POLITECNICO DI MILANO

Trajectory Planning from Multibody System Dynamics

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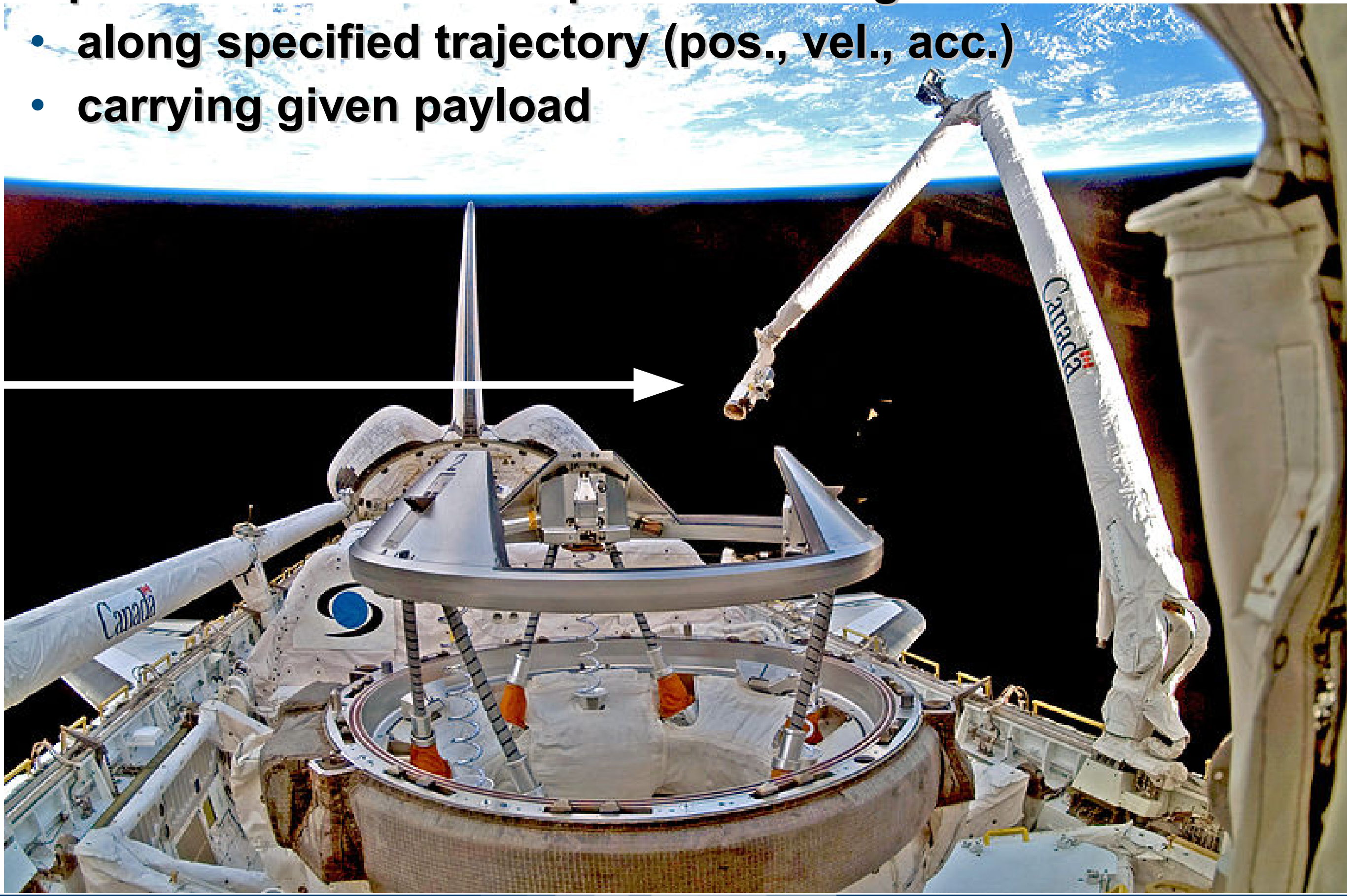
Manipulators

Manipulator:

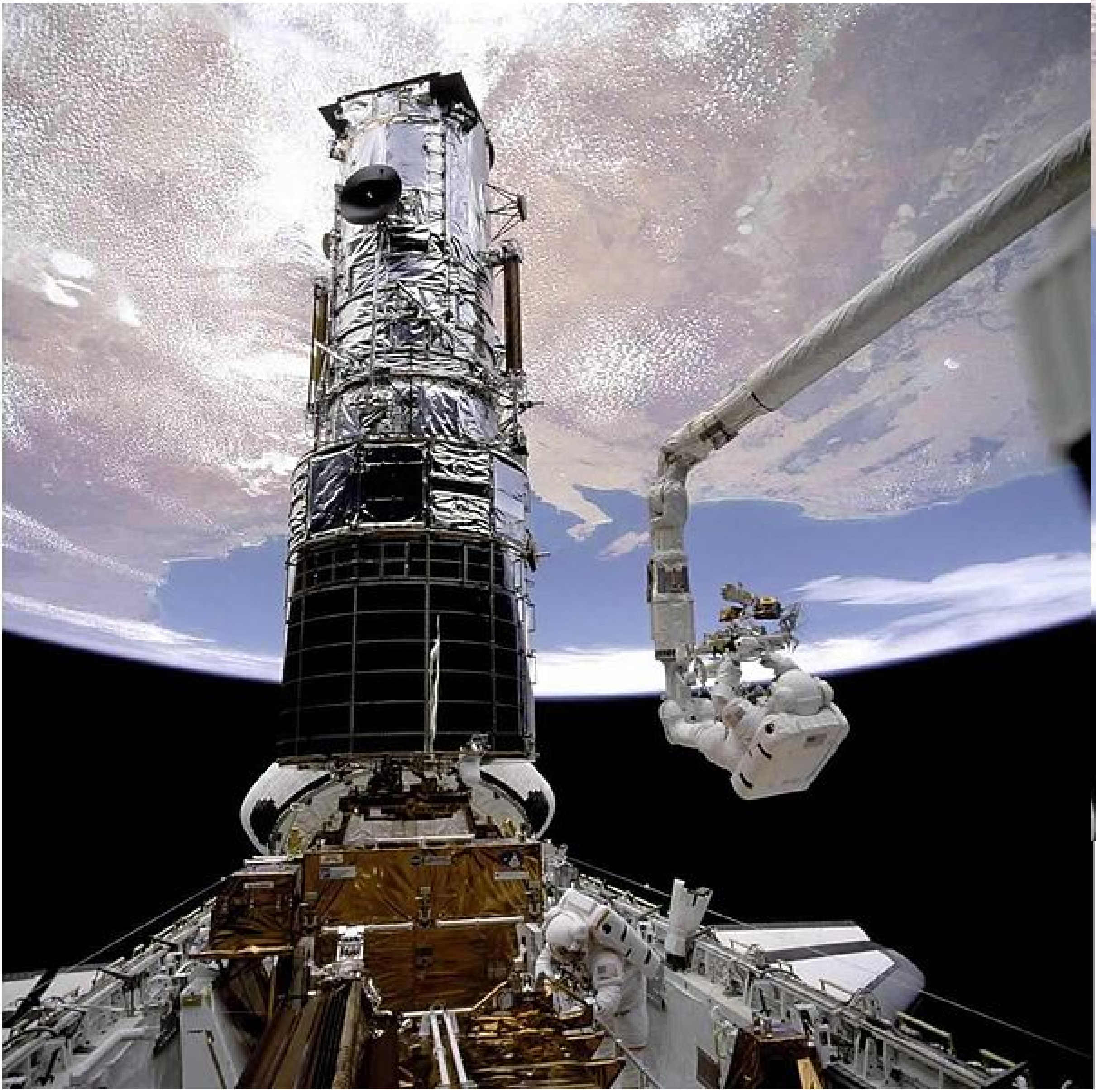
- chain of links commanded by motors
- purpose: place end effector in specified configuration
  - along specified trajectory (pos., vel., acc.)
  - carrying given payload

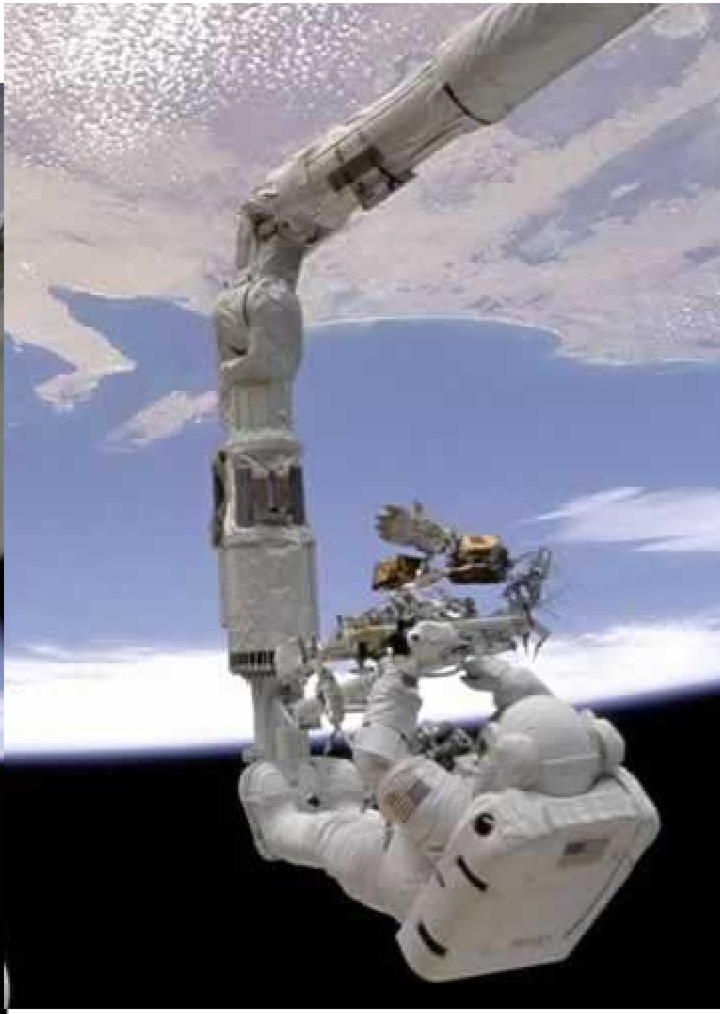
end effector



“Canadarm” (from NASA)

Manipulators

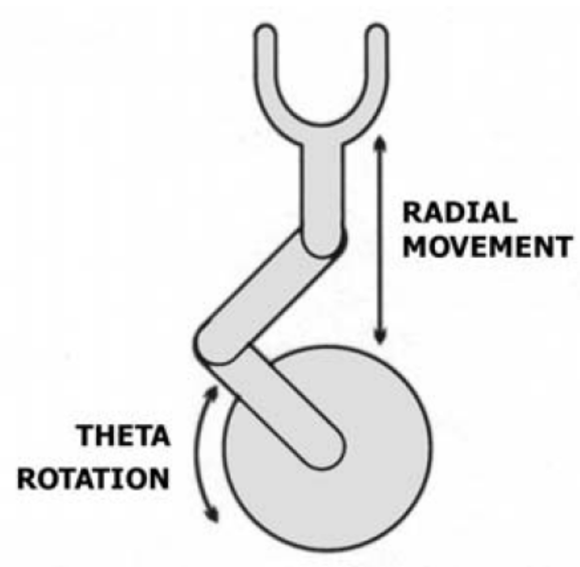


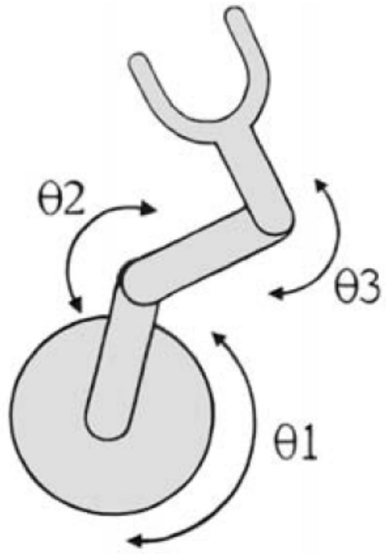


"Canadarm" (from NASA)

Manipulators

Industrial robots: wafer-handling manipulators





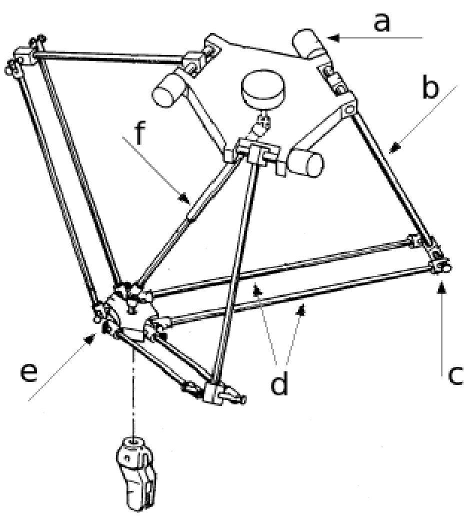
R-Theta — **Selectively Compliant Articulated Robot Arm (SCARA)**

