COORDINATION OF DIRECTIONAL OVERCURRENT RELAY TIMING USING LINEAR PROGRAMMING

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Abstract - A successive linear programming methodology is presented to treat more effectively those applications where a local structure change is performed to a system already in operation, and where the modification of the settings of already existent relays is not desirable. The dimension of the optimization problems to be solved is substantially reduced, and a sequence of small linear programming problems is stated and solved in terms of the time dial settings, until a feasible solution is reached. With the proposed technique, the number of relays of the original system to be reset is reduced substantially. It is found that there is a tradeoff between the number of relays to be reset and the optimality of the settings of the relays.

Keywords - Power system protection, protective device coordination, directional overcurrent relays.

I. INTRODUCTION

The planning and operation of power systems frequently requires the addition of system branches (e.g., transmission lines, transformers, etc.), and modifications of the system structure, in order to follow the variable characteristics of the load. These changes in the transmission network modify the response of the system due to fault conditions and therefore require the continuos verification and modification of the characteristics and distinct features of the protective apparatus of the system. Hence, a system structure change, such as the addition of a branch or a set of branches, requires not only the setting of the new relays, but also may imply the resetting of the existing relays in order to satisfy the new requirements of the power system.

Several effort has been devoted to the computer coordination of directional overcurrent relays. A complete summary of past work, as well as a survey of the existing software is presented in [1]. In particular, for a system already in operation, the problem of coordinating the operation of this type of devices, trying to modify only a small subset of the relays of the system, in response to changes in the system structure is treated in [2], and its solution by means of the application of linear programming techniques is presented in [3],[4],[5].

The modification of a large number of settings is costly and undesirable, therefore a feasible solution with higher relay operation times but with less relays to be reset may be preferred instead.[6] In this work, that dilemma is faced by means of the solution of a series of linear programming problems of reduced dimension, sacrificing part of the "optimality" of the original formulation, and leading to a "sub optimal" solution with higher operation times but less relays to be reset.

If the optimization approach suggested in [7] is applied to the case of an expansion in the transmission system, the linear programming problem will not only have a large dimension but also its solution may and will probably modify the settings of several relays in the system in order to obtain the minimum operation times that satisfy all the constraints.

With the proposed methodology, the linear programming problem is initially reduced to its minimum dimension. Then, depending upon the solution, the problem is augmented successively by including in the formulation certain relays of the original system. After the reduced problem has been solved using linear programming, the coordinated operation of the relays is then verified for the whole system, and the procedure is repeated until the selectivity condition is met for the complete system.

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II. STATEMENT OF THE PROBLEM

The coordination problem of directional overcurrent relays in interconnected power systems, can be stated as an optimization problem, where the sum of the operation times of the relays of the system is minimized, subject to the following constraints [7]:

.- coordination criteria,
.- bounds on the settings,
.- bounds on the operation times,
.- relay operation characteristics.

This optimization problem can be stated as:

$$\min \sum_i \sum_j \sum_k W_{ijk} T_{ijk} \tag{1}$$

subject to:

$$\mathbf{h(T)} \leq \mathbf{0} \quad \text{(coordination criteria)} \tag{2}$$

$$\mathbf{S}_{min} \leq \mathbf{S} \leq \mathbf{S}^{max} \quad \text{(bounds on the settings)} \tag{3}$$

$$\mathbf{T} = \mathbf{f(S)} \quad \text{(relay characteristics)} \tag{4}$$

where T_{ijk} is the operation time of relay i of branch j for fault k, and W_{ijk} is the correspondent weighting factor.

The vector **S** in general includes two types of settings: the vector of pickup currents **Ip** and the vector of time dial settings **TDS**. This article is focused on the calculation of the latter one.

The formulation also permits the consideration of constraints on the operation times, which appear, for example, when coordinating the overcurrent relay timing with the operation of the distance relays. These restrictions were omitted from the statement of the problem, in order to simplify the explanation of the methodology.

The treatment of the "transient configurations" that take place during the total fault clearing process, while only one of the main relays has operated, is presented in [8].

III. PROPOSED METHODOLOGY

A partition of the set of relays **R** is performed, creating two subsets of relays:

$\mathbf{R}_n$={relays which are candidates to be reset, associated to the vectors $\mathbf{TDS}_n$ and $\mathbf{T}_n$ },

$\mathbf{R}_o$={relays which are not candidates to be reset, associated to the vectors $\mathbf{TDS}_o$ and $\mathbf{T}_o$ }.

