

## AN ON -LINE RELAY COORDINATION ALGORITHM FOR ADAPTIVE PROTECTION USING LINEAR PROGRAMMING TECHNIQUE

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**Abstract:** *An adaptive system for protecting a distribution network should determine and implement relay settings that are most appropriate for the prevailing state of the power system. This paper presents a technique for determining coordinated relay settings. The technique uses the Simplex two-phase method; Phase I determines whether the constraints selected for illustrating the conditionality between primary and back up relays are feasible, and Phase II finds the optimal relay settings. A looped distribution system, protected by directional overcurrent relays, was used for testing the technique. The tests were conducted in a laboratory environment; some results from those tests are reported in the paper.*

### I. INTRODUCTION

The key issue in selecting the relay settings is to achieve the minimum possible operating times while maintaining coordination among all relays. Usually, finding the coordinated settings takes several iterations before a satisfactory solution is achieved. Traditionally, a trial and error procedure is employed for setting relays in multiloop networks. In the past few years, several mathematical techniques have been reported. Knable [1] proposed a technique to break all the loops at the, so called, break points and locate the relays from which to start the coordination procedure. Since looped circuits are normally protected by directional overcurrent relays located at both ends, the loops formed in the clockwise and anti-clockwise directions are considered for determining the break points. Dwarakanath and Nowitz [2] suggested a

systematic approach for determining the relative sequence setting of the relays in a multiloop network. They used a linear graph theory approach which provided a directional loop matrix. A minimal set of break points spanning all loops of the system graph were obtained from this matrix. Damborg et al. [3] extended the graph theoretic concepts and proposed a systematic algorithm for determining a relative sequence matrix corresponding to a set of sequential pairs which reduced the number of iterations. Jenkins et al. [4] proposed a functional dependency concept for topological analysis of the protection scheme. They expressed the constraints on the relay settings through a set of functional dependencies. Relay coordination was carried out through the identification of a break point set (BPS) and a relative sequence matrix. The choice of the initial settings of the BPS relays was used to select the settings of the remaining relays. A parametric optimization approach was reported by Urdeanneta et al. [5] that optimized the time multiplier settings (TMS) using the Simplex method. Optimal values of the pick-up currents for selected TMS were then determined by using a generalized reduced gradient technique.

Both the graph theoretic and functional dependency approaches provide a solution which is the best of the alternative settings considered, but not necessarily an optimal solution. In this paper, a relay coordination algorithm is described. It uses the Simplex two-phase method; Phase I detects whether all the selected operating conditions between the primary and backup relays are valid, and Phase II finds the optimum relay settings. The operating conditions that are detected in Phase I to be "not valid" are excluded at the beginning of the Phase II.

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The optimization technique was implemented in the Power System Research laboratory at the University of Saskatchewan [7]. Each relay was implemented on a TMS320C25 DSP board placed in a host personal computer. One personal computer performed the role of a substation computer and another acted as the central control computer.

All PCs were interconnected by communication links. Relaying DSPs were interconnected by a local area network and the PC acting as a central computer was connected to the PC acting as a substation computer by an RS 232 link. The software was tested in the laboratory to prove the viability of the concepts. The objective of this paper is to present some of the optimization concepts and their use in the project.

## II. THE ALGORITHM

The operating times of overcurrent relays protecting a looped distribution network as a function of the location of fault is shown in Fig. 1. The fault currents are generally maximum when the system has maximum generating sources connected to it. The inverse time overcurrent relays are set for this condition. When the generation is less, the fault current levels are lower and the relays take longer to clear the faults. To reduce the shock to the system due to the fault, it would be desirable to change the settings of relays so that the relays take minimum possible operating time without jeopardizing their coordination for all possible states of the system. This can be achieved by using adaptive protection that implements optimal setting of the relays during all operating states of the power system.