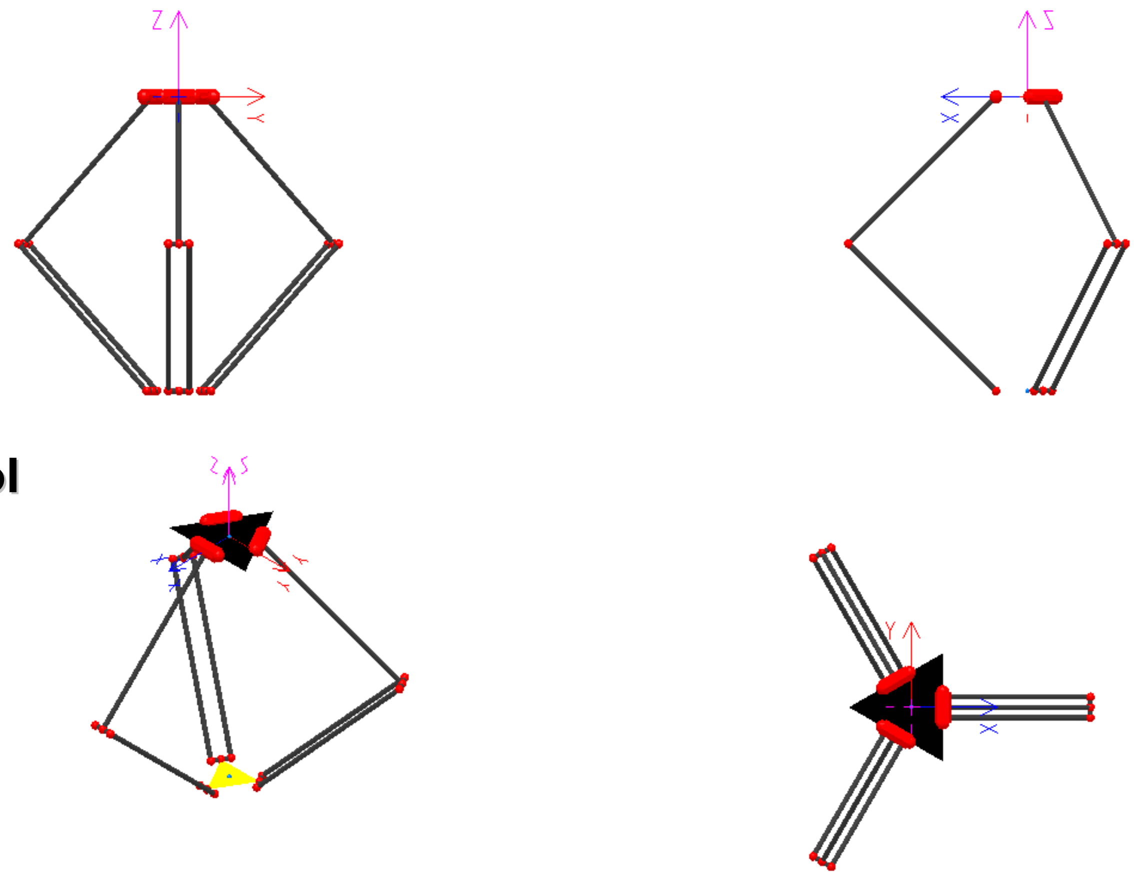
2-dof — **3-dof, limited footprint, no workspace limitation**

(from Innovative Robotics)

Examples of manipulator multibody modeling with MBDyn

Delta robot

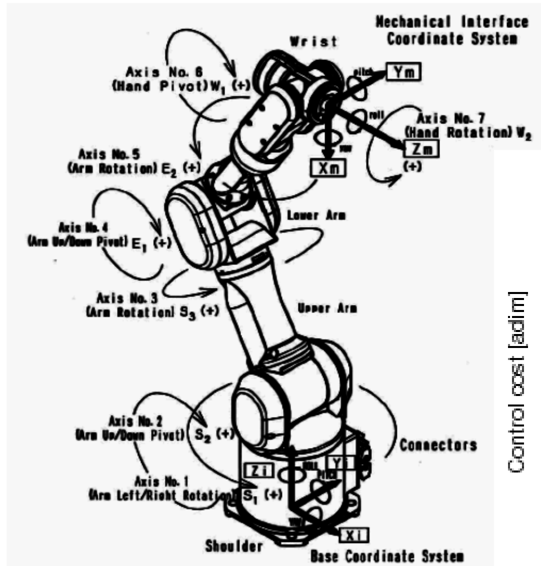


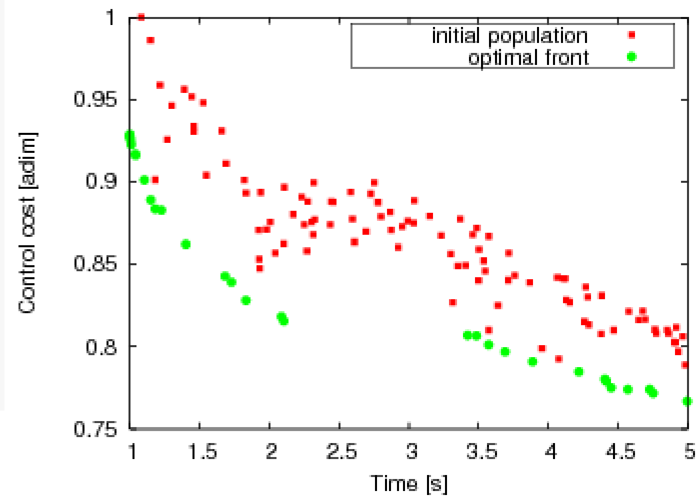


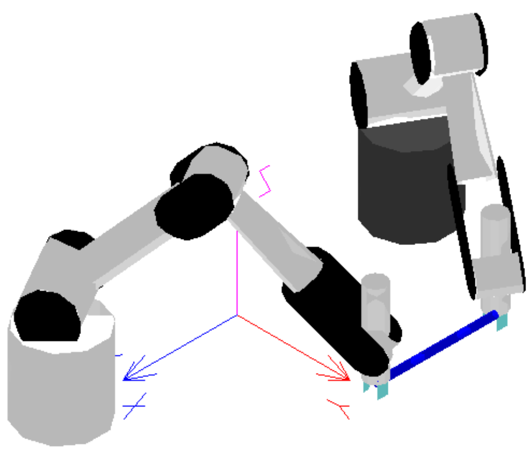
inverse dynamics for computed torque control

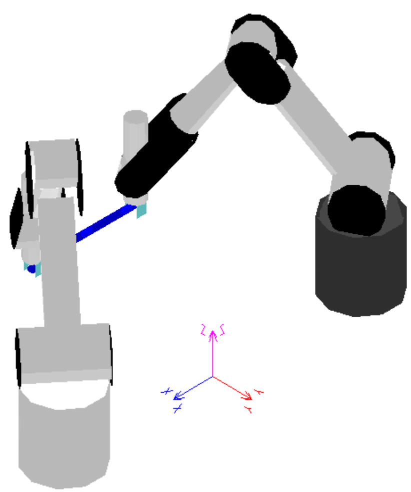
Examples of manipulator multibody modeling with MBDyn

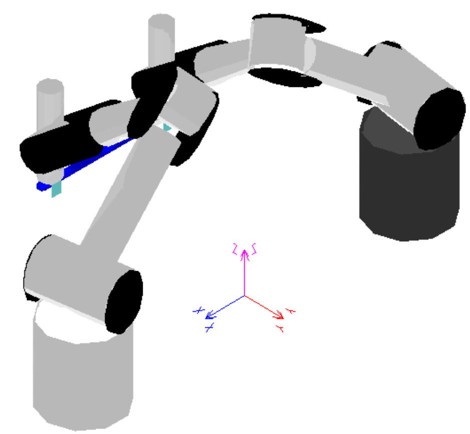
Robotics: PA-10 inverse kinematics with path optimization of cooperating robots





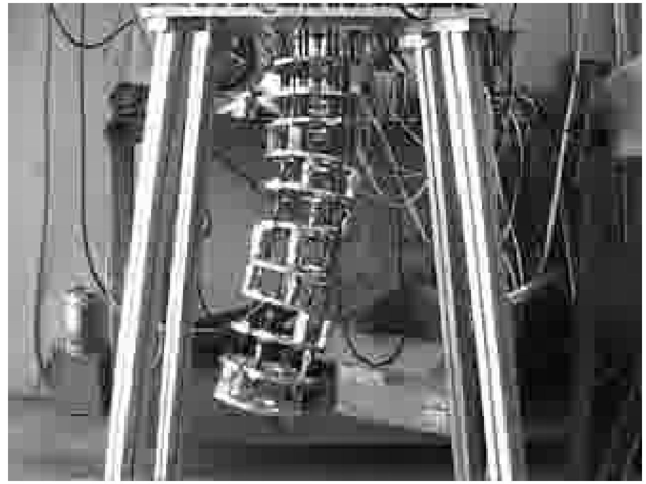


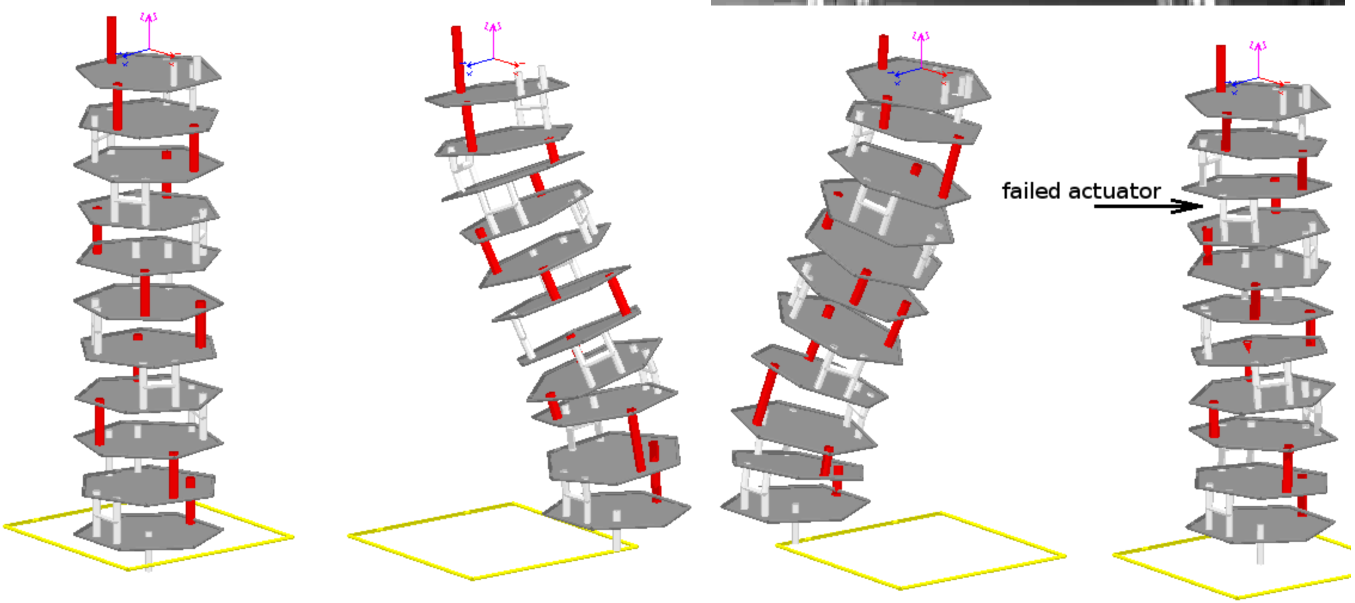


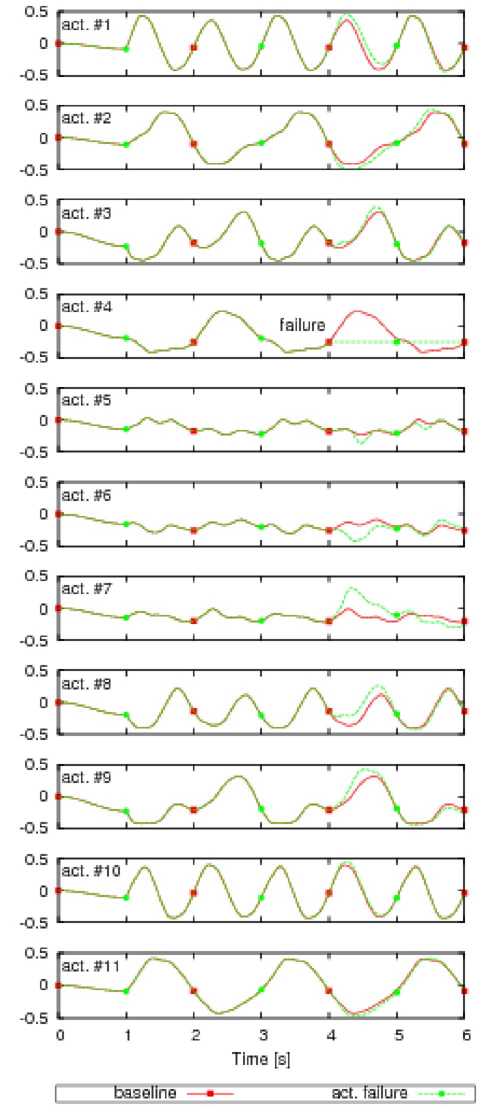


Examples of manipulator multibody modeling with MBDyn

Robotics:
biomimetic robot real-time motion planning by inverse kinematics with fault detection







