OPTIMAL COORDINATION OF DIRECTIONAL OVERCURRENT RELAYS IN INTERCONNECTED POWER SYSTEMS

Alberto J. Urdaneta, Member, IEEE — Universidad Simón Bolívar, Caracas, Venezuela

Ramón Nadira, Member, IEEE — Scientific Systems, Inc, Cambridge, MA

Luis G. Pérez Jiménez — Universidad Simón Bolívar, Caracas, Venezuela

Abstract - This paper presents a new methodology based upon the principles of optimization theory, to treat the problem of optimal coordination of directional overcurrent relays in interconnected power systems. With the application of the proposed technique, this coordination problem is stated as a parameter optimization problem which in general is of a large dimension, especially when many different system configurations and perturbations are to be considered. Several optimization procedures, including direct methods and decomposition techniques, for solving this large scale coordination problem are described, and results of optimally coordinating directional overcurrent relays in power systems with up to 30 buses are presented.

INTRODUCTION

The problem of coordinating protective relays in electric power systems consists of selecting their suitable settings such that their fundamental protective function is met under the requirements of sensitivity, selectivity, reliability, and speed [1,2,3,4]. These requirements must be met for a variety of system conditions and configurations, and can be translated into conditions such as: (i) a variety of fault conditions must be detected by the appropriate relays, (ii) the relays located closer to the fault should have priority of operation, (iii) if a primary relay fails, a backup relay should operate, and (iv) the operation of the relay should be as fast as possible to prevent equipment damage, and must occur only in the presence of abnormal operating conditions which jeopardize the system integrity.

A great deal of effort has been devoted to the automation of the solution of the coordination problem of directional overcurrent relays in power systems. The algorithms currently in use [5,6,7,8,9,10], have basically automated the traditional interactive solution philosophy, wherein the engineer runs different cases for distinct faults and configurations until an acceptable solution is reached. In general, these algorithms evaluate the performance of the predicted settings of the protective relays by the automatic construction of the characteristic curves of the devices, and make use of several techniques aimed at improving computational performance. For instance, some of these algorithms use separate routines to calculate the fault conditions (e.g. short circuit currents) by means of simple matrix manipulations, avoiding repeated inversions of the Node Admittance Matrix [11,12]. Others [5,7], only select and store the pairs of fault currents required for the calculations. In [13], an interactive algorithm based upon linear graph theory is suggested, and successful results are reported in [14,15]. However, the solution found by these procedures is not optimal in any strict sense, but simply the best of the tried possible solutions.

In this paper, the problem of coordinating directional overcurrent relays in power systems is stated and solved in the framework of optimization theory. The proposed approach determines the "optimal" solution to this coordination problem in a cost-effective and efficient way, by stating the problem as a parameter optimization problem, and solving it using efficient optimization techniques. Several such techniques were tested, including direct techniques and decomposition techniques coupled with hierarchical coordination procedures. It is pertinent to mention here that the optimization methodology presented in this paper can also be applied to the problem of optimal coordination of protective relays other than directional overcurrent relays (e.g. distance relays [16]). The general optimization approach and its particularization to the case of directional overcurrent relays are presented below.

THE OPTIMAL COORDINATION PROBLEM OF PROTECTIVE RELAYS

The coordination problem of protective relays in power systems is stated here as an optimization problem of the following general form:

$$\min_{s \in S} \; [\, z(s,p) \,] \tag{1}$$

where z represents a suitable performance index, s represents the protective device settings, S is the set of permissible settings, and p represents the perturbations or fault conditions. This coordination problem is a very large dimensional problem, especially when a large number of perturbations is to be considered. One way of obtaining a solution to this problem is by using the *extended minimax* approach proposed in [16], by means of which the problem in Eq. (1) is written as:

$$\min_{s \in S} \; [\, \max_{p \in P} z(s,p) \,] \tag{2}$$

where the subproblem $[\max z(s,p)$ s.t. $p \in P]$ is assumed to have multiple local solutions $p_k^* \in P^*$, $k = 1,\ldots, np$. $P^* = \{p_1^*, \ldots, p_{np}^*\}$ is the set of np worse or more "relevant" perturbations; this set, which might be determined with the help of system expertise, contains the boundaries and other relevant points of the perturbation space. The problem of determining P^* is not treated here and is assumed to have been solved beforehand.

Each of the relevant perturbations $p_k^* \in P^*$, $k = 1,\ldots, np$, defines a separate optimization problem with its own objective and constraint set, so that in essence, the problem stated in Eq. (2) can be formulated as a multiple objective optimization problem in terms of the settings, as follows:

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$$\min \ [\, z\,(s, p_1^*), \ldots, z\,(s, p_{np}^*)\,]$$
$$s.t. \quad s \in S_1, \ldots, s \in S_{np} \tag{3}$$

where S_k, $k = 1, \ldots, np$, represents the feasible set of settings for the relevant perturbations or scenarios. Assuming one objective per perturbation, this problem can be formulated as a multiple criteria nonlinear optimization problem of the form:

$$\min \ [\, z_1(s, T), \ldots, z_{np}(s, T)\,]$$

s.t. (a) $h(T) \le 0$ (coordination criteria)

(b) $s_{\min} \le s \le s_{\max}$ (bounds on the relay settings)

$T_{\min} \le T \le T_{\max}$ (bounds on the operation times)

(c) $T = f(s)$ (relay characteristics) (4)

The particularization of this general problem to the case of directional overcurrent relays is presented below.