The relay coordination algorithm, which is based on a parameter optimization technique, optimizes an objective function of operating times of the primary relays subject to keeping the operation of the backup relays coordinated. Ideally, all primary relays should operate in minimum possible time but this can not be achieved because it would be impossible to keep the backup protection

properly coordinated. One possible approach to achieving minimum shock to the system due to faults would be to minimize a weighted sum of the operating times of all primary relays hoping that the operating times of individual primary relays would be close to the minimum individual operating times that might be possible. Since the lines are short and approximately of equal length, equal weights (=1) were assigned for operating times of all the relays. For a network consisting of  $m$  relays, the operating times of the primary relays for near-end faults can be expressed as:

$$z = \sum_{i=1}^m t_{i,i}, \quad (1)$$

where:  $t_{i,i}$  is the operating time of the primary relay at  $i$  for near-end fault at  $i$ .

The operating times of the back-up relays must be more than the sum of the operating times of the primary relays and the coordination margin. This can be expressed as

$$t_{bi,i} \geq t_{i,i} + \Delta t, \quad \text{for } i = 1 \text{ to } m, \quad (2)$$

where:  $t_{i,i}$  is the operating time for the primary relay for a near-end fault,  
 $t_{bi,i}$  is the operating time for the backup relay for the same near-end fault and  
 $\Delta t$  is the coordination time interval (CTI).

In the application reported in this paper, overcurrent relays conformed to the following IEC characteristic [6]:

$$t = \frac{k \times TMS}{I_{mpu}^n - 1} \quad (3)$$

where:  $k$  is a constant,  
 $n$  is a characteristic index,  
 $I_{mpu}$  is the multiple of pick-up current and  
 $TMS$  is the time multiplier setting.

Since the pickup currents of the relays are pre-determined from the system requirements, Equation 3 becomes

$$t = a \times TMS, \quad (4)$$

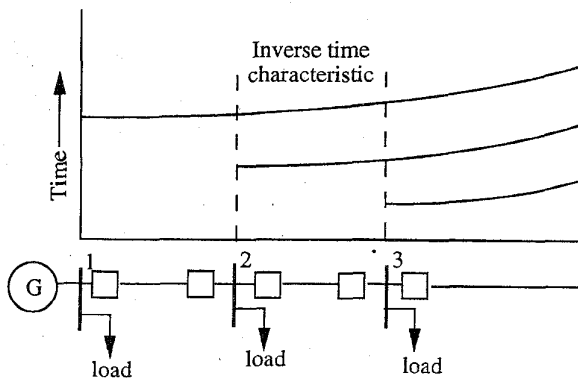


Fig. 1. Operating time of overcurrent relays as a function of the location of fault.

where: 
$$a = \frac{k}{I_{mpu}^n - 1}.$$

By making this substitution in Equation 1, the objective function becomes

$$z = \sum_{i=1}^m a_i TMS_i. \quad (5)$$

In this equation,  $a_i$ 's are known; values of  $TMS_i$  are determined by minimizing  $z$  and satisfying the coordination between the primary and backup relays. This equation is optimized using the well known Simplex two-phase method [9] subject to the condition that the operation of the back up relays remains properly coordinated. Phase - I determines whether all the selected constraints are valid. If constraint is not valid, it is excluded at the beginning of Phase - II.

Phase I of the two-phase Simplex method finds a feasible solution and Phase II finds the optimal solution. In Phase I, the initial extreme point is moved from the origin to the feasible region. In Phase II, pivoting is done from the initial extreme point to an optimal extreme point.

To begin with, the inequalities described by Equation 2 are converted to equalities by introducing non-negative surplus variables,  $x_{s1}, x_{s2}, \dots, x_{sm}$  and non-negative artificial variables  $x_{t1}, x_{t2}, \dots, x_{tm}$ . Since artificial variables are all non-negative, they are all zero only when their sum is zero. The artificial variables are eliminated by minimizing a function

$$w = x_{t1} + x_{t2} + \dots + x_{tm}. \quad (6)$$

During the minimization of  $w$ , the original objective function  $z$  is also updated. The process by which  $w$  is minimized is called Phase I.

If the value of  $w$  is zero at the end of phase I, the values of all artificial variables are zero and a feasible solution to the original problem has been achieved. At this stage all artificial variables become non-basic. If  $w$  cannot be reduced to zero, it is concluded that no feasible solution exists. One or more artificial variables, which are still in the basis, have positive values. In such a case, the infeasible constraints corresponding to the remaining positive artificial variables are withdrawn from the original problem. Phase I is repeated to obtain the feasible solution. Phase II of the Simplex method then optimizes the objective function. The

procedure for minimizing the object function,  $z$ , is similar to the procedure used for minimizing the function,  $w$ .

