Optimal Coordination of Directional Overcurrent Relays Considering Different Network Topologies Using Interval Linear Programming

Abbas Saberi Noghabi, Habib Rajabi Mashhadi, *Member, IEEE*, and Javad Sadeh, *Member, IEEE*

***Abstract*—In real power systems, the network topology is subjected to uncertainty due to single-line outage contingencies, maintenance activities, and network reconfigurations. These changes in the network topology may lead to miscoordination of directional overcurrent relays (DOCRs). To overcome this drawback, corresponding to each primary/backup relay pair, a set of inequality coordination constraints which is related to different network topologies should be satisfied. In this paper, a new approach based on the interval analysis is introduced to solve the DOCRs coordination problem considering uncertainty in the network topology. The basic idea is to convert the set of inequality constraints corresponding to each relay pair to an interval constraint. In this situation, the DOCR coordination problem is formulated as an interval linear programming (ILP) problem. Using well-known mathematical theorems, the obtained ILP problem, which has no equality constraints, can be converted to standard linear programming (LP). As a result, the number of coordination constraints is significantly reduced in the proposed methods. The application of the proposed method to the IEEE 14- and 30-bus test systems proves the ability of the interval method in modeling topology uncertainty in the large-scale coordination problem.**

***Index Terms*—Different network topologies, interval analysis, interval linear programming (ILP), overcurrent relay coordination.**

I. Introduction

DIRECTIONAL overcurrent relays (DOCRs) are commonly used as an economic alternative for the primary protection of distribution and subtransmission systems and as secondary protection of transmission systems [1]–[3]. The problem of relay coordination consists in determining the pickup current setting (I_P) and the time dial setting (TDS) for overcurrent relays in order for the protective system to react fast and be selective and reliable due to different fault conditions [1]. In this problem, the main objective is to minimize the operating time of DOCRs so that the set of coordination constraints is satisfied.

The DOCRs coordination problem can be formulated as a mixed integer nonlinear programming (MINLP) when the pickup current settings are considered integer variables and the time dial settings are continuous. This problem for a large multiloop-interconnected network has been recognized as a difficult optimization problem [1]. In addition, the network topology is subjected to uncertainty due to maintenance activities, network reconfigurations, and switching actions after faults. The change in the network topology may lead to miscoordination of directional overcurrent relays because of variation of the fault current level [2]–[4]. In order to consider the effects of the different network topologies, corresponding to each topology, a set of inequality constraints should be added in the main coordination problem. In this situation, the studied optimization problem experiences many coordination constraints [5], [6]. A hybrid GA method is introduced in [5] to solve the coordination problem in the case of different network topologies. In this method, the pickup current settings of all relays are coded into the genetic string as discrete variables. The LP is used, as a subroutine for fitness calculation, to determine optimal TDS of relays as continuous variables for each genetic string.

Due to several advantage of the LP, the DOCRs coordination problem is commonly modeled and solved by using linear optimization for given values of pickup currents [2]–[4]. In recent years, several attempts have been made to improve the computation efficiency of LP in solving the DOCRs coordination problem [7]–[9]. The presolution filtering techniques are proposed in [7] in order to reduce the dimension of the coordination problem and the number of constraints. A preprocessing method is introduced in [8] to reduce the number of constraints and detect those constraints which cause the coordination problem to be infeasible. In [9], a new algorithm is developed based on LP to reduce the effects of coordination constraints on the minimization of overall operating times with the optimal selection of pickup current.

Recently, interval analysis techniques have found interesting applications to model parametric uncertainty in power system operation [10]–[12]. In fact, the interval method provides a useful tool for modeling "unknown but bounded" uncertainties. In [10], interval computations are used to solve the load-flow problem considering uncertainty in transmission line and transformer parameters. The power-flow analysis in the electric energy market is studied in [11] based on interval uncertainties in parameters of network elements. In [12], a novel fault-location scheme is proposed based on the interval analysis and evidence theory for railway power transmission lines.

Considering different network topologies, the relay coordination problem can be formulated as a linear program with a large number of inequality constraints. Corresponding to each

Manuscript received July 22, 2009; revised October 29, 2009. Current version published June 23, 2010. Paper no. TPWRD-00545-2009.

The authors are with the Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad 9177948944, Iran (e-mail: ab_sa82@stu-mail.um.ac.ir; h_mashhadi@um.ac.ir; sadeh@um.ac.ir).