The Optimal Coordination Problem of Directional Overcurrent Relays

Overview. A typical inverse time overcurrent relay consists of two elements, an instantaneous unit, and a time overcurrent unit. The overcurrent unit has two values to be set, the pickup current value (Ip), and the time dial setting (TDS). The pickup current value is the minimum current value for which the relay operates. The time dial setting defines the operation time (T) of the device for each current value, and is normally given as a curve T vs. M, where M (i.e. the multiple of the pickup current), is the ratio of the relay current, I, to the pickup current value, i.e., $M = \frac{I}{Ip}$. In general, overcurrent relays respond to a characteristic function of the type,

$$T = f\,(TDS, Ip, I) \tag{5}$$

which, under simplistic assumptions, can be approximated by [2]:

$$T = K_1 \frac{TDS}{[(\frac{I}{Ip})^{K_2} + K_3]} \tag{6}$$

where K_1, K_2, and K_3, are constants that depend upon the specific device being simulated. A more precise formula for approximating the characteristics is [14,17]:

$$T = (PTDS)(PIp) \tag{7}$$

where:

$$PTDS = (K_{10} + K_{11}TDS + K_{12}TDS^2 + K_{13}TDS^3), \tag{8}$$

$$PIp = (A_0 + \frac{A_1}{(M-1)} + \frac{A_2}{(M-1)^2} + \frac{A_3}{(M-1)^3} + \frac{A_4}{(M-1)^4}) \tag{9}$$

and $K_{10}, K_{11}, K_{12}, K_{13}, A_0, A_1, A_2, A_3,$ and A_4 are scalar quantities which characterize the particular device being simulated.

The calculation of the two settings, TDS and Ip, is the essence of the directional overcurrent relay coordination study. It is very important to mention that in general, directional overcurrent relays allow for continuous time dial settings but discrete (rather than continuous) pickup current settings. In this study, however, both Ip and TDS were assumed to be continuous variables, in order to avoid the use of mixed nonlinear-integer programming techniques. The discrete Ip solutions are obtained in this study by rounding-off the continuous Ip solutions to the nearest discrete values. The application of mixed nonlinear-integer programming techniques to this coordination problem is a subject of current research.

Statement of the Problem. The general coordination problem in Eq. (4) can be directly particularized to the problem of selecting the settings for a coordinated operation of directional overcurrent relays. In this case, $s = [TDS, Ip]$, and the functions z_k, $k = 1, \ldots, np$, represent suitable objectives to be achieved, e.g. minimizing the equipment stress. One way of estimating this equipment stress is by the classical calculation of the energy dissipated by the equipment as heat. This energy is approximately proportional (for constant current magnitude) to the square of the equipment current magnitude multiplied by time. One way of indirectly minimizing this equipment stress, is by making each z_k a weighted aggregation of the operation times of the relays in zone k as follows:

$$z_k(TDS, Ip, T) = \sum_i \sum_j \omega_{ijk} T_{ijk} \tag{10}$$

where T_{ijk} is the operation time of relay i of zone j (i.e. relay R_{ij}) for a fault in zone k, and the weights ω_{ijk} may depend upon the probability of a given fault occurring in each of the zones of the protective relays. For the sake of simplicity, the following additive value function is used to aggregate the different objectives:

$$\sum_k z_k(TDS, Ip, T). \tag{11}$$

It can be shown ([16]) that the objective functions, when chosen as in Eq. (10), are not in conflict; therefore, the solution to the coordination problem with an additive value function as the objective (as in Eq. (11)) is not dependent upon the choice of the weights ω_{ijk}.

In this case of directional overcurrent relays, the constraints equations of the problem in Eq. (4) are:

(a) **coordination criteria**, $h(T) \le 0$, which for a given configuration can be described by:

$$T_{nmk} \ge T_{ijk} + \Delta T_{mj} \tag{12}$$

where T_{nmk} is the operation time of the first backup of R_{ij} for a fault in protection zone k, and ΔT_{mj} is the coordination time interval for zones m and j. This coordination time interval depends upon the operation times of the power circuit breakers, the operation criteria, and other system parameters. ΔT_{mj} can be assumed as given data for the purpose of calculating the relay settings, and is sometimes assumed to be a constant (C) for the entire system.

For the transient configurations that occur when only one relay of a zone has operated, the coordination criteria must still assure a coordinated operation, independently of the tripping sequence, that is:

$$T'_{nmk} \ge T'_{ijk} + C', \tag{13}$$

where the superscript ($'$) indicates transient configuration quantities;

(b) **bounds on the relay settings and operation times:**

$$TDS_{ij\ \min} \le TDS_{ij} \le TDS_{ij\ \max} \tag{14}$$
$$Ip_{ij\ \min} \le Ip_{ij} \le Ip_{ij\ \max} \tag{15}$$
$$T_{ijk\ \min} \le T_{ijk} \le T_{ijk\ \max}\,; \tag{16}$$

(c) **relay characteristics**, $T = f\,(TDS, Ip)$, or more specifically:

$$T_{ijk} = f_{ij}(TDS_{ij}, Ip_{ij}, I_{ijk})$$
$$T'_{ijk} = f_{ij}(TDS_{ij}, Ip_{ij}, I'_{ijk}) \tag{17}$$

where f_{ij} represents the relay characteristics (Eq. (6) or (7)),

TDS_{ij} is the time dial setting of relay R_{ij}, Ip_{ij} is the pickup current of relay R_{ij}, and I_{ijk} represents the current seen by relay R_{ij} for a fault in location k.

It is important to mention that it is theoretically possible to state the problem in Eq. (4) in a reduced form, that is, exclusively as a function of the settings TDS and Ip, thereby eliminating the equality constraints of the problem represented by Eq. (17).

OPTIMAL SELECTION OF THE SETTINGS OF DIRECTIONAL OVERCURRENT RELAYS: FIXED CONFIGURATIONS

When considering a fixed system configuration, the optimal coordination problem of directional overcurrent relays has to take into account several perturbations but only one configuration, which limits significantly the dimensionality of the resulting optimization problem. Solving this problem implies finding the coordinated settings TDS and Ip for all the directional overcurrent relays in the system, which will satisfy constraints (12)-(17) above for all the np system perturbations. We propose here solving this problem by taking advantage of the natural partition of the settings into the TDS settings and the Ip settings. That is, the overall problem is solved by an iterative procedure in which the time dial settings TDS are computed for a given set of pickup currents Ip, and vice versa, until convergence is achieved. One such possible iterative procedure which is based upon the Gauss-Seidel approach is as follows:

Step 0: Select an initial set of pickup currents Ip^*,

Step 1: Find TDS^* as the solution of: $\min_{TDS} z(TDS, Ip^*)$ subject to the relevant restrictions,

Step 2: Find Ip^* as the solution of: $\min_{Ip} z(TDS^*, Ip)$ subject to the relevant restrictions,

Step 3: Repeat steps 1 and 2 with the new values for the settings, until convergence is achieved, that is, until the maximum change in the settings is smaller than a given tolerance.