At each iteration, the convergence of the algorithm is verified by the evaluation of the coordination constraints for the whole system. If the coordination constraints of the complete system are not satisfied, then the optimization problem is augmented by the incorporation of a subgroup $\mathbf{R}_{no}$ of relays of the original system $\mathbf{R}_o$ into $\mathbf{R}_n$. The rest of the relays of $\mathbf{R}_n$ that does not belong to $\mathbf{R}_{no}$, form the subgroup $\mathbf{R}_{nn}$; so that the two subgroups $\mathbf{R}_{no}$ and $\mathbf{R}_{nn}$ constitute a partition of $\mathbf{R}_n$.

$\mathbf{R}_{nn}$= { Relays which were candidates to be reset at the previous iteration } ,

$\mathbf{R}_{no}$= { Recently incorporated relays } .

For the first iteration it is assumed that $\mathbf{R}_{nn}$ is the set correspondent to the relays of the added system, and $\mathbf{R}_{no}$ consists of those relays of the neighboring branches; therefore $\mathbf{R}_o$ is the set correspondent to the relays of the original system, except for those in $\mathbf{R}_{no}$. If this initial formulation results in a non feasible solution for the complete system, then the problem is augmented for the next step.

At each iteration, the optimization problem is stated only in terms of the variables associated to those relays belonging to $\mathbf{R}_n$, and which are candidates to be reset:

$$\text{minimize} \quad \mathbf{W}_n^T \cdot \mathbf{T}_n \quad , \tag{5}$$

subject to:

$$\mathbf{h}(\mathbf{T}_n) \leq \mathbf{0} \quad , \tag{6}$$

$$\mathbf{TDS}_{n\ min} \leq \mathbf{TDS}_n \leq \mathbf{TDS}_n^{max} \quad , \tag{7}$$

$$\mathbf{T}_n = \mathbf{f}\ (\mathbf{TDS}_n) \quad ; \tag{8}$$

where the minimum bounds on the settings $\mathbf{TDS}_{n\ min}$ of those relays in $\mathbf{R}_{no}$, are set equal to the original settings $\mathbf{TDS}_o$ (i.e., before the modification of the system):

$$\mathbf{TDS}_{no\ min} = \mathbf{TDS}_o \quad , \tag{9}$$

and those associated to the relays of $\mathbf{R}_{nn}$ are set equal to the minimum allowed by the device model; so that their correspondent settings are permitted to take values below their original or present settings:

$$\mathbf{TDS}_{nn\ min} = [\text{Minimum allowed by the correspondent device}] \ . \tag{10}$$

The proposed methodology is illustrated by FIG. 1.

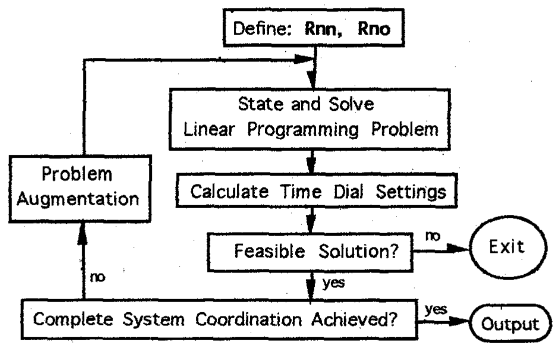

FIG.1. Proposed Solution Scheme.

A. Problem Augmentation

The step of augmenting the reduced problem may be performed in several ways, depending on the preferences of the decision maker in relation with the tradeoff between the number of relays to be reset and the magnitude of the operation times of the relays. In general it implies the incorporation of certain relays into the subset $\mathbf{R}_n$ and its desincorporation from $\mathbf{R}_o$.

One of the possible alternatives for the problem augmentation is the incorporation into $\mathbf{R}_n$ of all the relays of the neighboring branches. Its application to the relay coordination of a realistic system is studied in this article. This approach allows the reduction of the settings of some of the relays of the original system. In this case, the definitions of $\mathbf{R}_{nn}$, $\mathbf{R}_{no}$ and $\mathbf{R}_n$ are illustrated by FIG.2 and FIG.3.