Digital Object Identifier 10.1109/TPWRD.2010.2041560

primary/backup relay pair, a set of inequality coordination constraints which is related to different network topologies should be satisfied. In this paper, a new approach based on interval analysis is introduced to model this problem as interval linear programming. The basic idea is to convert the set of inequality constraints corresponding to each relay pair to an interval constraint. Using this approach, the uncertainty in the constraints' coefficients which occurs due to a change in the network topology is taken into account by a new ILP formulation. Based on the well-known mathematical theorems, the obtained ILP problem, which has no equality constraints, can be converted to a standard LP. The number of coordination constraints in the new formulation is significantly reduced and will be the same as the number of coordination constraints in the fixed network topology formulation. The proposed method is applied to the IEEE 14-bus and 30-bus test systems and the results show the ability of the interval method to model the uncertainty due to network topology variation.

The rest of this paper is organized as follows. In Section II, the ILP approach is described. In Section III, the coordination problem in the fixed and different network topologies is presented. In Section IV, the DOCR coordination problem considering topology uncertainty is formulated as an ILP problem. The proposed method is used to solve the DOCR coordination problem for IEEE 14-bus and 30-bus test systems in Section V. Finally, the conclusion is presented in Section VI.

II. ILP Optimization

Interval analysis provides a useful tool for modeling "unknown but bounded" uncertainties when upper and lower limits on the uncertainties are assumed to be known. In this section, basic concepts of interval analysis and a detailed survey of ILP, including interval inequality constraints, are presented [13]. For the sake of clarity, the interval matrix and variable are marked with superscript "I" in order to be distinct from exact values.

Definition 1: If $A=(a_{ij})$ and $B=(b_{ij})$ are two matrices in $R^{m\times n}$ and for all i and j $a_{ij}\le b_{ij}$, then we called $A\le B$.

Definition 2: If $\overline{A}$ and $\underline{A}$ are two matrices in $R^{m\times n}$ and $\underline{A}\le\overline{A}$, then the set of matrices $A^I=[\underline{A},\overline{A}]=\{A:\underline{A}\le A\le\overline{A}\}$ is called an interval matrix, while the matrices $\overline{A}$ and $\underline{A}$ are called its bounds. Similarly, an interval vector b^I is a one-column interval matrix $b^I=[\underline{b},\overline{b}]=\{b:\underline{b}\le b\le\overline{b}\}$ where $\underline{b},\overline{b}\in R^m$ and $\underline{b}\le\overline{b}$.

Definition 3: The interval linear inequality

$$A^IX\le b^I \tag{1}$$

is defined as a family of all linear inequality $AX\le b$, where $A\in A^I$ and $b\in b^I$. The exact vector $X\in R^n$ is called the solution of interval linear inequality.

Now, based on these definitions, the concept of solvability for an interval linear inequality (1) can be defined as follows.

1) Equation (1) is called weakly solvable if for each $A\in A^I$ and $b\in b^I$, the equation $AX\le b$ has a solution, which is generally dependent on A and b.
2) Equation (1) is called strongly solvable if X_0 exists, satisfying $AX_0\le b$ for each $A\in A^I$ and $b\in b^I$ (i.e., all of the equations $AX\le b$ have a solution in common), and X_0 is a strong solution of (1).

The following theorems give us a systematic method to check solvability and to find the strong solution of an interval linear inequality.

Theorem 1: The interval linear inequality is weakly solvable if and only if it is strongly solvable.

Theorem 2: The interval linear inequality (1) is strongly solvable if and only if the inequalities

$$\overline{A}X_1-\underline{A}X_2\le\underline{b} \tag{2a}$$
$$X_1\ge 0\quad X_2\ge 0 \tag{2b}$$

have a common solution and $X=X_1-X_2$ is a strong solution of (1).

The set of all strong solutions of (1) is denoted by X_S as

$$X_S=\{X_1-X_2:\overline{A}X_1-\underline{A}X_2\le\underline{b},\quad X_1\ge 0,\quad X_2\ge 0\}. \tag{3}$$

Based on these preliminaries, it is time to state the main problem (i.e., the ILP problem):

Definition 4: The ILP problem is defined as follows:
Find strong solution X
So that

$$\text{Minimize}\quad (C^I)^TX \tag{4a}$$
$$\text{Subject to}:\quad A^IX\le b^I \tag{4b}$$

where A^I, b^I, and C^I are the interval matrix and vectors.