In general, the optimality of Gauss-Seidel-type approaches like the one presented here is not guaranteed, especially when the decision space is divided into subspaces, (i.e. the TDS and the Ip subspaces) and each of the solutions of the decision subspaces is found while the variables in the other subspaces are kept constant. In this particular case, however, the situation is different. In order to illustrate this, the following heuristic explanation is provided. Since the objective function z is strictly unimodal in the range of allowable settings, some of the constraints (12)-(17) must be active at the optimum, if this optimum exists. Now, given that the settings are grouped by TDS and Ip pairs (one pair for each relay), then, if at any step of the Gauss-Seidel procedure there is an active lower bound, say on a TDS, then the objective function could be decreased by marginally violating that particular TDS constraint, according to the Kuhn-Tucker conditions for optimality [18]. However, a similar effect would be obtained at the next step of the procedure by reducing the corresponding pickup current Ip until either its lower bound or a time coordination constraint is reached. On the other hand, if the bounds on the subset of settings are not active, then the coordination constraints, which are constraints on the operation times, must be active. If this is the case, then the operation times can not be reduced any further in the feasible time region, and the resulting point is the optimum.

In what follows, we will present the approaches for computing TDS for a given Ip, and for computing Ip when TDS is known, whenever the configuration of the system under consideration is assumed to be fixed and known beforehand.

Optimal Selection of the Time Dial Settings

The optimal coordination problem of directional overcurrent relays is a nonlinear optimization problem (for fixed Ip) due to the fact that the time dial settings TDS are related to the operation times T_{ijk} in a nonlinear fashion. Therefore, whenever it is possible to relate these variables using a linear expression, the problem of finding TDS reduces to a linear programming problem. This is the case when the overcurrent relays are represented by characteristics of the type indicated in Eq. (6), and the pickup currents are assumed to be known. In this case, the equations of the relays can be rewritten in the following form which relates TDS and T_{ijk} linearly:

$$T_{ijk} = K_{ijk}\, TDS_{ij} \tag{18}$$

where:

$$K_{ijk} = \frac{K_1}{\left[\left(\frac{I_{ijk}}{Ip_{ij}}\right)^{K_2} + K_3\right]} \tag{19}$$

and therefore, the problem is simplified to a linear programming problem.

It is important to mention that the problem can still be stated as a linear programming problem even if the relays are to be represented by the more precise characteristic equations of the type presented in Eq. (7). In this case, a linear programming problem can be solved in terms of the variables $PTDS$, which can then be used to compute TDS from Eq. (8).

Illustrative Example. This example was designed to demonstrate the calculation of the time dial settings of the overcurrent relays of the system shown in Fig. 1, using the linear programming approach presented above. Notice that in the particular case of Fig. 1, each protection zone corresponds with one of the transmission lines. In this example, it was assumed that the relevant perturbations were three-phase faults which occurred at the middle of the transmission lines (to reduce the dimensionality of the example), and only the complete configuration was considered. The system data and the pickup current values for this system are contained in Table I. A coordination time interval of 0.2 s was adopted, together with a minimum allowed TDS of 0.1 (in a scale of 0.0 to 1.1). The objective function weights were all set equal to one.

For the configuration of the system shown in Fig. 1, the

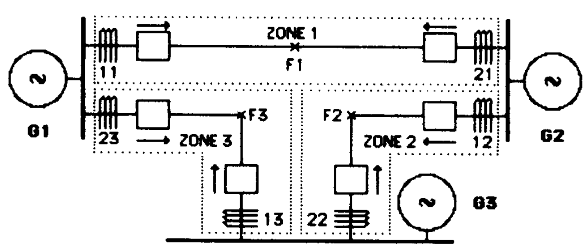

Figure 1. Three-Bus System Example.

Table I
Three Bus System Example Data

GENERATOR DATA (could be identified as equivalents):						
Generator 1:		100 MVA		69 kV		20 %
Generator 2:		25 MVA		69 kV		12 %
Generator 3:		50 MVA		69 kV		18 %
TRANSMISSION DATA:						
Line 12:		50 km		$z = 5.5 + j\ 22.85\ \Omega$		
Line 23:		40 km		$z = 4.4 + j\ 18.00\ \Omega$		
Line 13:		60 km		$z = 7.6 + j\ 27.00\ \Omega$		
RELAY DATA:						
All relays were assumed identical and with characteristic functions approximated by: $T_{ijk} = 0.14 \times TDS_{ij}/[(I_{ijk}/Ip_{ij})^{0.02} - 1]$						
Relay Number	11	21	12	22	13	23
C.T. Ratio	300/5	200/5	200/5	300/5	200/5	400/5
TAP	5	1.5	5	4	2	2.5
Ip (Amps.)	300	60	200	240	80	200

Table II
Simplex Algorithm Results (all units in seconds)

Optimal Objective Function Value = 1.9250		
$T_{111} = 0.3641$	$T'_{111} = 0.3550$	$TDS_{11} = 0.1000$
$T_{211} = 0.2856$	$T'_{131} = 0.5550$	$TDS_{21} = 0.1364$
$T_{222} = 0.3390$	$T'_{221} = 0.6464$	$TDS_{12} = 0.1000$
$T_{122} = 0.3216$	$T'_{211} = 0.2902$	$TDS_{22} = 0.1000$
$T_{133} = 0.3011$	$T'_{112} = 0.7459$	$TDS_{13} = 0.1298$
$T_{233} = 0.3144$	$T'_{122} = 0.3133$	$TDS_{23} = 0.1000$
$T_{131} = 1.1593$	$T'_{232} = 0.6094$	
$T_{221} = 0.8409$	$T'_{222} = 0.3304$	
$T_{112} = 0.9663$	$T'_{123} = 0.6751$	
$T_{123} = 1.0661$	$T'_{132} = 0.2951$	
$T_{213} = 1.0697$	$T'_{213} = 0.5073$	
$T_{232} = 0.8202$	$T'_{233} = 0.3073$	

coordination problem can be stated as the linear programming problem included in Appendix A, where for the sake of simplicity, and considering the fact that all the objectives have the same units (seconds), an additive value function was used to minimize the sum of the operation times of the relays in their own zone. The linear programming solution obtained for this small problem using the simplex method is shown in Table II. Several cases were tested varying the bounds on the settings and the coordination time interval. The method proved to be reliable with good convergence characteristics in all cases.