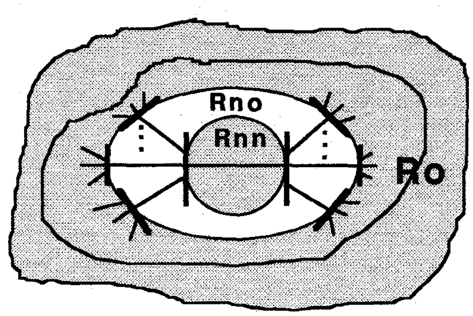

FIG.2. First Iteration

Another alternatives are, for example, an approach somehow similar to that suggested in ref. [2], the incorporation into $\mathbf{R}_n$ of the relays whose settings belong to the critical constraints, or the incorporation into $\mathbf{R}_n$ of the relays that operate as first backup of those devices already in $\mathbf{R}_n$. These alternative procedures for the problem augmentation where not explored in this work.

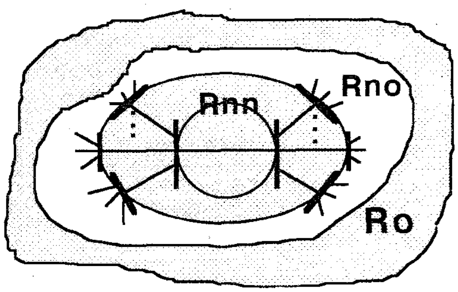

FIG.3. Second Iteration

B. Convergence Verification

This algorithm is repeated until the time coordination constraints are satisfied for the whole system. If the constraints of the entire system are not satisfied then the iterative procedure has to be continued, but if by the contrary, all the coordination restrictions are verified, then the procedure is stopped and a solution has been found.

If the procedure leads at the final iteration to a set $\mathbf{R}_n$ equal to $\mathbf{R}$, then the terminal reduced problem turns out to be similar to the complete formulation (3)(4)(5)(7)[8].

Finally, if either the reduced linear programming problem or if the terminal problem has no feasible solution (e.g., the coordination constraints can not be satisfied for the complete system with all the relays being considered in the formulation), then the system can not be coordinated with the specified parameters, and according to the general methodology proposed in ref.[9], the optimization problem has to be re-stated by the expert system at the top layer of the solution process. In this case the system experts may choose, among others, to vary the pickup current settings, to modify certain types of relays being used, or to change the coordination time interval of certain constraints.

C. Linear Programming

This type of algorithms is bound to solve optimization problems where a linear objective function is minimized, subject to a set of linear equality and/or inequality constraints.

The simplex method, applied in this work, is a well known solution technique [10]; although interior point algorithms have also proved to be very efficient [11].

The application of linear programming techniques is highly desirable, because of their robustness, their speed and the high dimension of the problems that they can handle.

Although stated as above, the optimization problem is nonlinear, if it is stated in terms of the variables PTDS, taking advantage of the linear relation between this variables and the relay operation times for a given set of pickup currents, [12] it can be simplified to a linear programming problem if the set of pickup currents is preset and assumed fixed, and the problem is solved only in terms of the time dial settings. In that case, the operation time of relay ijk is:

$$T_{ijk} = PIp_{ijk} \cdot PTDS_{ij} \quad , \qquad \textbf{(11)}$$

where:

$$PTDS_{ij} = B_0 + B_1 . TDS_{ij} + B_2 . TDS_{ij}^2 + B_3 . TDS_{ij}^3 \ , \qquad \textbf{(12)}$$

$$PIp_{ijk} = A_0 + A_1/(M_{ijk}-1) + A_2/(M_{ijk}-1)^2 + A_3/(M_{ijk}-1)^3 + A_4/(M_{ijk}-1)^4 \ , \qquad \textbf{(13)}$$

and:

$$M_{ijk} = I_{ijk} / Ip_{ij} \quad , \qquad \textbf{(14)}$$

so that, for a constant set of pickup currents, the vector of operation times:

$$\mathbf{T} = \mathbf{PIp}^{T} . \ \mathbf{PTDS} \quad , \qquad \textbf{(15)}$$

where $\mathbf{PIp}$ is a constant vector whose elements are calculated prior to using equations (15),(16).