Theorem 3: The optimum value (optimistic/pessimistic) of the ILP problem (4) and its strong solution can be obtained by solving the following standard linear programming problem:

$$\text{Minimize}\quad (\overline{C}^T or \underline{C}^T)X \tag{5a}$$
$$\text{Subject to}:\quad \overline{A}X_1-\underline{A}X_2\le\underline{b} \tag{5b}$$
$$X_1\ge 0\quad X_2\ge 0 \tag{5c}$$
$$X=X_1-X_2. \tag{5d}$$

In the rest of this paper, the DOCRs coordination problem is formulated as an ILP problem and its solution is found by using the previously shown theorems.

III. DOCRs Coordination Problem

In the DOCRs coordination problem, the main objective is to determine the pickup current setting and TDS of each relay so that the sum of the operating times of the primary relays for near-end faults is minimized and the set of coordination constraints is satisfied. In this section, the coordination problem in fixed and different network topologies is presented.

A. Fixed Network Topology Formulation

Mathematically, the coordination problem in the fixed network topology (namely main topology) can be formulated as an optimization problem as follows:

$$\text{Minimized}:\quad J=\sum_{i=1}^{n}t_i \tag{6}$$
$$\text{Subject to}:\quad t_j-t_i\ge \text{CTI}\qquad \forall(i,j)\in\Omega. \tag{7}$$
$$TDS_i^{\min}\le TDS_i\le TDS_i^{\max}\qquad i=1,\ldots,n \tag{8}$$

where n is the number of relays, and Ω is the set of primary/backup relay pairs in the main network topology. Corresponding to each primary/backup relay pair, there is an inequality in the form of (7), in which t_i and t_j indicate the operating times of primary relay i and backup relay j, respectively, for the near-end fault. Coordination time interval (CTI) is usually assumed between 0.2 and 0.5 s. The $\mathrm{TDS}_i^{\min}$ and $\mathrm{TDS}_i^{\max}$ are the lower and upper bounds of TDS of the ith relay, respectively.

In this paper, only coordination constraints based on the near-end faults are considered in the problem formulation. However, the constraints corresponding to far-end fault and instantaneous element can be easily formulated in the proposed approach.

In general, the operating time of the ith overcurrent relay depends on the pickup current setting (I_{Pi}), TDS (TDS_i) and fault current passing through the relay (I_{fi}) as follows:

$$t_i = F_i(I_{f_i}, I_{P_i}, \mathrm{TDS}_i). \tag{9}$$

The relay characteristic is mathematically modeled by using curve-fitting techniques. Usually, the aforementioned relay characteristic is approximated by the following multiplicative form [3], [4]:

$$t_i = f_i(I_{f_i}, I_{P_i}) \times g_i(\mathrm{TDS}_i). \tag{10}$$

In (10) , $f_i(I_{fi}, I_{Pi})$ is a nonlinear function of I_{Pi} and I_{fi}. The fault currents passing through the primary and backup relays (I_{fi}, I_{fj}) are calculated when a solid three-phase short circuit occurs on the front of the primary relay (i.e., the near-end fault). The value of I_{fi} for a fixed network topology is constant and the I_{Pi} is usually determined based on the maximum load current that has passed through the relay. Based on these assumptions, the value of $f_i(I_{fi}, I_{Pi})$ is constant and can be replaced by the c_i coefficient as follows:

$$t_i = c_i \times g_i(\mathrm{TDS}_i). \tag{11}$$

Using (11), the objective function and coordination constraints in the DOCR coordination problem can be expressed as a linear function of $g_i(\mathrm{TDS}_i)$ variables as follows:

$$\text{Minimized:} \quad J = \sum_{i=1}^{n} c_i x_i \tag{12}$$

$$\text{Subject to:} \quad c_j x_j - c_i x_i \geq \mathrm{CTI} \tag{13}$$

$$x_i^{\min} \leq x_i \leq x_i^{\max}. \tag{14}$$

To simplify the notation, the $g_i(\mathrm{TDS}_i)$ is replaced by x_i. In this case, the coordination problem can be formulated in the form of standard linear programming, as follows:

$$\text{Minimized:} \quad J = C^T X = \sum_{i=1}^{n} c_i x_i \tag{15}$$

$$\text{Subject to:} \quad AX \leq b \tag{16a}$$

$$x_i^{\min} \leq x_i \leq x_i^{\max} \quad i = 1, \ldots, n \tag{16b}$$

where C is a $n \times 1$ vector, which consists of the coefficients of the objective function in (12). A is the $m \times n$ matrix consisting of the coefficients of operating time in (13), and m is the number of coordination constraints in the main topology. In this paper, the matrix A is called the coordination constraint matrix, b is the $m \times 1$ vector which is related to the coordination time interval, and X is the $n \times 1$ vector that contains of all the x_i variables. Once the auxiliary x_i variables are determined, the corresponding TDS_i can be easily calculated.