Optimal Selection of the Pickup Current Settings

For a given *TDS*, the optimal coordination problem of directional overcurrent relays reduces to a nonlinear optimization problem, whose variables are the pickup currents (when using relay characteristics of the type described by Eq. (6)). Several nonlinear optimization algorithms could be selected to solve this problem. We have initially selected the generalized reduced gradient nonlinear optimization technique [18] for testing purposes. Other state-of-the-art optimization techniques (e.g. MINOS/Augmented [19]) could also be used.

Notice that if the system configuration is assumed to be fixed and known beforehand, and the relays are assumed to be represented by the characteristic equations indicated in Eq. (7), this problem of determining the pickup currents for fixed *TDS* can be stated in terms of a linear programming problem whose variables are the *PIp*'s. The solution to this linear programming problem would then be used together with the relay currents I_{ijk} (corresponding to the respective T_{ijk}) to compute *Ip* from Eq. (9).

Illustrative Example. The Gauss-Seidel algorithm presented above has been applied to the optimal coordination problem of the directional overcurrent relays in the system depicted in Fig. 1, with the same test conditions as described above. The problem was solved iteratively, initially in terms of *TDS* using the simplex method for linear programming, and then, with this intermediate solution, in terms of the pickup currents using the generalized reduced gradient approach. The test results are summarized in Fig. 2. Two different groups of results (say series 1, and 2) are presented in this figure. Series 1 and 2 differ in the values of the lower bounds on the *TDS*, which were 0.05 and 0.1 (in a scale of 0 to 1.1), respectively. At the first iteration of Series 1, most of the lower bounds on *TDS* were not active but the coordination constraints (i.e. the restrictions on the operation times) were binding; therefore, the objective function (sum of operation times) could not be significantly reduced anymore by varying the pickup currents. On the other hand, Series 2 had several of the lower bounds on *TDS* active at the first iteration and therefore the corresponding operation times were reduced by modifying the pickup currents at the second iteration.

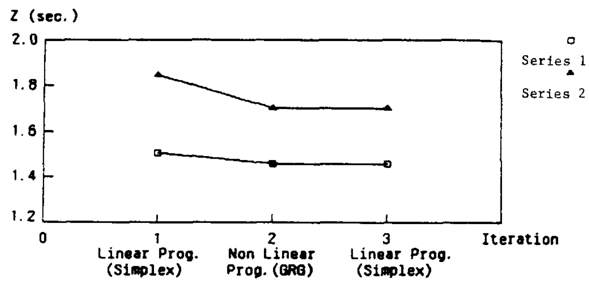

Figure 2. Objective Function Value vs. Iteration Number.

OPTIMAL SELECTION OF THE SETTINGS OF DIRECTIONAL OVERCURRENT RELAYS: MULTIPLE CONFIGURATIONS

When several different system configurations are considered relevant, the optimal coordination problem of directional overcurrent relays has to take into account the variables and constraints associated with several different configurations. For example, if for the system in Fig. 1, the four configurations shown in Fig. 3 are considered relevant, the objective function could be written as:

$$\min\ [\ (T_{1111} + T_{2111} + T_{1221} + T_{2221} + T_{1331} + T_{2331}) + \\ + (T_{1112} + T_{2112} + T_{1222} + T_{2222}) \\ + (T_{1113} + T_{2113} + T_{1333} + T_{2333}) \\ + (T_{1224} + T_{2224} + T_{1334} + T_{2334})\ ] \tag{20}$$

where T_{ijkl} indicates the operation time T_{ijk} for configuration l. Theoretically, this problem (with the associated set of constraints) could be solved using the approach described in the previous section. However, the particular structure of this problem permits the application of decomposition techniques and hierarchical coordination procedures, which make it possible to obtain solutions more efficiently than with the direct techniques. In this section, we present several possible decomposition schemes (including configuration decomposition, fault location decomposition, and configuration-fault location decomposition), as well as a procedure for coordinating the

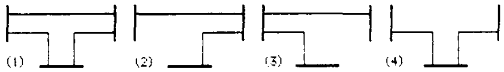

Figure 3. Relevant Configurations.

solutions obtained by these decomposition techniques.

Configuration Decomposition Scheme

In this scheme, the overall problem is partitioned into one lower level subproblem for each relevant configuration (Fig. 4). The variables involved in each subproblem at the lower level are the relay settings and the operation times for only one specific configuration. Therefore, the dimensionality of each subproblem at the lower level is considerable smaller than the dimensionality of the overall problem (i.e. when all the configurations are considered simultaneously). The subproblems at the lower level are coupled by the fact that the settings have to be the same for all configurations.

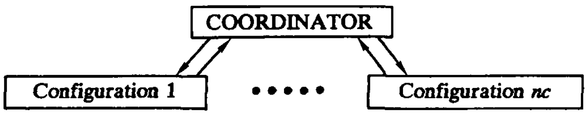

Figure 4. Configuration Decomposition Scheme.

To illustrate the characteristics of the problem structure, the overall coordination problem will be written in the following reduced form (without equality constraints):

$$\min z(s)$$
$$s.t.\ g_l(s) \le 0\ , \qquad l = 1, ..., nc \tag{21}$$

where nc is the number of relevant configurations, g_l represents the set of constraints in Eq. (12)-(16) for each configuration l, and z takes into account all the relevant perturbations for each configuration. Notice that in the above problem, the variables s are the same for all configurations. Problem (21) can also be formulated as follows:

$$\min \sum_l z_l(s_l)$$
$$s.t.\ g_l(s_l) \le 0, \qquad l = 1, ..., nc$$
$$s_1 = s_2 = \cdots = s_{nc} \tag{22}$$

in which case there is a different set of variables s_l for each configuration l. However, the last restriction guarantees that at a solution of problem (22) the settings for all configurations are the same.