1) Objective Function: With these considerations, the objective function may be expressed in terms of $\mathbf{PTDS}_n$ as:

$$\min \ \mathbf{PIp}_n^{T} . \ [\mathbf{W}]_n . \ \mathbf{PTDS}_n \quad , \qquad \textbf{(16)}$$

where $\mathbf{PIp}_n$ is a constant vector, and $[\mathbf{W}]_n$ is a matrix of weighting factors. However, it can be shown that with the objective function chosen in this way, the solution to the coordination problem is not dependent upon the choice of the weights. In this case, the objective function can be reduced to the expression:

$$\min \ \mathbf{PTDS}_n \qquad \textbf{(17)}$$

Also, due to the monotonic characteristic of the PTDS function in terms of the TDS variable for the range under study, and due to the fact that at the optimum, independently of which type of constraint is active, the relation $\partial TDS_{ij} / \partial TDS_{kl} \geq 0$ is verified for all i,j,k,l , the same results are obtained if ΣTDS is minimized instead of ΣPTDS, although with a nonlinear formulation.[7]

2) Coordination Constraints: The coordination constraints, described by (7), can be written as:

$$\mathbf{T}_{BACKUP\ n} - \mathbf{T}_{MAIN\ n} - \mathbf{\Delta T} \leq \mathbf{0} \quad , \qquad \textbf{(18)}$$

where $\mathbf{T}_{MAIN\ n}$ and $\mathbf{T}_{BACKUP\ n}$ are vectors of dimensions NC x 1 , whose elements are the relay operation times for the main and the backup relays, respectively. In general, each fault under study generates one row of equation (19) for each of the main relays of the protective zone where the fault is located. In terms of $\mathbf{PTDS}$, a linear expression is obtained that can be perfectly handled by the linear programming algorithm:

$$[\mathbf{K}_{BACKUP}]_n \mathbf{PTDS}_n - [\mathbf{K}_{MAIN}]_n \mathbf{PTDS}_n - \mathbf{\Delta T} \leq \mathbf{0} \ , \qquad \textbf{(19)}$$

where: $T_{MAIN\ n} = [K_{MAIN}]_n \mathbf{PTDS}_n$, (20)

$T_{BACKUP\ n} = [K_{BACKUP}]_n \mathbf{PTDS}_n$, (21)

and $[\mathbf{K}_{MAIN}]_n$ and $[\mathbf{K}_{BACKUP}]_n$ are constant matrices of dimensions $NC_n \times NR_n$, whose elements are calculated using equations (15),(16).

3) Bounds on the Settings: The bounds on the settings can also be expressed in terms of **PTDS:**

$$\mathbf{PTDS}_{n\ min} \leq \mathbf{PTDS}_n \leq \mathbf{PTDS}_n^{max} , \quad (22)$$

where the vector elements are calculated using equation (14):

$$PTDS_{ij\ min} = PTDS_{ij}(TDS_{ij\ min}) , \quad (23)$$

$$PTDS_{ij}^{\ max} = PTDS_{ij}(TDS_{ij}^{\ max}) , \quad (24)$$

4) Linear Programming Problem: The problem expressed by (6)(7)(8)(9)(10) can be expressed as a linear programming problem:

$$\min\ \mathbf{PTDS_n} , \quad (25)$$

subject to:

$$[\mathbf{K}_{MAIN}]_n \mathbf{PTDS}_n - [\mathbf{K}_{BACKUP}]_n \mathbf{PTDS}_n - \Delta \mathbf{T} \leq \mathbf{0} , \quad (26)$$

$$\mathbf{PTDS}_{n\ min} \leq \mathbf{PTDS}_n \leq \mathbf{PTDS}_n^{max} . \quad (27)$$

Once the linear programming problem has been solved in terms of **PTDS**, then the corresponding time dial settings have to be calculated for each of the relays whose settings have been modified, finding the roots of the polynomial given by (12).

IV. TEST CASE

The proposed procedure was tested on the system of FIG. 3 with 18 buses, 48 branches (twenty five 69 kV lines, ten 230 kV lines and thirteen 230/69 kV power transformers) and 74 directional overcurrent relays. The time dial settings were assumed to vary continuously and the relay coordination was performed considering the configurations that occur when only the first primary relay has operated and the second primary relay has not operated yet.

The initial settings for all the relays of the original system were calculated using the optimization methodology suggested in [7]. The results ($\mathbf{TDS_0}$) are shown in the second column of Table I.

This original system, with the set of optimal settings previously calculated, was modified with the addition of a branch between buses 9 and 11 (FIG.2), and the successive methodology, proposed in this article, was applied to the new system, with the added branch (L49).

