Metamaterials
Term Project Report

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Winter Semester 2024/2025
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Part A: Dispersion Curve Mode Shapes

Complete the model by adding the appropriate boundary conditions for the 2D unit cell in order to perform Floquet-Bloch type simulations.

The necessary item to complete for this step was to link the sides of each unit cell which would be connected to the equivalent sides of adjacent unit cells. It is apparent that four linkages must be made, which are shown by the highlighted areas of the images below:

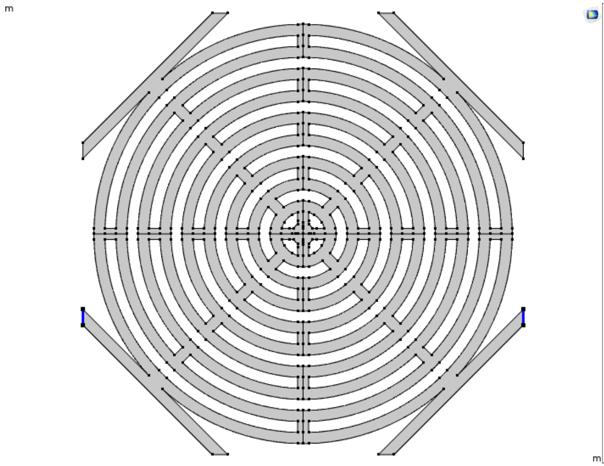

Figure 1 - Low Sides

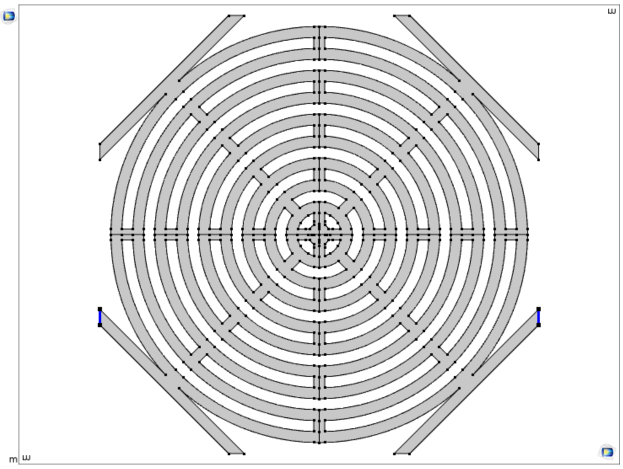

Figure 2 - High Sides

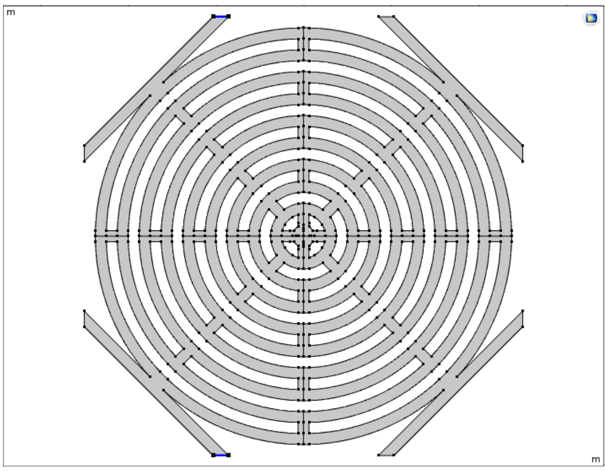

Figure 3 - Top and Bottom at Left

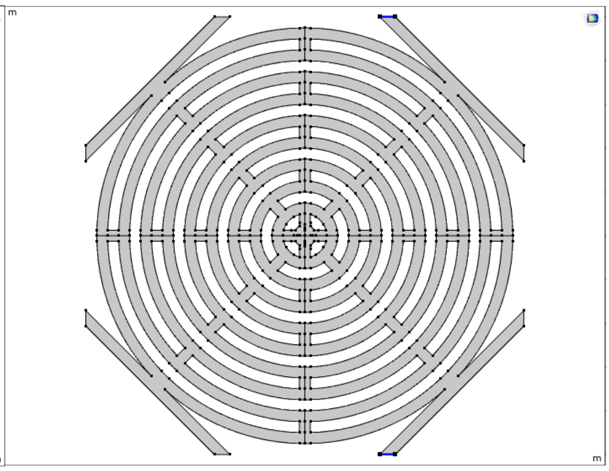

Figure 4 - Top and Bottom at Right

For each linkage – in Comsol these are input as periodic conditions – Floquet-Bloch periodicity must be selected, and the wavevectors are set as the defined parameters k_x and k_y. As these two values are varied in magnitude, they can be combined as components of the principal directions *x* and *y* to define the magnitude and direction of the total wavevector.