Problems (21) and (22) are actually solved by decomposing the overall problem into one lower level subproblem per configuration, and then iteratively solving each separate subproblem until convergence is achieved. The subproblem solutions are to be related by means of a coordination procedure. One such possible procedure will be described below.

Fault Location Decomposition Scheme

The partitioning of the problem according to the location of the faults is also an available option. The scheme is illustrated in Fig. 5. The problem structure is similar to that of the configuration decomposition scheme but with $k = 1, ..., np$. The variables of each of the subproblems at the lower level are the relay settings and operation times of those devices that must operate (either as main or backup relays) for one particular fault, for all the considered configurations.

Figure 5. Fault Location Decomposition Scheme.

It is interesting to mention that the structure of the coordination problem is such that this fault location partition scheme may be applied even in the case of fixed configurations. This fact leads to the following two-level decomposition scheme.

Configuration-Fault Location Decomposition Scheme

This combined approach, which is illustrated in Fig. 6, results in lower level subproblems which only involve the relay settings and operation times of the devices that must operate (either as main or backup relays) for one particular fault and for one particular configuration, thus reducing significantly the dimensionality of the optimization subproblems to be solved. Furthermore, this configuration-fault location scheme can be partitioned even further than illustrated in Fig. 6. That is, at the lowest level of Fig. 6, the problem can be partitioned into one subproblem for each main relay and for each backup relay, resulting in the four-level scheme illustrated in Fig. 6 and 7. In this case, each of the subproblems at the lower level can be formulated as:

$$\min T_{BACKUP}$$
$$\text{s.t. } T_{BACKUP} - T_{MAIN} \ge C$$
$$T_{BACKUP} = f_{BACKUP}(s_{BACKUP})$$
$$s_{\min} \le s_{BACKUP} \le s_{\max} \tag{23}$$

where in the case of directional overcurrent relays, $s_{BACKUP} = [TDS_{BACKUP}, Ip_{BACKUP}]$.

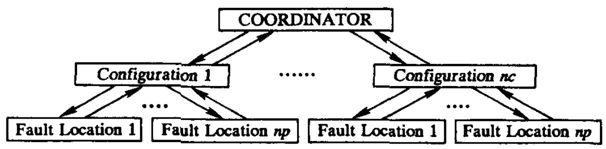

Figure 6. Combined Configuration-Fault Location Decomposition Scheme.

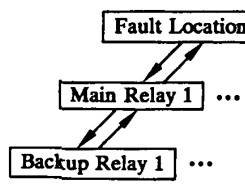

Figure 7. Partitioning by Relay Pairs.

In general, however, the solution to the problem in Eq. (23) is not unique. A solution to such problem can be found by the application of a procedure similar to the Gauss-Seidel technique described above, by means of which TDS_{BACKUP} is computed for a given value of Ip_{BACKUP}; and the value of Ip_{BACKUP} is then recalculated when it is necessary in order to meet the coordination criteria.

Another way of solving Eq. (23) is by the application of the classical solution procedure, that is, by increasing the settings of the backup relays in fixed steps until coordination is achieved. Therefore, the classical procedure of coordinating directional overcurrent relays can be seen as solving the overall optimization problem (whose objective function is the sum of the relay operation times) via a decomposition approach like the one illustrated in Fig. 6 and 7 which relaxes several of the problem constraints and coordinates the subproblem solutions using a given particular scheme.

Coordination Procedure

Selection of the Time Dial Settings. Several coordination procedures for determining the time dial settings may be applied [20,21]. Among these, the following procedure was applied to find the optimal *TDS* (or *PTDS* depending on the equation used for the relay characteristics, i.e. Eq. (6) or (7)), for a prefixed *Ip*:

(1) find: $z_1(s_1^*) = \min_s z_1(s)$, subject to: $s \in S_1$

(2) find: $z_k(s_k^*) = \min_s z_k(s)$
subject to: $s \in S_k,\ s \geq s_{k-1}^*,\ s \geq s_{k-2}^*, \ldots, s \geq s_1^*$; for $k = 2, \ldots, np$

(3) find: $z_k(s_k^*) = \min_s z_k(s)$
subject to: $s \in S_k,\ s \geq s_n^*$; for $n = 1, \ldots, np;\ n \neq i$

(4) repeat (3) until convergence is achieved, i.e. when $s_1^* = s_2^* = \cdots = s_{np}^*$.

This procedure is equivalent to a modification of the lower bounds on the settings according to the results obtained from the subproblems solved previously, and can be classified as a *restriction coordination* procedure [20]. In this procedure, the iterations continue until the interactions balance, i.e. until $s_1^* = s_2^* = \cdots = s_{np}^*$; then, according to the "interaction balance principle" the optimum is reached. The restriction coordination procedure is based on the fact that the overall optimum is always contained in the modified feasible set, since $s^* \geq s_n^*$ (element by element). This fact is shown in Appendix B for the case of the minimization of the sum of the relay operation times.

Selection of the Pickup Currents. A procedure similar to that suggested for the calculation of the *TDS*, can be applied to calculate the pickup currents in order to avoid the use of general nonlinear programming algorithms, using equations of the type of Eq. (7) and the decomposition technique illustrated in Fig. 6.

At the lower level of the combined configuration-fault location decomposition procedure, for each configuration and for each fault location, the problem is a linear programming problem in terms of the variables Plp_{ijkl}. With the "optimum" Plp_{ijkl} obtained for each fixed k,l, Eq. (9) has to be solved for Ip_{ij} (using the relay current values corresponding to the respective T) before feeding back the information to the higher levels.

Test Cases

Several test cases were run in order to evaluate the potential and effectiveness of the developed methodologies. Three test systems were selected for these runs, a three bus system, a six bus system, and a thirty bus system. For the sake of simplicity, in all the test cases presented here, the operation times of the backup relays are not included in the objective function of the optimal coordination problems, even though the developed approach could easily handle objective functions which consider such backup operating times. The rationale for this is that the operating times of the primary relays and those of the backup relays are not in conflict when considered as separate objectives, and therefore a reduction of one leads necessarily to a reduction of the other.