Manipulators: classification

Manipulators:

- **end effector prescribed degrees of freedom:** n
- **manipulator number of degrees of freedom:** f
- **number of motors:** c

Classification:

- **When** $n = c = f$ **the problem is purely kinematic**
- **When** $n < c = f$ **the problem is redundant**
- **When** $n = c < f$ **the problem is underactuated**

Multibody dynamics

Basic equations:

$$M(x)\ddot{x} = f(x, \dot{x}, t)$$

- **mechanics of unconstrained system of bodies**
- **subjected to configuration-dependent loads**

Can be obtained from many (equivalent!) approaches:

- ***Newton-Euler*: linear/angular equilibrium of each body**
- ***d'Alembert-Lagrange*: virtual work of active forces/moments**
- ***Gauss*, *Hertz*, *Hamilton*, ...: variational principles**

Multibody dynamics

Constrained system: kinematic constraints

- **holonomic**

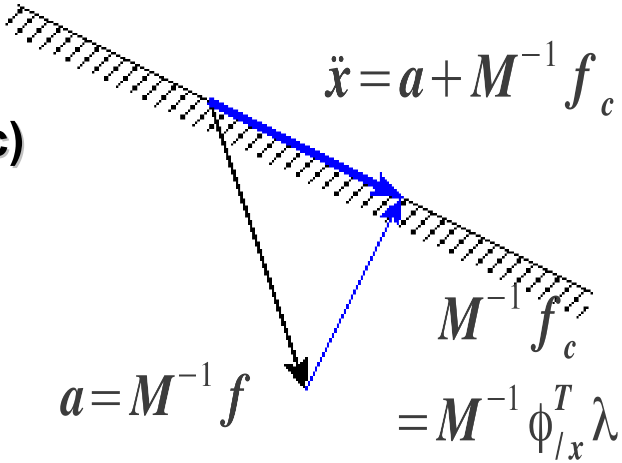
$$\phi\left(x, t\right) = 0$$

- **non-holonomic (not integrable to holonomic)**

$$\psi\left(x, \dot{x}, t\right) = 0$$

usually

$$A\left(x, t\right)\dot{x} = b\left(x, t\right)$$



- **algebraic relationship between kinematic variables**
- **explicitly dependent on time: rheonomic**
- **scleronomous otherwise**

Multibody dynamics

Redundant coordinate set:

$$M(x)\ddot{x}=f(x,\dot{x},t)-\phi_{/x}^{T}\lambda$$
$$\phi(x)=0$$

Minimal coordinate set:

$$\hat{M}(q)\ddot{q}=\hat{f}(q,\dot{q},t)$$

- **requires capability to write** $q \setminus x=\theta(q) \rightarrow \phi(x)=\phi(\theta(q))\equiv 0$
- **easier in differential form:**

$$\dot{x}=\theta_{/q}\dot{q} \rightarrow \dot{q}=\theta_{/q}^{+}\dot{x}$$
$$\phi_{/x}\dot{x}=\phi_{/x}\theta_{/q}\dot{q}\equiv 0 \rightarrow \phi_{/x}\theta_{/q}\equiv 0$$

- **Lagrange multipliers intrinsically eliminated**

Multibody dynamics

MBDyn uses redundant coordinate set at first order

$$\boldsymbol{M}(\boldsymbol{x})\dot{\boldsymbol{x}}=\beta$$

$$\dot{\beta}=\boldsymbol{f}(\boldsymbol{x},\dot{\boldsymbol{x}},t)-\phi_{/\boldsymbol{x}}^{T}\lambda$$

$$\phi(\boldsymbol{x})=0$$

One can easily show how formulations in the following can be generalized to redundant coordinate set at first order

For the sake of clarity, minimal coordinate set is used in the following

Motion Prescription

Manipulator problem: prescribed motion (at position level):

$$\hat{M}(q)\ddot{q} = \hat{f}(q, \dot{q}, t) + \hat{B}^T c$$
$$\varphi(q) = \alpha(t)$$

- **assume by now number of motor torques *c* equal to number of prescribed degrees of freedom *n***
- **problem structure similar to that of "passive" constraints**
- **BUT:**

$$M(x)\ddot{x} = f(x, \dot{x}, t) - \phi_{/x}^T \lambda$$
$$\phi(x) = 0$$

$$\hat{M}(q)\ddot{q} = \hat{f}(q, \dot{q}, t) + \hat{B}^T c$$
$$\varphi(q) = \alpha(t)$$

Motion Prescription

Lagrange multipliers:

$$M(x)\ddot{x}=f(x,\dot{x},t)-\phi_{/x}^{T}\lambda$$
$$\phi(x)=0$$
$$\phi_{/x}\ddot{x}=b'$$
$$\ddot{x}=M^{-1}\left(f-\phi_{/x}^{T}\lambda\right)$$
$$\phi_{/x}M^{-1}f-b'=\phi_{/x}M^{-1}\phi_{/x}^{T}\lambda$$
$$\lambda=\left(\phi_{/x}M^{-1}\phi_{/x}^{T}\right)^{-1}\left(\phi_{/x}M^{-1}f-b'\right)$$

invertible under broad assumptions

Motor torques:

$$\hat{M}(q)\ddot{q}=\hat{f}(q,\dot{q},t)+\hat{B}^{T}c$$
$$\varphi(q)=\alpha(t)$$
$$\varphi_{/q}\ddot{q}=\ddot{\alpha}$$
$$\ddot{q}=\hat{M}^{-1}\left(\hat{f}+\hat{B}^{T}c\right)$$
$$\varphi_{/q}\hat{M}^{-1}\hat{f}+\varphi_{/q}\hat{M}^{-1}\hat{B}^{T}c=\ddot{\alpha}$$
$$c=\left(\varphi_{/q}\hat{M}^{-1}\hat{B}^{T}\right)^{-1}\left(\ddot{\alpha}-\varphi_{/q}\hat{M}^{-1}\hat{f}\right)$$

invertible? **(related to the concept of differential flatness)**

$\left(\phi_{/x}\dot{x}\right)_{/x}\dot{x},\left(\varphi_{/q}\dot{q}\right)_{/q}\dot{q}$ ***omitted for clarity***

Motion Prescription: Fully Determined

When n. prescribed degrees of freedom = n. motors, rotor torques:

$$\hat{B}^T \equiv I$$

$$c = \left(\varphi_{/q} \hat{M}^{-1} \hat{B}^T\right)^{-1} \left(\ddot{\alpha} - \varphi_{/q} \hat{M}^{-1} \hat{f}\right)$$
$$= \hat{M} \varphi_{/q}^{-1} \ddot{\alpha} - \hat{f}$$

invertible?

Boils down to *purely kinematic problem*:

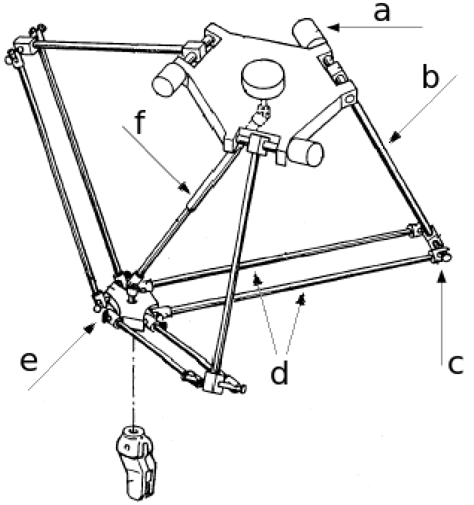
$$\varphi_{/q} \ddot{q} = \ddot{\alpha} \quad \rightarrow \quad \ddot{q} = \varphi_{/q}^{-1} \ddot{\alpha}$$

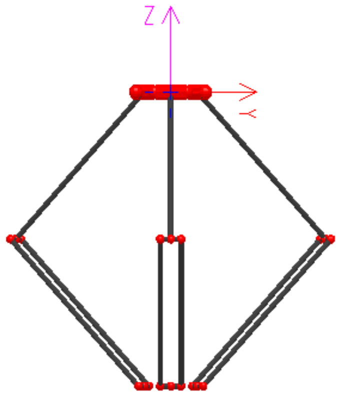
which formally implies (the last problem may need iterative solution)

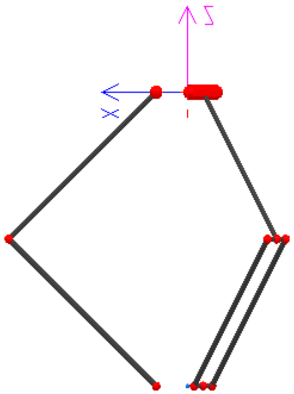
$$\varphi_{/q} \dot{q} = \dot{\alpha} \quad \rightarrow \quad \dot{q} = \varphi_{/q}^{-1} \dot{\alpha}, \quad q = \varphi^{-1}(\alpha)$$

Examples of manipulator multibody modeling with MBDyn

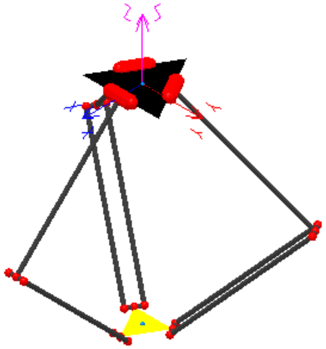
Delta robot: ***3 dof, 3 prescribed motion eqs.***

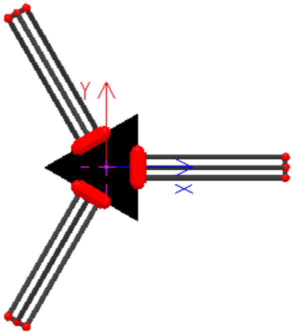






inverse dynamics for computed torque control





Motion Prescription: Underdetermined

When n. prescribed dof < n. motors, problem is *underdetermined*:

$$\hat{B}^T \equiv I$$

$$c = \left(\varphi_{/q} \hat{M}^{-1} \hat{B}^T\right)^+ \left(\ddot{\alpha} - \varphi_{/q} \hat{M}^{-1} \hat{f}\right)$$

$$= \hat{M} \varphi_{/q}^+ \ddot{\alpha} - \hat{f}$$

Pseudo-invertible (when full rank)!

NOTE: we are considering a LOCAL optimization

GLOBAL optimization is a totally different problem

- **multibody dynamics can be a tool in support of optimization**
- **local optimization can be used in real time**

Motion Prescription: Underdetermined

When n. prescribed dof < n. motors, problem is *underdetermined*;

Which pseudo-inverse?

- **Moore-Penrose Generalized Inverse:**

$$\varphi_{/q}\dot{\boldsymbol{q}}=\dot{\alpha} \;\rightarrow\; \dot{\boldsymbol{q}}=\varphi_{/q}^{+}\dot{\alpha}=\varphi_{/q}^{T}\left(\varphi_{/q}\varphi_{/q}^{T}\right)^{-1}\dot{\alpha}$$

- **Inertia-weighted GI (often “better”, heavier parts move less):**

$$\varphi_{/q}\ddot{\boldsymbol{q}}=\ddot{\alpha} \;\rightarrow\; \hat{\boldsymbol{M}}\ddot{\boldsymbol{q}}=\varphi_{/q}^{T}\mu \;\rightarrow\; \ddot{\boldsymbol{q}}=\hat{\boldsymbol{M}}^{-1}\varphi_{/q}^{T}\mu \;\rightarrow\; \varphi_{/q}\hat{\boldsymbol{M}}^{-1}\varphi_{/q}^{T}\mu=\ddot{\alpha}$$

$$\rightarrow\; \mu=\left(\varphi_{/q}\hat{\boldsymbol{M}}^{-1}\varphi_{/q}^{T}\right)^{-1}\ddot{\alpha} \;\rightarrow\; \ddot{\boldsymbol{q}}=\hat{\boldsymbol{M}}^{-1}\varphi_{/q}^{T}\left(\varphi_{/q}\hat{\boldsymbol{M}}^{-1}\varphi_{/q}^{T}\right)^{-1}\ddot{\alpha}$$

in any case, minimum (weighted) norm solutions; then one can add arbitrary (position /) velocity (/ acceleration) in the nullspace of $\varphi_{/q}^{+}$

$$\dot{\boldsymbol{q}}=\varphi_{/q}^{+}\dot{\alpha}+\check{\omega},\quad \check{\omega}=\left(\boldsymbol{I}-\varphi_{/q}^{+}\varphi_{/q}\right)\omega$$

Motion Prescription: Underdetermined

When n. prescribed dof < n. motors, problem is *underdetermined*;

Problem can be split in staggered sequence of:

- configuration (nonlinear)

$$\varphi(\boldsymbol{q})=\alpha\,,\quad \varphi_{/q}\Delta\boldsymbol{q}=\alpha-\varphi(\boldsymbol{q}),\quad \Delta\boldsymbol{q}=\varphi_{/q}^{+}(\alpha-\varphi(\boldsymbol{q}))$$

- velocity (linear)

$$\varphi_{/q}\dot{\boldsymbol{q}}=\dot{\alpha}\,,\quad \dot{\boldsymbol{q}}=\varphi_{/q}^{+}\dot{\alpha}$$

- acceleration (linear)

$$\varphi_{/q}\ddot{\boldsymbol{q}}=\ddot{\alpha}\,,\quad \ddot{\boldsymbol{q}}=\varphi_{/q}^{+}\ddot{\alpha}$$

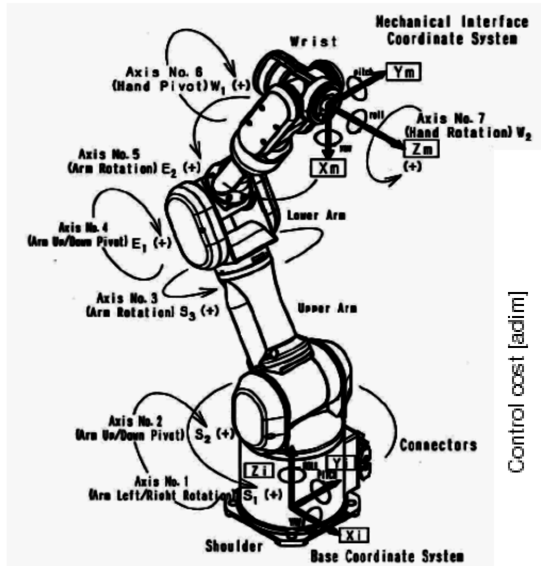
All problems share same matrix (only needs be updated during config.)

