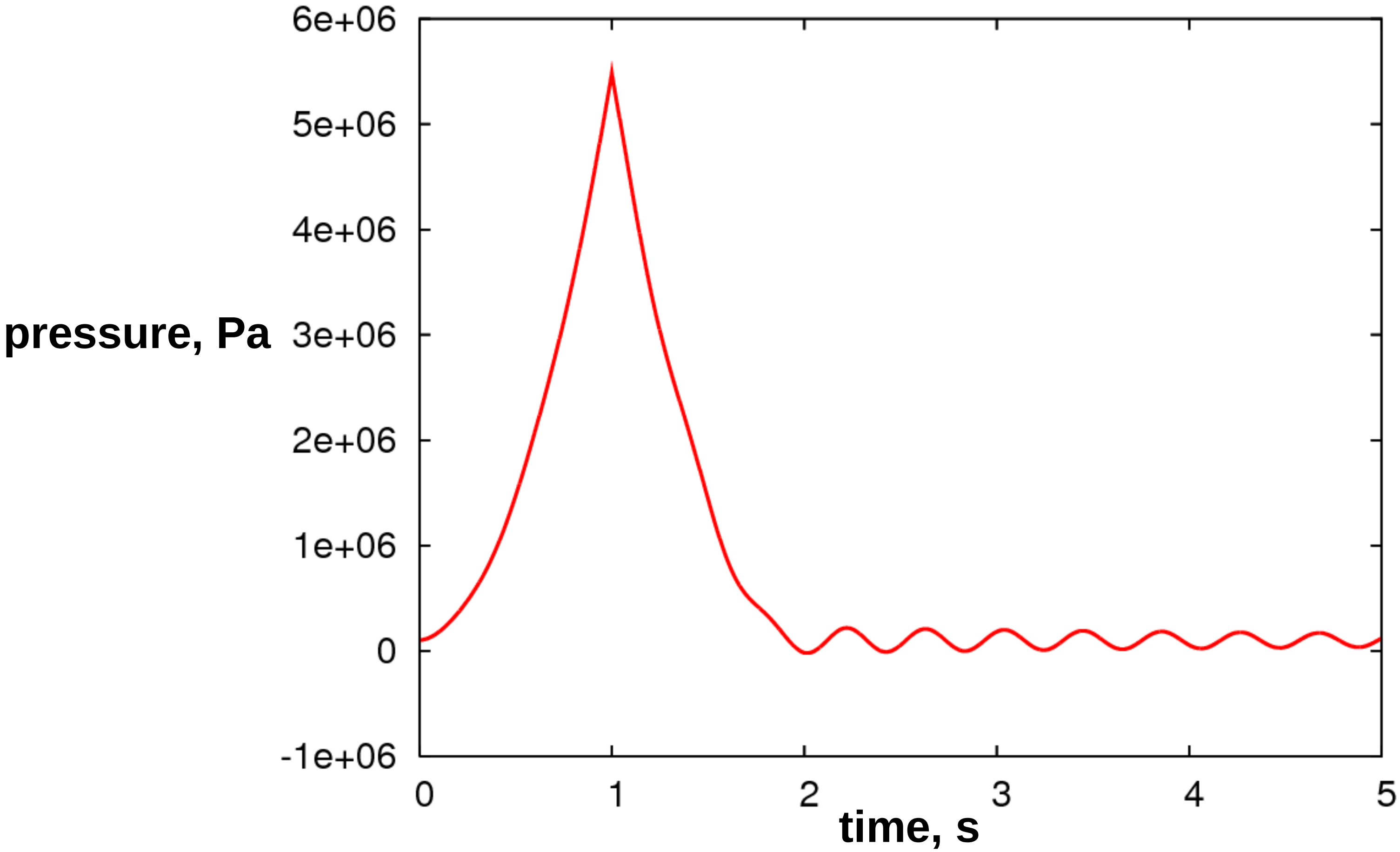
- **Internal bending moment close to actuator connection**



Example: Hydraulically Actuated Beam

MBDyn

- **Pressure at imposed flow node**



Consclusions

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- Multibody: natural environment for integration of multiphysics
- Need for further development: complete library, build test suite, validate components
- The software is free: try it, and feed back!

  http://www.mbdyn.org/

  mbdyn-users@mbdyn.org

Questions?