### III. CASE STUDIES

The developed algorithm was used to obtain in an on-line mode the  $TMS$  values for protecting the distribution network of the City of Saskatoon using the approach described in the this paper; more details are given in Reference 7. The details of the network configuration are provided in Appendix A. Each line of the network is equipped with overload, phase fault directional overcurrent and instantaneous overcurrent relays. Overload relays are connected to the ct's that see currents up to three times their ratings. When the currents are more than those limits, phase fault overcurrent ct's become effective [8]. The instantaneous units reduce the operating time for near-end faults.

While testing the algorithm for adaptive relaying functions, the following four operating conditions were selected:

1. Maximum system load and maximum system generation with line 1-20 closed (MXLG-1).
2. Minimum system load and maximum system generation with line 1-20 closed (MNLG-1).
3. Maximum system load and maximum system generation with line 1-20 open (MXLG-2).
4. Minimum system load and maximum system generation with line 1-20 open (MNLG-2).

For each operating condition, pre-fault currents and voltages are obtained from the system measurements, and some are obtained from the state estimator. The adaptive protection software simulates faults at the near and far ends of the lines. The software determines the secondary pickup currents for overload, phase fault overcurrent and instantaneous overcurrent relays from the criteria mentioned in the next section.

### IV. RELAY SETTING CRITERIA

The relay settings for adaptive protection are based on the selected power system operating conditions. For comparison, a non-adaptive approach has also been considered and the settings are selected based on the worst case scenario to prevent unnecessary trippings.

### A. Overload Pick-up

In the non-adaptive approach, the relays are set at 2.0 times the maximum load current. When the adaptive approach is used, the overload settings equal to two times the load being catered could be used. The load will be usually changing with the passage of time. The objective is that if the communication between the relays and the central computer fails, the protection system should continue to operate properly. To ensure this, the overload settings are not allowed to become less than two times the present load and also not less than 1.5 times the maximum load anticipated over the course of the next twenty four hours.

### B. Overcurrent Pickup

The pick-up settings of the phase fault overcurrent relays are determined considering both the load and fault currents. With the decrease in the load current, the pick-up settings are reduced but are kept at more than the maximum line current experienced over a 24 hr period.

### C. Instantaneous Overcurrent Settings

The tap settings of the instantaneous relays are selected to be 1.3 times the far-end fault currents. Since the fault currents change with the change in system generation and circuit topology, the settings for instantaneous relays are also changed.

In the case of the non-adaptive approach, the instantaneous tap settings are determined considering the maximum far-end fault currents and are kept fixed.

## V. RESULTS

After selecting the pickup settings and calculating the fault currents, the coordination software minimized the operating time for the near end faults to determine the optimum time multiplier settings. A coordination time interval of 0.2 sec was used. Table 1 shows the calculated pickup and time multiplier settings of overload, overcurrent and instantaneous overcurrent relays. The table also includes the load currents, fault currents and ct ratios. Figs. 2 and 3 show the operating characteristics for relays 1-1 (at Ave C S/S) and 5-1 (at Pl. Hill S/S) which protect the line between Ave C and Pl. Hill substations. Fig. 4 shows the operating characteristics for relay 1-4 (at Ave C S/S) which protects line 1-20. Fig. 5 shows similar characteristics for relay 4-1 (at Cowley S/S) which protects the line between Cowley and Pl. Hill substations. All characteristics are shown for adaptive and non-adaptive approaches using the developed algorithm. It is clear from these figures that the technique provides better solution to the distribution network protection

compared to settings provided by the traditional non-adaptive approach..

Table 1: Load, fault currents and settings of the relays.