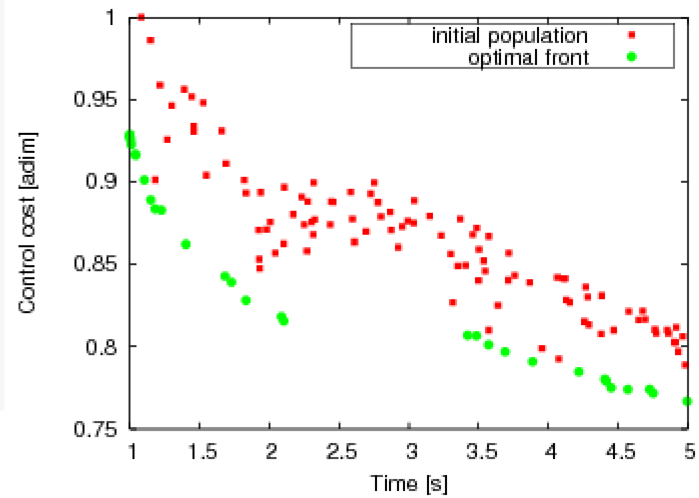
Different pseudo-inverses can be used in different phases: ergonomy, minimum kinetic energy change, minimum torque, ...

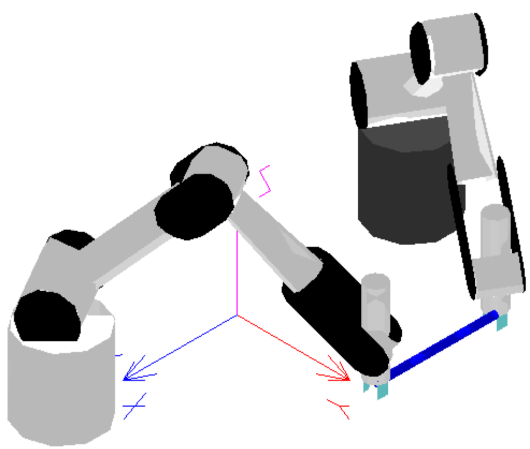
Examples of manipulator multibody modeling with MBDyn

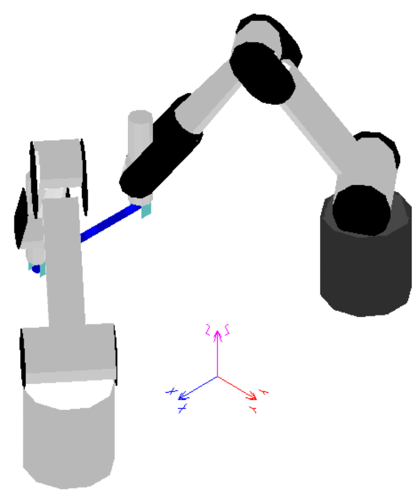
Robotics: PA-10

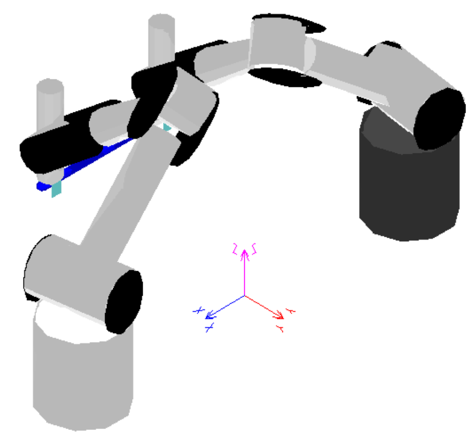
7 dof, up to 6 prescribed motion eqs.







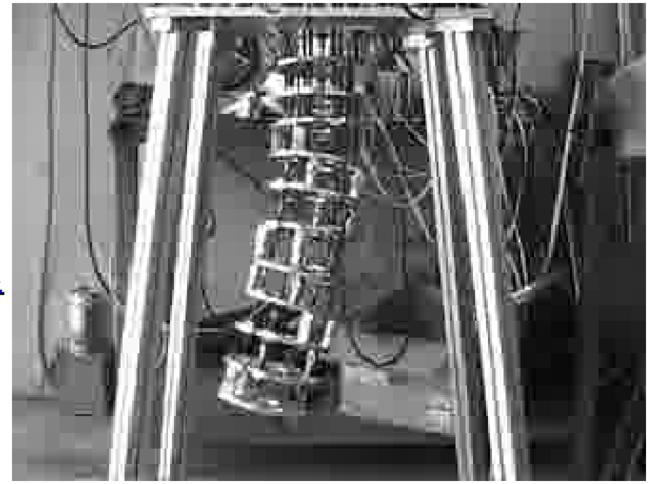


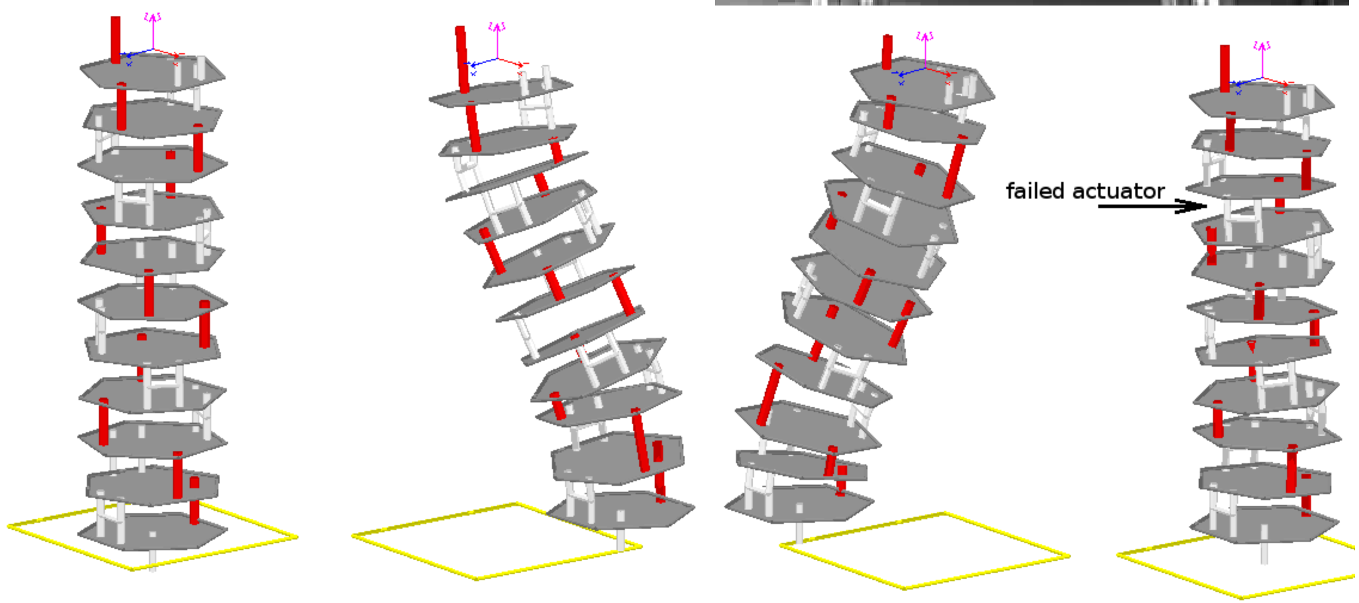


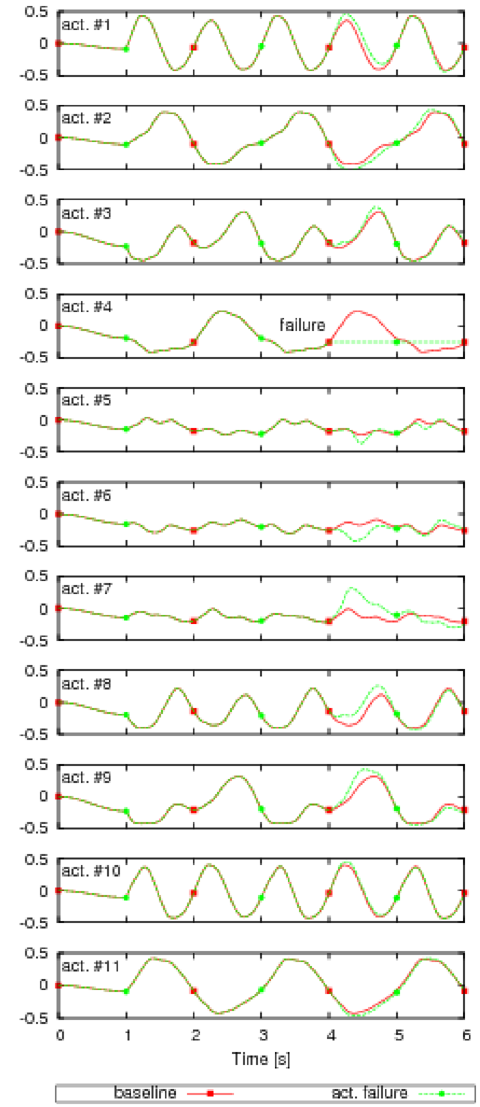
Examples of manipulator multibody modeling with MBDyn

biomimetic robot

11 dof, up to 6 prescribed motion eqs.



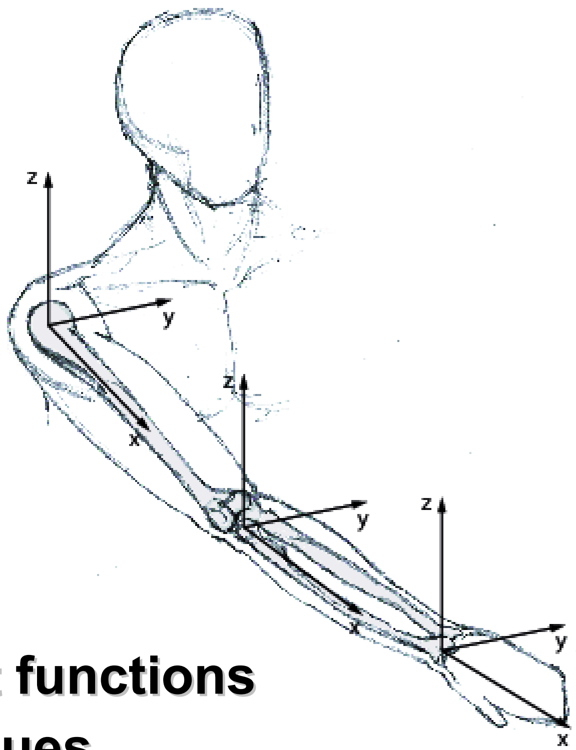




Examples of manipulator multibody modeling with MBDyn

Human arm

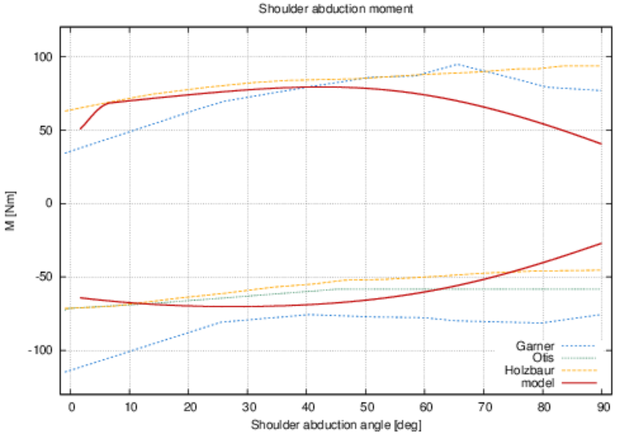
7 dof, up to 6 prescribed hand motion eqs.