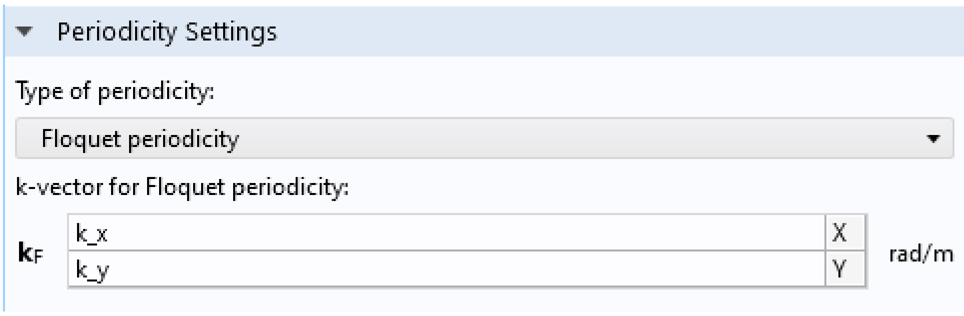

Figure 5 - Selection of Periodicity and Wavevector Components

Calculate the first 10 dispersion curves of the Irreducible Brillouin Contour (IBC) and show them in a dispersion diagram.

The Irreducible Brillouin Contour (IBC) for this unit cell is the same as for the IBC for the unit cell used during the tutorial of this course. There are four axes of symmetry: the two principal axes x and y, as well as the diagonals (See Figure 6). 25 points along each of the three sides of the IBC (See Figure 7) are selected, with the maximum value of the wavevector in each direction being set as π/a, which is chosen because the Floquet-Bloch periodicity condition ensures that the results will repeat for every 2 wavevectors, and the axes of symmetry ensure that only half of those wavevectors need to be calculated.

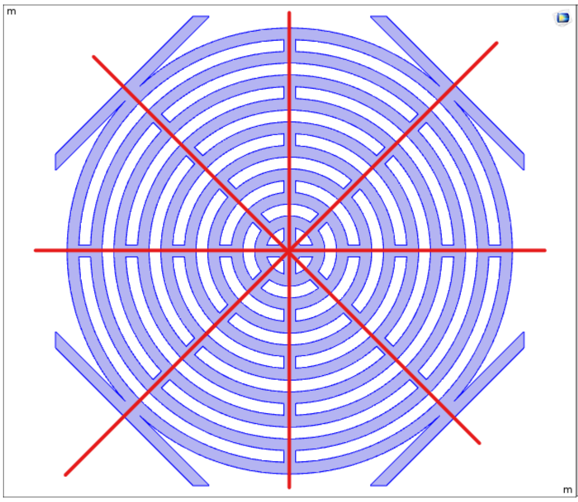

Figure 6 - Unit Cell Axes of Symmetry

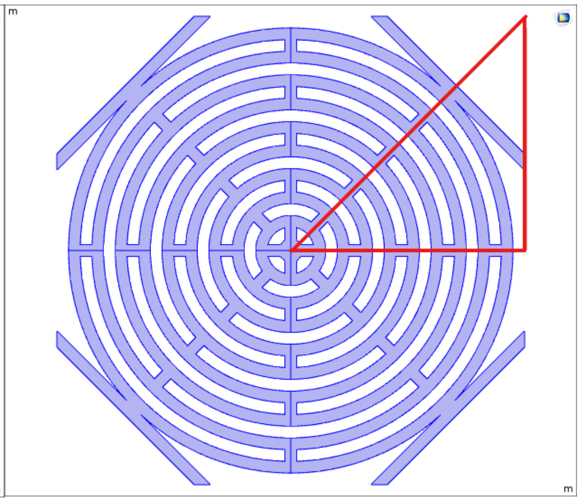

Figure 7 - Irreducible Brillouin Contour

In Comsol, an eigenfrequency study is utilised, with the instruction to calculate the first ten eigenfrequencies for the given wavevectors, which are varied through a parametric study. The results are then plotted in Figure 8, where each group of 25 solutions display one side of the IBC.

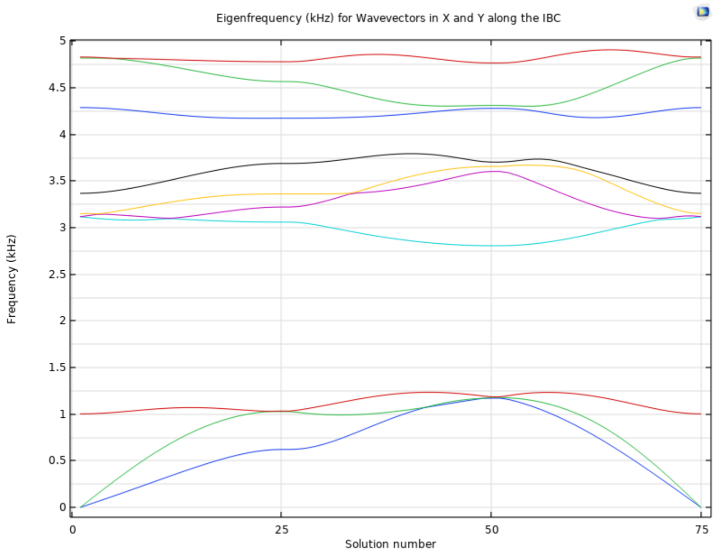

Figure 8 - Dispersion Diagram along the Irreducible Brillouin Contour

Perform a mesh convergence study until you reach a mesh size for which the frequencies of the third dispersion curve change not more than 30 Hz for the next smaller mesh size. Use the parameter "meshSize" in the model.

See Figures 9, 10, and 11 for three dispersion curves with the mesh size reduced. Whereas the dispersion diagrams from above utilise a mesh size of 5mm, Figure 9 reduces this value by a factor of 10 to 0.5mm. Figure 10 then halves it further to 0.25mm, and Figure 11 halves it again to 0.125mm.