A) First Iteration: Initially, the linear programming problem involves the relays of the added branch (L49) which are R149 and R249, with their TDS_{min} set equal to the minimum allowed by the correspondent relay models; as well as those relays of the original system which belong to the neighboring branches (L1, L2, L11, L12, L7, L8, T3, T4) which are: R11, R21, R12, R22, R111, R211, R112, R212, R17. R27, R18, R28, R138, R238, R139 and R239, with their

TABLE I
Test Case Results

Relay N°	ORIGINAL SYSTEM. TDS_0	MODIFIED SYSTEM $TDS^{(1)}$	$TDS^{(2)}$	$TDS^{(3)}$	$TDS^{(4)}$
11	0.131	0.131	0.134*	0.135*	0.135*
21	0.500	0.532	0.509*	0.530*	0.530*
12	1.654	1.654	1.533*	1.535*	1.535*
22	0.500	0.500	0.500*	0.530*	0.530*
13	0.100	0.100	0.100	0.100	0.100
23	0.836	0.836	0.836	0.817*	0.817*
14	0.100	0.100	0.100	0.100	0.100
24	0.836	0.836	0.836	0.817*	0.817*
15	0.100	0.100	0.100	0.100	0.100
25	0.836	0.836	0.836	0.817*	0.817*
16	0.100	0.100	0.100	0.100	0.100
26	0.836	0.836	0.836	0.817*	0.817*
17	1.491	1.491	1.508*	1.508*	1.508*
27	1.725	1.725	1.637*	1.637*	1.637*
18	1.491	1.491	1.508*	1.508*	1.508*
28	1.725	1.725	1.637*	1.637*	1.637*
19	2.160	2.160	2.160	2.171*	2.171*
29	1.868	1.868	1.868	1.848*	1.848*
110	2.110	2.110	2.110	2.122*	2.122*
210	0.195	0.195	0.195	0.193*	0.193*
111	2.019	2.019	2.100*	2.102*	2.102*
211	0.957	1.178	1.367*	1.371*	1.371*
112	2.019	2.019	2.100*	2.102*	2.102*
212	0.957	1.178	1.367*	1.371*	1.371*
113	1.413	1.413	1.413	1.401*	1.401*
213	2.234	2.234	2.234	2.189*	2.189*
114	1.413	1.413	1.413	1.401*	1.401*
214	2.234	2.234	2.234	2.189*	2.189*
115	1.056	1.056	1.056	1.056	1.057*
215	1.404	1.404	1.404	1.404	1.395*
116	0.137	0.137	0.137	0.137	0.138*
216	1.694	1.694	1.694	1.694	1.685*
117	2.131	2.131	2.131	2.131	2.131
217	2.268	2.268	2.268	2.268	2.268
118	2.131	2.131	2.131	2.131	2.131
218	2.268	2.268	2.268	2.268	2.268
119	2.763	2.763	2.763	2.763	2.763
219	0.109	0.109	0.109	0.109	0.109
120	2.763	2.763	2.763	2.763	2.763
220	0.109	0.109	0.109	0.109	0.109
121	0.179	0.179	0.179	0.179	0.179
221	0.146	0.146	0.146	0.146	0.146
122	0.179	0.179	0.179	0.179	0.179
222	0.146	0.146	0.146	0.146	0.146
123	0.142	0.142	0.142	0.142	0.142
223	2.135	2.135	2.135	2.135	2.135
124	0.142	0.142	0.142	0.142	0.142
224	2.135	2.135	2.135	2.135	2.135
136	0.782	0.782	0.782	0.782	0.775*
236	0.100	0.100	0.100	0.100	0.100
137	0.782	0.782	0.782	0.782	0.775*
237	0.100	0.100	0.100	0.100	0.100
138	0.957	0.957	0.918*	0.918*	0.918*
238	0.500	0.500	0.500	0.500	0.500
139	0.957	0.957	0.918*	0.918*	0.918*
239	0.500	0.500	0.500	0.500	0.500
140	1.053	1.053	1.053	1.053	1.043*
240	0.500	0.500	0.500	0.500	0.500
141	1.050	1.050	1.050	1.050	1.040*
241	0.100	0.100	0.100	0.100	0.100
142	1.305	1.305	1.305	1.305	1.295*
242	0.500	0.500	0.500	0.500	0.500
143	1.368	1.368	1.368	1.368	1.358*
243	0.500	0.500	0.500	0.500	0.500
144	0.529	0.529	0.529	0.529	0.529
244	0.500	0.500	0.500	0.500	0.500
145	0.704	0.704	0.704	0.704	0.704
245	0.500	0.500	0.500	0.500	0.500
146	0.704	0.704	0.704	0.704	0.704
246	0.500	0.500	0.500	0.500	0.500
147	1.250	1.250	1.250	1.250	1.250
247	0.500	0.500	0.500	0.500	0.500
148	1.250	1.250	1.250	1.250	1.250
248	0.500	0.500	0.500	0.500	0.500
149	——	0.500	0.500	0.500	0.500
249	——	0.961	1.024	1.026	1.026
ΣTDS	——	Non Feasible	75.221	75.089	75.028