The coordination problem, which is described in this section, is formulated based on the fixed or main network topology. However, the transmission network topology may be changed due to planned outages, such as maintenance activities or unplanned outages including single-line contingencies. These changes in the network topology may lead to miss-coordination of directional overcurrent relays. To overcome this drawback, the effects of changes in the network topology are considered in the problem formulation in the next section.

B. *Different Network Topologies Formulation*

Obviously, when the network topology is changed, the short-circuit currents which passed through the relays are varied. Consequently, the operating times of primary and backup relays will be changed. To consider the effects of these changes, the new set of coordination constraints corresponding to each network topology should be added to the coordination problem.

Considering different network topologies, (7) is rewritten as follows:

$$t_j^s - t_i^s \geq \mathrm{CTI} \quad \forall (i,j) \in \Omega_s \quad \Omega_s \subset \Omega \quad \forall s \in S \tag{17}$$

where t_i^s and t_j^s indicate the operating times of primary and backup relays, respectively, in the sth network topology. In this equation, S is the set of all topologies which have been obtained under single-line outage contingencies of the main topology. The Ω_s is the set of primary/backup relay pairs in the sth network topology. Assuming the linear relation between x_i variables and the relay operation times, as described in part A of this section, (17) becomes:

$$c_j^s x_j - c_i^s x_i \geq \mathrm{CTI} \quad \forall (i,j) \in \Omega_s \quad \forall s \in S. \tag{18}$$

Also, the objective function in the case of different network topologies can be considered as follows:

$$J = \sum_s w^s J^s = \sum_s \sum_{i=1}^{n} w^s c_i^s x_i = \sum_{i=1}^{n} c_i' x_i \tag{19a}$$

$$c_i' = \sum_s w^s c_i^s \quad i = 1, 2, \ldots, n \tag{19b}$$

where w^s is a weighting coefficient corresponding to the sth network topology. Consequently, the coordination problem considering different network topologies can be stated as

$$\text{Minimized:} \quad J = C'^T X \tag{20}$$

$$\text{Subject to:} \quad A^s X \leq b \quad \forall s \in S \tag{21a}$$

$$x_i^{\min} \leq x_i \leq x_i^{\max} \quad i = 1, \ldots, n \tag{21b}$$

where C' is the $n \times 1$ vector consisting of c_i' coefficients which is obtained from (19b). The matrix A^s consists of the coefficients of operating time in (18) for the sth network topology.

IV. DOCRs Coordination Problem Using the ILP Approach

Considering different network topologies in the coordination problem, many coordination constraints corresponding to each primary/backup relay pair should be added to the optimization problem. Numerically, in IEEE 30-bus test system, the total number of coordination constraints considering different network topologies is increased to 5050 constraints compared to 160 constraints at the fixed network topology formulation. To solve the optimization problem in this section, a new method is presented based on the ILP formulation.

As discussed in the previous section, corresponding to each network topology, an inequality coordination constraint related to each primary/backup relay pair is added to the coordination problem. All of the constraints related to one pair can be expressed as only one coordination constraint with uncertain coefficients. In order to take this uncertainty into account, which is unknown but bounded, the constraint coefficient is modeled as an interval. Consequently, all of the coordination constraint matrices $A^s = (a^s_{kl})$, introduced in the previous section, are replaced by a matrix with interval elements, denoted as A^I, which is defined as follows:

$$A^I = [\underline{A},\overline{A}] = \{A : \underline{A} \leq A \leq \overline{A}\} \tag{22}$$

where

$$\overline{A} = (\overline{a}_{kl}) \qquad \overline{a}_{kl} = \max_s (a^s_{kl}) s \in S \tag{23}$$

$$\underline{A} = (\underline{a}_{kl}) \qquad \underline{a}_{kl} = \min_s (a^s_{kl}) s \in S. \tag{24}$$

The matrices $\overline{A}$ and $\underline{A}$ consist of the maximum and minimum elements of the coordination constraint matrices (A^s), respectively. The matrix A^I is the interval coordination constraint matrix which is composed of the set of all matrices between $\overline{A}$ and $\underline{A}$. Since $A^s \subset A^I$ is for every topology s, the solution, therefore, of interval linear inequality $A^I X \leq b$ satisfies the inequalities $A^s X \leq b$ for every s.

Therefore, the overall DOCR coordination problem considering topology uncertainty can be formulated as an ILP problem as (4). The interval vectors C^I and b^I in (4) are replaced by exact vectors C' and b, respectively. As explained in Section II, the strong solution of the ILP problem with inequality constraints is obtained by solving the standard LP as (5).