Relay	Operat. Cond.	Load Curr. (A)	Fault Currents (Amps)		Phase Fault & O/L cts	Secondary Pickup/ Tap Settings			TMS for O/C & O/L
			Near-end Faults	Far-end Faults		O/L Relay	O/C Relay	Inst. Relay	
1-1	MXLG-1	398	46815	4941	2000/5	7.00	4.75	16.0	0.088
	MNLG-1	181	42546	4440	&	6.00	4.50	14.5	0.084
	MXLG-2	646	40648	5703	800/5	8.00	4.25	18.5	0.085
	MNLG-2	337	36503	5273		7.00	3.75	17.0	0.086
1-4	MXLG-1	404	42642	3875	2000/5	6.75	5.00	13.0	0.084
	MNLG-1	192	38804	3692	&	6.00	4.50	12.0	0.086
	MXLG-2	-	-	-	600/5	-	-	-	-
	MNLG-2	-	-	-		-	-	-	-
1-6	MXLG-1	522	42490	8741	2000/5	9.25	4.50	28.5	0.089
	MNLG-1	223	38940	8670	&	8.25	4.00	28.0	0.086
	MXLG-2	557	38328	8947	600/5	9.25	4.00	29.0	0.088
	MNLG-2	298	34605	8638		8.25	3.50	28.0	0.100
2-1	MXLG-1	241	15331	6738	1000/5	9.00	3.75	44.0	0.148
	MNLG-1	111	13998	6403	&	6.75	3.75	42.0	0.140
	MXLG-2	362	15246	6764	400/5	9.00	3.75	44.0	0.149
	MNLG-2	154	13971	6471		8.00	3.75	42.0	0.141
2-2	MXLG-1	81	18574	4806	1000/5	7.25	3.75	31.0	0.088
	MNLG-1	34	17369	4669	&	6.25	3.50	30.0	0.088
	MXLG-2	164	18493	4828	200/5	8.25	3.75	31.5	0.100
	MNLG-2	78	17278	4693		7.25	3.50	30.5	0.106
3-1	MXLG-1	100	16179	3526	1000/5	6.00	3.25	23.0	0.108
	MNLG-1	48	14784	3407	&	5.00	3.00	22.0	0.088
	MXLG-2	200	16207	3560	300/5	6.75	3.25	23.0	0.089
	MNLG-2	97	14812	3442		6.00	3.00	22.5	0.090
3-4	MXLG-1	210	14915	4544	1000/5	7.00	3.00	29.5	0.147
	MNLG-1	103	13540	4357	&	6.25	2.75	28.5	0.157
	MXLG-2	113	14887	4577	300/5	6.25	3.00	30.0	0.149
	MNLG-2	53	13502	4387		5.25	2.75	28.5	0.159
4-1	MXLG-1	153	14509	1806	1000/5	7.75	3.00	12.0	0.221
	MNLG-1	75	13240	1739	&	6.50	2.75	11.5	0.267
	MXLG-2	261	14596	3361	300/5	8.75	3.00	22.0	0.096
	MNLG-2	123	13299	3180		7.75	2.75	21.0	0.100
4-4	MXLG-1	158	15596	3497	1000/5	8.00	3.25	23.0	0.092
	MNLG-1	76	14360	3375	&	7.00	3.00	22.0	0.100
	MXLG-2	79	14029	3446	200/5	7.00	3.00	22.5	0.100
	MNLG-2	30	12838	3319		6.00	2.75	21.5	0.108
5-1	MXLG-1	389	16499	2725	1000/5	9.75	4.00	18.0	0.073
	MNLG-1	167	15380	2265	&	8.50	4.00	15.0	0.068
	MXLG-2	326	11941	4395	400/5	9.75	4.00	29.0	0.084
	MNLG-2	50	10958	3983		7.50	4.00	26.0	0.076
5-2	MXLG-1	322	13185	1251	1000/5	6.75	6.25	8.0	0.035
	MNLG-1	157	12314	1109	&	5.75	6.25	7.5	0.033
	MXLG-2	612	14130	3512	800/5	7.75	6.25	23.0	0.038
	MNLG-2	298	13171	3212		6.75	6.25	21.0	0.037

## VI. CONCLUSIONS

This paper has described an on-line relay setting and coordination technique that can be used to obtain settings of relays provided in a distribution network. The algorithm, based on a linear programming technique using the Simplex two-phase approach, optimizes the operating times of the relays. The developed technique has the ability to identify the infeasible constraints (conditions) and to isolate them

from the final phase of the computations. The algorithm was applied to a loop distribution network. Some of the test results have been presented and are discussed in the paper.