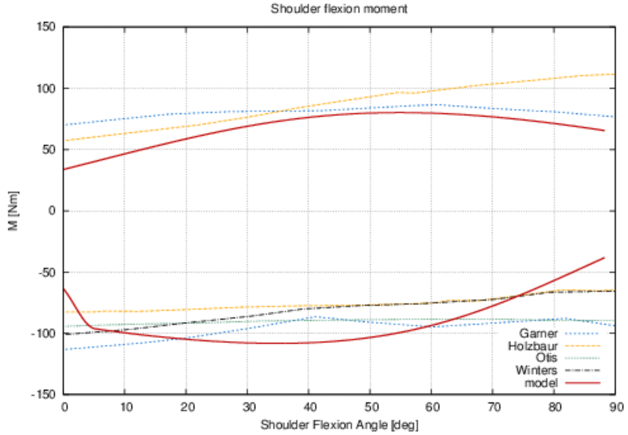
- **inverse kinematics with ergonomy cost functions**
- **inverse dynamics to compute joint torques**
- **optimization to compute muscular activation**

Examples of manipulator multibody modeling with MBDyn

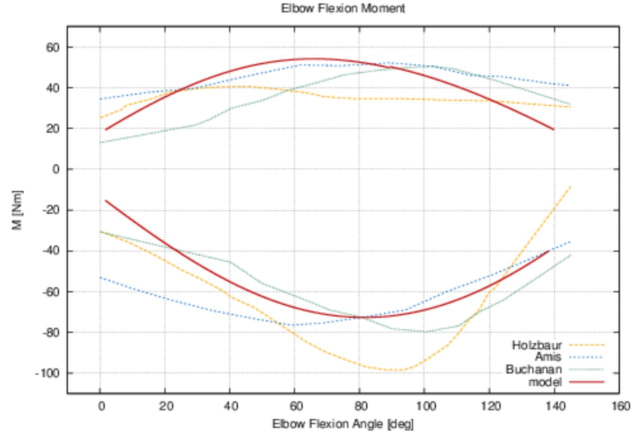
shoulder abduction



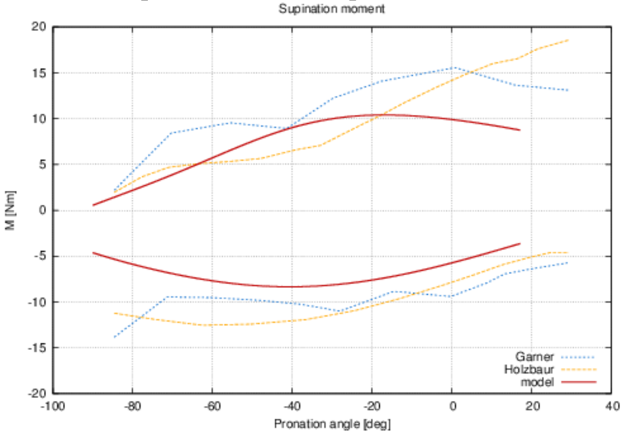
shoulder flexion



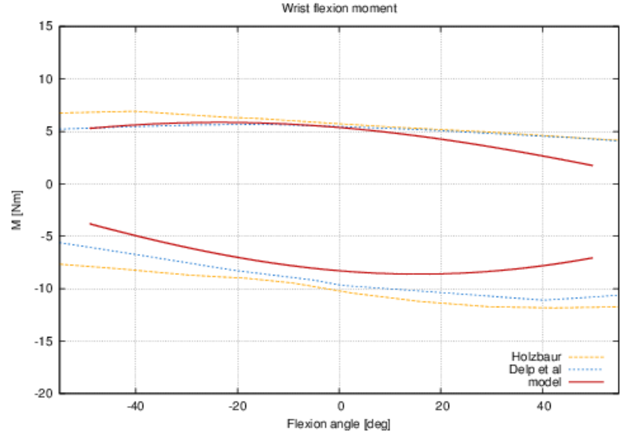
elbow flexion



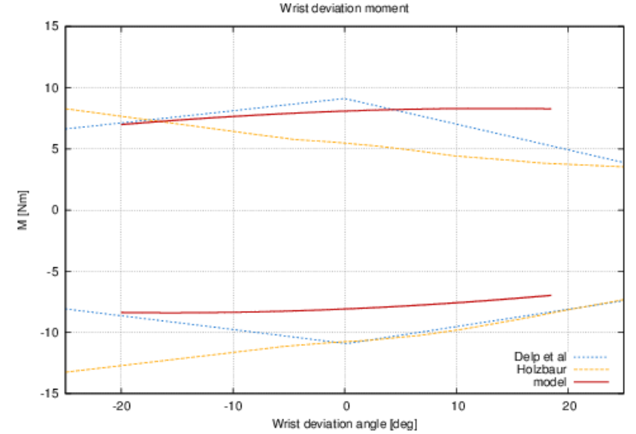
prono-supination



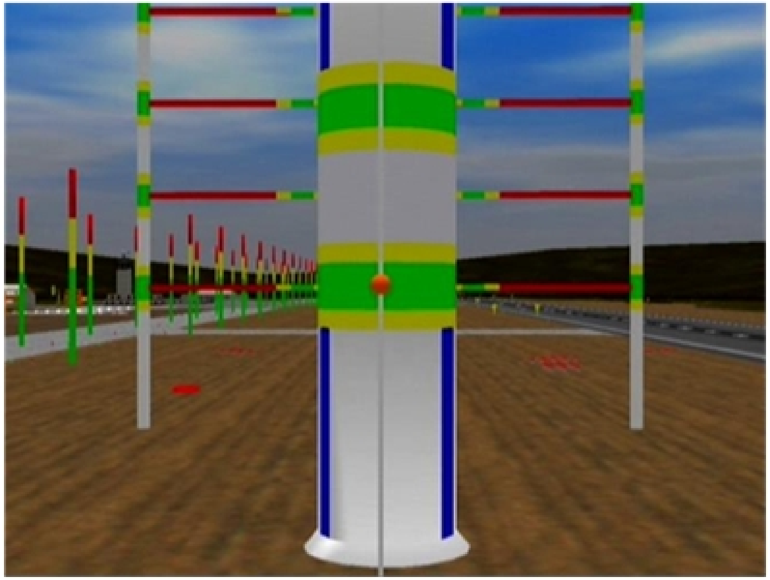
wrist flexion

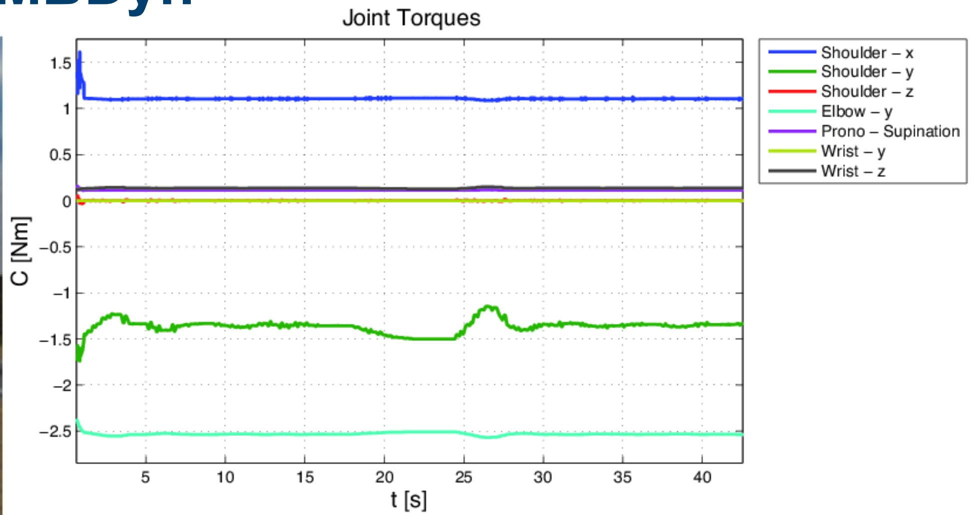


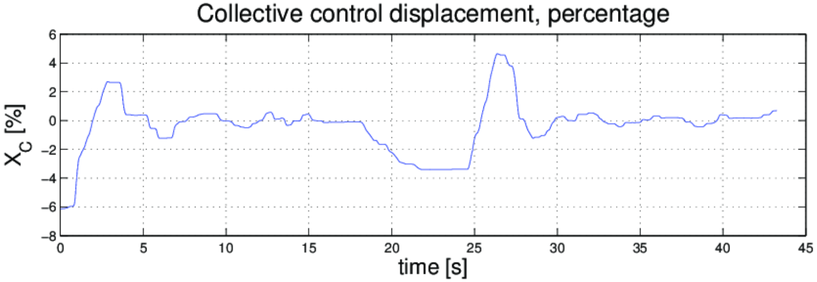
wrist deviation

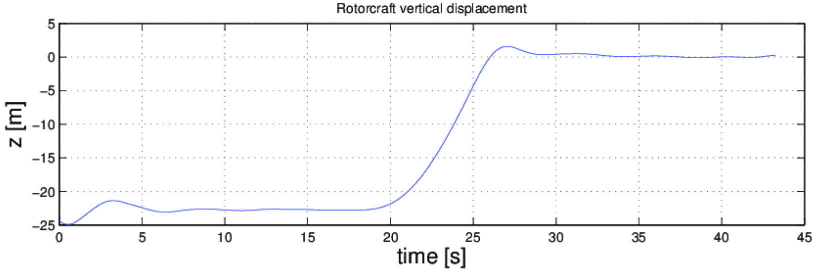


Examples of manipulator multibody modeling with MBDyn

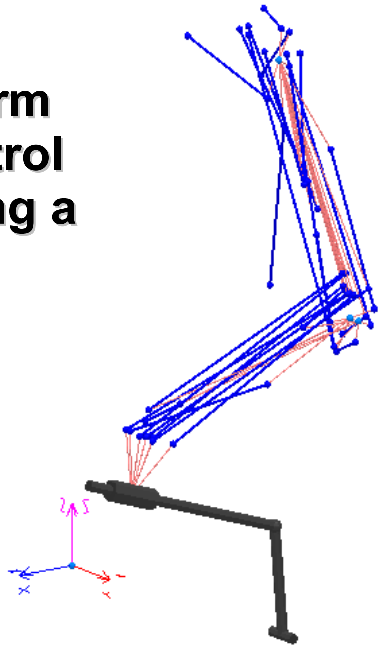








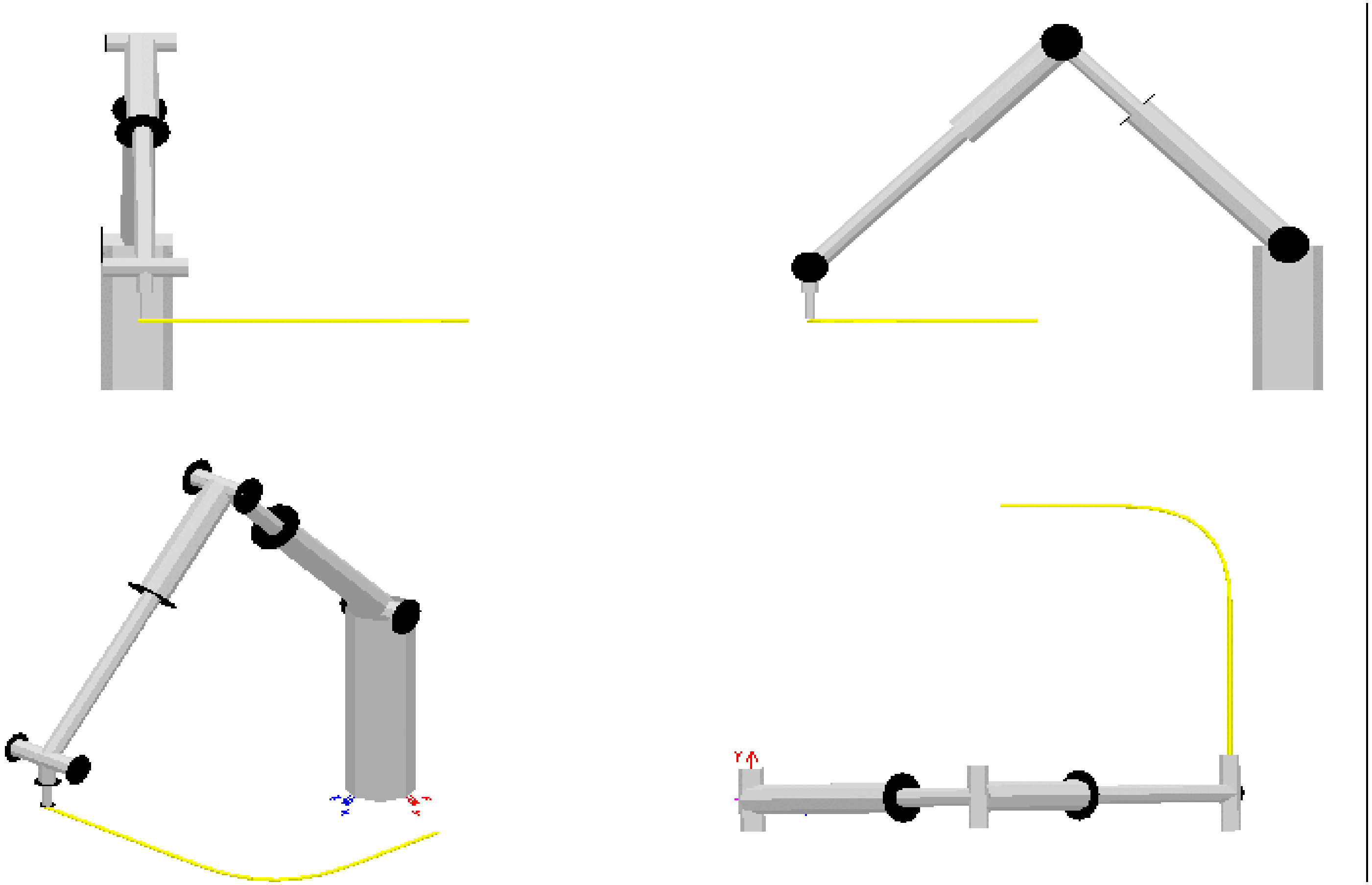
helicopter pilot's left arm holding collective control inceptor and performing a vertical repositioning maneuver