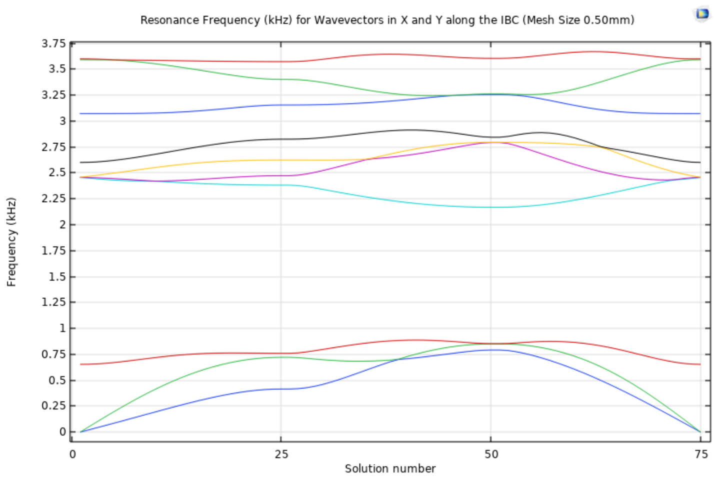

Figure 9 - Mesh Size 0.5mm

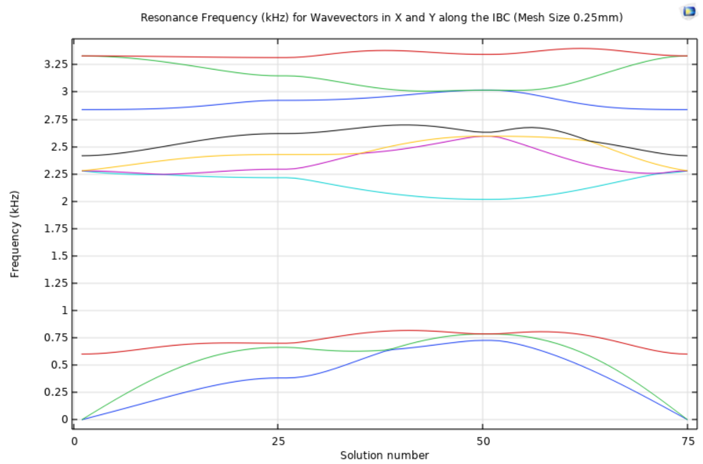

Figure 10 - Mesh Size 0.25mm

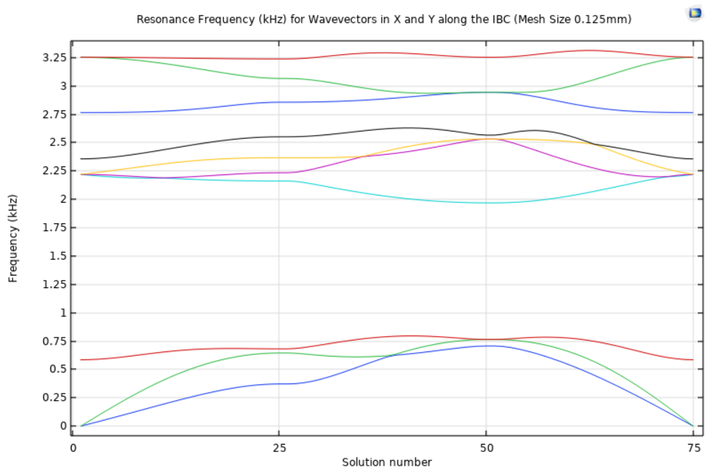

Figure 11 - Mesh Size (0.125mm)

A large decrease can be observed in the maximum eigenfrequencies between the mesh sizes of 5mm and 0.5mm (a decrease of 25%), and a further decrease can be observed going to 0.25mm (13%), but there is little to no decrease when the mesh size decreases from 0.25mm to 0.125mm. Thus, the optimal mesh size can be assumed to be around 0.25mm. The calculation time more than doubles for the 0.125mm mesh size, and with similar results, it is not necessary to use additional computational resources for the calculation.

Highlight the full and the directional band gaps in the dispersion diagram which you obtain from the smallest (=converged) mesh size.

See Figure 12. Two full band gaps can be seen: one at 0.88-2.00kHz and one from 2.75-2.80kHz. The upper and lower limits of these are marked by the thick red lines. In the first side of the IBC, there are 4 directional band gaps; three of which that start or end at one of the unidirectional band gaps. There are three in the second side of the IBC, with one of them starting at the end of the second unidirectional band gap, and a very small one on the third side of the IBC, directly below the second unidirectional band gap. This is shown visually in Figure 12, and the values are shown in Table 1.