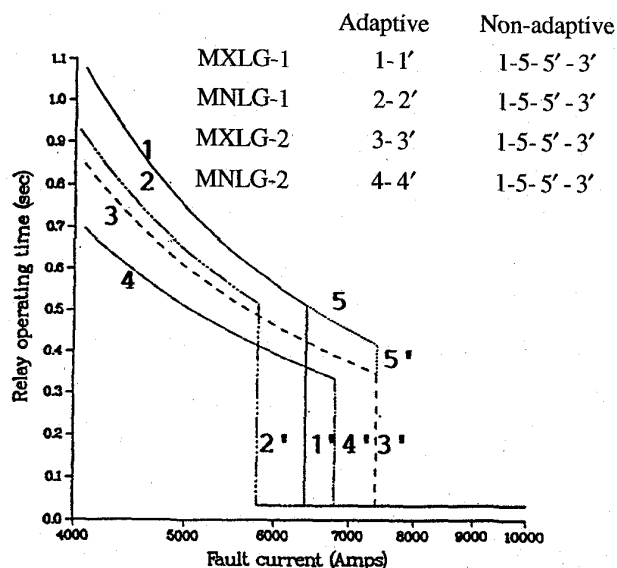


Fig 2: Operating characteristics of relay 1-1 with and without adaptive settings.

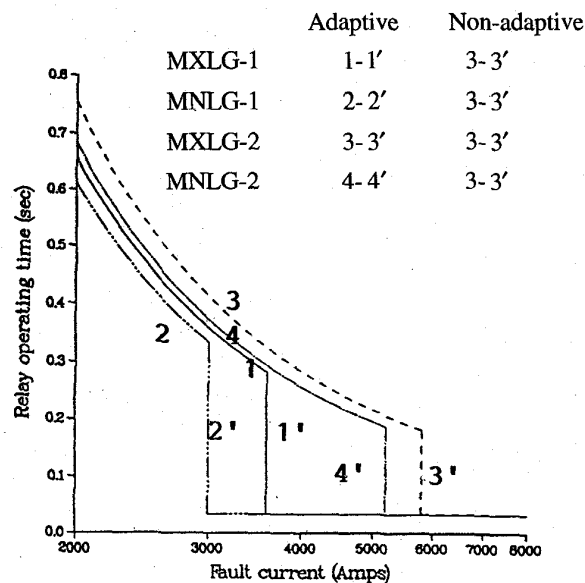


Fig 3: Operating characteristics of relay 5-1 with and without adaptive settings.

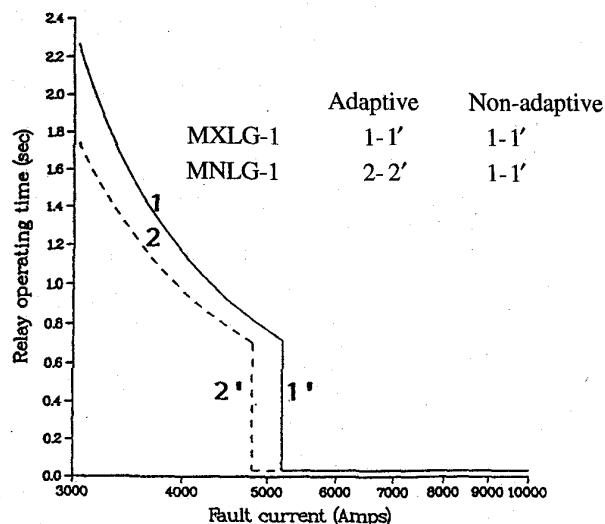


Fig 4: Operating characteristics of relay 1-4 with and without adaptive settings.

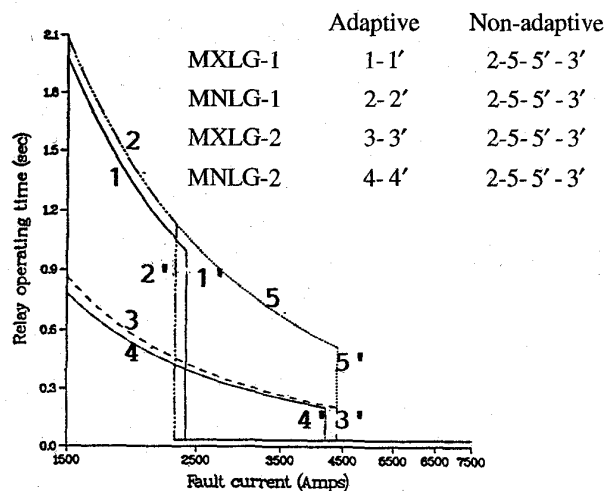


Fig 5: Operating characteristics of relay 4-1 with and without adaptive settings.