Examples of manipulator multibody modeling with MBDyn

PA 10 robot doing corner smoothing trajectory

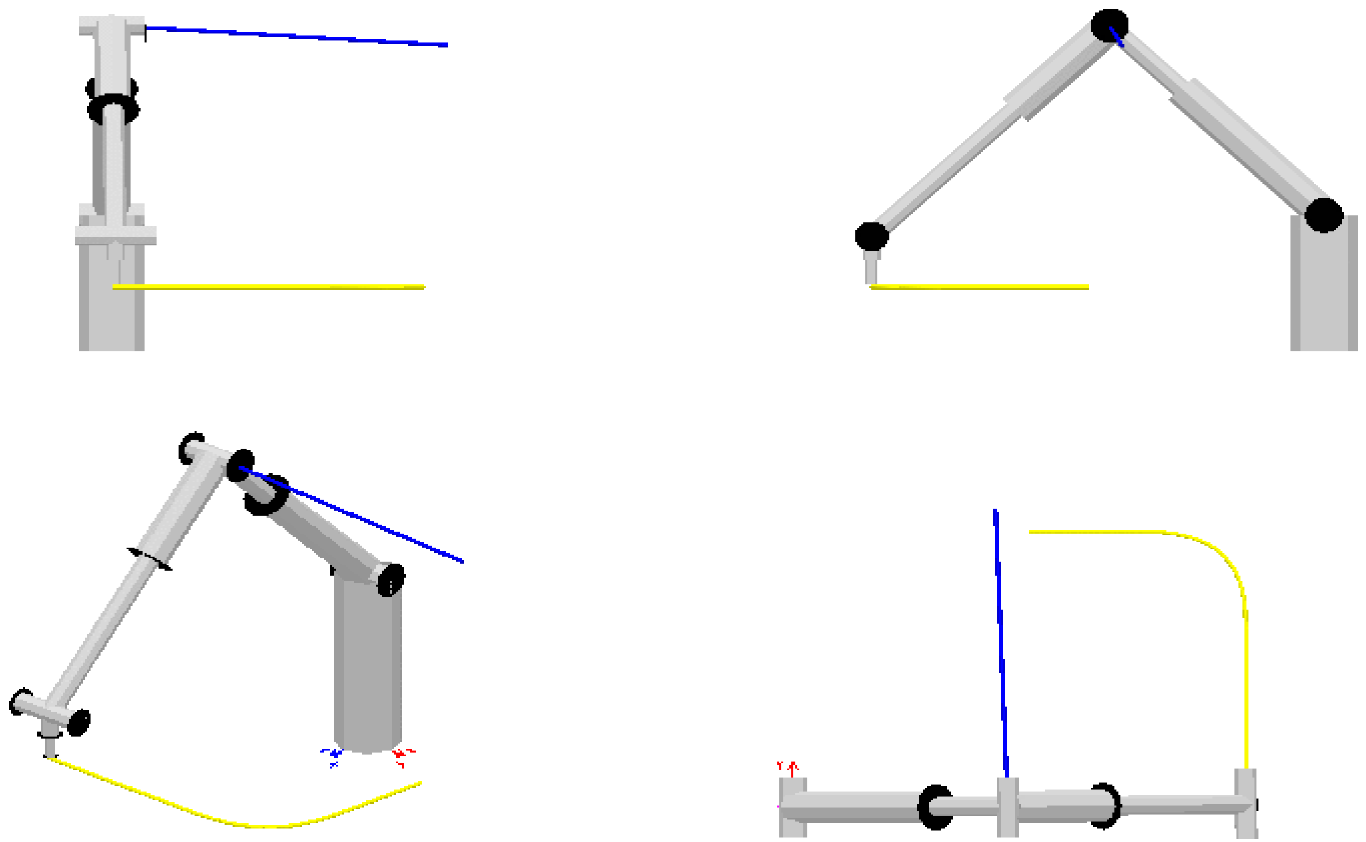
7 dofs, 5 prescribed motion eqs.



Examples of manipulator multibody modeling with MBDyn

PA 10 robot doing corner smoothing trajectory and obstacle avoidance

7 dofs, 5 prescribed motion eqs.



Motion Prescription: Underactuated

When n. prescribed dof = n. motors < n. dofs, problem is *underactuated*:

$$\hat{B}^T \neq \varphi_{/q}^T$$

$$c = \left(\varphi_{/q} \hat{M}^{-1} \hat{B}^T\right)^{-1} \left(\ddot{\alpha} - \varphi_{/q} \hat{M}^{-1} \hat{f}\right)$$

$$= \hat{P}^{-1} \left(\ddot{\alpha} - \varphi_{/q} \hat{M}^{-1} \hat{f}\right)$$

invertible?

Motion Prescription: Underactuated

When n. prescribed dof = n. motors < n. dofs, problem is *underactuated*:

$$\hat{P} = \varphi_{/q} \hat{M}^{-1} \hat{B}^T$$

Consider a QR decomposition $\varphi_{/q}^T = Q\,R = [Q_1 Q_2] \begin{bmatrix} R_1 \\ 0 \end{bmatrix} = Q_1 R_1$

then $\hat{P} = \varphi_{/q} \hat{M}^{-1} \hat{B}^T = R_1^T Q_1^T \hat{M}^{-1} \hat{B}^T$

consider now the equality

$$\hat{B}^T = \hat{M}\,Q\,Q^T \hat{M}^{-1} \hat{B}^T = \hat{M}\,[Q_1 Q_2] \begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix} \hat{M}^{-1} \hat{B}^T = \hat{M}\left(Q_1 Q_1^T + Q_2 Q_2^T\right) \hat{M}^{-1} \hat{B}^T = \hat{B}_\perp^T + \hat{B}_\parallel^T$$

parallel to constraint manifold

then $\hat{P} = \varphi_{/q} \hat{M}^{-1} \hat{B}^T = R_1^T Q_1^T \hat{M}^{-1} \hat{B}^T = R_1^T Q_1^T \hat{M}^{-1} \hat{B}_\perp^T$

normal to constraint manifold

Motion Prescription: Underactuated

When n. prescribed dof = n. motors < n. dofs, problem is ***underactuated***:

If $\hat{P} = \varphi_{/q} \hat{M}^{-1} \hat{B}^T$ is singular, tangent realization of control is needed.

Several techniques have been proposed, all essentially based on differential flatness (staggered differentiation and substitution to affect constraint equation via control forces through other than inertia forces)

"clever" approach: when elastic forces are present,

$$\hat{M} \ddot{q} = \hat{B}^T c - \hat{K} q$$

numerical solution using implicit scheme: $\Delta q = (h b_0)^2 \Delta \ddot{q}$

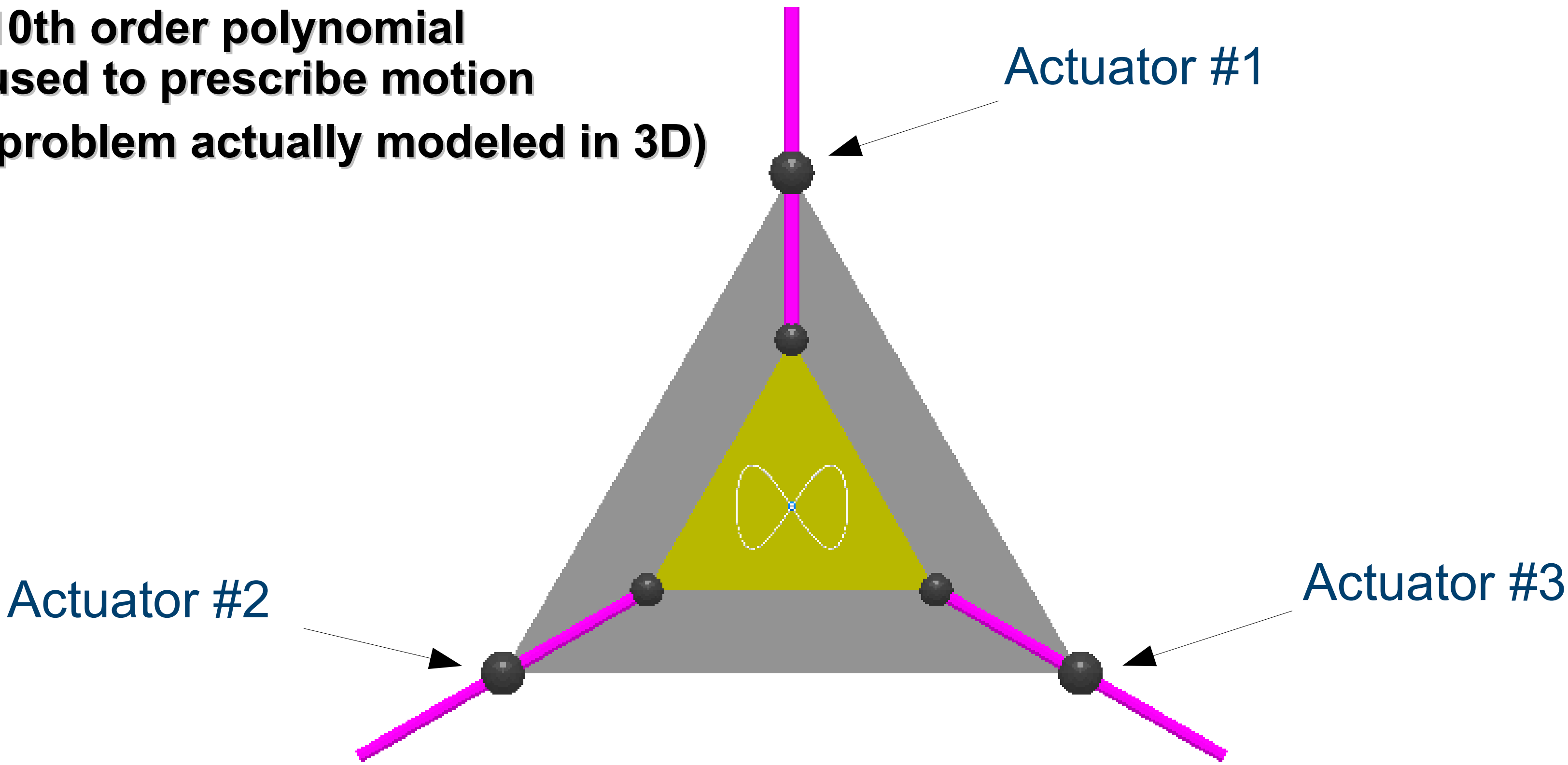
$$\left(\hat{M} + (h b_0)^2 \hat{K}\right) \Delta \ddot{q} = \hat{r}$$

now $\hat{P}^* = \varphi_{/q} \left(\hat{M} + (h b_0)^2 \hat{K}\right)^{-1} \hat{B}^T$ non-singular when matrix pencil is not!