Test 1: Three Bus System. This case was designed to initially test the proposed partitioning procedures. Two decomposition schemes, the configuration decomposition scheme (Fig. 4), and the fault location decomposition scheme (Fig. 5), were applied to the problem of finding *TDS* (for fixed *Ip*) for the three bus system of Fig. 1 with the same system conditions indicated in Table I. For the first scheme (say, Case 1A), the four configurations of Fig. 3 were considered, but the transient configurations were not taken into account, while for the second scheme (say, Case 1B), only one configuration with three fault locations were considered (see Fig. 1). The coordination procedure applied in this test case was similar but more conservative than the one explained above (the bounds on the settings were kept constant throughout a complete iteration). These settings were updated only after each iteration was completed (a complete iteration in this context means completely solving a set of lower level subproblems for a given set of bounds on the settings).

In each case, the results of finding *TDS* for fixed *Ip* using linear programming at the lower level, were compared at each iteration with the results obtained by the direct approach (shown in Table II). The maximum differences between the elements of the solution vectors are illustrated in Fig. 8. Notice that both decomposition schemes converge to the solution obtained by the direct approach after 5 iterations.

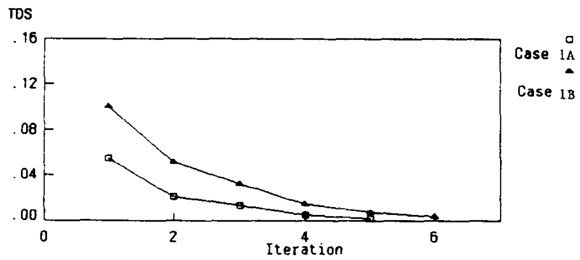

Figure 8. Global and Decomposition Solutions: Maximum Differences vs. Iteration Number.

Test 2: Six Bus and Thirty Bus Systems. In this test case, the configuration-fault location decomposition scheme (Fig. 6) was applied to the problem of selecting *TDS*, for a constant *Ip*, for a six bus, and a thirty bus test systems. Two optimization approaches, linear programming (simplex algorithm) and geometric programming [22] were implemented and tested for solving the problem at the lowest level (in terms of *TDS* and in the context of the proposed Gauss-Seidel procedure). Although the problem characteristics seemed appropriate for the application of the geometric programming algorithm, actual testing showed that this method has convergence problems, which were attributed to the non-convex properties of the problem [16]. Therefore, the simplex algorithm was the method of choice for solving the subproblems at the lowest level. The coordination procedure explained earlier was used in each case. The system data for each of these two systems is very extensive which makes it impossible to present here. Some details can be found in [16].

The following conditions were adopted in each case: (i) only first backup conditions were considered, (ii) the relays were assumed identical and were simulated using equations of the type of Eq. (7), (iii) the relay coefficients were calculated using a two step minimum squares algorithm, as presented in [17], calculating initially the coefficients of the expression PIp, and then those of the polynomial $PTDS$, and (iv) the faults considered in these systems were assumed to be located very close to each of the system buses.

The results for these test cases are shown in Fig. 9, where the maximum differences between the elements of the updated solution vector and those of the one at the previous iteration are plotted against the iteration number. Different cases were tested, including distinct fault types such as single phase to ground, phase to phase, and phase to phase to ground. The method in general showed good performance.

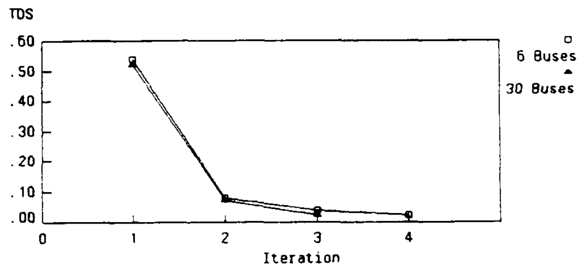

Figure 9. Combined Decomposition Scheme: Maximum Differences vs. Iteration Number.

ON-LINE AND EMERGENCY CONTROL

The coordination methodology presented here has the potential to be applied in on-line and emergency control applications. In order to illustrate this, consider for instance, the scheme of Fig. 4. In that case, the optimum objective function value obtained separately for each of the lower level configuration subproblems, is better than or equal to the overall optimum (which satisfies the constraints of all the configurations). Therefore, as a control strategy, it could be possible to effect a change of the relay settings in real time, according to a given present configuration, and to the particular needs of each case. A possible way of actually achieving this would be to obtain in advance the optimal solutions for the set of previously chosen relevant configurations. In case the system is operating in a configuration other than a relevant one, the overall optimum may be used as the control to be applied, while the proper solution, corresponding to that precise configuration, is being found on line.

The classical procedure of coordinating protective relays is limited to the minimization of the sum of operation times; however, this objective need not always be convenient. For example, if the system is in an insecure or emergency state, the operation times of certain relays will preferably be maximized to avoid or to delay unnecessary cascade outages that could lead the system into an emergency or restorative state. The formulation proposed in this paper can handle this and other objectives, and provides insight and a deeper understanding of the overall coordination problem.

CONCLUSIONS

This paper has presented an approach based upon the minimax optimization approach to optimally coordinate protective relays in power systems. With the application of the proposed methodology, this coordination problem is stated as a nonlinear parameter optimization problem that can be solved using conventional optimization techniques.

The coordination problem was particularized to the case of directional overcurrent relays, and solved using direct optimization techniques, as well as several decomposition approaches and partitioning schemes. The paper demonstrates that the classical iterative procedure for solving the coordination problem of directional overcurrent relays is a solution which is limited to the minimization of a specific objective, (i.e. the sum of the relay operation times), and which partitions the optimization problem into subproblems using a particular scheme to coordinate the subproblem solutions.

These conceptual contributions enlighten the problem from a different angle, opening a new area for research where perhaps better and faster solutions can be found.