Thus, based on the aforementioned description, solving the DOCR coordination problem considering different network topologies using the ILP formulation is summarized in the following procedure:

Step 1) Determine the pickup current of each relay ($I_{P_i}, i = 1, \ldots, n$).

Step 2) Determine the primary/backup relay pairs (Ω_s) for the sth network topology.

Step 3) Calculate the near-end fault currents passing through all the primary and backup relays (I_{fi}, I_{fj}) based on the sth network topology.

Step 4) Make A^s for the sth network topology.

Step 5) Repeat Steps 2) to 4) for each network topology $s \in S$.

Step 6) Calculate interval matrix A^I and exact vector C'.

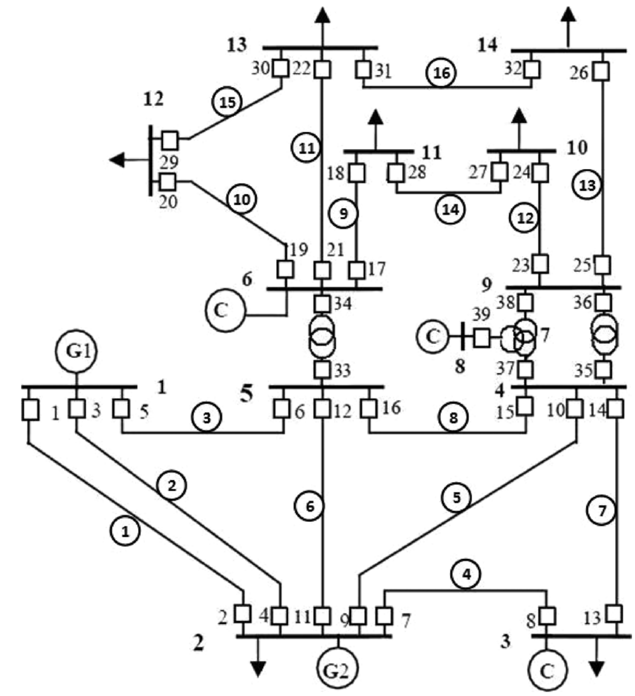

Fig. 1. Single-line diagram of the IEEE 14-bus test system.

Step 7) Solve the ILP problem to obtain the optimal TDS values, using (5).

V. Case Studies

In order to evaluate the proposed method, in this section, the developed formulation is applied to two different test systems: IEEE 14-bus and 30-bus test systems [15]. It is assumed that all of the lines and transformers in these test systems are equipped with the overcurrent relays having IEC standard inverse-type characteristics ($k_1 = 0.14$ and $k_2 = 0.02$) [8]. The CTI is assumed to be 0.2 s for each primary/backup relay pair and the TDS ranges are continuously from 0.05 to 1.

A. IEEE 14-Bus Test System

The single-line diagram of the 14-bus IEEE test system is shown in Fig. 1, which consists of 16 lines, 3 transformers, and 39 overcurrent relays. The pickup current settings of the relays in this test system, which are determined based on the maximum load currents, are shown in the second column of Table I.

In following two cases are examined for this test system:

Case 1) coordination problem considering fixed network topology;

Case 2) coordination problem considering different network topologies using the ILP method.

At first, the coordination problem on the 14-bus IEEE test system is solved in Case 1) and the optimal values of TDS are calculated and shown in the third column of Table I. The summation of TDS is shown in the last row of this table. The number of coordination constraints in this case is equal to 105 constraints, which is the same as the number of primary/backup relay pairs in the main network topology.

The changes in the network topology result in violation of many coordination constraints when the settings of case 1 are

TABLE I
SETTINGS OF OVERCURRENT RELAYS FOR THE IEEE 14-BUS TEST SYSTEM

Relay No.	Pickup Current (A)	TDS Case 1	TDS Case 2
1	450	0.0500	0.0680
2	500	0.0669	0.1287
3	500	0.0500	0.0679
4	450	0.0668	0.1286
5	200	0.1291	0.2040
6	400	0.0500	0.0614
7	900	0.0500	0.0500
8	400	0.0506	0.0887
9	600	0.0500	0.1052
10	500	0.0500	0.0520
11	600	0.0505	0.1044
12	300	0.0500	0.0918
13	100	0.2241	0.3772
14	500	0.0500	0.0611
15	800	0.0850	0.1216
16	800	0.0500	0.0760
17	300	0.3148	0.4182
18	300	0.1705	0.2358
19	200	0.2838	0.3892
20	60	0.1571	0.2362
21	700	0.2125	0.2861
22	140	0.2001	0.2903
23	900	0.2004	0.2576
24	300	0.1911	0.2711
25	400	0.2193	0.3178
26	250	0.1996	0.2765
27	400	0.2088	0.2688
28	600	0.1738	0.2392
29	250	0.1800	0.2737
30	300	0.1509	0.1987
31	400	0.2100	0.2798
32	100	0.2578	0.4129
33	200	0.1859	0.2426
34	900	0.0940	0.1632
35	300	0.0594	0.0823
36	200	0.1563	0.2646
37	300	0.0862	0.1197
38	200	0.0500	0.1057
39	150	0.1363	0.1848
	$\sum$ TDS	5.2216	7.6014