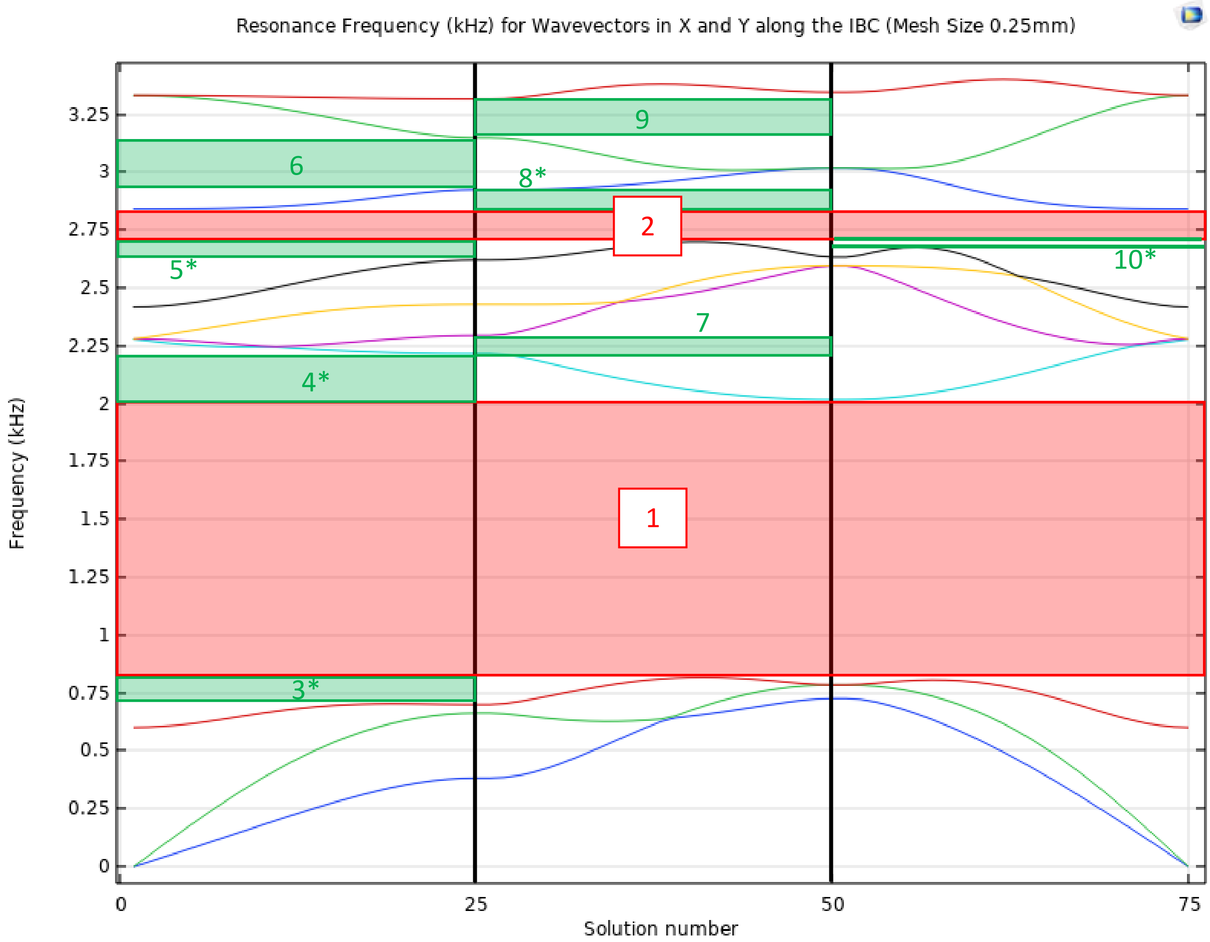

Figure 12 - Band Gap Locations

Calculate the gap to mid-gap ratio of all band gaps that you find including complete and directional band gaps.

Table 1 – Band Gap Locations and Gap to Mid-Gap Ratios

Name	f_o **[Hz]**	f_c **[Hz]**	**BG% [%]**
Unidirectional Band Gaps			
1	817.5	2018	84.7
2	2699	2840.	5.1
Directional Band Gaps, IBC Plane 1			
3*	703.4	*817.5	15.0
4*	*2018	2216	9.4
5*	2620	*2699	3.0
6	2924	3149	7.4
Directional Band Gaps, IBC Plane 2			
7	2216	2294	3.5
8*	*2840.	2924	2.9
9	3149	3316	5.2
Directional Band Gaps, IBC Plane 3			
10*	2676	*2699	0.9

*Indicates that the start/end of this directional band gap is the start/end of one of the unidirectional band gaps.

Generate two mode shape plots for the unit cell. The plots must show the norm of the displacement field ("solid.disp" in COMSOL) for the following two points of the second dispersion curve: The wavenumber in y-direction is equal to 0 and the wavenumber in x-direction corresponds to a wavelength which is (a) 4 times the size of the unit cell, and (b) 2 times the size of the unit cell.

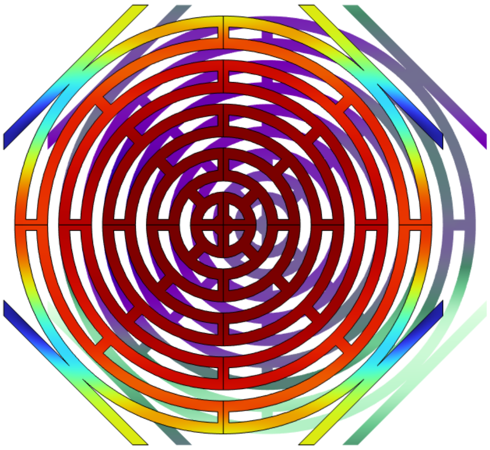

Figure 13 - Mode Shape Plot for a Wavenumber k_x Corresponding to a Wavelength 4 Times the Unit Cell (31.416 rad/m)