## VII. REFERENCES

- [1] Knable, A.H., 'A Standardised Approach to Relay Coordination', IEEE Winter Power Meeting, 1969.
- [2] Dwaraknath, M.H. and Nowitz, L., 'An Application of Linear Graph Theory for Co-ordination of Directional Overcurrent Relays', Electric Power Problems: The Mathematical Challenge, Proceedings of the SIAM Conference, Seattle, Washington, 1980, pp. 104-114.



## **BIOGRAPHIES**

**Bijoy Chattopadhyay** (M'93) received his B.E. from the Regional Engineering College, Durgapur, India in 1978. He obtained his M.Sc and Ph.D degrees from the University of Saskatchewan, Saskatoon, Canada in 1986 and 1993 respectively. He worked as a design and project engineers in a power consultancy company and as a distribution planning engineer in a power utility company.

Dr. Chattopadhyay is currently manager of software engineering at Mehta Tech, Inc. His areas of research interest are power system analysis, application of microprocessors for power system monitoring, protection and control, and substation automation.

**Mohindar S. Sachdev** (M'67, SM'73, F'83) was born in Amritsar, India, in 1928. He received the B.Sc. degree from the Banaras Hindu University, India, the M.Sc. degrees from the Punjab University, Chandigarh, India and the University of Saskatchewan, Saskatoon, Canada and, the Ph.D. and the D.Sc. degrees from the University of Saskatchewan, Saskatoon, Canada.

He worked for the Punjab P.W.D. Electricity Branch and the Punjab State Electricity Board, India from 1950 to 1968 in System Operation, Design and Planning. In 1968, he joined the University of Saskatchewan where he is currently Professor of Electrical Engineering. His areas of interest are power system analysis and power system protection.

Dr. Sachdev is a Fellow of the Institution of Engineers (India) and a Fellow of the Institution of Electrical Engineers, London (England). He is a Registered Professional Engineer in the Province of Saskatchewan and a Chartered Engineer in the UK.

**Tarlochan S. Sidhu** (M'90, SM'94) received the B.E. (Hons.) degree from the Punjabi University, Patiala, India in 1979 and the M.Sc. and Ph.D. degrees from the University of Saskatchewan, Saskatoon, Canada in 1985 and 1989 respectively. He worked for the Regional Computer Center, Chandigarh, India from 1979 to 1980 and developed software for computer-based systems. He also worked for the Punjab State Electricity Board, India from 1980 to 1983 in distribution system operation and thermal generating station design. After obtaining the Ph.D. degree, he joined Bell-Northern Research Ltd., Ottawa, Canada and worked on a software development project for about one year. He joined, in 1990, the University of Saskatchewan where he is presently Associate Professor of Electrical Engineering.

His areas of research interest are power system protection and control and applications of microprocessors and neural networks for power system monitoring, protection and control.

### Discussion

**A.J. Urdaneta, L.G. Pérez** (Universidad Simón Bolívar), **H. Restrepo** (GERS Ltda. Consultores) :

The authors are to be commended for a very interesting article.

The application of the Simplex method for linear programming to calculate the time dial settings for a coordinated operation of directional overcurrent relays, was proposed in ref. [7] of the paper. The authors are responsible for the first on-line application of this methodology, in a real time adaptive protection scheme. We would appreciate the authors comments on the following points.

I.- The transient configurations that take place when only one of the main relays has operated (first main relay) and the second one (second main relay) has not operated yet, must be considered in the mathematical formulation of the linear programming problem, as a key issue for achieving a feasible relay coordination scheme,[A] in order to obtain real operation times that match those predicted by the algorithm, and a completely selective protective scheme. This point is specially critical when determining the settings of the instantaneous units.

In the on-line application of the paper, it seems that the transient configurations were not included in the formulation.