Examples of manipulator multibody modeling with MBDyn

Inspired from Betsch et al., 2008 & 2010

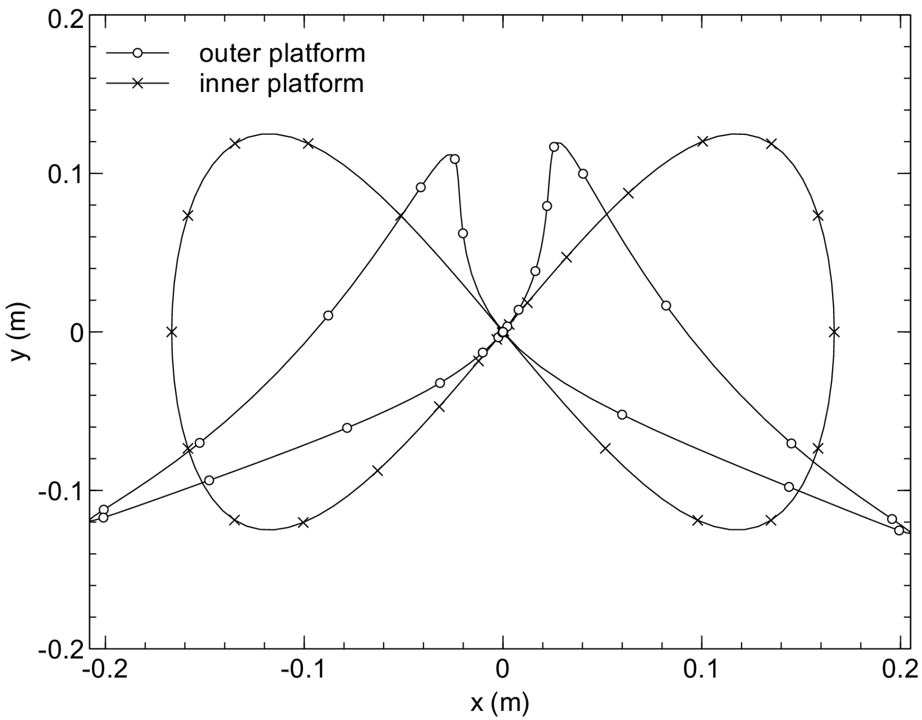
- **three torque motors**
- **links can slide through motors**
- **prescribed lemniscate ("eight"-shaped) trajectory of smaller triangle**
- **10th order polynomial used to prescribe motion**
- **(problem actually modeled in 3D)**

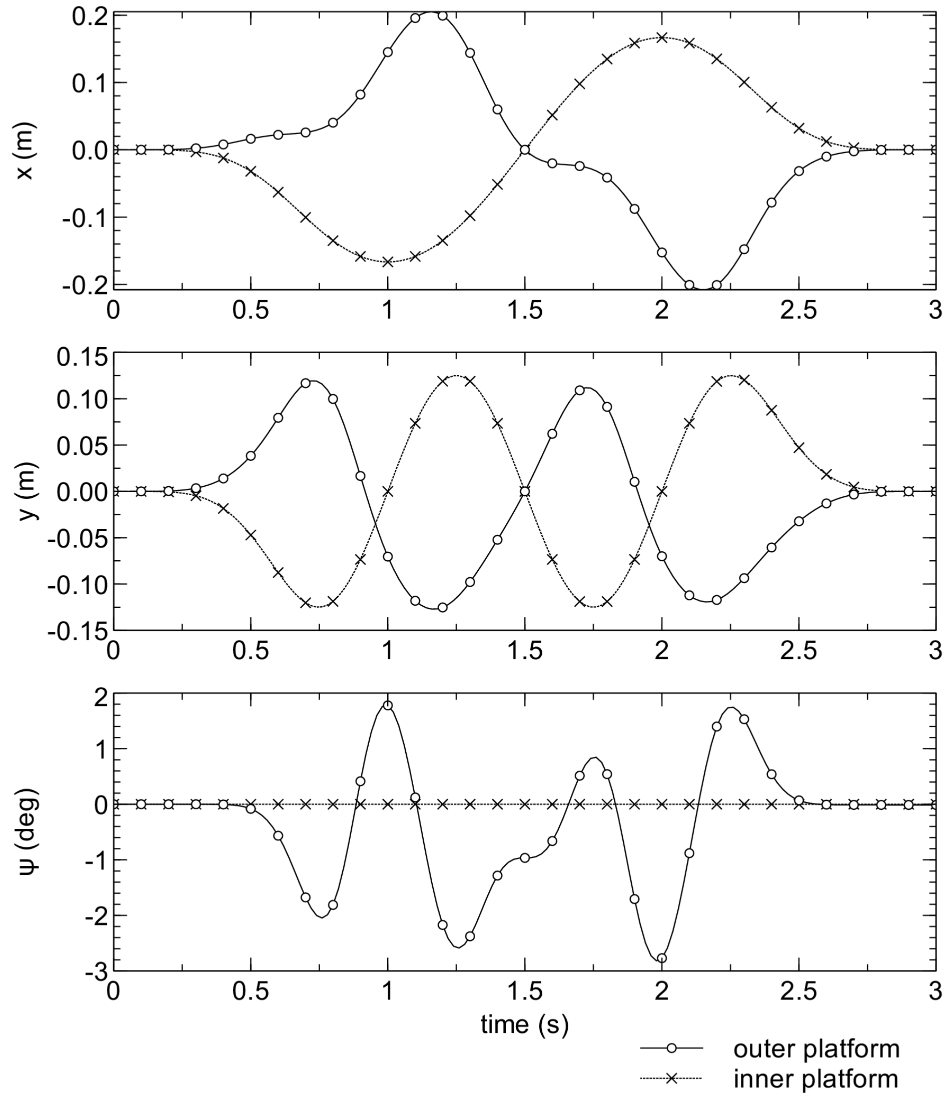


<http://www.aero.polimi.it/masarati/Download/mbdyn/images/triangle2.gif>

Examples of manipulator multibody modeling with MBDyn

Feedforward verification with predicted motor rotations: trajectories of triangles

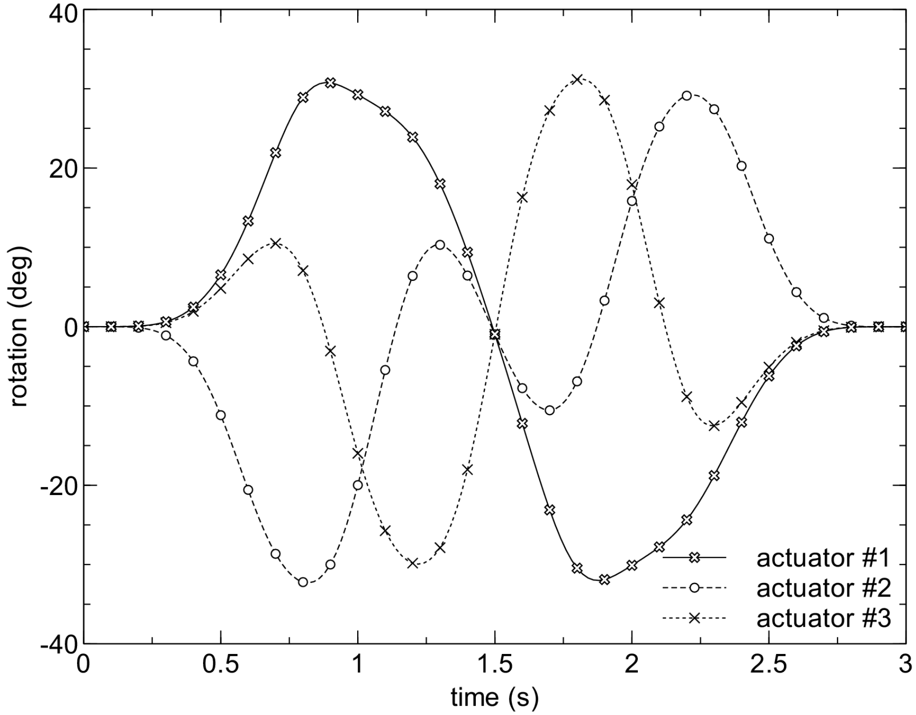


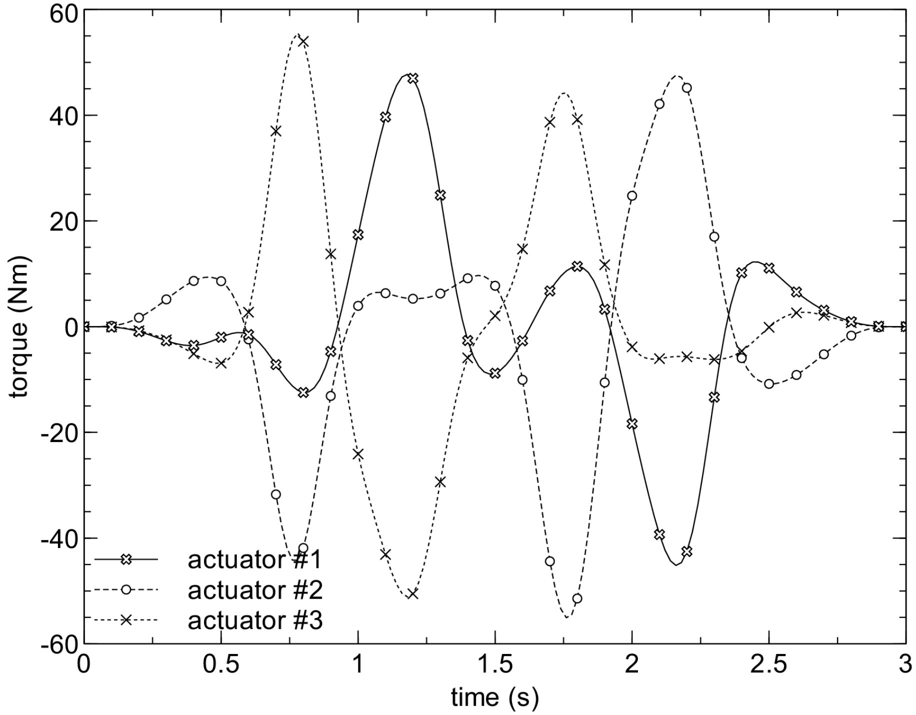


P. Masarati, M. Morandini, A. Fumagalli, "Control Constraint Realization Applied to Underactuated Aerospace Systems", ASME 2011 IDETC/CIE August 28-31, 2011, Washington DC (DETC2011-47276).

Examples of manipulator multibody modeling with MBDyn

Feedforward verification with predicted motor rotations: motor rotations and torques





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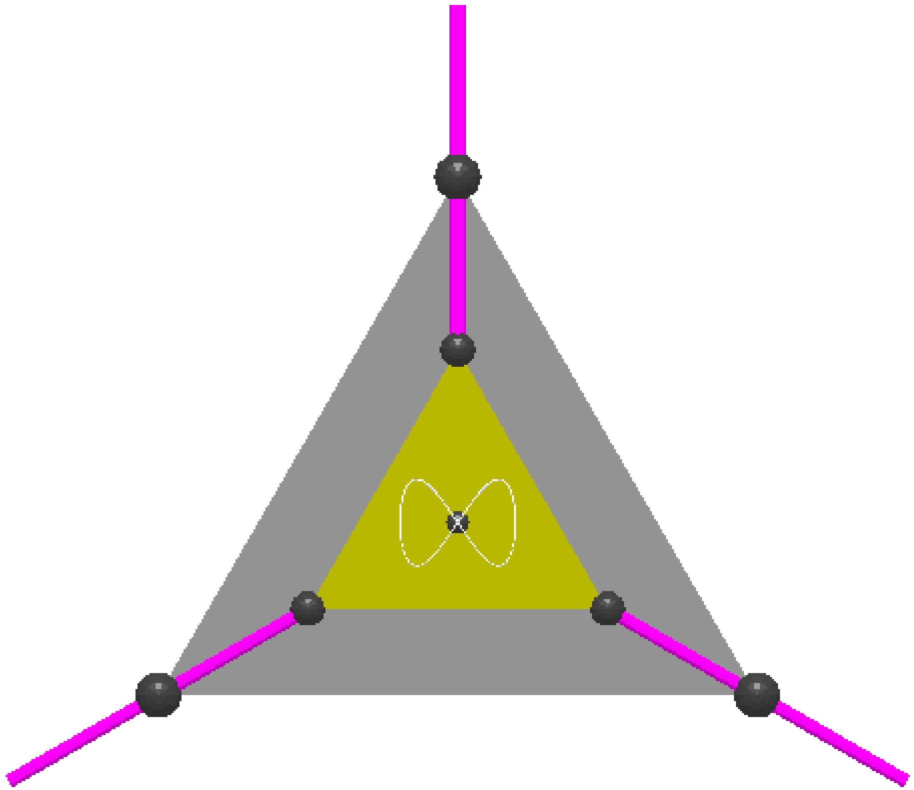
Examples of manipulator multibody modeling with MBDyn

Feedforward verification with predicted motor rotations: animation



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Feedforward/feedback

Motion planning: determine joint motion from end effector motion

- planned joint motion can be prescribed through localized control
- feedforward can improve quality of tracking

Torque demand as a function of acceleration: $c = \hat{M}\ddot{q} - \hat{f}$

when acceleration for torque demand is desired acceleration: $c_{ff} = \hat{M}\ddot{q}_d - \hat{f}$

when acceleration for torque demand is

$$\ddot{q} = \ddot{q}_d + K_D\left(\dot{q}_d - \dot{q}\right) + K_P\left(q_d - q\right)$$

torque becomes

$$c_{fb} = \hat{M}\left(\ddot{q}_d + K_D\left(\dot{q}_d - \dot{q}\right) + K_P\left(q_d - q\right)\right) - \hat{f}$$