ACKNOWLEDGMENTS

The authors are grateful to Dr. T.E. Dy Liacco and Dr. K.A. Loparo for their continuous advice throughout this work. Thanks are also due to the Center for Automation and Intelligent Systems Research of Case Western Reserve University for permitting the use of its computer facilities. Finally, the first author would like to acknowledge the financial support of the Fundación Gran Mariscal de Ayacucho of the Venezuelan government, with his gratitude.

GLOSSARY

C	Coordination Time Interval
ij	(Subscript) of relay i of zone j
ijk	(Subscript) of ij for fault k
$ijkl$	(Subscript) of ijk for configuration l
I	Vector of Relay Currents
Ip	Vector of Pickup Currents
(j)	(Subscript), j-th element of a vector
M	Multiple of the Pickup Current
nc	Number of Relevant Configurations
np	Number of Relevant Perturbations
p	Vector of Perturbations
P	Set of Feasible Perturbations
$PTDS$, PIp	Defined by Eq. (7),(8), and (9)
s	Vector of Relay Settings
S_k	Set of Feasible Settings for Perturbation k
T	Vector of Operation Times
TDS	Vector of Time Dial Settings
z, f, g, h	Real Valued Functions

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APPENDIX A

The problem of determining the optimal *TDS* for the configuration of the system shown in Fig. 1 can be formulated as the following linear programming problem (all units are in seconds):

$$\min T_{111} + T_{211} + T_{222} + T_{122} + T_{133} + T_{233}$$

s.t.

$$\begin{array}{ll}
T_{131} - T_{111} \ge 0.2 & T_{232} - 8.2020 \times TDS_{23} = 0 \\
T_{221} - T_{211} \ge 0.2 & T_{123} - 10.6610 \times TDS_{12} = 0 \\
T_{112} - T_{122} \ge 0.2 & T_{213} - 7.8420 \times TDS_{21} = 0 \\
T_{232} - T_{222} \ge 0.2 & T'_{111} - 3.5500 \times TDS_{11} = 0 \\
T_{123} - T_{133} \ge 0.2 & T'_{211} - 2.0540 \times TDS_{21} = 0 \\
T_{213} - T_{233} \ge 0.2 & T'_{122} - 3.1330 \times TDS_{12} = 0 \\
T'_{131} - T'_{111} \ge 0.2 & T'_{222} - 3.3040 \times TDS_{22} = 0 \\
T'_{221} - T'_{211} \ge 0.2 & T'_{133} - 2.2730 \times TDS_{13} = 0 \\
T'_{112} - T'_{122} \ge 0.2 & T'_{233} - 3.0730 \times TDS_{23} = 0 \\
T'_{232} - T'_{222} \ge 0.2 & T'_{131} - 4.2750 \times TDS_{13} = 0 \\
T'_{123} - T'_{133} \ge 0.2 & T'_{221} - 6.4640 \times TDS_{22} = 0 \\
T'_{213} - T'_{233} \ge 0.2 & T'_{112} - 7.4590 \times TDS_{11} = 0 \\
T_{111} - 3.6410 \times TDS_{11} = 0 & T'_{232} - 6.0940 \times TDS_{23} = 0 \\
T_{211} - 2.0940 \times TDS_{21} = 0 & T'_{123} - 6.7510 \times TDS_{12} = 0 \\
T_{122} - 3.2160 \times TDS_{12} = 0 & T'_{213} - 3.7190 \times TDS_{21} = 0 \\
T_{222} - 3.3900 \times TDS_{22} = 0 & TDS_{11} \ge 0.1 \\
T_{133} - 2.3190 \times TDS_{13} = 0 & TDS_{21} \ge 0.1 \\
T_{233} - 3.1440 \times TDS_{23} = 0 & TDS_{12} \ge 0.1 \\
T_{131} - 8.8730 \times TDS_{13} = 0 & TDS_{22} \ge 0.1 \\
T_{221} - 8.4090 \times TDS_{22} = 0 & TDS_{13} \ge 0.1 \\
T_{112} - 9.6330 \times TDS_{11} = 0 & TDS_{23} \ge 0.1
\end{array}$$

APPENDIX B

In this appendix, we show that in the restriction coordination procedure, the overall optimum is always contained in the modified feasible set, i.e. that $s^* \ge s_i^*$ (element by element). Consider the reduced problem (i.e. the problem without equality constraints) in terms of only one of the two subsets of settings (either *TDS* or *Ip*). This problem has basically two types of constraints: (a) the lower bounds on the settings, $s_{(j)} \ge b$, for all (j), and (b) the coordination constraints, $c\, s_{(j)} \ge d\, s_{(k)} + C$, where $c, d > 0$, and the relays corresponding to $s_{(j)}$ and $s_{(k)}$ are one a backup of the other. Then at the optimum, independently of which type of constraint is active, $\frac{\partial s_{(k)}}{\partial s_{(j)}} \ge 0$, for $i \ne j$. Now assume there is an element $s^*_{(j)}$ of s^* such that $s^*_{(j)} \le s_{i(j)}$ (the corresponding element of s_i). Then $s^* \le s_i^*$ element by element, and $z_i(s)^* = a\, s^* \le z_i(s_i) = a\, s_i^*$, because the elements of the vector (a) are all non negative when minimizing the sum of the relay operation times. Therefore, the contradiction that s_i^* is not optimal is reached.

Discussion

J. E. Stephens (Illinois Power Co., Decatur, IL): Optimal settings for directional overcurrent relays in an interconnected power system are those which result in minimum relay time *for the required coordination.* Coordination may not be required for all possible situations.

No mention is made of the instantaneous overcurrent relay setting which usually provides much of the terminal relaying and is important to the setting of the backup time overcurrent relay and coordination. Two fault conditions are critical to the coordination of two overcurrent relays and provide the basis for the time setting of the backup time delay relay.

1) The fault just beyond the downstream breaker in the configuration that produces maximum fault current (minimum relay time) in the backup breaker whose relays are being set.
2) The fault through the downstream breaker of a magnitude equal to the instantaneous overcurrent (IT) relay setting for that breaker in the configuration that produces maximum fault current in the backup relay being set.

All other configurations and fault conditions are not relevant for that coordination pair.