considered for the overcurrent relays. The numbers of violated constraints in the coordination problem due to single-line outage contingencies are shown in Table II. As shown in this table, the maximum percentage of violated constraints is 23.3 when line 8 is removed from the main topology.

In order to avoid the violation of the coordination constraints, the set of coordination constraints corresponding to each network topology should be considered in the overall DOCRs coordination problem as Case 2). In this case, the number of coordination constraints, which is equal to the sum of the primary/backup relay pair numbers in each network topology, is increased to 1556 constraints. The proposed algorithm is applied to the overall coordination problem in Case 2). The interval coordination matrix (A^I) is determined and the coordination problem is formulated as an ILP problem. Using (5), the ILP problem with only inequality constraints is reduced to an LP problem with just 105 coordination constraints, which is equal to the number of coordination constraints in the main topology, and 78 decision variables. The obtained LP problem is solved and the optimal values of TDS variables are shown in the fourth column of Table I. Although the optimal values of TDS in Case 2) have increased compared to Case 1), the solution obtained from the proposed interval method in Case 2) makes the protection relay robust against uncertainty in the network topology due to single-line outage contingencies.

In order to show the robustness of the interval proposed method, the results of Cases 1) and 2) are compared for an arbitrary relay pair. In Table III, the operating times of one primary/backup relay pair (22, 32), as an example, for every single-line outage contingency are presented.

Column 1 of this table shows the line number which is out of service. The line outage zero indicates the main network topology. The fault currents passing through the backup and primary relays corresponding to each network topology are presented in columns 2 and 3. The operating times of backup and primary relays corresponding to the optimal solution of Case 1) are shown in columns 4 and 5, respectively. The sixth column of this table shows the “constraint value” [16], defined in (25),

TABLE II
NUMBER OF VIOLATED CONSTRAINTS ARISING DUE TO THE SINGLE-LINE OUTAGE

Line outage No.	Coordination Constraints No.	Miss-coordination Constraints No.	Percentage of Miss-coordination
1	93	14	15.1
2	93	14	15.1
3	91	11	12.1
4	95	7	7.4
5	88	16	18.2
6	87	8	9.2
7	94	13	13.8
8	90	21	23.3
9	89	12	13.5
10	95	14	14.7
11	91	17	18.7
12	88	12	13.6
13	84	15	17.9
14	91	14	15.4
15	96	14	14.6
16	86	14	16.3

TABLE III
OPERATING TIMES OF PRIMARY/BACKUP RELAY PAIR, (22, 32), DUE TO SINGLE-LINE OUTAGE CONTINGENCIES

Line Outage No.	Near-End Fault Current (A)		Case 1			Case 2		
			Operating Times(s)		Constraint Value	Operating Times(s)		Constraint Value
	Backup	Primary	Backup	Primary		Backup	Primary	
0	1554	3068	0.640	0.440	0	1.025	0.638	0.187
1	1554	3068	0.640	0.440	0	1.025	0.638	0.187
2	1554	3068	0.640	0.440	0	1.025	0.638	0.187
3	1535	3030	0.643	0.442	0.001	1.03	0.641	0.189
4	1553	3066	0.640	0.440	0	1.025	0.638	0.187
5	1526	3028	0.644	0.442	0.002	1.032	0.641	0.191
6	1534	3026	0.643	0.442	0.001	1.03	0.641	0.189
7	1504	2990	0.648	0.444	0.004	1.038	0.644	0.194
8	1586	3084	0.635	0.439	**-0.004**	1.017	0.637	0.18
9	1670	3141	0.623	0.436	**-0.013**	0.998	0.633	0.165
10	1599	1678	0.633	0.550	**-0.117**	1.014	0.798	0.016
12	1670	3141	0.623	0.436	**-0.013**	0.998	0.633	0.165
14	1670	3141	0.623	0.436	**-0.013**	0.998	0.633	0.165
15	1599	1678	0.633	0.550	**-0.117**	1.014	0.798	0.016