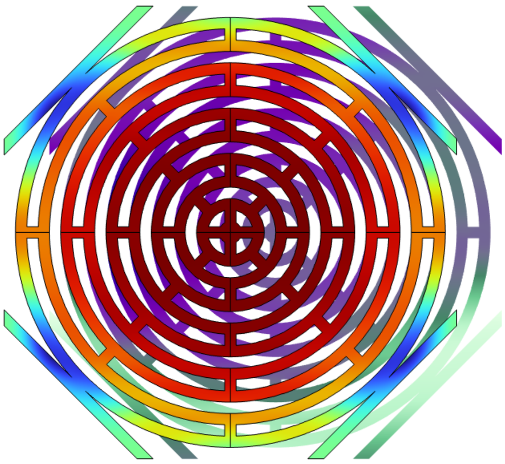

Figure 14 - Mode Shape Plot for a Wavenumber k_x Corresponding to a Wavelength 2 Times the Unit Cell (62.832 rad/m)

Part B – Optimizing the Unit Cell

List the changes you made with respect to the original unit cell

Figures 15-18 show the dispersion diagrams for the optimised unit cells with each of the four materials. Since only two parameters can be changed – the material and the thickness of the unit cell beams – a material was first chosen, and then the tBeam parameter was varied. Starting at -0.125mm, tBeam was gradually increased until the first band gap was as large as possible (both in terms of absolute range and BG%). It is also important to note that the other band gaps should remain within the 100-3000Hz range, so this needed to be checked for each result, and the tBeam parameter was modified further until the band gaps were all within the range of interest. The final tBeam value for each material is listed in Table 2 below.

Table 2 – tBeam Value for Each Material to Maximise the First Total Band Gap within the Frequency Range of Interest of 100-3000Hz

Material	Value of tBeam [mm]
PMMA	- 0.040
ABS	- 0.040
PLA	- 0.125
PP	- 0.125

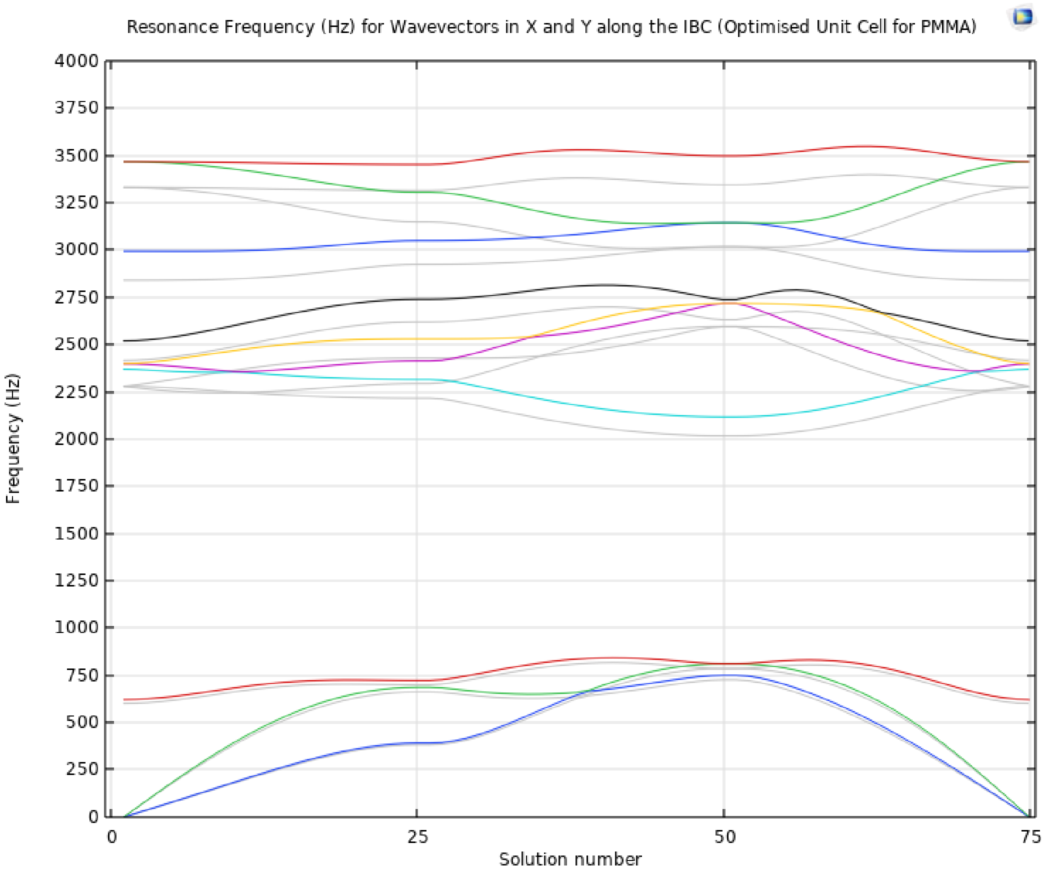

Figure 15 - Optimised Unit Cell with PMMA (tBeam = -0.04mm)