Note: .- (*) indicates the modified settings
.- $\mathbf{TDS^{(4)} = TDS^{(5)}}$

respective settings TDS_{min} set equal to the previous or original setting TDS_O :

$$\mathbf{R_{nn}}^{(1)} = \{ R149,\ R249 \} \quad , \qquad \mathbf{(28)}$$

$\mathbf{R_{no}}^{(1)}$={ R11, R21, R12, R22, R111, R211, R112, R212, R17, R27, R18, R28, R138, R238, R139, R239 }. **(29**

With the problem stated in this reduced way, the linear programming solution for the first iteration of the successive methodology resulted in settings that does not satisfy the coordination constraints of the whole system, therefore the selectivity condition for the operation of the relays of the complete system is not verified, and a new iteration of the successive methodology is required.

The linear programming results ($\mathbf{TDS^{(1)}}$) are shown in Table I.

B) Second Iteration: The linear programming problem is augmented, defining the new set $\mathbf{R_{nn}}^{(2)}$ as integrated by those relays belonging to $\mathbf{R_{nn}}^{(1)}$, and to $\mathbf{R_{no}}^{(1)}$:

$$\mathbf{R_{nn}}^{(2)} = \mathbf{R_{nn}}^{(1)} \cup \mathbf{R_{no}}^{(1)} \qquad \mathbf{(30)}$$

$\mathbf{R_{nn}}^{(2)}$={R149, R249, R11, R21, R12, R22, R111, R211, R112, R212, R17, R27, R18, R28, R138, R238, R139,R239 } **(31)**

and the new set $\mathbf{R_{no}}^{(2)}$ is defined as composed by the relays of $\mathbf{R_o}^{(1)}$ which, for the previous iteration belonged to the neighboring branches of those associated to $\mathbf{R_{no}}^{(1)}$:

$\mathbf{R_{no}}^{(2)}$ = {R13, R23, R14, R24, R15, R25, R16, R26, R19, R29, R110, R210, R113, R213, R114, R214, R117, R217, R118, R218, R136, R236, R137, R237} **(32)**

The linear programming solution results in time dial settings ($\mathbf{TDS^{(2)}}$), presented in Table I, which satisfy the coordination constraints of the complete system, verifying the selectivity condition for the operation of the relays.

This solution guarantees the correct functioning of the protective devices, and may be implemented if feasibility is the main objective of the decision maker. However, it is not optimal, in the sense that it does not assure the minimum operation times.

B) Further Iterations: The objective function may be reduced by pursuing further augmentations of the optimization problems.

For example, if a third iteration is performed, augmenting the linear programming problem according to the proposed methodology, the sets $\mathbf{R_{nn}}^{(3)}$and $\mathbf{R_{no}}^{(3)}$ will result as follows:

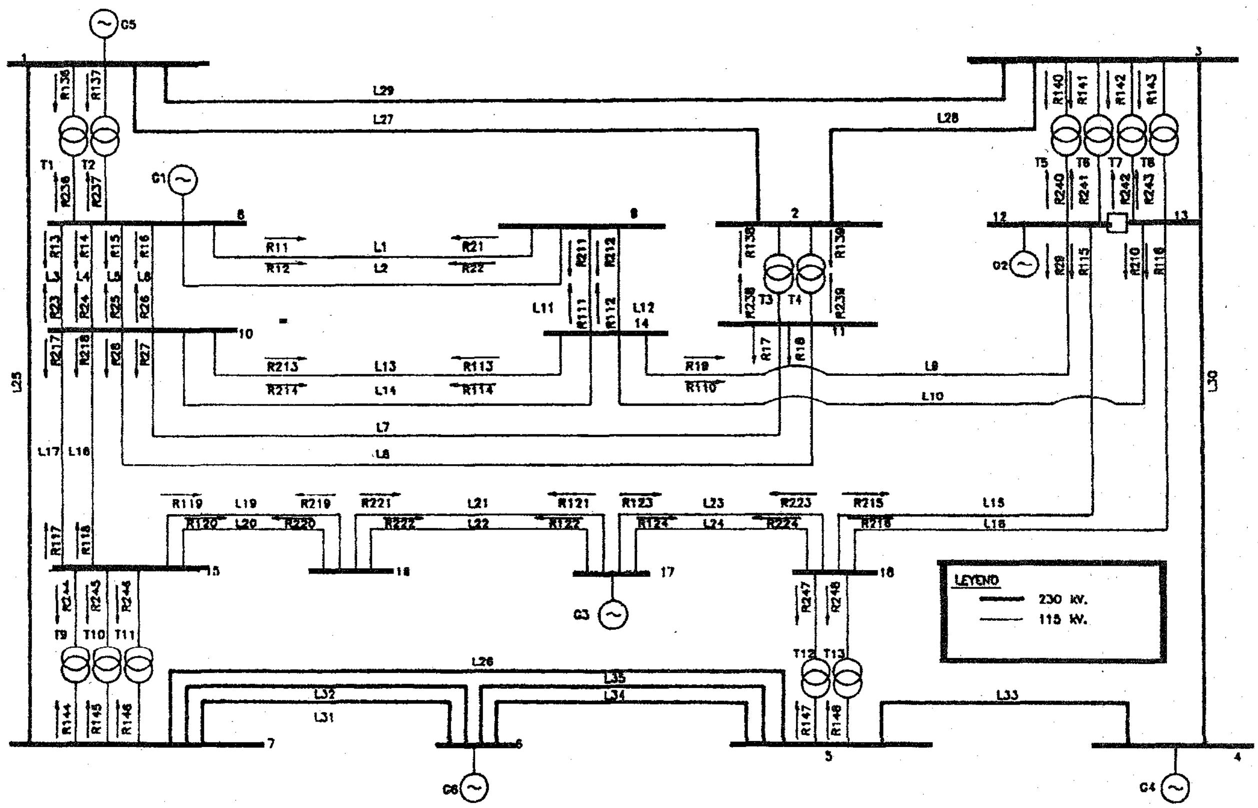

FIG. 4. One Line Diagram of the Test Case System

$\mathbf{R_{nn}}^{(3)}$ = { R149, R249, R11, R21, R12, R22, R111, R211, R112, R212, R17, R27, R18, R28, R138, R238, R139, R13, R23, R14, R24, R15, R25, R16, R26, R19, R29, R110, R210, R113, R213, R114, R214, R117, R217, R118, R218, R136, R236, R137, R237 } **(33)**

$\mathbf{R_{no}}^{(3)}$={ R115, R215, R116, R216, R119, R219, R120, R220, R140, R240, R141, R241, R142, R242, R143, R243, R144, R244, R145, R245, R146, R246 } **(34)**

The resultant relay settings ($\mathbf{TDS}^{(3)}$), shown in Table I, not only verify the selectivity condition with a coordinated operation for the complete system, but also reduce the objective function value, obtaining faster operation times than those obtained for the second iteration of the successive methodology. However it requires the resetting of 26 relays, while the previous one only requires the resetting of 13 devices.

Table I also shows the results obtained for the fourth and fifth iteration of the proposed successive methodology ($\mathbf{TDS}^{(4)}$). It should be pointed out that for the fifth iteration the optimization problem includes the complete system and therefore the linear programming formulation is identical to that proposed in ref.[1], including all the relays of the original system, as well as those of the added system into the set $\mathbf{R_{nn}}$. In this case, the settings of a total of 36 devices were modified.

The plot of the sum of the time dial settings ($\sum\mathbf{TDS}$) versus the number of relays to be reset, obtained for each iteration of the proposed methodology for the test case, is shown in FIG. 4.