What were the differences between the predicted operation times and the real operation times of the relays for the second main relays and its correspondent backups?

II.- The second issue that we would like to raise, is related to the selection of the weighting factors in the objective function.

Due to the particular characteristics of the mathematical formulation, it can be shown [ref. [7] of the paper] that the solution to the optimization problem is not dependent upon the choice of the weights, as long as they are positive real numbers. This particular characteristic of the optimization problem is perhaps more clearly explained if the relay operation times are assumed as independent objective functions in the optimization formulation, due to the fact that this independent objectives do not compete one with each other, and therefore, the reduction of one necessarily leads to the reduction of the others. The constraints that relate (or couple) the operation times of different units, are the coordination constraints. These constraints, are such that always  $\partial T_{ijk} / \partial T_{lmn} > 0$  if they are active ( $=0$  if they are not active), for all  $i,j,k,l,n,m$ .

Also, it is not complicated to show that the minimization of the operation times of the main relays only, leads to exactly the same setting solution than the minimization of all the operation times, associated to the operation as main and as backup units. Therefore, the choice of the value of  $w=1$  for all the weights is very appropriate.

Finally, we would like to congratulate the authors for the application of linear programming techniques to the solution of the adaptive coordination problem of directional overcurrent relays, in an on-line environment.

### References

[A] Restrepo,H., Urdaneta,A.J., Márquez, S., Fajardo, J., "Optimal Coordination of Directional Overcurrent Relays:

Transient Configurations", Proceedings of the IEEE-CSEE (Chinese Society of Electrical Engineers), International Conference on Power System Technology (ICPST'94), Beijing, China, October, 18-21, 1994.

Manuscript received February 24, 1995.

ALEXANDER P. APOSTOLOV, (Rochester - Integrated Systems Division), Rochester, NY :

The authors have presented a very interesting paper on on-line relay coordination using linear programming technique. The algorithm described is based on the assumption that the lines are short and approximately of equal length, i.e. equal weights ( $=1$ ) are assigned for the operating times of all relays. However, in many cases it is necessary to coordinate relays protecting long lines with relays protecting short lines or vice versa. How are such system configurations going to affect the performance of the algorithm described ?

The instantaneous overcurrent setting is described as based on the far-end fault current. However, for mutually coupled transmission lines it is possible to have maximum external fault current for a short circuit on the parallel line with a breaker opened by the instantaneous protection of the faulted line. How is such sequential operation considered by the algorithm ?

In systems which use breaker failure protection it is necessary to coordinate the remote back-up relays with the breaker failure relay. Is this considered in the algorithm ?

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**B. Chattopadhyay (Mehta Tech. Inc.), M.S. Sachdev and T.S. Sidhu, Power System Research Group, University of Saskatchewan, Canada.** We thank the discussers for their interest in the paper. Our response to their questions and comments is as follows:

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1. Directional overcurrent relays remain coordinated during transient configurations arising from the tripping of one of the circuit breakers controlling a line. The instantaneous relays also remain coordinated for such transients. If a situation is encountered when the relays loose their coordination during a transient, the requirements of the condition can be incorporated in the process.

The purpose of the study reported in this paper was to determine optimized coordinated relay settings for the distribution network of the City of Saskatoon. This was achieved and the characteristics of some of the relays are shown in the paper. The operating times of the relays were neither predicted nor checked from simulations. This can, however, be done by applying, to the relays, currents calculated by the EMTDC via D/A converters. This procedure is, however, not essential to prove that the operation of the relays will remain coordinated.



2. We thank the discussers for confirming our observation that the selecting equal weights in the objective function is a valid approach.

A. Apostolov

1. We do not see any problems in applying the proposed algorithm to systems that contain long as well as short lines.

2. The proposed algorithm was tested using the looped distribution network of the City of Saskatoon. Overcurrent and directional overcurrent relays are used to protect the radial and looped lines of this network. The loop networks that contain parallel lines with sources at intermediate buses

can not be protected by directional overcurrent relays only; it becomes necessary to use other relaying schemes, such as distance relays. The sequential operation described by the discussers was, therefore, not studied.

3. It is possible to include breaker-failure relays while determining the optimized coordinated settings of the relays. This can be accomplished by including appropriate constraints while minimizing the objective function.

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