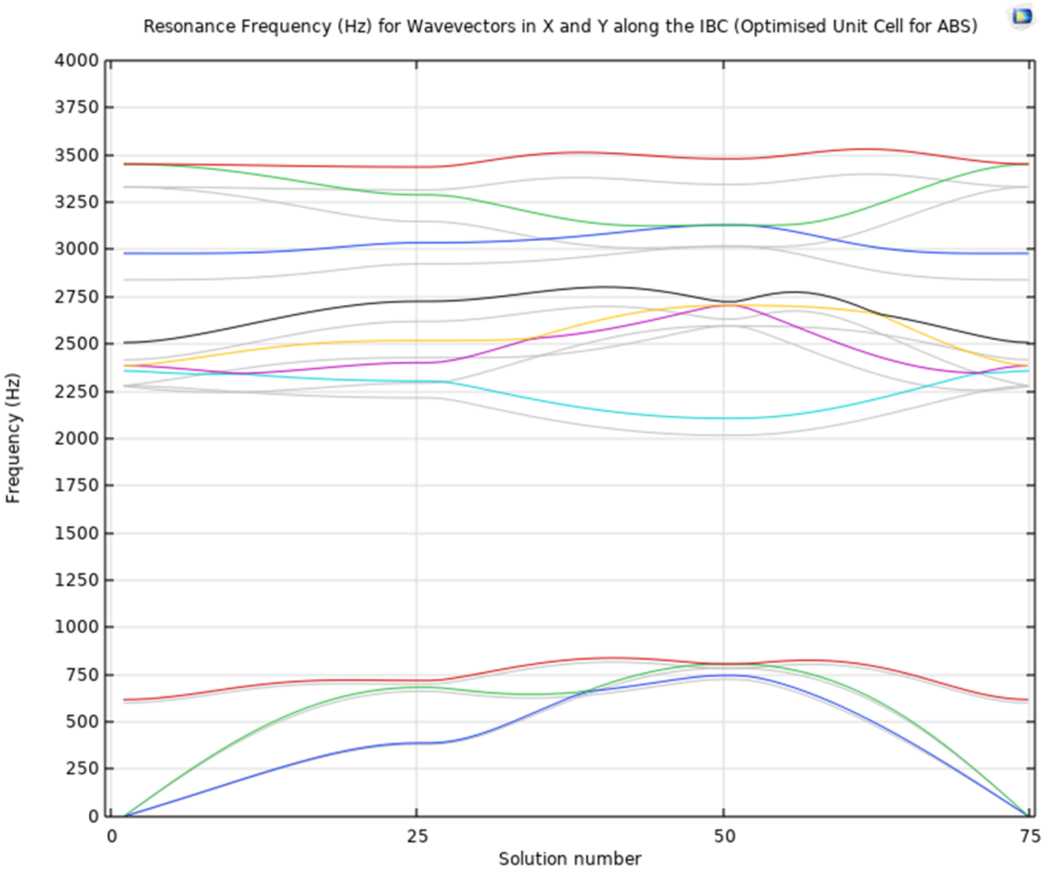

Figure 16 - Optimised Unit Cell with ABS (tBeam = -0.04mm)

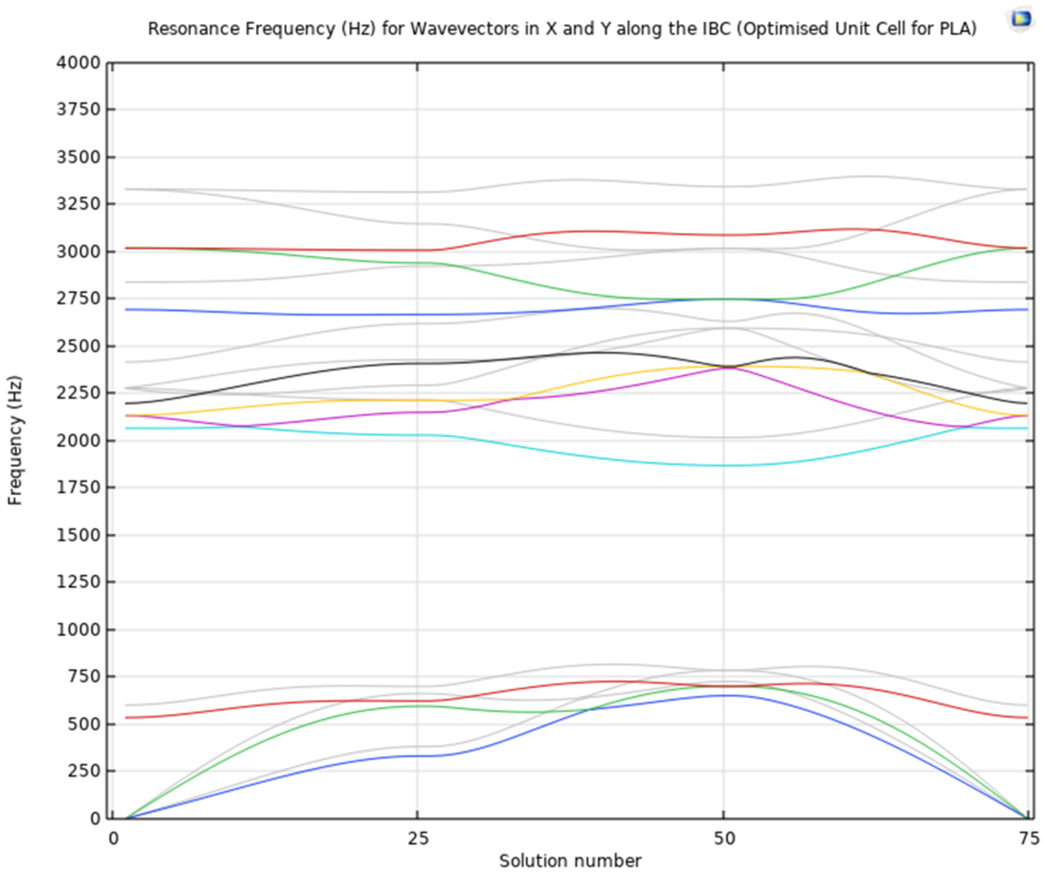

Figure 17 - Optimised Unit Cell with PLA (tBeam = -0.125mm)