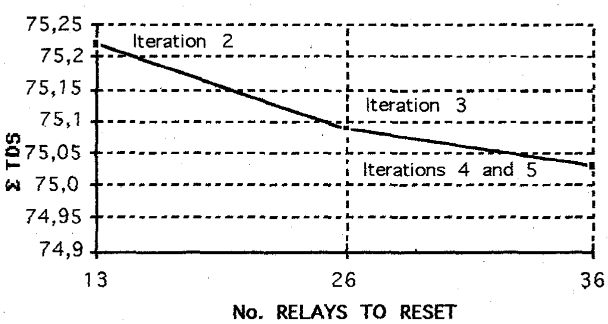

FIG.4. Sum of Time Dial Settings vs. Number of Relays to Reset

This two objectives (Sum of the Time dial Settings and Number of Relays to be Reset) are in conflict, so that for the set of non inferior or Pareto Optimal solutions, in order to reduce one of them the other one must be increased. According to the preferences of the decision maker any of the non inferior solutions may be chosen.

If the problem is coordinated using a different methodology, and the obtained solution lays in the upper zone of FIG.3, then it is an inferior solution because one of the two objectives (the sum of the operation times or the number of relays to be reset) may be reduced without increasing the other one, preserving the coordinated operation of the protective devices.

V. CONCLUSIONS

The proposed methodology, based upon the successive solution of small dimension linear programming problems, provides a new and efficient tool to solve the coordination problem of directional overcurrent relays in interconnected power systems, calculating the time dial settings for a given set of pickup currents, when a local structure change has been performed to the system. With this methodology, a reduced number of relays of the original system is to be reset, depending on the values of the original settings of the relays before the addition, on the magnitude of the introduced structure changes, and on the preferences of the decision maker. In this conditions, there is a tradeoff between the optimality of the global solution for the settings of the relays, and the number of relays to be reset.

VI. ACKNOWLEDGMENTS

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VII. GLOSSARY

f, h	real valued function vectors
[K]	constant 2NR x NR real matrix
M	fault current multiple of the pickup current (14)
NC	number of coordination constraints
NR	number of relays
PIp, PTDS	polynomials defined by (14), (15)
R	set of relays
S	vector of settings
T	relay operation time
T	vector of relay operation times.
TDS	time dial setting.
TDS	vector of time dial settings.
Ip	pickup current setting.
Ip	vector of pickup current settings.
W	weighting factor.
W	diagonal matrix of weighting factors.
ΔT	coordination time interval.
ΔT	vector of coordination time intervals.
Indexes:	
backup	backup relays quantities
ij	relay i of branch j (i=1,2 sending and receiving ends, respectively)
ijk	of ij for fault k
min	minimum
max	maximum
main	main relay quantities
n	associated to relays candidates to be reset
nn	associated to relays candidates to be reset whose minimum settings are fixed at the minimum allowed by the correspondent device.
no	associated to relays candidates to be reset whose minimum settings are fixed at the present or original values.
o	associated to relays not to be reset
(1),(2),...	iteration number.
T	transposed.
[ ]	indicates matrices.

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DISCUSSION

ALEXANDER P. APOSTOLOV, (Rochester - Integrated Systems Division), Rochester, NY :

The authors have presented an interesting paper on the application of linear programming for overcurrent relays coordination. It will be beneficial if the authors can comment on the following questions:

1. How is the difference in the accuracy of electromechanical and microprocessor based overcurrent relays taken into consideration ?
2. How are the inverse-time characteristics for different relays modeled ?
3. How does the different breaker speed at different locations affect the coordination ?

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The modeling of the inverse-time characteristics is perhaps the key point for the application of the linear programming algorithm. This is performed by means of equations [11],[12],[13],[14] of the paper. The specific parameters for each particular relay type, are previously calculated by minimizing the squared deviations with respect to the nominal characteristic of each device. In the case of devices with sharp changes in the characteristic, it is possible to increase the order of the PIp and PTDS expressions, in order to improve the exactitude of the equations, although in other cases, such as for some computer relays, the use of a linear expression for the PTDS expression is accurate enough for the purpose of the algorithm.

With regard to points (1) and (2) of the discussion, both, the difference in the accuracy of the relays as well as the different breaker speed, may be taken into consideration when selecting the value of the coordination time interval of each of the coordination equations. This parameter may not be the same for the entire system, and could be adjusted individually for each of the coordination equations, depending on the accuracy of the particular relays and on the speed of the circuit breakers that are specifically involved in each case.

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