TABLE IV
SETTINGS OF OVERCURRENT RELAYS FOR THE IEEE 30-BUS SYSTEM

Relay No.	TDS	Relay No.	TDS
1	0.1461	39	0.4441
2	0.2047	40	0.4238
3	0.1942	41	0.2341
4	0.0749	42	0.3877
5	0.2601	43	0.4822
6	0.1757	44	0.1845
7	0.1286	45	0.4077
8	0.1635	46	0.2779
9	0.1293	47	0.5253
10	0.0878	48	0.1169
11	0.2253	49	0.4771
12	0.1901	50	0.3357
13	0.2314	51	0.4263
14	0.2660	52	0.1840
15	0.2400	53	0.5338
16	0.1141	54	0.3547
17	0.1327	55	0.5456
18	0.0972	56	0.3848
19	0.3081	57	0.4440
20	0.3534	58	0.4583
21	0.3144	59	0.2427
22	0.1405	60	0.4326
23	0.2695	61	0.0500
24	0.1787	62	0.0500
25	0.0500	63	0.1650
26	0.0500	64	0.4994
27	0.2861	65	0.2562
28	0.1156	66	0.0606
29	0.4689	67	0.1732
30	0.2156	68	0.0500
31	0.4446	69	0.1676
32	0.2236	70	0.0500
33	0.3159	71	0.0941
34	0.1841	72	0.0902
35	0.4853	73	0.2505
36	0.4008	74	0.1886
37	0.5524	75	0.4068
38	0.3433	76	0.1428

where the negative values indicate a violation of the associated constraints

$$\text{constraint value} = t_j - t_i - \text{CTI}. \tag{25}$$

It is seen that by assuming the setting based on Case 1), the violation of six coordination constraints is caused by single-line outage contingencies. Similarly, the operating times of backup and primary relays and the "constraint value" corresponding to the optimal solution of Case 2) are presented in columns 6, 7, and 8 of Table III, respectively. As seen from this table, all violated constraints are removed–that means the solution in Case 2) for relay pair (22, 32) is robust against the single-line outage.

B. IEEE 30-Bus Test System

The second test system is the IEEE 30-bus system. It consists of two subsystems: 1) a 132-kV subtransmission system and 2) a 33-kV distribution system with 76 overcurrent relays [2].

The number of primary/backup relay pairs for the main topology is 160. Considering the single-line outage, the number of coordination constraints is increased to 5050. This is a very large scale LP problem which is solved by using the proposed interval method. The optimal TDS variables are shown in columns 2 and 4 of Table IV. The number of coordination constraints in the proposed ILP method is reduced to 160 constraints which is equal to the number of coordination constraints in the main network topology. Hence, the coordination constraint reduction in the proposed interval method is the salient advantage of this method in the large-scale power system.

VI. CONCLUSION

In this paper, considering different network topologies, a new method based on the interval analysis is proposed for the DOCRs coordination problem. The relay coordination problem can be formulated as a linear programming problem with a large number of inequality constraints when different network topologies are considered. In the proposed algorithm, the set of inequality constraints corresponding to each relay pair is converted to an interval constraint. Using this technique, the coordination problem is modeled as an ILP problem with interval inequality constraints. The obtained ILP problem, which has no equality constraints, is converted to a standard LP. The presented results clearly show the advantage of the proposed interval method in solving the DOCR coordination problem which is robust against topological uncertainty. Furthermore, the number of coordination constraints is drastically reduced when the proposed method is applied to the 14-bus and 30-bus IEEE test systems.