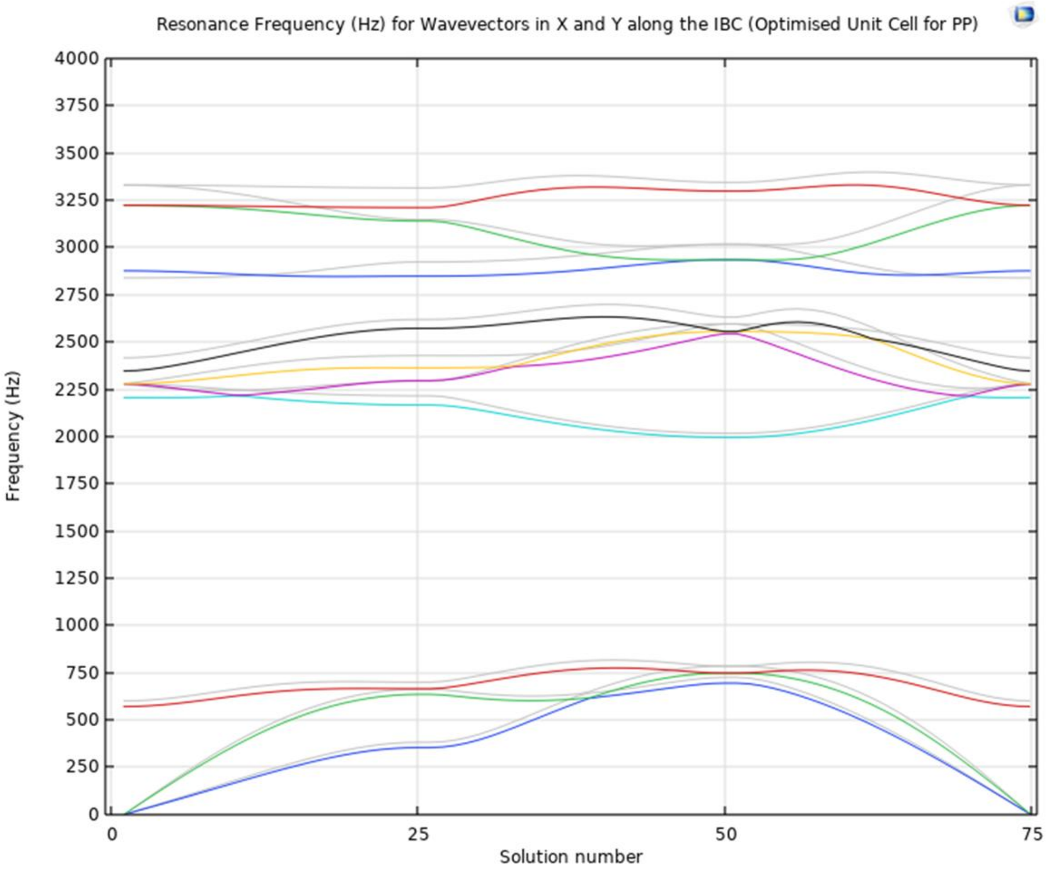

Figure 18 - Optimised Unit Cell with PP (tBeam = -0.125mm)

Show the dispersion diagram of your optimized unit cell and highlight the band gaps (complete and directional) that lie in the frequency range of interest.

Figures 15-18 show the dispersion diagrams for each of the four materials in comparison to the original unit cell. The light grey lines are those of the original unit cell, and the coloured lines are those of the optimised cells. To choose which one is the overall most optimal unit cell, the band gap ratio of the two total band gaps are used. Table 3 summarises this value for each material.

Table 3 – Gap-to-Mid-Gap Ratio for the Total Band Gaps of the Optimised Unit Cells in Each Material

Material	Used tBeam [mm]	First BG% [%]	Second BG% [%]
PMMA	- 0.040	86.1	6.2
ABS	- 0.040	86.1	6.2
PLA	- 0.125	88.1	7.8
PP	- 0.125	88.1	7.8

Interestingly, PMMA and ABS have nearly the same results to one another; as do PLA and PP. But the later two have larger BG% for the two total band gaps. In the data, however, the two are not identical. The first total band gap for PLA starts at a lower frequency than for PP, however, so for this reason PLA is chosen as the optimal material for the unit cell. Figure 19 shows the band gaps, and Table 4 summarises the ranges and gap-to-mid-gap ratios for each of them.