The first criteria also provides the basis for setting the IT of the backup relay. The operation of the instantaneous relay may be significant or critical to obtain the minimum time setting of the backup relay. If short lines in a network system require unacceptable overcurrent relay times, then distance relays may be required to insure instantaneous tripping for faults on a major portion of the line.

Reference is made in the paper to determining relay pickup currents for *fixed* time dial settings (TDS). If a relay tap setting is varied to obtain better coordination, a change in TDS would usually also be required. In practice, relay pickup settings would usually be selected based on load current requirements and minimum line end fault conditions. They may then be adjusted to improve coordination for a second iteration of time dial settings.

In the illustration example of Fig. 1, the faults were taken at the middle of each transmission line. Apparently, relay settings were determined and coordinated on these fault current values, but *line end faults* are required for the critical values on which the relay settings and coordination must be based. Also, instantaneous relays would have a significant effect on the time delay settings required.

At the beginning of the discussion of the test cases it is stated, "in all the test cases presented here, the operation times of the backup relays are not included in the objective function of the optimal coordination problems ····." This statement appears to be in direct opposition to the stated purpose of the investigation since the time overcurrent relays are the backup relays for which optimal settings are being determined to coordinate with the downstream relays. Can the authors clarify this statement?

Manuscript received February 17, 1987.

R. T. Casey (Georgia Power Co., Atlanta, GA): The authors are to be commended for their new methodology of coordinating directional overcurrent relays in power systems using the principles of optimization theory. This technical approach to the coordination of directional relays could increase productivity and improve system reliability.

Some specific questions arise regarding the parameters and procedures that were used to determine your settings.

1) Does your program or technique allow calculations to be done on one individual transmission line without affecting the existing settings on other lines? If not, large systems would have to reset every directional ground relay on their system after each study is done.
2) The relevent configurations, the different types of relays, and mutually coupled lines could increase the permutations necessary for correct analysis to an infinite number. Does this technique merit review for 100 bus systems or larger?
3) Throughout your paper you determine the coordination for the directional ground relay time overcurrent unit. What considerations have you incorporated to coordinate the directional ground relay instantaneous unit?
4) The case of two parallel lines terminated at the same stations at both ends creates coordination problems for directional and nondirectional ground relays. For this configuration the time overcurrent units at all four terminals must be set the same. The relay requiring the highest setting governs the setting for the other three. Would your "Optimal Selection of the Settings of Directional Overcurrent Relays: Fixed Configurations" produce a solution similar to the one above?

Finally, this concept does open a new area for research in directional ground relaying that should be pursued further.

Manuscript received March 2, 1987.

A. J. Urdaneta, R. Nadira, and **L. G. Perez Jimenez**: The authors would like to thank Mr. Stephens and Mr. Casey for their interest in the paper and their valuable comments. We will try to address their questions in the order they were raised. First, both discussers inquire about coordinating the instantaneous units of directional overcurrent relays. Even though this coordination problem was not specifically addressed in the paper, it is also possible to state it as an optimization problem which could be solved using the general optimization techniques presented. The main difference between the formulation of this problem and the one presented in the paper is that the coordination constraints for the instantaneous unit coordination problem are stated in terms of the currents and not in terms of the operation times.

We agree with Mr. Stephens that the best (optimal) coordination of directional overcurrent relays need not always lead to the best protective scheme. However, the proposed methodology may help determine whether the use of directional overcurrent relays is convenient or not.

With respect to the issue of determining the relay pickup currents (Ip) for fixed time dial settings (TDS), we would like to reiterate that the Gauss-Seidel procedure proposed in the paper for solving the optimal coordination problem is an iterative approach, which consists of two basic steps: 1) finding the optimum TDS^* for a given set of Ip^*, and 2) finding the optimum Ip^* for a given set of TDS^*. These steps are iterated until convergence is achieved. Thus optimality is only achieved when no feasible TDS^* and Ip^* solutions improve the objective function value. This can be viewed as an automatization of the manual procedure described by Mr. Stephens.

In the example of Fig. 1, the faults were purposely taken at the middle of the transmission lines; otherwise (as indicated in the text of the example) the dimensionality of the resulting optimization problem would have been relatively large, and therefore the results would have been somewhat difficult to present. However, we agree with Mr. Stephens that for this particular example the set P^* probably consists of a different set of perturbations.

Mr. Stephens also raises a question about the objective function used in the test cases. The optimization technique presented in the paper can accommodate several different objective functions; the function used in the test runs is just a particular case. However, it should be pointed out that if the operation time of a given relay is minimized when it acts as a main relay, then the operation times of its operation as a backup relay are also minimized. In other words, the operating times of the primary relays and those of the backup relays are not in conflict when considered as separate objectives, and therefore a reduction of one leads necessarily to a reduction of the other.

With respect to Mr. Casey's question about the technique allowing for calculations to be done on individual lines, we can say that the proposed technique is very flexible and can be adapted to the particular needs of each case. For instance, for the specific situation of a coordination on individual lines, one could include in the statement of the optimal coordination problem certain restrictions which would have the effect of fixing the settings of those relays which are to be kept unchanged. The current implementation of the methodology, however, has no special provisions for automatically solving for the settings of a specific line without modifying the rest of the system. Indeed, this would be a desirable feature of the program.

With regards to the applicability of the technique on large scale problems, it can be said that the coordination of directional overcurrent relays must be performed considering only the faults and configurations that are strictly necessary. That is, it is possible to solve the optimal coordination problem relaxing many of its constraints (knowing *a priori* that many such constraints are not active at the optimal solution) with the resulting effect of reducing the computational requirements. This consideration was not explored during the course of this study and would constitute a required improvement to the implementation of the technique, especially if it is to be tested on systems of realistic size.

Finally, with regards to the settings of directional overcurrent units on parallel lines, the optimization will lead in general to two different pairs of settings, one for each pair of directional overcurrent units of each station (the radial case is an example where the pairs of "optimal settings" are different). The particular settings will vary with the parameters of the lines and with the characteristics of the system at both extremes. However, if it is desired, a restriction could be added to the optimization problem which would force the settings of all four relays to be the same.

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