REFERENCES

[1] A. J. Urdaneta, R. Nadira, and L. G. Perez, "Optimal coordination of directional overcurrent relays in interconnected power systems," *IEEE Trans. Power Del.*, vol. 3, no. 3, pp. 903–911, Jul. 1988.
[2] A. Y. Abdelaziz, H. E. A. Talaat, A. I. Nosseir, and A. A. Hajjar, "An adaptive protection scheme for optimal coordination of overcurrent relays," *Elect. Power Syst. Res.*, vol. 61, no. 1, pp. 1–9, Feb. 2002.
[3] A. J. Urdaneta, L. G. Perez, and H. Restrepo, "Optimal coordination of directional overcurrent relays considering dynamic changes in the network topology," *IEEE Trans. Power Del.*, vol. 12, no. 4, pp. 1458–1464, Oct. 1997.
[4] A. J. Urdaneta, H. Restrepo, S. Marquez, and J. Sanchez, "Coordination of directional overcurrent relay timing using linear programming," *IEEE Trans. Power Del.*, vol. 11, no. 1, pp. 122–129, Jan. 1996.
[5] A. Saberi Noghabi, J. Sadeh, and H. Rajabi Mashhadi, "Considering different network topologies in optimal overcurrent relay coordination using a hybrid GA," *IEEE Trans. Power Del.*, vol. 24, no. 4, pp. 1857–1863, Oct. 2009.
[6] C. W. So and K. K. Li, "Time coordination method for power system protection by evolutionary algorithm," *IEEE Trans. Ind. Appl.*, vol. 36, no. 5, pp. 1235–1240, Sep. 2000.
[7] A. J. Urdaneta, L. G. Pérez, J. F. Gómez, B. Feijoo, and M. González, "Presolve analysis and interior point solutions of the linear programming coordination problem of directional overcurrent relays," *Elect. Power Energy Syst.*, vol. 23, no. 8, pp. 819–825, Nov. 2001.
[8] H. K. Karegar, H. A. Abyaneh, V. Ohis, and M. Meshkin, "Pre-processing of the optimal coordination of overcurrent relays," *Elect. Power Syst. Res.*, vol. 75, pp. 134–141, Aug. 2005.
[9] M. Ezzeddine and R. Kaczmarek, "Reduction of effect of coordination constraints in the linear optimization of operating times of overcurrent relays," in *Proc. 14th IEEE Mediterranean Electrotechnical Conf.*, Ajaccio, Corsica, May 2008, pp. 707–712.
[10] Z. Wang and F. L. Alvarado, "Interval arithmetic in power flow analysis," *IEEE Trans. Power Syst.*, vol. 7, no. 3, pp. 1341–1349, Aug. 1992.
[11] A. T. Šarić and A. M. Stanković, "An application of interval analysis and optimization to electric energy markets," *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 515–523, May 2006.
[12] Z. Xu, M. Liu, G. Yang, and N. Li, "Application of interval analysis and evidence theory to fault location," *Inst. Eng. Technol. Elect. Power Appl.*, vol. 3, no. 1, pp. 77–84, 2009.
[13] J. Rohn and J. Kreslova, "Linear interval inequalities," *Linear Multilinear Algebra*, vol. 38, no. 1, pp. 79–82, 1994.

[14] J. M. Gers and E. J. Holmes, *Protection of Electricity Distribution Networks*, ser. Inst. Elect. Eng. Power Energy, 2nd ed. London, U.K.: IEE, 2004, vol. 47, p. 71.
[15] Univ. Washington. Seattle, WA, Mar 2006. [Online]. Available: http://www.ee.washington.edu/research/pstca/pf30/pg_tca30bus.htm
[16] D. Birla, R. Maheshwari, and H. O. Gupta, "A new nonlinear directional overcurrent relay Coordination technique, and banes and boons of Near-end faults based approach," *IEEE Trans. Power Del.*, vol. 21, no. 3, pp. 1176–1182, Jul. 2006.

Abbas Saberi Noghabi was born in Gonabad, Iran, in 1977. He received the B.Sc. degree in electrical engineering from Abbaspour University, Tehran, Iran, in 1999, the M.Sc. degree in electrical engineering from K. N. Toosi University of Technology, Tehran, in 2001, and is currently pursuing the Ph.D. degree at Ferdowsi University of Mashhad, Mashhad.

His research interests are power system protection as well as power system operation and optimization methods.

Habib Rajabi Mashhadi (M'09) was born in Mashhad, Iran, in 1967. He received the B.Sc. and M.Sc. degrees (Hons.) in electrical engineering from the Ferdowsi University of Mashhad, Mashhad, and the Ph.D. degree in electrical and computer engineering from Tehran University, Tehran, under the joint cooperation of Aachen University of Technology, Aachen, Germany.

He is an Associate Professor of Electrical Engineering at Ferdowsi University. His research interests are power system operation and economics, power system planning, and biological computation.

Javad Sadeh (M'08) was born in Mashhad, Iran, in 1968. He received the B.Sc. and M.Sc. degrees (Hons.) in electrical engineering from the Ferdowsi University of Mashhad, Mashhad, Iran, in 1990 and 1994, respectively, and the Ph.D. degree in electrical engineering from Sharif University of Technology, Tehran, with the collaboration of the Electrical Engineering Laboratory of the Institut National Polytechnique de Grenoble (INPG), Grenoble, France, in 2001.

Since then, he has been an Assistant Professor at the Ferdowsi University of Mashhad. His research interests are power system protection, dynamics, and operation.