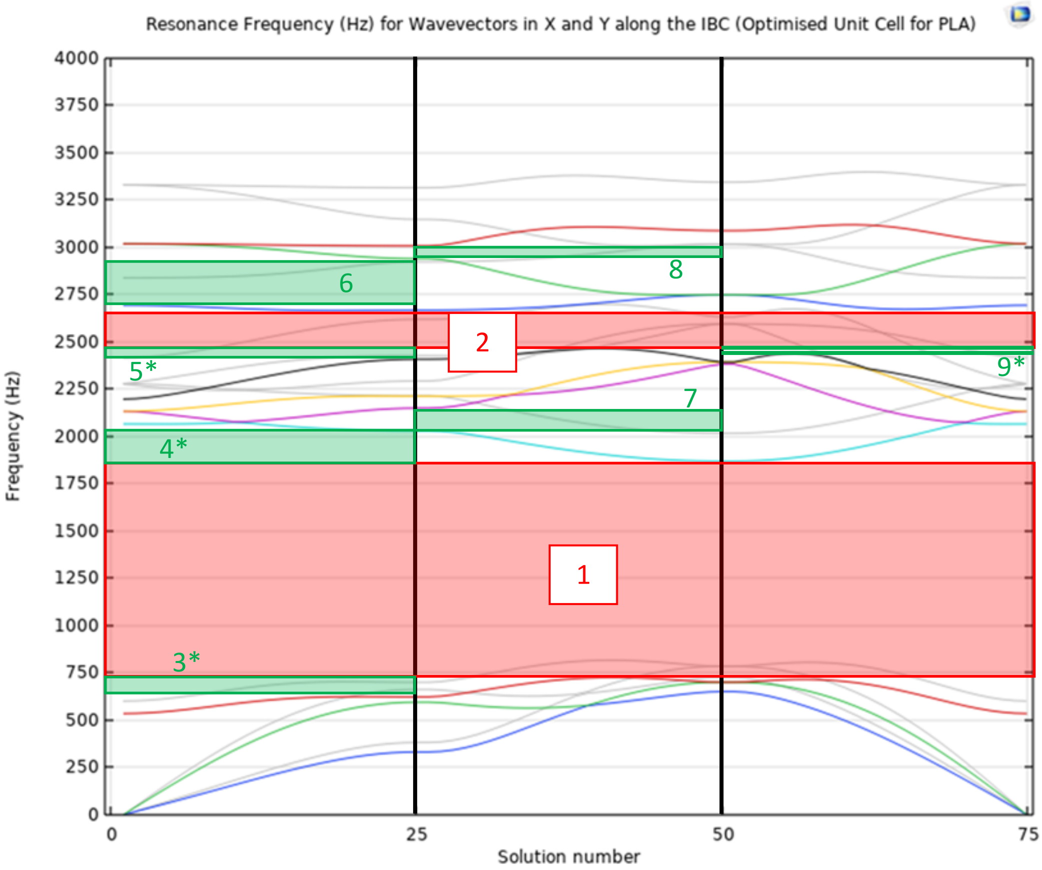

Figure 19 - Band Gaps for Optimised Unit Cell with PLA (tBeam = -0.125mm)

Table 4 – Band Gap Locations and Gap to Mid-Gap Ratios for Optimised Unit Cell with PLA

Name	f_o **[Hz]**	f_c **[Hz]**	**BG% [%]**
Unidirectional Band Gaps			
1	726.5	1870.	88.1
2	2466	2666	7.8
Directional Band Gaps, IBC Plane 1			
3*	624.6	*726.5	15.1
4*	*1870.	2030.	8.2
5*	2409	*2466	2.3
6	2695	2941	8.8
Directional Band Gaps, IBC Plane 2			
7	2030.	2150	5.8
8	2941	3008**	2.3
Directional Band Gaps, IBC Plane 3			
9*	2441	*2466	1.0

*Indicates that the start/end of this directional band gap is the start/end of one of the unidirectional band gaps.

**Indicates that the end of this band gap is outside the frequency range of interest.

Prove that a metamaterial made from the optimized unit cell absorbs vibrations better than a metamaterial based on the original unit cell.

Table 5 shows the comparison of the gap-to-mid-gap ratios for the band gaps in the same locations between the original and optimised unit cells. Note that some directional band gaps appear in different relative locations between the two compared cells, and so these band gaps are not included.

Table 5 – Comparison of Original to Optimised Unit Cell

Original Unit Cell (PMMA, tBeam = 0.00mm)		**Optimised Unit Cell (PLA, tBeam = -0.125mm)**	
Band Gap #	**BG% [%]**	**Band Gap #**	**BG% [%]**
Unidirectional Band Gaps			
1	84.7	1	88.1
2	5.1	2	7.8
Directional Band Gaps, IBC Plane 1			
3	15.0	3	15.1
4	9.4	4	8.2
5	3.0	5	2.3
6	7.4	6	8.8
Directional Band Gaps, IBC Plane 2			
7	3.5	7	5.8
9	5.2	8	2.3
Directional Band Gaps, IBC Plane 3			
10	0.9	9	1.0

From Table 5, most of the gap-to-mid-gap ratios are larger for the optimised unit cell over the original (with the exception of directional band gaps 4, 5, and 9 on the original unit cell, which are larger than the optimised one). Since the most important band gaps are the full band gaps, number 1 and 2 in the table, and these are both several percentage points higher for the optimised cell, this is proof that the optimised unit cell absorbs vibrations better than the original unit cell.