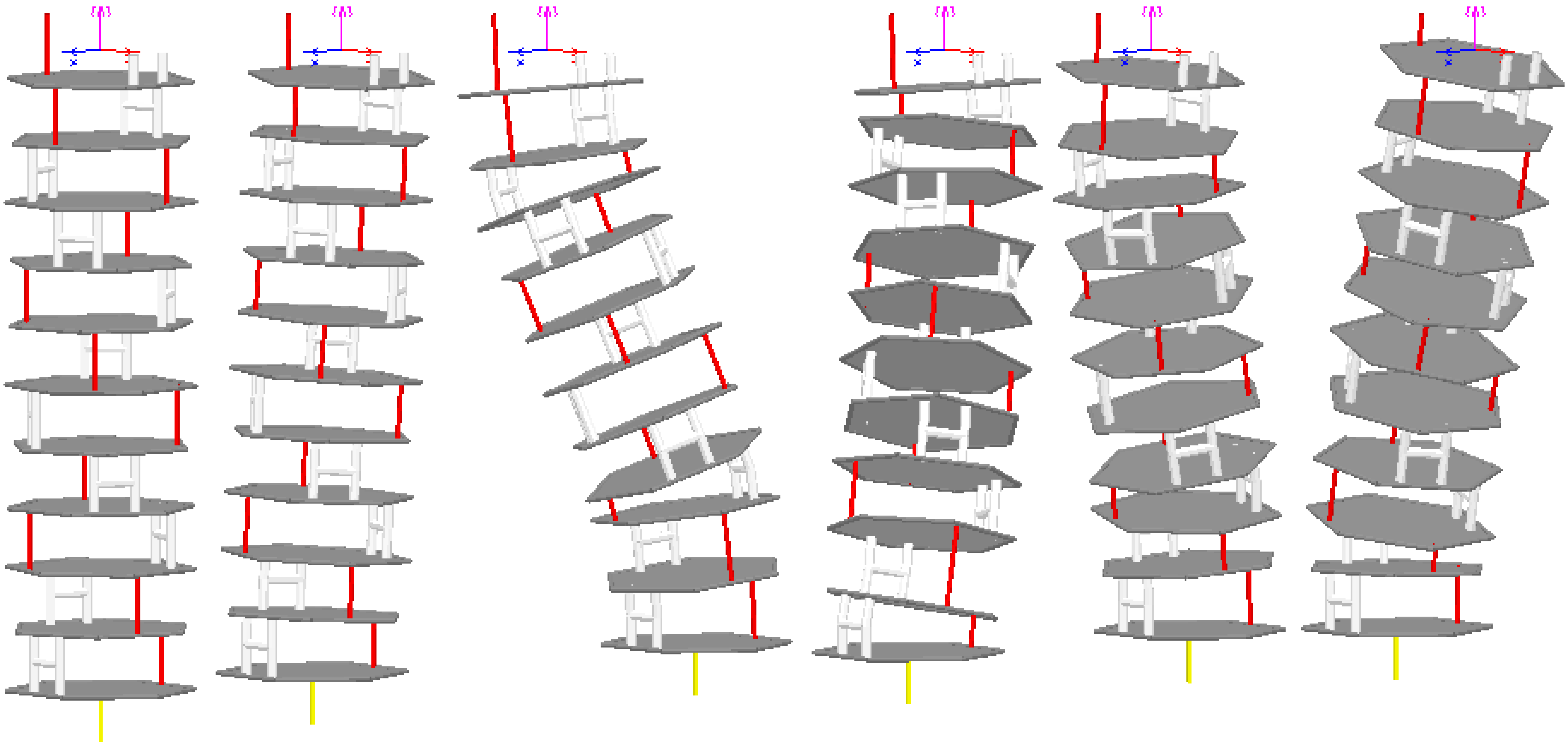
and dynamics become

$$\hat{M}\left(\left(\ddot{q}_d - \ddot{q}\right) + K_D\left(\dot{q}_d - \dot{q}\right) + K_P\left(q_d - q\right)\right) = 0$$

appropriate choice of coefficients yields asymptotic error cancellation

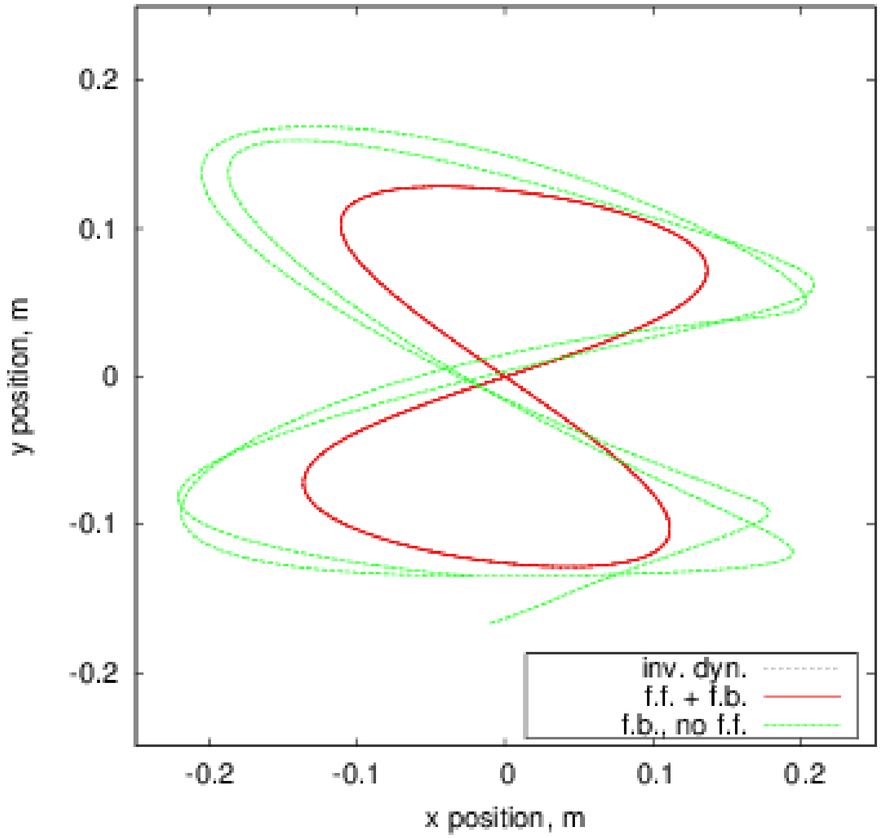
Feedforward/feedback

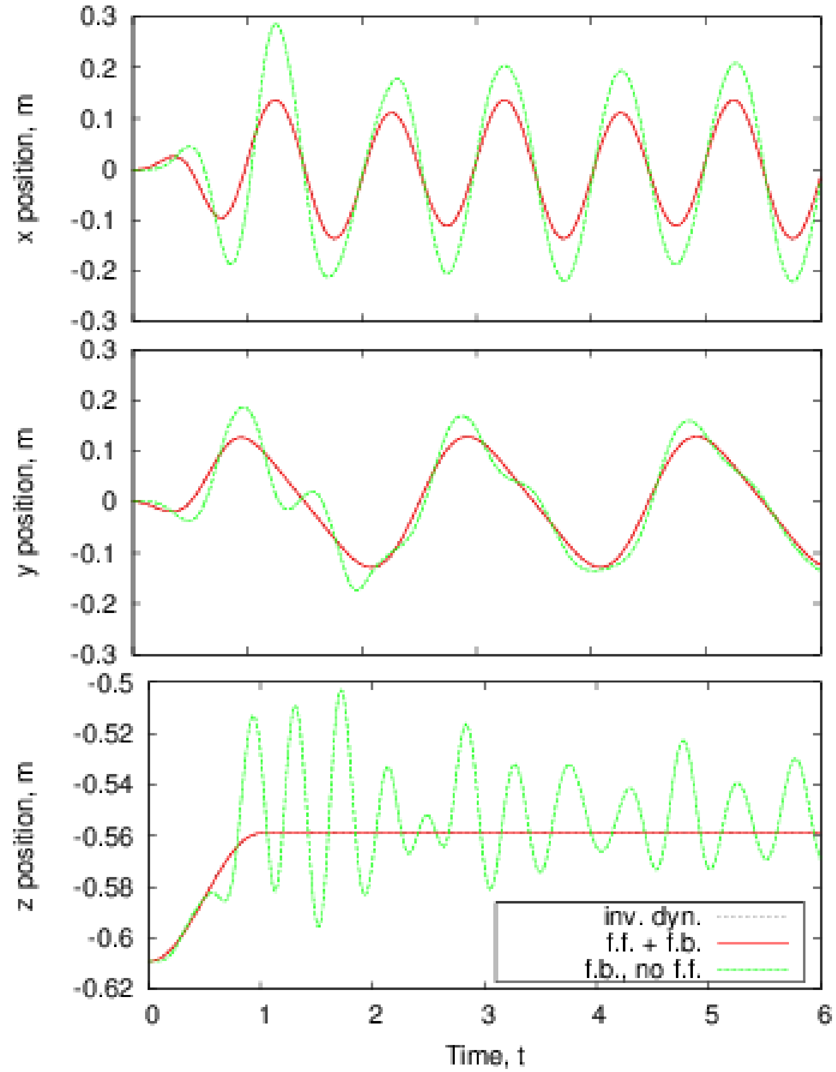
Biomimetic manipulator: ***11 dof, 5 prescribed motion eqs.***



Feedforward/feedback

Biomimetic manipulator: verification with and without feedforward (same gains)

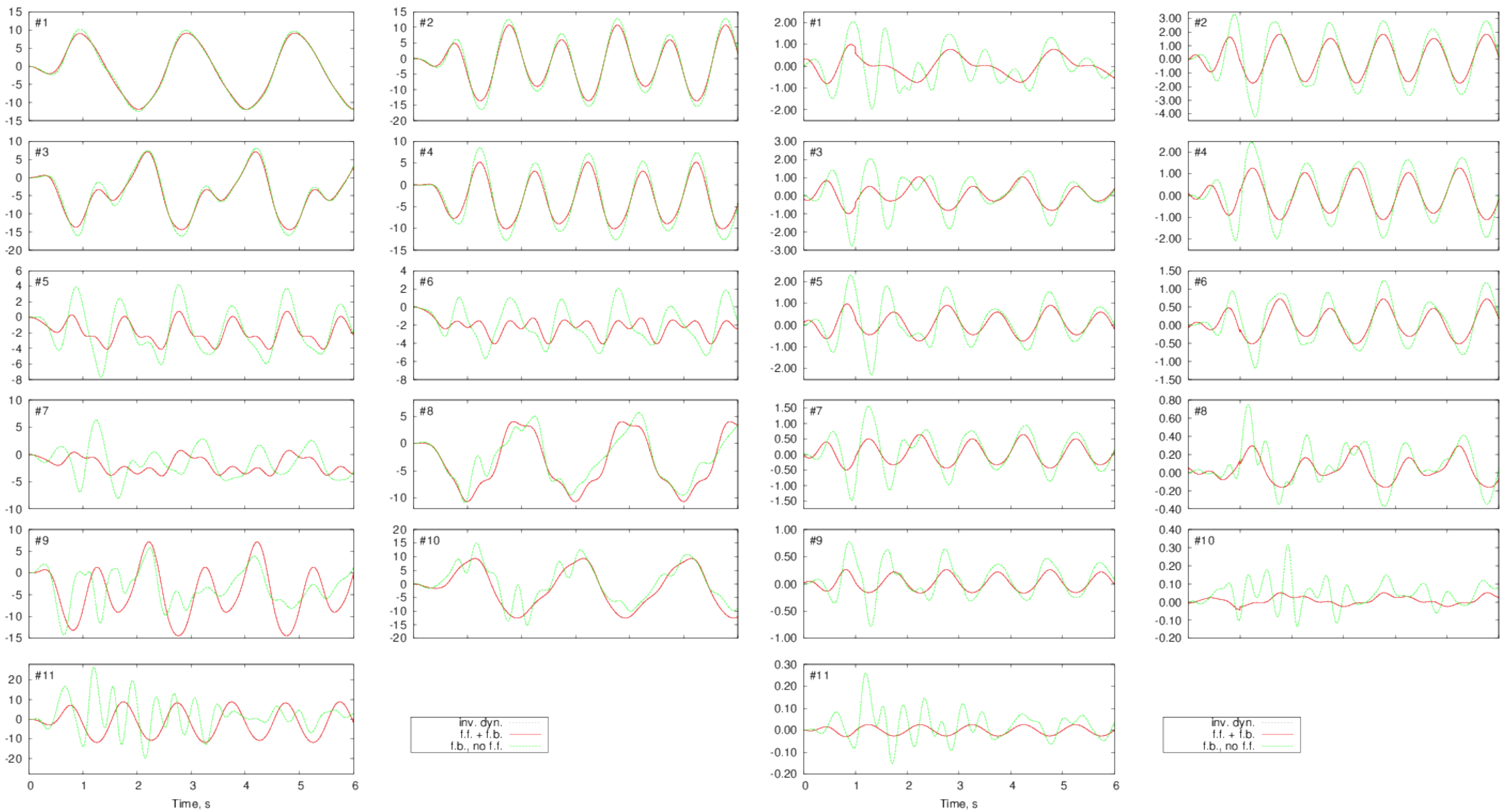




Feedforward/feedback

Biomimetic manipulator: verification with and without feedforward

angles **torques**



Questions?